On In Situ “Calibration” of Shipboard ADCPs*

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(Manuscript received 30 March 1988, in final form 14 July 1988)

ABSTRACT

Methods for in situ calibration of acoustic-Doppler current profilers (ADCPs) are considered for measurement of absolute current profiles from a moving ship. Errors are of two types: sensitivity and alignment. Least square error estimates are given for experimental determination of both factors, for use in the “water track” or “bottom tracking” mode. Errors in the estimation of either factor may lead to large errors in derived water velocities, although the major contributions of the two factors arise from different sources and are approximately orthogonal to one another.

1. Introduction

The coordinate system \((x, y)\) is the true coordinate frame in which the position and velocity of the ship are determined. The \((x', y')\) frame is that of the multibeam Acoustic Doppler Current Profiler (ADCP) whose individual Doppler velocities are decomposed into east and north using the ship’s gyro. The \((x', y')\) frame is rotated counterclockwise by an unknown angle \(\alpha\) from the \((x, y)\) frame. The angle two represents an unknown error in the ADCP/gyro reference frame which must be determined experimentally. For generality, velocities are presented in geographic coordinates rather than in transducer or beam geometry.

Furthermore, the ADCP data must be “scaled up” by a factor \((1 + \beta)\), where \(\beta\) is generally small. This may be due to a nonzero trim to the transducer/ship, as opposed to the yaw which is represented by the angle \(\alpha\), or small errors in the beam geometry as discussed in section 5. Scaling of the Doppler currents may also be required to remove an overall system bias. Both \(\alpha\) and \(\beta\) need to be experimentally determined; this has been done in a nonuniform way by a number of investigators (Joyce, Wunsch, Pierce 1986; Kosro 1985; Didden 1987; Trump 1986). Presented is a suggested scheme for ADCP “calibration” which may take advantage of bottom tracking by the ADCP if water depths are not too large. The notation (use of \(\beta\)) follows that in Kosro (1985).

Let \((u', v')\) represent the east and north velocity components in the \((x', y')\) coordinate frames. Subscripts “\(w, d, s\)” refer to the “water, Doppler, ship”. Velocities in the two coordinate frames are related by

\[
\begin{pmatrix}
  u' \\
  v'
\end{pmatrix} = 
\begin{pmatrix}
  \cos \alpha & \sin \alpha \\
  -\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
  u \\
  v
\end{pmatrix}
\]  

(1)

and

\[
\begin{pmatrix}
  u \\
  v
\end{pmatrix} = 
\begin{pmatrix}
  \cos \alpha & -\sin \alpha \\
  \sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
  u' \\
  v'
\end{pmatrix}.
\]  

(2)

The absolute value of the velocity of the scatterers, typically taken to be the water velocity, \((u_w)\) is the sum of the Doppler velocity \((u_d)\) and the ship’s velocity \((u_s)\). In the \((x, y)\) frame

\[
u_w = u_s + (1 + \beta) u_d
\]

(3)

where \(\beta\) is the adjustment factor noted above. Doppler measurements are made in the local, rotated frame (that of the ADCP/gyro). Thus the water velocities in the “true” \((x, y)\) coordinate frame are

\[
u_w = u_s + (1 + \beta) (u_d' \cos \alpha - v_d' \sin \alpha)
\]

\[
v_w = v_s + (1 + \beta) (u_d' \sin \alpha + v_d' \cos \alpha).
\]

(4)

Calibration requires determination of \(\alpha, \beta\) in (4). Formulas are derived in sections 2 and 3 followed by a discussion of data sensitivity to errors in \(\alpha, \beta\) and some physical motivation for choosing these two (and only two) parameters for in situ calibration.

2. No bottom tracking

With no bottom tracking, assumptions must be made about the water velocity. If the same piece of water is criss-crossed two or more times, then the water velocity on any leg can be assumed to be the same as
the ensemble average of all legs (denoted by brackets) to within some noise level, \( \epsilon \)

\[
\begin{align*}
u_w &= \langle u_w \rangle + \epsilon_u \\
v_w &= \langle v_w \rangle + \epsilon_v.
\end{align*}
\]

(5)

Defining

\[
\delta u = u - \langle u \rangle
\]

(6)

for each of the different velocity components in (4), and using (5) gives

\[
\begin{align*}
\frac{\delta u_w}{\delta v_w} &= \left( \begin{array}{c} \delta u_x \\ \delta v_x \end{array} \right) + (1 + \beta) \left( \begin{array}{cc} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{array} \right) \left( \begin{array}{c} \delta u_x' \\ \delta v_x' \end{array} \right) \\
&= \left( \begin{array}{c} \epsilon_u \\ \epsilon_v \end{array} \right).
\end{align*}
\]

(7)

If \( \epsilon^2 = \epsilon_u^2 + \epsilon_v^2 \), then we seek to solve (7) so as to minimize the error in the noise kinetic energy, \( \epsilon \), or the dot product of (7) with itself:

\[
\begin{align*}
(\delta u_x')^2 + (\delta v_x')^2 + (1 + \beta)^2 [(\delta u_x')^2 + (\delta v_x')^2] \\
&+ 2(1 + \beta) \{[\delta u_x' \delta u_x + \delta v_x' \delta v_x] \cos \alpha \\
&+ [\delta u_x' \delta v_x - \delta v_x' \delta u_x] \sin \alpha \} = \epsilon^2.
\end{align*}
\]

(8)

Least square estimators are found for \( \alpha, \beta \) subject to the conditions that

\[
\frac{\partial \langle \epsilon^2 \rangle}{\partial (\alpha, \beta)} = 0.
\]

(9)

The \( \alpha \)-condition yields

\[
\tan \alpha = \frac{\langle \delta u_x' \delta v_x - \delta v_x' \delta u_x \rangle}{\langle \delta u_x' \delta u_x + \delta v_x' \delta v_x \rangle}.
\]

(10)

The \( \beta \)-condition and (10) yield

\[
1 + \beta = -\frac{\langle \delta u_x' \delta u_x + \delta v_x' \delta v_x \rangle}{\langle (\delta u_x')^2 + (\delta v_x')^2 \rangle \cos \alpha}.
\]

(11)

If the residual current, or "errors" in (7) are independent of ship velocity \( (u_x, v_x) \), then using (7) and (10); (11) can be rewritten as

\[
1 + \beta = \left[ \frac{\langle \delta u_x' \delta v_x + \delta v_x' \delta u_x \rangle}{\langle (\delta u_x')^2 + (\delta v_x')^2 \rangle} \right]^{1/2}
\]

(12)

which shows the relationship of the sensitivity to the horizontal kinetic energy of the ship and the observed Doppler current. In general (11) should be preferable to (12) because it does not require any additional assumptions about the noise.

As an example, data will be taken from a calibration "run" on the R/V ENDEAVOR in June of 1982. Doppler data have been averaged vertically between 60 and 100 m over 7 range bins and temporally over 20 minutes. On the first leg the vessel steamed northward followed by a southward leg (Table 1). The procedure allows for additional legs at different ship speeds, including on-station measurements. For the data in Table 1 the results using (10) and (11) are as follows:

\[
\alpha = -1.33^\circ, \quad 1 + \beta = 0.993.
\]

Prior to doing the calculations, the Doppler data were adjusted to account for sound speed variations using the salinity data from a surface sample and the ADCP transducer temperature from a thermistor. In this example the local coordinate system must be rotated clockwise by an angle of 1.33° and the ADCP data must be scaled by a multiplicative factor of 0.993 to best calibrate the measurements. This example, though rather simple and not using the "ensemble" method, is illustrative of the technique applied to single estimation of \( \alpha, \beta \).

3. With bottom tracking

If water depths are shallow enough, the absolute Doppler velocity of the scatterers (ocean bottom) should be zero when in the bottom track mode and the Doppler velocities should be equal and opposite to the ship velocity. An ensemble average (over a segment of cruise track) of (4) should vanish. In this case, the values of \( \alpha \) and \( \beta \) are similar in form to (10) and (11) and are given by

\[
\tan \alpha = \frac{\langle u_x v_x - v_x u_x \rangle}{\langle u_x u_x + v_x v_x \rangle}.
\]

(10a)

The \( \beta \)-condition and (10a) yield

\[
1 + \beta = -\frac{\langle u_x u_x + v_x v_x \rangle}{\langle (u_x')^2 + (v_x')^2 \rangle \cos \alpha}.
\]

(11a)

and as in the previous section (11a) can be written approximately as

\[
1 + \beta = \left[ \frac{\langle u_x^2 + v_x^2 \rangle}{\langle (u_x')^2 + (v_x')^2 \rangle} \right]^{1/2}.
\]

(12a)

Since the error terms, \( \epsilon \), for bottom tracking do not contain spatial or temporal variations of the water motion, the noise in the bottom track estimates should be

<table>
<thead>
<tr>
<th>( v_x )</th>
<th>( \delta v_x )</th>
<th>( u_x )</th>
<th>( \delta u_x )</th>
<th>( v_x' )</th>
<th>( \delta v_x' )</th>
<th>( u_x' )</th>
<th>( \delta u_x' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.120</td>
<td>5.100</td>
<td>-.064</td>
<td>.008</td>
<td>-.5208</td>
<td>-.5133</td>
<td>.162</td>
<td>.111</td>
</tr>
<tr>
<td>-5.079</td>
<td>-5.100</td>
<td>-.078</td>
<td>-.008</td>
<td>5.059</td>
<td>5.133</td>
<td>-.061</td>
<td>-.111</td>
</tr>
</tbody>
</table>
smaller than for water tracking and the resulting calibrations for $\alpha$, $\beta$ should be better.

In a recent study Pollard and Reed (1988) have used the formulas for $\alpha$, $\beta$ and compared results from water and bottom tracking. Their results indicate no significant differences in $\alpha$ but some systematic differences in $\beta$ (of order 1%) which are attributed to differences in the signal processing in the water and bottom-tracking modes.

The bottom track method can average over more observations (i.e., it can be used without course changes while the ship is steaming) and can also be applied in cases where TRANSIT satellite fixes are used for navigation. In this case the bottom Dopplers are vector averaged over the time interval between fixes and the resultant bottom Dopplers are used together with the estimated ship velocity between the two satellite fixes.

4. Sensitivity to errors in $\alpha$, $\beta$

If estimates of $\alpha$, $\beta$ are used to correct the measured Doppler velocities, the correct water velocities are simply

$$u_w = u_t + u'_t$$
$$v_w = v_t + v'_t$$

where

$$u'_t = (1 + \beta)(u'_t \cos \alpha - v'_t \sin \alpha)$$
$$v'_t = (1 + \beta)(u'_t \sin \alpha + v'_t \cos \alpha).$$

Errors in $\alpha$, $\beta$ of $\Delta \alpha$, $\Delta \beta$ will produce errors in estimated water velocity which can be determined from (13) and (14) to be

$$\Delta u_w = \frac{(\Delta \beta/(1 + \beta))u'_t}{4\sigma} - \Delta \alpha u'_t$$
$$\Delta v_w = \frac{(\Delta \beta/(1 + \beta))v'_t}{4\sigma} - \Delta \alpha v'_t.$$ (15)

These can be estimated by assuming, for example, that the vessel is moving through quiescent water with a velocity $(u_t, 0)$. If the errors $\Delta \alpha$, $\Delta \beta$ are small then

$$u'_t \approx -u_t$$
$$v'_t \approx 0.$$ (16)

Inserting (16) into (15) gives

$$\Delta u_w \approx -\frac{(\Delta \beta/(1 + \beta))u_t}{4\sigma}$$
$$\Delta v_w \approx -\Delta \alpha u_t.$$ (17)

From (17) it can be seen that sensitivity errors, $\Delta \beta$, result in false water velocities in the direction of ship motion while alignment errors, $\Delta \alpha$, are at right angles to the ship motion. If the ship is moving eastward at a speed of 5 m s$^{-1}$, then errors in $\Delta \beta/(1 + \beta)$ and $\Delta \alpha$ of $\pm 0.5\%$ and $\pm 0.3\%$, respectively will result in corresponding errors in $\Delta u_w$, $\Delta v_w$, of $\pm 2.5$ cm s$^{-1}$. While these may be acceptable for some applications, for others they are not. Joyce et al. (1986) discussed alignment errors and their effect on using ADCP data to estimate the transport and velocity structure of the Gulf Stream. Biases due to $\Delta \alpha$ could be removed post facto by using mass conservation and two sections which crossed the Gulf Stream in opposite directions, since errors change sign when the ship steams a reciprocal course.

5. Physical basis for $\alpha$, $\beta$

As noted earlier, $\alpha$ can be considered as an alignment error in the Doppler transducer/ship gyro relative to the "true" heading. Time dependence in estimated $\alpha$ has been reasonably associated with the ship’s gyro while the mean value is a result of the installation of the ADCP transducer and mean errors in the gyro.

Sensitivity error ($\beta \neq 0$) may arise because the orientation of the acoustic beams is not correct. Considering for example a vertical plane containing two acoustic beams, as in a 4-beam Janus configuration, Rowe and Young (1979) have shown that in that vertical plane the Doppler velocities in the horizontal and vertical ($u_d$, $v_d$) can be given by

$$u_d = \frac{\Delta \sigma_1 - \Delta \sigma_2}{4\sigma} \frac{c}{\sin \phi}$$
$$w_d = \frac{\Delta \sigma_1 + \Delta \sigma_2}{4\sigma} \frac{c}{\cos \phi}$$

where $\Delta \sigma_1$, $\Delta \sigma_2$ are the Doppler shifts from a transmitted frequency $\sigma$, $c$ is the sound velocity and $\phi$ the magnitude of the beam orientation from vertical. If the pair of beams have orientations which are not equal to $-\phi$, $\phi$, then (18) must be modified. Pitch errors will result in a rotation error in the vertical with equal (but opposite) changes in angles $\Delta \phi$ for each of the beams. The resulting offset is generally small for horizontal velocities because 1) the vertical velocity of the water is usually small and 2) errors are proportional to $(1 - \cos \Delta \phi)$.

If the orientation errors $\Delta \phi$ are not equal and opposite, then systematic sensitivity errors can be significant. If $\Delta \phi$ represents a small average beam error for the two Janus beams, then one can show that (18) become, ignoring rotation effects due to ship trim

$$u_d \approx \frac{\Delta \sigma_1 - \Delta \sigma_2}{4\sigma} \frac{c}{\sin \phi} (1 - \Delta \phi \cot \phi)$$
$$w_d \approx \frac{\Delta \sigma_2 + \Delta \sigma_2}{4\sigma} \frac{c}{\cos \phi} (1 + \Delta \phi \tan \phi).$$ (19)

Sensitivity of the horizontal Doppler velocity in (3) can be compared to (19) with the result that

$$\beta = -\Delta \phi \cot \phi.$$ (20)

A value of $|\Delta \phi| = 0.3^\circ$ results in a $|\beta|$ of 1%. With 4-beam systems, (20) would suggest that there might be two different $\beta$'s, one from each pair of beams. As
noted in section 4, however, the sensitivity of $\beta$ depends strongly upon the direction of projection of the acoustic beams parallel to the ship's motion. Thus practical sensitivity "calibration" for ship-mounted transducers need only worry about a single effective $\beta$. Finally, one observes in (19) that incorrect attention to horizontal variations in sound velocity will result in changes in $\beta$.

6. Discussion

Circumstances will govern whether or not water or bottom tracking is used for in situ calibration. Water tracking can be employed anywhere, but it suffers from the requirement that the water velocity on any segment of track be the same as for the ensemble average of either a longer segment of track or different segments criss-crossing the same general region. Noise can be suppressed by vertical and temporal averaging. Both methods described are limited ultimately by navigation errors in the ship velocity. In order to reduce the need for repeated in situ calibrations, care must be taken that sound speed variations along the track are properly accounted for before the foregoing calculations are made. Errors in estimation of $\alpha$ can create constant biases in the athwart-ship velocity component. If the ADCP is used as a reference level for geostrophic velocity calculations with hydrographic stations, this bias can result in unacceptably large errors in water column volume transport. The use of GPS navigation and bottom-tracking offer the possibility of improved in situ calibration for the future and should be actively pursued.

Two advantages in using the aforementioned calibration "constants" are that they arise from independent sources and that they produce errors approximately orthogonal: one ($\alpha$) perpendicular to the ship motion and one ($\beta$) parallel. Both are linearly related to ship speed, however, and one must be aware of the trade off between ship speed and desired velocity accuracy for effective use of shipboard ADCPs.

Acknowledgment. Support for this work was provided by grants to WHOI from NSF (OCE 85 01176) and NASA (NAGW-1026). The author wishes to acknowledge comments by A. Plueddemann and an anonymous reviewer, as well as being able to view a preliminary draft of the manuscript by Pollard and Reed.

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