Decomposition of Residual Circulation in Estuaries

PENG CHENG

College of Ocean and Earth Sciences, Xiamen University, Xiamen, Fujian, China

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ABSTRACT

The residual currents in estuaries are produced by a variety of physical mechanisms. To understand the contribution of each individual mechanism to the creation of residual circulation, it is necessary to separate the effect of one particular mechanism from the others. In this study, a method based on dynamics is developed to decompose the residual circulation into individual components corresponding to different forcing mechanisms. Specifically, residual flows are partitioned based on the separate contributions by river discharge, horizontal density gradient, internal tidal asymmetry, advection, semi-Stokes transport, and wind. The method includes the effects of the earth's rotation and can be applied for general conditions. Under the precondition that the ratio between width and length of the estuary is small, the continuity equation can be simplified such that the method only requires the data at a cross-estuary section to decompose residual currents. This makes the method practicable for real estuaries. Results from a generic numerical model are used to illustrate the decomposition method and to demonstrate its validity for weakly stratified estuaries.

1. Introduction

Residual circulation in estuaries is forced by a wide range of different physical mechanisms, specifically river discharge, along-estuary density gradient, nonlinear advection, asymmetric tidal mixing, and wind. Traditionally, gravitational circulation driven by the along-estuary density gradient has been considered as the dominant component of estuarine exchange flow (Pritchard 1956). The classical analytical solutions, which are based on a linear momentum balance between along-estuary density gradient and friction, have been widely applied to describe residual circulation in estuaries (e.g., Hansen and Rattray 1965; Chatwin 1976; Officer 1976; MacCready 2004). However, a number of studies have shown that many processes in addition to the along-estuary density gradient can make important contributions to the creation of residual circulation (Jay 1991; Wong 1994; Lerczak and Geyer 2004). To discern the roles of various processes in estuarine dynamics, it is advantageous to separate total residual currents into different components corresponding to their driving forces.

The widely used methods to decompose residual estuarine currents such as empirical orthogonal function are largely based on statistical grounds. A difficulty in applying those methods is to extract the physical meaning of the decomposed modes. Several analytical models have been developed to dynamically separate residual currents according to the driving mechanisms (Li and O’Donnell 2005; Winant 2008; Huijts et al. 2009). Because of the simplified assumptions, however, those analytical models are restricted to tidally dominated estuaries in which the first-order tidal currents are not affected by horizontal density gradients or winds. Also, the simplification of a time-independent vertical eddy viscosity excludes tidal variations in turbulent mixing.

Burchard and Hetland (2010) developed a decomposition method that solves the tidally averaged momentum equation to obtain the residual flow as the sum of the contributions from tidal straining, gravitational circulation, wind straining, and depth-mean residual flow due to freshwater runoff. This method successfully quantified residual currents in irrotational tidal estuaries with a one-dimensional numerical model, and has been extended to a cross-estuary section by adding the decomposition of the cross-channel velocity (Burchard et al. 2011). Another method, which is an extension of the analytical models of Ianniello (1977), McCarthy (1993), and Cheng et al. (2010), was developed to decompose along-estuary residual currents (Cheng et al. 2011, 2013). Essentially, the two decomposition methods followed a similar strategy. Cheng’s method solves both...
tidally averaged momentum and continuity equations for each component of residual currents such that the rigid-lid condition and local runoff function assumed in Burchard’s method are not needed. Also including the continuity equations can explicitly describe river-induced and Stokes-transport-induced flows. Furthermore, solving the continuity equation makes the decomposition method independent to numerical model configuration; thus, it could be applied to observational data if high-quality estimates of eddy viscosity can be obtained over multiple tidal cycles.

While Burchard’s method is restricted to irrotational estuaries, Cheng’s method is restricted to along-estuary sections. In this study, the method of Cheng et al. (2013) is further extended to cross-estuary sections including Coriolis force; therefore, it can decompose three-dimensional residual flows in real estuaries. The main purpose of this work is to describe the decomposition method and demonstrate its validity for weakly nonlinear systems.

The remainder of this paper is structured as follows: In section 2, the development of the decomposition method is presented. In section 3, two generic numerical experiments are undertaken to illustrate the decomposition method. One experiment examines tidally dominated estuaries, and the other examines the effects of a down-estuary wind event. In section 4, the limitations of the decomposition method are discussed and the conclusions are drawn. In the appendix, a perturbation analysis is given to demonstrate that for elongated estuaries (i.e., the ratio of width and length of the estuary is small), the continuity equation can be simplified such that the residual water surface slopes can be obtained analytically.

### 2. Development of decomposition method

The momentum and continuity equations under a sigma coordinate system are given by

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial \sigma} - fu = -g \frac{\partial \eta}{\partial x} - \frac{g}{\rho_0} \left[ \frac{\partial}{\partial \sigma} \left( D \int_0^\infty \rho \, d\sigma' \right) + \rho \frac{\partial D}{\partial \sigma} \right] + \frac{1}{D} \frac{\partial}{\partial \sigma} \left( K_m \frac{\partial u}{\partial \sigma} \right) + \frac{1}{D \frac{\partial}{\partial \sigma}} \left( K_m \frac{\partial u}{\partial \sigma} \right),
\]

(1a)

where \( u(x, y, z, t), v(x, y, z, t), w(x, y, z, t), \omega(x, y, \sigma, t) \) are the velocity components in the along-estuary, transverse, and vertical directions, respectively. Here \( D = H + \eta, H(x, y) \) is the mean water depth, \( \eta(x, y, t) \) is the water surface elevation, \( t \) is time, \( \rho(x, y, z, t) \) is water density, \( \rho_0 \) is a reference density, and \( K_m(x, y, z, t) \) is the vertical eddy viscosity. When the vertical sigma space is not uniform, \( D \) should be replaced with \( \partial z/\partial \sigma \). The choice of the sigma coordinate is to overcome the difficulty in calculating tidally averaged quantities around mean sea level affected by tidal fluctuations of the water surface.

Variables \( u(x, y, z, t), v(x, y, z, t), K_m(x, y, z, t) \) and \( 1/D^2 \) are decomposed into two parts: tidal mean (represented with an overbar) and tidal variation (represented with a prime):

\[
\begin{align*}
\bar{u} &= u, \\
\bar{v} &= v, \\
\bar{K}_m &= K_m + K'_m, \quad \text{and} \\
1/D^2 &= Z + Z'.
\end{align*}
\]

(2a) (2b) (2c) (2d)

Note that using Taylor expansion, the first-order approximation of \( 1/D^2 \) can be written as \( 1/D^2 = 1/H^2 - 2\eta/H^3 \). However, calculating \( Z \) and \( Z' \) directly is not complicated. Substituting Eq. (2) for Eq. (1), and taking the average of Eq. (1) over a tidal cycle leads to

\[
\bar{Z} \frac{\partial}{\partial \sigma} \left( \bar{K}_m \bar{\frac{\partial u}{\partial \sigma}} \right) + \bar{f} \bar{v} = \frac{\partial \bar{\eta}}{\partial x} + \frac{g}{\rho_0} \left[ \frac{\partial}{\partial \sigma} \left( D \int_0^\infty \rho \, d\sigma' \right) + \rho \frac{\partial D}{\partial \sigma} \right] + \frac{1}{D} \frac{\partial}{\partial \sigma} \left( \bar{K}_m \bar{\frac{\partial u}{\partial \sigma}} \right) + \frac{1}{D} \frac{\partial}{\partial \sigma} \left( \bar{K}_m \bar{\frac{\partial u}{\partial \sigma}} \right) + \frac{1}{D} \frac{\partial}{\partial \sigma} \left( \bar{K}_m \bar{\frac{\partial u}{\partial \sigma}} \right),
\]

(3a)
The last four terms on the right-hand side of Eqs. (3a) and (3b) result from the expansion of vertical shear stress terms. Those terms represent the covariance of tidal fluctuations of eddy viscosity and vertical shear, and are considered to drive a residual flow that is referred to as the internal tidal asymmetry (ITA)-induced flow (Jay 1991). The tidally averaged current velocity at a sigma layer is the sum of an Eulerian velocity and a semi–Stokes velocity (see appendix A of Cheng et al. 2013). Thus, the continuity equation has two terms in each bracket, representing the Eulerian transport and the semi–Stokes transport, respectively. The decomposition method further separates the Eulerian velocity into multiple components, and uses a Stokes return flow to account for the semi–Stokes transport.

The residual currents are considered to have six components: 1) the river-induced flow \([\overline{u_R}(x, y, z), \overline{v_R}(x, y, z)]\); 2) the density-driven flow \([\overline{u_D}(x, y, z), \overline{v_D}(x, y, z)]\); 3) the advection-induced flow \([\overline{u_A}(x, y, z), \overline{v_A}(x, y, z)]\); 4) the ITA-induced flow \([\overline{u_I}(x, y, z), \overline{v_I}(x, y, z)]\); 5) the Stokes return flow \([\overline{u_R}(x, y, z), \overline{v_R}(x, y, z)]\); and 6) the wind-driven flow \([\overline{u_W}(x, y, z), \overline{v_W}(x, y, z)]\). Here, the advection component combines horizontal and vertical advection terms together, but it is straightforward to further separate the total advection into along-estuary and cross-estuary components. The residual currents and the corresponding residual sea levels, therefore, are written as

\[
\begin{align*}
\overline{u} & = \overline{u_R} + \overline{u_D} + \overline{u_A} + \overline{u_I} + \overline{u_S} + \overline{u_W}, \\
\overline{v} & = \overline{v_R} + \overline{v_D} + \overline{v_A} + \overline{v_I} + \overline{v_S} + \overline{v_W}, \\
\eta & = \overline{\eta_R} + \overline{\eta_D} + \overline{\eta_A} + \overline{\eta_I} + \overline{\eta_S} + \overline{\eta_W}.
\end{align*}
\]  

Substituting Eq. (4) for Eq. (3) yields a pair of momentum and continuity equations for each component of the residual circulation as shown:

\[
Z \frac{\partial}{\partial \sigma} \left( \overline{K_m} \frac{\partial \overline{\eta}}{\partial \sigma} \right) - f \overline{\eta} = \frac{g}{\rho_0} \left[ \frac{\partial}{\partial y} \left( D \frac{\partial \rho \partial \sigma'}{\partial \sigma} \right) + \rho \sigma \frac{\partial D}{\partial y} \right] + \frac{\partial \overline{\Pi}}{\partial x} + \frac{\partial \overline{\eta}}{\partial y} + \frac{\partial \overline{\overline{\Pi}}}{\partial \sigma},
\]  

and

\[
\frac{\partial}{\partial x} \left( \int_{-1}^{0} \overline{H u_R} \, d\sigma + \int_{-1}^{0} \overline{\eta u_R} \, d\sigma \right) + \frac{\partial}{\partial y} \left( \int_{-1}^{0} \overline{H \eta u_R} \, d\sigma + \int_{-1}^{0} \overline{\eta \eta u_R} \, d\sigma \right) = 0.
\]  

Here, the subscript \(i\) represents each component of the residual circulation, and \(F_{xR}\) and \(F_{yR}\) are the driving forces for each component at the \(x\) and \(y\) directions, respectively. Note that the continuity of the Stokes return flow has a different form to the other components:

\[
\frac{\partial}{\partial x} \left( \int_{-1}^{0} \overline{H u_R} \, d\sigma + \int_{-1}^{0} \overline{\eta u_R} \, d\sigma \right) + \frac{\partial}{\partial y} \left( \int_{-1}^{0} \overline{H \eta u_R} \, d\sigma + \int_{-1}^{0} \overline{\eta \eta u_R} \, d\sigma \right) = 0.
\]  

The semi–Stokes transport (\(\int_{-1}^{0} \overline{u_D} \, d\sigma\) and \(\int_{-1}^{0} \overline{v_D} \, d\sigma\)) is part of Stokes transport (\(\overline{u_R} \sigma = 0\) and \(\overline{v_R} \sigma = 0\)). To obtain the Stokes-transport-induced flow, one can replace the semi–Stokes transport with the Stokes transport in the continuity equation.

The driving forces are

\[
F_{xR} = F_{yR} = 0,
\]

for river-induced flow:

\[
F_{xD} = \frac{g}{\rho_0} \left[ \frac{\partial}{\partial x} \left( D \frac{\partial \rho \partial \sigma'}{\partial \sigma} \right) + \rho \sigma \frac{\partial D}{\partial x} \right],
\]

and

\[
F_{yD} = \frac{g}{\rho_0} \left[ \frac{\partial}{\partial y} \left( D \frac{\partial \rho \partial \sigma'}{\partial \sigma} \right) + \rho \sigma \frac{\partial D}{\partial y} \right],
\]

for density-driven flow;
for advection-induced flow:

\[ F_{xA} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial \sigma}, \quad \text{and} \]

\[ F_{yA} = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial \sigma}, \]

(9a, 9b)

for ITA-induced flow:

\[ F_{xI} = -\left[Z \frac{\partial}{\partial \sigma} \left(K_m' \frac{\partial u}{\partial \sigma} \right) + Z' \frac{\partial}{\partial \sigma} \left(K_m' \frac{\partial u_{t}}{\partial \sigma} \right) \right], \quad \text{and} \]

\[ F_{yI} = -\left[Z \frac{\partial}{\partial \sigma} \left(K_m' \frac{\partial v}{\partial \sigma} \right) + Z' \frac{\partial}{\partial \sigma} \left(K_m' \frac{\partial v_{t}}{\partial \sigma} \right) \right], \]

(10a, 10b)

for wind-driven flow. The boundary conditions used to solve the governing equations for Eqs. (7)–(11) are no shear at the surface and no slip at the bottom. For the wind-driven flow, the surface shear stress is equal to the wind stress (i.e., \( \tau_{ux} \) and \( \tau_{uy} \) for the along- and cross-estuary components, respectively).

Following Ianniello (1977) and McCarthy (1993), the decomposition method separates the continuity equation and the water surface slope into multiple components corresponding to various residual flows. The idea behind this treatment is that individual residual flow is mass conserved, and the residual water surface slope not only accounts for river discharge, but also contributes to mass conservation for each residual flow, and is therefore needed in momentum equations. By this way, the decomposed residual currents can be compared to those analytical solutions developed for individual residual flow—for example, the estuarine gravitational circulation (Hansen and Rattray 1965) and the wind-driven circulation in semiclosed embayment (Wong 1994; Winant 2004).

The residual flow velocities can be obtained by solving Eqs. (5a) and (5b) numerically if the water surface slopes are known. To determine the water surface level, integrating Eqs. (5a) and (5b) vertically results in

\[ \theta [\eta] - f [\eta] = -g \frac{\partial \eta}{\partial x} - [F_{xI}] + \theta \tau_{xI}, \quad \text{and} \]

(13a)

\[ \theta [\eta] + f [\eta] = -g \frac{\partial \eta}{\partial y} - [F_{yI}] + \theta \tau_{yI}. \]

(13b)

Here the square bracket represents vertical integration—that is, \( [\cdot] = \int_{0}^{h} d \sigma \), \( \theta = (Z \rho_{h}/\rho_{0}) \), \( \tau_{xI}(x, y) \), and \( \tau_{yI}(x, y) \) are surface shear stresses at \( x \) and \( y \) directions for each component, respectively. The tidally averaged bottom shear stress is parameterized with a drag factor \( r \) (Geyer 2010), \( r = a_{0} \rho_{h} C_{D} U_{T} \), where \( C_{D} \) is the drag coefficient (with a typical value of 0.0025), \( U_{T} \) is the tidal velocity magnitude, and \( a_{0} \) is a dimensionless constant related to the effectiveness of turbulent momentum flux.

The general expressions of the depth-mean velocities read as follows:

\[ \eta_{I} = -Pg \frac{\partial \eta_{i}}{\partial x} - PQ \frac{\partial \eta_{i}}{\partial y} - P[F_{xI}] - Q[F_{yI}] \]

+ \( P \eta_{xI} + Q \eta_{yI} \) and

\[ \eta_{y} = P \frac{\partial \eta_{y}}{\partial x} - P \frac{\partial \eta_{y}}{\partial y} + Q[F_{xI}] - P[F_{yI}] \]

\[ -Q \eta_{xI} + P \eta_{yI}, \]

(14a, 14b)

where \( P = \theta r(f^{2} + \theta^{2} r^{2}) \), and \( Q = f(f^{2} + \theta^{2} r^{2}) \). Substituting the depth-mean velocities into the continuity equation [e.g., Eq. (5c)] gives the equation of water surface level:

\[ Pg \left( \frac{\partial^{2} \eta_{I}}{\partial x^{2}} + \frac{\partial^{2} \eta_{I}}{\partial y^{2}} + \frac{\partial P}{\partial x} \frac{\partial Q}{\partial y} \frac{\partial \eta_{I}}{\partial x} + \frac{\partial Q}{\partial x} \frac{\partial P}{\partial y} \frac{\partial \eta_{I}}{\partial y} \right) \]

\[ = -\frac{\partial}{\partial x} (P[F_{xI}]) + \frac{\partial}{\partial y} (Q[F_{xI}]) - \frac{\partial}{\partial x} (Q[F_{yI}]) + \frac{\partial}{\partial y} (P[F_{yI}]) \]

\[ + \frac{\partial}{\partial x} (P \eta_{xI}) - \frac{\partial}{\partial y} (Q \eta_{xI}) + \frac{\partial}{\partial x} (Q \eta_{yI}) + \frac{\partial}{\partial y} (P \eta_{yI}). \]

(15)

The boundary conditions to solve Eq. (15) are considered: the cross-estuary transport vanishes at the banks of the estuary—namely, \( [\eta] = 0 \); the along-estuary transport at the head of the estuary is equal to river discharge for river-induced flow and vanishes for the other components; and the water surface level must be specified at the mouth of the estuary, and can be simply set to zero.
Equation (15) can be solved numerically with a standard second-order finite difference scheme. This method requires gridded data covering the entire estuary, and is feasible for numerical model output. However, field surveys are always restricted to limited observation sites that are not enough to support the numerical solution of Eq. (15). To apply this method to real estuaries with observational data, it is necessary to seek an alternative way to obtain water surface slopes instead of directly computing the water surface level through solving Eq. (15). On the basis of a perturbation analysis (see details in the appendix), under the precondition that the ratio of width ($B$) and the length ($L$) of the estuary (i.e., $\alpha = B/L$) are small (or has an order of 0.1), the along-estuary residual water surface slope ($\partial \overline{h}/\partial x$) is independent to width, and the continuity can be simplified to

$$\int_0^B H[\overline{\pi}] \, dy = 0 \quad \text{and} \quad [\overline{\pi}] = 0. \tag{16a}$$

For river-induced flow, $\int_0^B H[\overline{\pi}_i] \, dy = R$, where $R$ is river discharge, and for Stokes return flow, the simplified continuity reads

$$\int_0^B H[\overline{\pi}_S] \, dy = -\int_{-1}^0 \overline{u} \overline{v} \, ds \, dy \quad \text{and} \quad [\overline{\pi}_S] = -\int_{-1}^0 \overline{u} \overline{v} \, ds. \tag{17a}$$

Using the simplified continuity, the water surface slopes can be obtained analytically. Therefore, the decomposition method only requires observational data or model output at a transverse section to separate residual currents. Once the horizontal velocities of each component of the residual currents are computed, the corresponding vertical residual velocity ($\overline{w}_i$) is available through the continuity equation

$$\frac{\partial \overline{w}_i}{\partial \sigma} + D \frac{\partial \overline{u}_i}{\partial x} + D \frac{\partial \overline{v}_i}{\partial y} = 0, \tag{18}$$

and the sigma coordinate transformation

$$\overline{w}_i = \overline{w}_i + \overline{u} \frac{\partial \overline{z}}{\partial x} + \overline{v} \frac{\partial \overline{z}}{\partial y}. \tag{19}$$

3. Numerical experiments

The decomposition method requires information on tidal current velocity, river discharge, wind stress, density field, and vertical eddy viscosity to calculate residual current velocities. An idealized numerical experiment is used to provide an example of the application of this decomposition method, as all of the required information is available in the model output. Although the method has the potential to be applied to a wide range of parameter space, we only present an elongated estuary with weak stratification and strong tidal currents. Because weakly stratified estuaries are weakly nonlinear systems, the existing analytical theories established for tidal currents and axis-wind-driven circulation (e.g., Wong 1994; Winant 2004, 2008) can be used to examine the model results. In addition, this type of estuary is chosen in order to show the importance of gravitational circulation as well as tidally induced and wind-driven residual currents.

a. Configuration of numerical model

A generic numerical experiment was carried out using the Regional Ocean Modeling System (ROMS), which is a free-surface, hydrostatic, primitive equations ocean model that uses stretched, terrain-following vertical coordinates and orthogonal curvilinear horizontal coordinates on an Arakawa C grid (Haidvogel et al. 2000; Shchepetkin and McWilliams, 2005). The model domain (Fig. 1) is designed as an estuary–shelf system, similar to that used in Cheng and Valle-Levinson (2009), but with explicit tides. The part of the domain corresponding to the estuary is straight, 500 km long, and has no along-channel bottom slope to minimize bottom bathymetry effects. The long estuarine channel is used to allow gradually damping of tides and to reduce the reflection of tides at the head of the channel, which is treated as a wall in the model configuration. The cross-channel section is 1.5 km wide and has a triangular shape with the maximum depth of 15 m at thalweg and the minimum depth of 5 m at banks. The continental shelf is 100 km wide and has a fixed cross-shelf slope of 0.05%. 

FIG. 1. A schematic of the numerical model domain and bathymetry.
The model grid is 240 (along estuary) × 80 (along shelf) × 20 (vertical) cells. The river has 200 grid cells along the channel and 31 cells across the channel. The along-channel grid size increases exponentially from the mouth of the estuary (50 m) to its head (about 12 km), providing a highly resolved region near the mouth of the estuary. The cross-channel and vertical grids are uniformly discretized. A freshwater discharge with a section-averaged velocity of 0.01 m s$^{-1}$ is imposed at the head of the estuary channel, while a semidiurnal tide ($S_2$) with an amplitude of 1.4 m is specified at the eastern open boundary. The inflowing river water is prescribed to have zero salinity and a temperature of 15°C, identical to the background temperature set throughout the entire domain. The salinity of the coastal ocean is 35 psu. A two-equation turbulence closure, $k$–$\omega$, is used to calculate vertical mixing. The model ran for 120 days to reach steady state and the analysis focuses on a cross section in the middle of the estuary where the section-averaged salinity is half of that at the estuary mouth. The first experiment represents a tidally dominated estuary and no wind forcing was prescribed, while the second experiment continued from the first experiment with a down-estuary wind event added after the 120-day tidal simulation.

b. Tidally dominated estuary

The tidally averaged salinities at the cross section show denser water in the thalweg and fresher water over shoals (Fig. 2a). Salinities are generally larger over the right side (facing the river) than over the opposite side. The tidally averaged stratification is relatively weak and asymmetrically distributed across the estuary (Fig. 2b). This lateral asymmetry of stratification results from the combined effect of asymmetric tidal mixing and the earth’s rotation. During flood tides, tidal currents tend to divert toward the right side of the cross section and mixing is relatively high from tidal straining and the strong inflow. During ebb tides, tidal currents tend to divert toward the left side of the cross section and mixing is relatively weak from tidal straining and fresher outflow. Over a tidal cycle, “net” mixing appears stronger on the right side than on the opposite side.

The cross-estuary distributions of along-estuary residual currents are shown in Fig. 3. The lateral asymmetry in stratification influences the transverse structure of the along-estuary residual currents, resulting in stronger currents on the left-side slope because the magnitude of residual currents is inversely related to the vertical eddy viscosity. Under the influence of the earth’s rotation, the distributions of the residual currents are asymmetric on the cross section. The river-induced and Stokes return (Figs. 3a and 3e) flows are unidirectional and have similar transverse structures, as they are driven by the barotropic pressure gradient. The Stokes return flow is relatively large because of the strong tide specified in the experiment. Density-driven flow (Fig. 3b) shows a laterally sheared two-layer circulation caused by the lateral variability of bottom bathymetry and the influence of stratification (Wong 1994; Kasai et al. 2000; Valle-Levinson 2008). The magnitude of the density-driven flow is smaller than that of the advection-induced, ITA-induced, and Stokes return flows, indicating that the density-driven flow is not the dominant component in the residual currents in this weakly stratified estuary. The advection-induced flow (Fig. 3c) generally reinforces the density-driven flow, supporting the conclusion of Lerczak and Geyer (2004) that lateral advection acts as a driving term for the estuarine exchange flow under weakly stratified conditions. The ITA-induced flow (Fig. 3d) is laterally sheared with seaward flows in the thalweg and landward flows over shoals. The general patterns of advection-induced and ITA-induced flows are quite similar to the results of the numerical experiment with large Simpson number by Burchard et al. (2011). The discrepancies mainly come from prescription of Coriolis forcing and relatively stronger stratification in this study. These results indicate the need for further studies to understand the effects of stratification and the earth’s rotation on residual estuarine currents. The sum of the five components of the residual currents obtained using the decomposition method (Fig. 3f) has a very similar transverse distribution to that obtained by directly averaging modeled currents.
(Fig. 3g), clearly demonstrating the self-consistency of the decomposition method. The difference between Figs. 3f and 3g is small with relatively large values in the upper water column (Fig. 3h). The calculation of residual currents is sensitive to the vertical eddy viscosity that has large variations over the thalweg. The calculation errors might be relevant to the distribution of the vertical eddy viscosity.

The lateral residual circulation is relatively complicated in terms of its physical interpretation (Fig. 4). This study focuses on methodology, so it only provides a tentative interpretation. The lateral river-induced and Stokes return flows (Figs. 4a and 4e) result from the Coriolis effects on an unidirectional flow that generates a counterclockwise lateral circulation through tilting (or the vertical shear) of \( f \bar{u} \). Because the lateral semi–Stokes transport (i.e., \( \int f \bar{u} d\sigma \)) is negligible, the lateral mass balance is simplified to \( [\bar{n}] = 0 \) in the calculation of lateral Stokes return flow (the same treatment was applied for the second experiment). The lateral density-driven, advection-induced, and ITA-induced flows result from the combined effect of tilting of their longitudinal counterparts (\( f \bar{n} \)) and their lateral forcing [i.e., Eq. (5b)]. The lateral density-driven circulation (Fig. 4b) is generally clockwise, mainly responding to the higher salinities on the right side of the section (Fig. 2a). The lateral advection-induced flow (Fig. 4c) shows two counterclockwise-rotating cells over the slopes, largely responding to the rotation of \( u_A \). The lateral ITA-induced flow (Fig. 4d) has a two-cell pattern with convergence at the surface and divergence near the bottom, responding to the effects of both lateral ITA and the rotation of \( \bar{u} \). The total lateral residual currents generally show a counterclockwise cell in the upper water column and a clockwise cell in the lower water column (Figs. 4f and 4g). The upper-layer cell mainly
comes from the combination of the advection-induced and ITA-induced flows, while the lower layer cell comes from the contribution of the density-driven flow. The differences between Figs. 4f and 4g are small with relatively large values over the thalweg that might result from the variation of the vertical eddy viscosity (Fig. 4h).

c. Effects of down-estuary wind

On the basis of the experiment of a tidally dominated estuary, a down-estuary wind event was added following the 120-day simulation. The wind event lasted 6 days and had transition periods at the first and last days (Fig. 5). During the middle period of the event, the wind was steady and had a speed of 10 m s\(^{-1}\), equivalent to a shear stress of 0.14 N m\(^{-2}\) according to the formula of Large and Pond (1981). The model output in the fifth day of the event (e.g., day 125) was selected for analysis. Tidally averaged salinities, vertical eddy viscosities, and residual currents were computed at the same transverse section analyzed in the experiment of a tidally dominated estuary in order to explore the effects of wind. The simulation ended after the wind event, giving no chance to examine the adjustment of the estuary responding to the wind event. Previous studies have shown that the stratification in a partially mixed estuary recovers quickly to prior wind conditions within a day (Li and Li 2012). It is expected that the estuary adjustment in this experiment is quick.

Compared to the tidally dominated estuary (Fig. 2a), stratification significantly increases at the transverse section (Fig. 6a) when a down-estuary wind exists. The water column becomes fresher near the surface and becomes saltier near the bottom, clearly showing the effect of wind-induced straining on estuarine stratification (Scully et al. 2005). Down-estuary winds push freshwater seaward, reducing surface salinities; at the same time, the wind-induced return flow near the bottom brings salty water into the estuary, enhancing bottom salinities. As a response to the enhanced stratification, the eddy viscosity
generally decreases at the section (Fig. 6b). In the tidally dominated estuary, the vertical pattern of vertical eddy viscosity shows an asymmetric parabolic distribution with the lowest values near the surface (Fig. 2b), while during the wind event, the minimum eddy viscosity moves downward and relatively higher eddy viscosities appear in the surface layer, indicating wind-induced mixing in the upper water column of the cross section. Therefore, down-estuary winds affect estuarine stratification through two ways: increasing stratification of the entire water column and increasing mixing near the surface.

Analytical models have shown that in a channel with lateral variability of bottom bathymetry, the wind-driven flow moves along the wind direction over shoals and moves in the opposite direction in the thalweg (Wong 1994; Winant 2004). The vertical pattern of the wind-driven flow at the cross section (Fig. 7f) supports the previous analytical studies. In comparison to the experiment without a wind event (Fig. 3b), the density-driven flow is generally enhanced (Fig. 7b) due to reduced vertical eddy viscosities and larger along-estuary density gradients resulted from increased landward salt fluxes that were transported by the wind-driven flow. The density-driven flow also exhibits more pronounced lateral variability with landward flow tending to be intensified over the right side. Because of the earth’s rotation, down-estuary winds tend to push freshwater toward the left bank, while the landward salt flux deflects toward the right bank (Fig. 6a). As a consequence, the lateral variability of salt transport causes larger along-estuary density gradients over the right slope of the cross section and, in turn, enhances the landward flow. The transverse patterns of the river-induced, advection-induced, ITA-induced, and Stokes return flows (Figs. 7a, 7c, 7d, and 7e) are similar to those without wind influences (Figs. 3a, 3c, 3d, and 3e), but they show less shear in the top water column, resulting from the wind-intensified mixing near the water surface.

The lateral river-induced, Stokes return, and wind-driven flows (Figs. 8a, 8c, and 8f) are not straightforward. The two lateral circulations are generally opposite of each other, consistent to the patterns of their longitudinal counterparts. The pattern of total lateral residual currents is quite complicated (Figs. 8a and 8b) and cannot be directly inferred from a particular component of lateral circulation. Generally, the lower water column, which is dominated by the tidal bottom boundary layer, shows a two-cell structure with divergence in the thalweg. The upper water column, which is modified by the wind-driven boundary layer, shows complex current patterns without straightforward interpretation.

**4. Discussion and conclusions**

The decomposition method relies on the assumption that the momentum and continuity equations can be divided into several components, each representing a forcing mechanism. This implies that those components are independent. However, different driving forces actually interact with each other. For example, the advection terms can be expanded into two parts, representing residual advection and tidal nonlinearities, defined as

$$
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \right)
+ \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \right)
+ \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \right)
$$

and

$$
\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} \right)
+ \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} \right)
+ \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} \right)
$$

respectively. (20a)

**FIG. 5.** Water surface wind stress as a function of time during the down-estuary wind event.
In tidally energetic estuaries where residual currents are much weaker than tidal currents, the advection-induced flow generally represents the effects of tidal nonlinearities [the second brackets on the right-hand side of Eqs. (20a) and (20b)]. In estuaries where the magnitudes of residual currents are comparable to or less than those of tidal currents, the advection-induced flow also contains the advection of momentum by residual currents [the first brackets on the right-hand side of Eqs. (20a) and (20b)] and, as a consequence, the physical meaning of advection-induced flow needs careful interpretations. An alternative is to further separate the advection-induced flow to residual-advection-induced and tidal-nonlinearities-induced flows in responding to the two components of the advection terms.

Asymmetries in tidal mixing lead to tidal variations in the vertical structure (or the vertical shear) of tidal currents, resulting in a residual flow that is referred to as asymmetric-tidal-mixing-induced flow (Cheng et al. 2010). The driving force for the asymmetric-tidal-mixing-induced flow is defined by the covariance between tidal fluctuations of eddy viscosity and vertical shear of the along-estuary flow (Jay 1991; Stacey et al. 2001). In narrow estuaries tidal straining (or strain-induced periodic stratification; Simpson et al. 1990) could be the dominant process generating asymmetries in vertical shear of tidal currents, while in wide estuaries the covariance might largely result from lateral advection (Burchard and Schuttelaars 2012). As asymmetric tidal mixing is only one of the mechanisms generating the asymmetric shear, it would be more general to refer to the asymmetric-tidal-mixing-induced flow as ITA-induced flow (Jay 1991; Cheng et al. 2013).

Except for the river-induced and the Stokes return flows, the solutions of residual currents are made up of two parts: the barotropic component and the component arising from the specific driving forcing. The barotropic component not only represents a local runoff term that accounts for continuity (Burchard et al. 2011) but also affects the spatial patterns of the residual currents (Cheng et al. 2013). Therefore, it is an essential component that needs to be included in residual flows.

Calculation errors exist in the decomposition method, as the comparison of total residual currents showed some discrepancies between the calculated (the sum of all components obtained from the decomposition) and the modeled tidally averaged currents. The errors result in part from the calculation of each term in the momentum equation using model output, which is inconsistent with the time step of the numerical calculation, and in part from the simplified surface and bottom boundary conditions. To reduce calculation errors, the diagnostic module of ROMS was used to obtain all separate terms in the momentum equations.

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In the development of the decomposition method, six specific driving forces of residual currents were considered [e.g., Eq. (5)]. Following the same strategy, however, it is not difficult to add other possible physical mechanisms into the momentum equations, such as wave radiation stress (Longuet-Higgins and Stewart 1964; Mellor 2008) and the effect of remote wind (Wang and Elliott 1978). The latter produces a unidirectional flow that can be treated in the same way as river discharge. The associated volume flux of changes in the subtidal water level at the estuary mouth caused by remote winds can be simply related to the product of the time rate of change in subtidal water level and the cumulative surface area up-estuary to the land limit of salt intrusion (Wong and Valle-Levinson 2002). To obtain equations on subtidal time scales, the momentum and continuity equations were averaged over a tidal cycle. This way to take time average might limit the decomposition method to explore the changes in residual currents on subtidal time scales, for example, the spring–neap variability. A possible solution is to take low-pass filtering of the momentum and continuity equations, and then to obtain time series of each component of residual circulation. An analogous method has been developed to decompose the subtidal salt flux in the Hudson River (Lerczak et al. 2006).

Although the decomposition method is illustrated using a numerical model in this study, it could potentially...
be applied to observational data if measured current velocities, density field, vertical eddy viscosities, and wind stress values are available in high spatial and temporal resolutions. The simplification of the continuity equation allows the decomposition to be practically applied for a transverse section without the need of observational data covering the entire estuary. Accurate measurements of vertical eddy viscosities are particularly required because the spatial patterns of residual currents are highly sensitive to the vertical distribution of the eddy viscosity. Turbulence measurement poses major challenges to observations at the present time. Under very weak stratification, the vertical distribution of eddy viscosity can be simplified and prescribed using a parabolic function or even a constant. However, well-developed stratification reduces eddy viscosities and amplifies residual currents in the upper water column, resulting in highly sheared flows. Therefore, the approximation of a constant or a parabolic function for the vertical eddy viscosity is inappropriate to predict the vertical profiles of residual currents in stratified estuaries.

Despite these limitations of the decomposition method, it provides a way to extract flow components driven by different forcing mechanisms. The decomposition method represents a significant step forward than those numerical experiments in which one or more forcing mechanisms are switched off in the numerical model because in real estuaries, all forcing mechanisms are present simultaneously and none of them can be removed. Although the method was validated with weakly stratified estuaries, it has the potential to be applied in estuaries with strong stratification. Further work is required to seek physical interpretations of the various flow components produced by the decomposition method and to explore flow regimes under different river discharge, tidal, and wind forcing conditions.
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APPENDIX

Simplification of Continuity Equation

Because numerically solving Eq. (15) to obtain the water surface level is not practicable with observational data in real estuaries, we tried to explore the type of estuaries for which the continuity equation can be simplified such that the water surface slopes can be calculated analytically. This was carried out by doing perturbation analysis of the continuity equation. The analysis reached two conclusions for elongated estuaries of which the ratio between width and length is small: 1) the longitudinal residual water surface slope is approximately independent of width and 2) the lateral residual flow is mass balanced. The two conclusions allow the continuity equation [Eq. (15)] to be solved analytically. The procedure of the perturbation analysis largely followed the work of Winant (2007, 2008).

We consider an estuary that is an elongated channel with a varying cross-channel bottom bathymetry and a constant width, $B$ (the caret denotes dimensional variables), on the $f$ plane. The length $L$ is much larger than the width, and the maximum depth is much smaller than $B$. Without losing generality, we assume constant vertical eddy viscosities and adopt $z$ coordinates. The dimensional governing equations for the estuary include

FIG. 8. Transverse distributions (looking landward) of lateral residual currents at the middle of the estuary (the same section in Fig. 4) during the down-estuary wind event: (a) river-induced flow, (b) density-driven flow, (c) advection-induced flow, (d) ITA-induced flow, (e) Stokes return flow, (f) wind-driven flow, (g) sum of the decomposed residual currents [(a)–(f)]; and (h) tidally averaged currents from model output. The magnitude of river-induced flow is multiplied by a factor of 2 for clarity.
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v = -g \frac{\partial \eta}{\partial x} + \hat{g} \frac{\partial \hat{z}}{\partial x} \hat{z} + \frac{\partial}{\partial z} \left( K_m \frac{\partial \eta}{\partial z} \right), \quad (A1a) \]

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + f u = -g \frac{\partial \eta}{\partial y} + \hat{g} \frac{\partial \hat{z}}{\partial y} \hat{z} + \frac{\partial}{\partial z} \left( K_m \frac{\partial \eta}{\partial z} \right), \quad (A1b) \]

\[ \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \left( \int_{-h}^{0} \hat{u} \, dz \right) + \frac{\partial}{\partial y} \left( \int_{-h}^{0} \hat{v} \, dz \right) = 0. \quad (A1c) \]

Nondimensional variables are introduced as follows for tidally dominated estuaries:

\[ t = \hat{t} \omega, \quad f = f / \omega, \quad x = \hat{x} \hat{L}, \quad y = \hat{y} \hat{B}, \]

\[ z = \hat{z} / \hat{H}, \quad h = \hat{h} / \hat{H}, \quad u = \hat{u} \hat{U}_0, \quad v = \hat{v} \hat{V}_0, \]

\[ w = \hat{w} \hat{W}_0, \quad \eta = \hat{\eta} / \hat{\eta}_0, \quad s = \hat{s} / \hat{s}_c, \quad (A2) \]

where,

\[ \omega = \frac{2 \pi}{T}, \quad e = \frac{\hat{\eta}_0}{\hat{H}}, \quad \hat{U}_0 = \hat{e} \omega \hat{L}, \quad \hat{V}_0 = \hat{e} \omega \hat{B}, \quad \hat{W}_0 = \hat{e} \omega \hat{H}, \]

\( \hat{h} \) is water depth of the estuary, \( \hat{H} \) is the maximum water depth, \( \hat{\eta}_0 \) is tidal amplitude, \( T \) is tidal period, and \( \hat{s}_c \) represents the maximum horizontal salinity difference. Combining Eqs. (A1) and (A2) yields the following nondimensional governing equations:

\[ \frac{\partial u}{\partial t} + e \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) - \alpha f v = - \frac{1}{\kappa^2} \frac{\partial \eta}{\partial x} + e \gamma \frac{\partial s}{\partial x} \hat{z} \frac{\partial^2 \hat{z}}{\partial \hat{z}^2} + \frac{\delta^2}{2} \frac{\partial^2 u}{\partial \hat{z}^2}, \quad (A3a) \]

\[ \frac{\partial v}{\partial t} + e \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) + \frac{1}{\alpha} f u = - \frac{1}{\alpha \kappa^2} \frac{\partial \eta}{\partial y} + e \gamma \frac{\partial s}{\partial y} \hat{z} \frac{\partial^2 \hat{z}}{\partial \hat{z}^2} + \frac{\delta^2}{2} \frac{\partial^2 v}{\partial \hat{z}^2}, \quad (A3b) \]

\[ \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \left( \int_{-h}^{0} \hat{u} \, dz \right) + \frac{\partial}{\partial y} \left( \int_{-h}^{0} \hat{v} \, dz \right) = 0, \quad (A3c) \]

where

\[ \delta = \sqrt{2K_m / \hat{\omega} \hat{H}^2}, \quad \alpha = \hat{B} / \hat{L}, \quad k = \hat{\omega} \hat{L} / \sqrt{\hat{g} \hat{H}}, \quad \gamma = \frac{\hat{g} \hat{s}_c}{\hat{e}^2}. \]

\( \alpha \) is the horizontal aspect ratio of the basin, \( \delta \) is the relative amplitude of bottom boundary layer to the maximum water depth measuring vertical mixing, \( \kappa \) is a relative measure of the length of the basin to the tidal wavelength (Winant 2007), and \( \gamma \) measures the relative importance of the baroclinic pressure gradient (McCarthy 1993). Taking the tidal average of Eq. (A3) gives

\[ \frac{\delta^2}{2} \frac{\partial^2 \eta}{\partial \hat{z}^2} + f \alpha v = \frac{1}{\kappa^2} \frac{\partial \eta}{\partial x} + \frac{e \gamma}{\kappa^2} \frac{\partial s}{\partial x} \hat{z} + \frac{e}{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}}. \quad (A4a) \]

\[ \frac{\delta^2}{2} \frac{\partial^2 \eta}{\partial \hat{z}^2} - \frac{1}{\alpha} f \pi = \frac{1}{\alpha^2 \kappa^2} \frac{\partial \eta}{\partial y} - \frac{e \gamma}{\alpha^2 \kappa^2} \frac{\partial s}{\partial y} \hat{z} + e \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right), \quad (A4b) \]

\[ \frac{\partial}{\partial \hat{x}} \left( \int_{-h}^{0} \hat{u} \, dz + e \eta u|_{\hat{z}=0} \right) + \frac{\partial}{\partial \hat{y}} \left( \int_{-h}^{0} \hat{v} \, dz + e \eta v|_{\hat{z}=0} \right) = 0. \quad (A4c) \]

Combining Eqs. (A4a) and (A4b) and defining a complex velocity \( Y = \pi + i \alpha \pi \), results in

\[ \frac{\delta^2}{2} \frac{\partial^2 Y}{\partial \hat{z}^2} - \alpha f \frac{\delta^2}{2} \frac{\partial^2 Y}{\partial \hat{z}^2} |_{\hat{z}=0} = \frac{1}{\kappa^2} \left( \frac{\partial \eta}{\partial x} + \frac{i \partial \eta}{\alpha \partial y} \right) - \frac{e \gamma}{\kappa^2} \left( \frac{\partial s}{\partial x} \hat{z} + \frac{i \partial s}{\alpha \partial y} \right) + \left[ \frac{e}{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}} \right], \quad (A5) \]

Following the same strategy of the decomposition method presented in section 2, the residual flow and water surface level have four components as shown:

\[ Y = Y_R + Y_D + Y_A + Y_S, \quad (A6a) \]

\[ \eta = \eta_R + \eta_D + \eta_A + \eta_S, \quad (A6b) \]

and Eq. (A5) is transferred to a general form for each component of the residual currents,

\[ \frac{\delta^2}{2} \frac{\partial^2 Y_i}{\partial \hat{z}^2} - \alpha f \frac{\delta^2}{2} \frac{\partial^2 Y_i}{\partial \hat{z}^2} |_{\hat{z}=0} = \left( F_{xi} + i \alpha F_{yi} \right). \quad (A7) \]

Here, for density-driven flow, \( F_{xi} + i \alpha F_{yi} = -(e \gamma / \kappa^2) \left[ \frac{\partial \eta}{\partial x} \hat{z} + \frac{i \partial \eta}{\alpha \partial y} \right] \); and for advection-induced flow, \( F_{xA} + i \alpha F_{yA} = \left( \frac{e}{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}} \right) \left[ \frac{1}{\kappa^2} \frac{\partial \eta}{\partial x} + \frac{i \partial \eta}{\alpha \partial y} \right] + i \alpha e \left[ \frac{e}{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}} \right] \). The solution to
Eq. (A7) is considered as \( Y_i = Y_i^\eta + Y_i^F \), driven by the water surface slope and the corresponding driving force, respectively. Hence, Eq. (A7) is further separated into the following two parts:

\[
\frac{\delta^2 \partial^2 Y_{iF}}{2 \partial z^2} - if Y_{iF} = F_{si} + i\alpha F_{si}.
\] (A8b)

The boundary conditions to solve the above-given equations are no shear at the surface and no slip at the bottom. The general form of the solution of \( Y_{i\eta} \) is

\[
Y_{i\eta} = (p_r + ip_i) \left( \frac{\partial \eta}{\partial x} + \frac{i \partial \eta}{\alpha \partial y} \right),
\] (A9)

where \( p \) is the velocity vertical profile function, and the subscripts \( r \) and \( i \) represent the real and imaginary parts, respectively. The mass transport driven by the residual forces is

\[
\left[ \eta_{i\eta} \right] + i\alpha \left[ \eta_{i\eta} \right] = \int_{-h}^{0} Y_{i\eta} \, dz
\] (A10)

and then

\[
\left[ \eta_{i\eta} \right] = P_r \frac{\partial \eta}{\partial x} - P_i \frac{\partial \eta}{\partial y}, \quad \text{and} \quad \left[ \eta_{i\eta} \right] = \frac{1}{\alpha} \left( P_r \frac{\partial \eta}{\partial x} + P_i \frac{\partial \eta}{\partial y} \right).
\] (A11a)

\[
\left[ \eta_{iF} \right] + i\alpha \left[ \eta_{iF} \right] = \int_{-h}^{0} Y_{iF} \, dz.
\] (A12)

Therefore, the total mass transport is the sum of the following two components:

\[
\left[ \eta_i \right] = \left[ \eta_{i\eta} \right] + \left[ \eta_{iF} \right] \quad \text{and} \quad \left[ \eta_i \right] = \left[ \eta_{i\eta} \right] + \left[ \eta_{iF} \right].
\] (A13a)

Substituting Eq. (A13) into the continuity equation (except for the Stokes return flow)

\[
\frac{\partial}{\partial x} \left( \int_{-h}^{0} \eta_i \, dz \right) + \frac{\partial}{\partial y} \left( \int_{-h}^{0} \eta_i \, dz \right) = 0
\] (A14)

results in

\[
\frac{\partial}{\partial y} \left( P_r \frac{\partial \eta}{\partial y} \right) + \alpha \left[ \frac{\partial}{\partial y} \left( P_i \frac{\partial \eta}{\partial y} \right) - \frac{\partial}{\partial x} \left( P_i \frac{\partial \eta}{\partial x} \right) \right] + \alpha^2 \frac{\partial^2}{\partial x^2} \left( P_r \frac{\partial \eta}{\partial x} \right) = 0.
\] (A15)

The boundary conditions across the estuary shows that at \( y = \pm 1 \), \( \left[ \eta_i \right] = 0 \), or

\[
P_r \frac{\partial \eta_i^{(0)}}{\partial y} + \alpha P_i \frac{\partial \eta_i^{(0)}}{\partial y} + \alpha^2 \left[ \eta_i^{(0)} \right] = 0.
\] (A16)

For an elongated estuary, the continuity equation is expanded as a sum of perturbation series in power of \( \alpha \):

\[
\eta_i = \eta_i^{(0)} + \alpha \eta_i^{(1)} + \alpha^2 \eta_i^{(2)} + \cdots.
\] (A17)

Including the expansion into Eq. (A15) gives an ordered set of problems. The lowest problem shows

\[
\frac{\partial}{\partial y} \left( P_r \frac{\partial \eta_i^{(0)}}{\partial y} \right) = 0;
\] (A18)

the boundary conditions on the lateral transport at the same order is

\[
P_r \frac{\partial \eta_i^{(0)}}{\partial y} = 0, \quad \text{at} \quad y \pm 1.
\] (A19)

This indicates \( \frac{\partial \eta_i^{(0)}}{\partial y} = 0 \), or equivalently, \( \eta_i^{(0)} \) is independent of \( y \).

To the order \( \alpha \) problem,

\[
\frac{\partial}{\partial y} \left( P_r \frac{\partial \eta_i^{(1)}}{\partial y} \right) + \frac{\partial}{\partial y} \left( P_i \frac{\partial \eta_i^{(0)}}{\partial y} \right) + \alpha \left[ \eta_i^{(0)} \right] = 0;
\] (A20)

the boundary conditions on the lateral transport at the same order is

\[
P_r \frac{\partial \eta_i^{(1)}}{\partial y} + P_i \frac{\partial \eta_i^{(0)}}{\partial x} + \alpha \left[ \eta_i^{(0)} \right] = 0, \quad \text{at} \quad y \pm 1.
\] (A21)

Note that \( \left[ \eta_i^{(0)} \right] \) is \( O(\alpha^{-1}) \) relative to \( \left[ \eta_i^{(0)} \right] \) [see Eqs. (A11) and (A12); also see Winant 2008]. With the boundary condition [Eq. (A21)], integrating Eq. (A20) across the width of the estuary gives
\[ P_i \frac{\partial \eta_i^{(1)}}{\partial y} + P_i \frac{\partial \eta_i^{(0)}}{\partial x} + \alpha \eta_i^f = 0. \]  
(A22)

This is equivalent to imposing \[ [\mathbf{v}] = 0 \] at \( O(\alpha) \); that is, the cross-estuary flow is mass balanced.

For the Stokes return flow, the continuity returns to
\[
\frac{\partial}{\partial x} \left( \int_{-h}^{0} \nu \, dz + S_x \right) + \frac{\partial}{\partial y} \left( \int_{-h}^{0} \nu \, dz + S_y \right) = 0, 
\]  
(A23)

where \( S_x = \varepsilon \eta u \vert_{z=0} \) and \( S_y = \varepsilon \eta v \vert_{z=0} \). The mass transport becomes
\[
[\mathbf{\eta}] = [\mathbf{\eta}_N] + [\mathbf{\eta}_S] + S_x \quad \text{and} \quad \mathbf{[\eta]} = [\mathbf{\eta}_N] + [\mathbf{\eta}_S] + S_y. 
\]  
(A24a)

\[
[\mathbf{\eta}] = [\mathbf{\eta}_N] + [\mathbf{\eta}_S] + S_x \quad \text{and} \quad \mathbf{[\eta]} = [\mathbf{\eta}_N] + [\mathbf{\eta}_S] + S_y. 
\]  
(A24b)

The same conclusions as the other components to the continuity equation can be obtained using the same analysis procedure.

The above-presented analysis is for tidally dominated estuaries. When a steady wind dominates the estuary, nondimensional variables are introduced as follows:

\[
f = \hat{f}f_0, \quad x = \hat{x}L, \quad y = \hat{y}B, \quad z = \hat{z}H, \quad h = \hat{h}H, \quad u = \hat{u}U_0, \quad v = \hat{v}V_0, \quad w = \hat{w}W_0, \quad \eta = \hat{\eta}/\eta_0, \quad s = \hat{s}s, 
\]  
(A25)

where

\[
\hat{U}_0 = \frac{\tau_0 \hat{H}}{\rho_0 K_m}, \quad \hat{V}_0 = \frac{\hat{B} L}{\hat{H}} \hat{U}_0, \quad \hat{W}_0 = \frac{\hat{H}}{\hat{L}} \hat{U}_0, 
\]

\[
\hat{\eta}_0 = \frac{\tau_0 \hat{L}}{\rho_0 \hat{g} \hat{H}}, \quad \hat{v} = \frac{\hat{v}_0}{\hat{H}}. 
\]

The definitions of \( \hat{U}_0 \) and \( \hat{\eta}_0 \) are referred to Winant (2004). Combining Eqs. (A1) (without the local accelerations) and (A25) yields the following nondimensional governing equations for steady-wind-driven estuaries:

\[
R_0 \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \alpha \hat{u} 
\]

\[
= -\frac{\delta^2 \partial \eta}{2 \partial x} + \frac{\delta^2 \partial \eta}{2 \partial x} + \frac{\delta^2 \partial^2 u}{2 \partial z^2} \quad \text{and} \quad (A26a) 
\]

\[
R_0 \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) + \alpha \hat{u} 
\]

\[
= -\frac{1}{\alpha^2} \frac{\delta^2 \partial \eta}{2 \partial y} + \frac{1}{\alpha^2} \frac{\delta^2 \partial^2 \eta}{\partial y \partial z} + \frac{\delta^2 \partial^2 \eta}{2 \partial z^2}. \quad (A26b) 
\]

where \( R_0 = \hat{u_0}/Lf_0 \) is the Rossby number, \( \delta = \sqrt{2K_m f_0 \hat{H}} \) is the Ekman number, and \( \alpha = B/L \) is the horizontal aspect ratio. The nondimensional equations are similar to the tidally averaged governing equations [i.e., Eqs. (A4a) and (A4b)], and a similar perturbation analysis will reach the same simplification of the continuity equation.

REFERENCES


