Optimum Estimation of Coastal Currents Using Moving Vehicles

Kuanyu Chen, a,b Chen-Fen Huang, a Zhe-Wen Zheng, c Sheng-Fong Lin, d Jin-Yuan Liu, e and Jenhwa Guo f

Abstract: Ocean acoustic tomography (OAT) deploys most moored stations on the periphery of the tomographic region to sense the solenoidal current field. Moving vehicle tomography (MVT), an advancement of OAT, not only samples the region from various angles for improving the resolution of mapped currents but also acquires information about the irrotational flow due to the sampling points inside the region. To reconstruct a complete two-dimensional current field, the spatial modes derived from the open-boundary modal analysis (OMA) are preferable to the conventional truncated Fourier series since the OMA technique describes the solenoidal and irrotational flows efficiently in which all modes satisfy the coastline and open boundary conditions. Comparisons of the reconstructions are presented using three different representations of currents. The first two representations explain only the solenoidal flow: the truncated Fourier series and the OMA Dirichlet modes. The third representation, accounting for the solenoidal and irrotational flows, uses all the OMA modes. For reconstructing the solenoidal flow, the OMA representation with the Dirichlet modes performs better than the Fourier series. A large difference appears near the bay mouth, where the OMA-Dirichlet reconstruction shows a better fit to the uniform currents. However, considerable uncertainty exists outside the bay mouth where the irrotational currents dominate. This can be improved by the third representation with the inclusion of the Neumann and boundary modes. The reconstruction results using field data were validated against the acoustic Doppler current profiler (ADCP) measurements. Additionally, incorporating constraints from ADCP measurements enhances the accuracy of the reconstruction.

Significance statement: This study contributes toward improving our understanding of accurately measuring oceanic circulation patterns over large areas without relying solely upon stationary sensors or satellite imagery. The study combines multiple sources, such as shipboard ADCP and tomographic techniques, to obtain a complete picture of what is happening beneath surface waters across entire regions under investigation. It has important implications for fields such as climate science, marine biology, and fisheries management, where accurate knowledge of the movement and distribution of water masses is crucial for predicting future trends and making informed decisions.

Keywords: Acoustic measurements/effects; In situ oceanic observations; Remote sensing

1. Introduction

Knowledge of coastal currents can benefit the management of coastal seas, which are the most biologically productive and vulnerable at the same time, as most of the fish is harvested from the coastal seas. In situ measurements of coastal currents are difficult to obtain due to their spatial and temporal complexities. For example, circulations in coastal seas are often formed by flow separation due to sudden changes in topography (Middleton 2001). Circulations also occur when a strong current contacts with the surrounding seawater of a slower velocity caused by topography. These features in the coastal current system cannot be easily observed by point measurements, such as acoustic Doppler current profiler (ADCP), due to their relatively small spatial scales.

Synoptic images of ocean states, such as ocean temperature and current fields, can be obtained by ocean acoustic tomography (OAT), proposed by Munk and Wunsch (1979). OAT has been applied to the coastal seas to obtain the horizontal structure of tidal currents in coastal seas (e.g., Park and Kaneko 2000; Yamoaka et al. 2002; Zhu et al. 2013). Reconstructing an accurate ocean image using OAT typically requires deploying many tomographic moorings on the periphery of the tomographic region to obtain a sufficient number of acoustic rays to sample the region.

Moving vehicle tomography (MVT) uses mobile platforms to increase the number of acoustic rays that sample the tomographic region from various angles and to improve the reconstructions of temperature (Cornuelle et al. 1989) and currents (Huang et al. 2019). Two types of currents are of interest: solenoidal flows and irrotational flows. Solenoidal flows can be obtained by both OAT and MVT, which both use travel-time measurements between two platforms. However, obtaining...
irrotational flows requires measurements within the tomographic region (Munk et al. 1995), and can only be obtained with MVT as the mobile platforms cruise within the region where current fields evolve gradually.

Reconstructing ocean current fields using tomographic techniques requires decomposing a continuous field into a chosen basis set. Many studies (e.g., Munk et al. 1995; Yamoka et al. 2002; Gaillard 1992; Huang et al. 2013) represented the solenoidal flow component with a truncated Fourier series. However, this basis set might be inefficient in coastal seas, as it does not satisfy the coastline boundary conditions and cannot adequately describe localized flow features such as fronts. An ocean representation with prior environmental information could accurately describe ocean physics. Thus, this study explores the open-boundary modal analysis (referred to as OMA) (Lekien et al. 2004) to represent both the irrotational and solenoidal flows in coastal seas using three types of coastline-fitting normal modes, i.e., the Dirichlet, Neumann, and boundary modes. The OMA representation has been applied to the spatial interpolation and filtering of radar surface current measurements in Bodega Bay and Monterey Bay (Kaplan and Lekien 2007). For the application of OAT, the OMA representation has been applied to coastal seas (M. Chen et al. 2020).

An optimal estimation of ocean states has been achieved for temperature fields by combing tomographic data and direct point measurements, e.g., XBT/conductivity–temperature–depth (CTD), in an inverse procedure (Cornuelle et al. 1993; Dushaw et al. 1993a,b; Dushaw and Sagen 2016). For the current reconstruction, the tomographic data measure the integrated current velocity projected along the ray path, and thus, the integrated measurements are sensitive to the relatively low wavenumber characteristics. While the ADCP point measurements measure the variation at a single location, and thus, the data can localize a detailed spatial feature (Cornuelle and Worcester 1996). As the sampling properties between the tomographic and point measurements complement each other, employing both data types in an observation system would lead to an optimal estimate of ocean states.

In this study, we demonstrate the optimal estimate of ocean currents using the data collected during a coastal MVT experiment conducted in WangHaiXiang Bay, Keelung, Taiwan, in 2017 (K. Chen et al. 2020), in which both integral and point measurements of currents were taken. WangHaiXiang Bay, with an average water depth of about 25 m, is a semienclosed bay characterized by strong tidal currents flowing parallel to the bay mouth. These tidal currents create cyclonic and anticyclonic circulations inside the bay during the flood and ebb tidal phases, respectively. Throughout the experiment, several CTD casts were conducted at the site. These casts revealed uniform depth profiles for both temperature and salinity, which are likely attributed to the relatively shallow water depth and efficient tidal mixing within the water column. A separate analysis of currents measured by the shipboard and bottom-mounted ADCPs (C.-R. Ho 2023, personal communication) indicates that the depth variation of flow is relatively mild. Thus, the primary objective of this study is to map the depth-averaged horizontal current fields in the bay. To further evaluate the performance of the Fourier and OMA representations, numerical experiments of reciprocal acoustic transmission are performed using the simulated currents from the Coastal Ocean Model off Keelung (referred to as KCOM) for the period of this MVT experiment. The synthetic data [differential travel times (DTTs)] are obtained using the geometry of the acoustic rays from the MVT experiment.

The rest of this paper is organized as follows. Section 2 briefly describes the synthetic ocean currents and the synthetic DTT data of the 2017 MVT experiment. Section 3 introduces the current field representations using the truncated Fourier series and OMA modes. Section 4 formulates the MVT inverse problem. Section 5 presents the application of OMA to the tomographic reconstruction of ocean currents in WangHaiXiang Bay and discusses the model resolution. The results of synthetic tomographic experiments and the 2017 MVT experiment for the current reconstructions are examined in section 6, followed by concluding remarks in section 7.

2. Forward problem

The DTTs obtained from reciprocal acoustic pulses are proportional to the line integral of the current velocity projected along the ray path (Worcester 1977). For the \( j \)th transceiver pair, the DTT \( d_j \) is related to the current velocity \( \mathbf{v} \) and a reference sound speed \( c \) (Munk et al. 1995):

\[
d_j = \frac{2}{c^2} \int_{\Gamma_j} \mathbf{v} \cdot \mathbf{r'} ds,
\]

where \( \mathbf{r'} \) is the unit vector tangent to the ray path \( \Gamma_j \) for the reference sound speed \( c \), and \( s \) is the arc length along the ray.

Applying the Helmholtz representation, a two-dimensional (2D) current field \( \mathbf{v}(x, y) \) can be decomposed into solenoidal and irrotational components

\[
\mathbf{v}(x, y) = \nabla \times (\psi \hat{z}) + \nabla \phi,
\]

where \( \psi \) is the streamfunction and \( \phi \) is the velocity potential, and \( \hat{z} \) is the unit vector along the \( z \) axis. Both \( \psi \) and \( \phi \) are scalar fields depending on \((x, y)\), defined within a horizontal domain. Substituting Eq. (2) into Eq. (1) and applying the gradient theorem, we obtain

\[
d_j = \frac{2}{c^2} \int_{\Gamma_j} [\nabla \times \psi(x, y) \hat{z}] \cdot \mathbf{r'} ds - \frac{2}{c^2} [\phi(r_{j2}) - \phi(r_{j1})],
\]

where \( r_{j1} \) and \( r_{j2} \) are the beginning and end points of the ray path \( \Gamma_j \). As shown by Norton (1988), the solenoidal flow can be uniquely reconstructed from the line integral of \( \nabla \times (\psi \hat{z}) \) along the ray paths within a bounded area and further provides a direct measure of a relative vorticity field \( \nabla \times (\nabla \times \hat{z}) \).

For the irrotational flow, only the values on \( \phi \) at the locations of each transceiver pair does the line-integral measurement provide, see Eq. (3). To obtain the irrotational flow \( \nabla \phi \) and the resulting divergence field \( \nabla^2 \phi \), the transceivers are required to be placed in the interior of the tomographic region (Munk et al. 1995). The MVT experiment is suitable for reconstructing the irrotational component, as the transceivers are towed by vehicles cruising inside the tomographic region.
3. Representation of the current field

To reconstruct the current field using the OAT method, two representations of the current field are discussed, including the truncated Fourier series and the spatial modes from the OMA.

a. Truncated Fourier series

Most OAT studies in coastal seas have used the truncated Fourier series (Yamoaka et al. 2002; Zhu et al. 2013) to represent the streamfunction for expressing the solenoidal effect. The streamfunction could be represented by the following Fourier series truncated at the maximum wavenumber $K_{\text{max}}$ (Gaillard 1992; Huang et al. 2013),

$$\psi(x, y) = \sum_{k=-K_{\text{max}}}^{K_{\text{max}}} \sum_{l=0}^{l(k)} m_{kl}^\cos \cos \left(\frac{2\pi lx}{L_{\text{inv}}} + \frac{2\pi ly}{L_{\text{inv}}} + \frac{2\pi ly}{L_{\text{inv}}}ight) + m_{00}^\sin \sin \left(\frac{2\pi lx}{L_{\text{inv}}} + \frac{2\pi ly}{L_{\text{inv}}}ight),$$

where $l(k) = (K_{\text{max}}^2 - k^2)^{1/2}$ with $K_{\text{max}}$ being high enough to depict the small-scale feature; $m_{kl}^\cos$ and $m_{kl}^\sin$ are the Fourier coefficients. The side length of the inversion domain $L_{\text{inv}}$ is twice larger than the tomographic area to avoid the periodic effect.

Substituting Eq. (4) into Eq. (1), the DTT for $j$th ray can be written as

$$d_j = \frac{-2}{c^2} \int_{\Gamma_j} \left(\cos \theta \frac{\partial \phi}{\partial \eta} + \sin \theta \frac{\partial \phi}{\partial \xi}\right) ds,$$

where $\theta$ is the angle between the $j$th ray path $\Gamma_j$ and $x$ axis. In this study, $\Gamma_j$ is the straight line connecting the $j$th transceiver pair.

b. Spatial modes using OMA

OMA uses eigenmodes to project the flow onto a horizontal domain based on the modal decomposition in streamfunction and velocity potential (Lekien et al. 2004; Kaplan and Lekien 2007). Three types of eigenmodes are derived. First, streamfunction modes $\psi_i$ satisfy Dirichlet boundary conditions (referred to as Dirichlet modes). Second, velocity potential modes $\phi_j$ satisfy Neumann boundary conditions (Neumann modes). Third, velocity potential modes $\phi_j$ satisfy the nonhomogeneous boundary conditions along the open boundary (boundary modes). Thus, a comprehensive 2D current field accounting for solenoidal and irrotational flows can be represented by a linear combination of those modes

$$v(x, y) = \sum_{i=1}^{N_\psi} m_i^\psi \nabla \times \psi_i(x, y) \hat{z} + \sum_{i=1}^{N_\phi} m_i^\phi \nabla \phi_i(x, y) + \sum_{i=1}^{N_{\phi^b}} m_i^{\phi^b} \phi_i(x, y),$$

in which $m_i^\psi$, $m_i^\phi$, and $m_i^{\phi^b}$ are the corresponding mode coefficients. To retain significant information, the current field is approximated with a finite number of each mode set $N_\psi$, $N_\phi$, and $N_{\phi^b}$.

Substituting Eq. (6) into Eq. (1), the DTT for the $j$th ray can be expressed as

$$d_j = \frac{-2}{c^2} \int_{\Gamma_j} \left(\cos \theta \frac{\partial \phi}{\partial \eta} + \sin \theta \frac{\partial \phi}{\partial \xi}\right) ds,$$

where the data vector $d$ is contaminated by the noise $n$ due to observation and modeling errors. For the Fourier representation, the $(j, i)$ element of the observation matrix $G_{\text{ftomo}}$, Eq. (5), is given by the projection of current contributed by the $i$th Fourier basis function on the ray path $\Gamma_j$. Similarly for the OMA representation in Eq. (7), with the additional discrete values of $\phi_i$ at the endpoints of each ray.

Incorporation of point measurements into line-integral data could further improve the current mapping. When using point measurements with the OMA representation, the $(j, i)$ element of the observation matrix $G_{\text{point}}$ is the $i$th OMA velocity mode sampled at the $j$th location. A combination of those two data types results in an augmented data vector (Dushaw and Sagen 2016)

$$d = [d_{\text{ftomo}}, d_{\text{point}}],$$

and observation matrix

$$G = \begin{bmatrix} G_{\text{ftomo}} \\ G_{\text{point}} \end{bmatrix}.$$
both current field representations. For the Fourier representation, a red-noise spectrum with a rollover length (Huang et al. 2013) is used to enforce more weight on smaller wavenumbers. As for the OMA representation, $C_m$ is a prior model covariance matrix obtained from the temporal variation of the modeled current fields. The regularization parameter $\beta$ is determined by the $L$-curve criterion (Hansen and O’Leary 1993) to find a balance between the weighted data residual norm and the weighted model norm.

The generalized least squares solution is

$$\hat{m} = G^d,$$  \hspace{1cm} (12)

where $G^d = (G^T C_d^{-1} G + \beta C_m^{-1})^{-1} G^T C_d^{-1}$ is a generalized inverse of the observation matrix $G$. Then, the estimated current fields in eastward and northward directions ($v_x$, $v_y$) at the spatial grid are

$$\hat{v}(x, y) = (v_x, v_y) = (E\hat{m}, N\hat{m}),$$  \hspace{1cm} (13)

where the matrices $E$ and $N$ are for mapping the estimated model parameter vector $\hat{m}$ to, respectively, the eastward and northward components of the current field. Each column of $E$ or $N$ is the differentiation of the corresponding mode. For the solenoidal part represented by the Fourier basis functions or the Dirichlet modes, the columns of $E$ and $N$ are the differentiation of the functions (for Fourier basis functions) or modes (for Dirichlet modes) with respect to $y$ and $x$, respectively. As for the irrotational part represented by the Neumann and boundary modes, the columns of $E$ and $N$ are the differentiation of the modes with respect to $x$ and $y$, respectively.

Assuming that there is no correlation between $m$ and $d$, the model uncertainty is described by the covariance matrix

$$\hat{P} = \langle (\hat{m} - m)(\hat{m} - m)^T \rangle$$  \hspace{1cm} (14)

$$= \frac{1}{\beta} (R - I) C_m (R - I)^T + G^T C_d G^T,$$  \hspace{1cm} (15)

where $R = G^T G$ is the model resolution matrix and $R \approx I$ indicates that the model can be virtually perfectly resolved. The first term in Eq. (15) indicates the uncertainty resulting from the limited resolution that characterizes the bias of the predicted model, and the second term is due to the propagation of data error.

The performance of different current field representations is evaluated by comparing the reconstructed currents with the true using the root-mean-square difference (RMSD) and fractional-residual-error variance (FREV) defined as follows:

$$\text{RMSD} = \langle \hat{v} - v_{\text{true}} \rangle^{1/2},$$  \hspace{1cm} (16)

$$\text{FREV} = \frac{\langle (\hat{v} - v_{\text{true}})^2 \rangle}{\langle v_{\text{true}}^2 \rangle},$$  \hspace{1cm} (17)

where $\langle \cdot \rangle$ denotes a spatial average over the tomographic area surrounded by the outer edge of all ray paths; $\langle v_{\text{true}}^2 \rangle$ is the
variability of the true tidal current speeds in the tomographic area, referred to as the tidal current variance. For the simulation study, \( v_{\text{true}} \) is the depth-averaged velocity calculated from the synthetic ocean. While for the experimental data, \( v_{\text{true}} \) is replaced by the depth-averaged ADCP measurements, i.e., \( v_{\text{true}} = v_{\text{adcp}} \).

5. Applications of OMA to WangHaiXiang Bay

To illustrate the OMA method presented in section 3b and its applicability in WangHaiXiang Bay, synthetic data are generated using the simulated currents from ocean models and the ray distribution (yellow lines in Fig. 1b) from the 2017 MVT experiment (K. Chen et al. 2020).

a. Synthetic ocean

A three-dimensional (3D) KCOM was developed with the related initial and lateral boundary conditions to simulate the tidal currents for synthetic tomographic experiments. With tidal forcing, the KCOM is based on the Princeton Ocean Model (POM) (Blumberg and Mellor 1987). The domain includes WangHaiXiang Bay (WB), Keelung Harbor (KH), and the adjacent waters, from 121°43.8′E to 121°52.2′E and from 25°7.2′N to 25°12.6′N, with a horizontal grid resolution of 50 m (Fig. 1a). For vertical grids, 26 sigma levels were used with the spacing decreasing near the surface and bottom to resolve the momentum flux in the upper and lower boundaries, respectively. The simulation started on 1 June 2017 and lasted for 30 days with an interval of 1 min.

The output of KCOM is validated by comparing the modeled sea surface height with the measurements at the Keelung Tidal Station (purple star in Fig. 1a) with a root-mean-square error less than 6 cm. The resulting depth-averaged current fields (Figs. 2a,b) indicate that, during the flood (ebb) tide, the strong tidal currents stream northwestward (southeastward) passing through the bay mouth with a cyclonic (anticyclonic) circulation formed inside the bay.

b. Synthetic DTTs

Numerical reciprocal acoustic propagation experiments were performed using the Gaussian-beam acoustic model (Porter and Bucker 1987) to simulate the acoustic propagation between paired transceivers in a moving medium. The effects of ocean currents on acoustic propagation are accounted for by adding or subtracting the 3D KCOM projected currents along the ray from the actual sound speed (Taniguchi and Huang 2014). The acoustic parameters used in the simulations are listed in Table 1.

From the simulated arrival patterns in reciprocal directions of the transceiver pair, the synthetic DTT is obtained by calculating the cross-correlation function (CCF) of the reciprocal arrival patterns (Huang et al. 2019). The DTT data are further contaminated by the additive zero-mean Gaussian noise with the standard deviation determined from the measured DTTs collected in the 2017 MVT experiment.

c. OMA modes for the WangHaiXiang Bay

The computational domain for the OMA analysis is specified as 5 km \( \times \) 5 km (blue solid line in Fig. 1a). The obtained modes are ordered from the lowest to the highest corresponding eigenvalues. A higher-order mode exhibits a relatively small-scale variability. The total number of OMA modes, which includes Dirichlet, Neumann, and boundary modes, is chosen based on an eigenvalue threshold. When reducing the eigenvalue threshold, more OMA modes are included. Assuming that the KCOM-modeled currents represent the true field, the fraction of the current variance captured by including additional modes increases and eventually levels off at 93.2%. This is achieved with a total of 72 modes, consisting of 24 Dirichlet modes \( (N_\phi = 24) \), 37 Neumann modes \( (N_\phi = 37) \), and 11 boundary modes \( (N_\phi = 11) \). Thus, a total of 72 OMA modes, which account for 93.2% of the tidal variability.

TABLE 1. Parameters used in the acoustic simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>18 kHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>4.5 kHz</td>
</tr>
<tr>
<td>Launching angles</td>
<td>-30° to 30°</td>
</tr>
<tr>
<td>Number of launching beams</td>
<td>5000</td>
</tr>
<tr>
<td>Sediment compressional speed</td>
<td>1600 m s(^{-1})</td>
</tr>
<tr>
<td>Sediment compressional attenuation</td>
<td>0.9 dB (\lambda)^(-1)</td>
</tr>
<tr>
<td>Sediment density</td>
<td>1.8 g cm(^{-3})</td>
</tr>
</tbody>
</table>

FIG. 2. Depth-averaged current fields for (a) flood and (b) ebb tides modeled by the KCOM.
current variance at the site, are included in the following current reconstruction.

Figure 3 displays a set of the selected modes. The corresponding velocity modes are tangent to the streamlines of the interior Dirichlet modes (first column), and they are orthogonal to the level sets of interior Neumann (second column) and boundary modes (third column). Note that the boundary modes are derived based on a discrete Fourier basis defined on the open boundary; \( \phi_6 \) corresponds to the constant boundary condition, \( \phi_7 \) to \( \phi_{12} \) correspond to the boundary conditions of the Fourier cosine functions, and \( \phi_8 \) to \( \phi_{13} \) correspond to those of the Fourier sine functions. Compared to \( \phi_7 \) (Fig. 3f), a relatively small spatial variability is observed for \( \phi_8 \) (Fig. 3e) as the open boundary of this mode satisfies the lowest Fourier sine function.

d. Resolution of the OMA modes

Due to the regularization applied to stabilize the current reconstruction, a bias exists between reconstructed and true currents. Here, we examine how much more reliable certain OMA modes are compared to the others using the relative diagonal values of the resolution matrix \( R \) (Fig. 4). Among all the three types of modes, Dirichlet \( \psi_6 \) (circles) has the best resolution with a value of 0.45. For the Neumann and boundary modes, \( \phi_{12} \) (triangles) and \( \phi_7 \) (squares) have the best resolution with a value of 0.42 and 0.09, respectively.

The reliability/resolvability of Dirichlet modes \( \psi \) is related to the projection of the corresponding velocity field onto the rays. A Dirichlet mode is better resolved for a given ray distribution when most rays have a large line integral of the
currents projected along the path. From the diagonal values of $R$ (circle in Fig. 4), better resolution is observed for $\psi_0$, $\psi_1$, and $\psi_3$, in descending order. The velocity pattern of $\psi_0$ (Fig. 3a) shows a circulation feature inside the bay, similar to those of $\psi_1$ and $\psi_2$ (not shown here). With those three transceivers, the rays are triangular shapes pivoting on the moored transceiver located at the lower right corner. Thus, a portion of the rays along these three triangular sides are parallel to the current circulation inside the bay. However, the modes $\psi_2$, $\psi_9$, and $\psi_{23}$ (Fig. 3b for the illustration of $\psi_{23}$) have a relatively poor resolution due to most of the rays being perpendicular to the velocity modes.

For the Neumann modes $\phi$, the resolution is related to the difference in $\phi$ at the locations of a transceiver pair. A Neumann mode is better resolved when most ray paths have a large difference in $\phi$ values. From the mode pattern of the best resolvable mode $\phi_{23}$ (Fig. 3c), we see that, except for the rays colored in magenta, most of the rays have blue and red colors at the endpoints of the ray. For the pattern of the poorly resolved Neumann mode, e.g., $\phi_{31}$ (Fig. 3d), most of the rays have a relatively small change in the color lightness at the endpoints.

Similar to the discussion for the Neumann modes, the resolvability of a boundary mode $\phi^b$ depends on the difference in the sampled potential values at the endpoints of a ray. The diagonal values of $R$ (square in Fig. 4) decrease from $\phi^b_{23}$ to $\phi^b_0$ and from $\phi^b_{21}$ to $\phi^b_{11}$. Compared to the Dirichlet and Neumann modes, a relatively small resolution value of 0.09 (square in Fig. 4) is observed for the best-resolved boundary mode $\phi^b_{23}$. For the corresponding mode pattern (Fig. 3e), the endpoints of each ray have less contrast in the color lightness compared to those of the Neumann modes. Most boundary modes (e.g., Fig. 3f for $\phi^b_h$) have a relatively poor resolution due to the small variation of the potential field inside the bay, resulting from the distant open boundary.

6. Results and discussion

All the DTT data collected during the experiment are used to improve the accuracy of the current estimate, provided that the ocean field is frozen over the data collection period. The reconstruction of the ocean currents using either synthetic or experimental data uses the following three different representations. The first two representations, accounting for only the solenoidal flow, are the truncated Fourier series and the OMA-derived Dirichlet modes (referred to as the OMA-Dirichlet). The third representation, including both the solenoidal and irrotational flows, is all three types of OMA-derived modes (referred to as the OMA-all).

a. Inversion of synthetic data

For the simulation study, the hourly DTT datasets based on the ray distribution of the 2017 MVT experiment are synthesized using the KCOM current field on 27 June 2017. For the solenoidal flow, the FREVs of the OMA-Dirichlet reconstruction (dashed line in Fig. 5b) are always smaller than those for the Fourier reconstruction (dotted line) over a period of 24 h. The time average of the FREV with the OMA-Dirichlet representation is 28.7%, compared to 48.2% with the Fourier. The OMA-Dirichlet representation results in an approximately 20% reduction of the time-averaged FREV as those modes satisfy the boundary condition and thus provide a more accurate representation of the oceanographic process.

The OMA-all representation is used to reconstruct the current field consisting of solenoidal and irrotational flows. With the additional inclusion of the Neumann and boundary modes that satisfy the boundary conditions, the OMA-all representation is expected to have better performance than the Fourier representation. Over the entire period of 24 h, the FREVs for the reconstructed currents using the OMA-all representation (solid line in Fig. 5b) are smaller than those using the Fourier (dotted line). The time-averaged FREV using the OMA-all representation is greatly reduced to 17.8%, compared to 48.2% using the Fourier representation.

The spatial distributions of the reconstructed currents at 7 and 16 h are shown in the top and bottom panels of Fig. 6, respectively. The current field at 7 h exhibits a lower spatial current variance of 0.06 m$^2$ s$^{-2}$ (downward arrow in Fig. 5a). Especially, the currents outside the bay are weakened and have slightly more spatial variations (see red arrows in the top panels of Fig. 6), which might result in considerable uncertainty in the current reconstruction. At this period, the current field reconstructed with the Fourier representation (black arrows in Fig. 6a) accounts for only 46.8% of the tidal current variance (FREV = 53.2%) due to a lack of crossing rays in resolving the current variations outside the bay. Using the OMA-Dirichlet representation, the reconstructed currents (Fig. 6a-2) explain 54.6% of the tidal current variance (FREV = 45.4%). The OMA-Dirichlet reconstructed currents can explain 7.8% more of the tidal current variance. This improvement might be due to incorporating the coastal boundary conditions inside the bay for the OMA-Dirichlet.
modes. Finally, using the OMA-all representation, the reconstructed currents (Fig. 6a-3) explain the maximum 67.7% of the tidal current variance (FREV = 32.3%) due to the inclusion of the Neumann and boundary modes. Of all three representations, the smallest RMSD of about 13 cm s\(^{-1}\) is observed using the OMA-all representation. Similarly, large FREVs are observed at about 2, 7, 13, and 19 h (Fig. 5b). This might result from small tidal current variances occurring at a period of half of the semidiurnal cycle (dashed line in Fig. 5a).

Compared to the current field at 7 h, the current field at 16 h corresponds to a high current variance of 1.31 m\(^2\) s\(^{-2}\) (rightward arrow in Fig. 5a). Outside the bay, the current field exhibits a strong uniform current flowing southeastward. Inside the bay, it circulates in a clockwise direction (see red arrows in the bottom panels of Fig. 6). Using the Fourier representation, the tomographic inversion (black arrows in Fig. 6b-1) reconstructs the circulation inside the bay, but cannot explain the uniform currents outside; the reconstruction accounts for only 44.2% of the tidal current variance (FREV = 55.8%). A poor fit to the true currents is observed outside the bay where the irrotational component dominates, resulting in a large RMSD with a value of 52 cm s\(^{-1}\). Using the OMA-Dirichlet representation (Fig. 6b-2), the circulation inside the bay can also be resolved. An improvement is observed near the bay mouth, with the FREV of 29.8% and RMSD of 38 cm s\(^{-1}\). Compared to the Fourier representation, the reconstructed field yields approximately 26% reduction of the FREV since most of the inverted currents inside the bay are well constrained by the coastline. Finally, using the OMA-all representation, the tomographic inversion (Fig. 6b-3) reconstructs both the circulation and the uniform currents well; the reconstruction accounts for up to 92.0% of the tidal current variance (FREV = 8.0%). The RMSD is significantly reduced to 20 cm s\(^{-1}\).

The reasons for the noticeable performance of the OMA-all representation are as follows. First, the spatial modes inherently satisfy the boundary conditions along the shoreline and open boundaries, thus imposing more prior information on the tomographic inversion. Second, the irrotational flow outside the bay can be appropriately accounted for by including the Neumann and boundary modes. Thus, the reconstruction using the complete OMA representation results in consistency with the true currents. However, the Fourier representation is inefficient due to the absence of the irrotational flow. Therefore, the OMA-all representation is preferable for mapping coastal currents in a shallow-water environment.

b. Inversion of experimental data

Data acquired during the 2017 MVT experiment are used to illustrate those three representations of the current field. As a reference, the current field near the bay mouth was
reconstructed (red arrows in Fig. 7d) by smoothing and filtering the shipboard ADCP measurements (white arrows) using the OMA-all representation. Note that the reconstructed currents cannot explain 9.8% of the ADCP-measured current variance. Below are the tomographic reconstructions using three different representations of the current field.

First, using the Fourier representation for the solenoidal flow, the reconstructed currents (red arrows in Fig. 7a) show a fair agreement with the ADCP measurements (white arrows) except for the currents near the northernmost and easternmost tomographic boundaries. Due to the absence of ray crossing, the reconstruction results in a 49.7% of the ADCP-measured current variance unexplained. The circulation is revealed with a predictive uncertainty of 7 cm s$^{-1}$ near the bay center. In contrast, considerable uncertainty is observed near the bay mouth since the Fourier representation has a better wavenumber resolution (K. Chen et al. 2020).

Second, using the OMA-Dirichlet representation for the solenoidal flow, the circulation inside the bay is also resolved (Fig. 7b). However, the estimated currents show a poor fit to the ADCP measurements outside the bay mouth due to lacking rays from various angles. Similar to the 49.7% unexplained variance using the Fourier representation, a FREV of 49.9% is obtained using the OMA-Dirichlet. In addition, the predictive uncertainty (background color in Fig. 7b) shows a relatively small uncertainty compared to that using the Fourier representation (Fig. 7a). This is because the OMA-Dirichlet representation provides more prior information about the boundary conditions along the coastline.

Third, using the OMA-all representation for the solenoidal and irrotational flows, a significant improvement in the reconstructed currents (Fig. 7c) is observed outside the bay, where the irrotational flow dominates. The uniform currents near the northeast area are better reconstructed using the Neumann and boundary modes for the irrotational flow. This improvement cannot be achieved using the truncated Fourier series or Dirichlet modes for the solenoidal flow alone. The FREV of OMA-all reconstruction is reduced to 23.0%, and the predictive uncertainty is reduced to 4 cm s$^{-1}$ near the circulation center.

Incorporating the ADCP measurements in the tomographic reconstruction process further improves the current field using the OMA-all representation. Compared to the previous reconstruction results, the predictive uncertainty is the minimum, especially outside the bay mouth where the ADCP measurements are available (Fig. 7e), since the point measurements can compensate for the lack of current information due to the few crossing rays in that region. The FREV of this optimally reconstructed current field is reduced to 11.2%. Combining the DTT and ADCP data with the OMA-all representation may yield an optimal estimate of the current field.
7. Concluding remarks

This study has demonstrated the optimum reconstruction of ocean currents using moving vehicles with the current representations satisfied the coastal boundary conditions and the combination of both tomographic (DTT) and point (ADCP) measurements. Simulation studies have been conducted using synthetic DTTs to evaluate the reconstruction performance using those three current representations: the Fourier, OMA-Dirichlet, and OMA-all representations. The OMA-Dirichlet representation performs better than the Fourier representation for the solenoidal flow, especially in the area near the shoreline and the uniform currents near the bay mouth. For the solenoidal and irrotational flows, the OMA-all representation achieves a complete and accurate current field, resolving both the circulation inside the bay and the uniform currents near the mouth. The data collected from the 2017 MVT experiment demonstrated the reconstruction method. The overall reconstructed currents show consistency with the ADCP measurements, indicating that using the complete basis functions to represent the ocean currents can improve the mapping.

Regarding the properties of tomographic line-integral data and ADCP-point measurements, the former provides an averaged current along the ray path (better resolution in the wave-number domain), and the latter focuses on the local currents (better resolution in the spatial domain). Different sampling properties of the tomographic and ADCP measurements are combined to compensate for each other. The combined reconstruction leads to an optimal comprehensive image of ocean currents in coastal seas.

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Data availability statement. The processed acoustic data are stored at the Institute of Oceanography, National Taiwan University. To access these data, one may reach out to C.F.H. for potential collaboration. The OMA was performed using an algorithm sourced from the open-access repository: https://github.com/rowg/hfrprogs.

REFERENCES


