A Diagnostic Numerical Model of the Quasi-Biennial Oscillation

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ABSTRACT

The long-period variations in zonally symmetric circulations of the tropical stratosphere are investigated. The zonally averaged momentum, continuity, and heat energy equations, subject to the assumptions of hydrostatic balance and a geostrophically balanced zonal wind, are used to formulate a numerical model which is integrated in time to study the evolution of the flow. Terms representing diabatic heating (thermal forcing) and divergence of eddy momentum flux (dynamical forcing) appear as input parameters whose distributions in space and time are specified in various ways to obtain diagnostic information on the dynamics of the observed quasi-biennial wind oscillation.

Experiments with thermal forcing through time varying diabatic heating indicate that an enormous variation in the radiative heating is required to produce the observed amplitude of the zonal wind oscillation, and that a thermally driven model is incapable of reproducing the observed characteristics of the downward propagating wind regimes. Dynamical forcing with reasonable amplitudes produced zonal wind changes of comparable magnitude to those observed in the atmosphere. However, experiments with a time varying momentum source in the region above 25 km were unable to simulate the downward propagation of the atmospheric wind regimes. It is concluded that the time variations of momentum fluxes which force the zonal wind oscillation exhibit a phase dependence on height.

1. Introduction

Development of a theory which can explain the cause, dynamics and structure of the quasi-biennial oscillation in the tropical stratosphere is one of the most provocative current problems in dynamic meteorology. It is perplexing that these relatively simple, zonally symmetric fluctuations which occur on a massive scale with large amplitudes should be so difficult to explain, even from a diagnostic point of view.

Fig. 1 illustrates the basic properties of the zonal wind oscillation:

1) Easterly and westerly wind regimes alternate faithfully with patterns repeating themselves at roughly 2-yr intervals.

2) Successive regimes propagate downward at an average rate of about 1 km month⁻¹. However, the leading edges of westerly regimes descend more rapidly than those of easterlies.

3) Amplitude is almost independent of height between 10 and 30 mb. Attenuation is increasingly evident as regimes descend below 50 mb.

4) The fluctuations cover a broad range of latitude and are remarkably coherent. There is no marked phase dependence on latitude.

5) In the long-term mean there is a small increase in easterlies with height at all latitudes. Mean easterlies also increase with latitude to a maximum at around 12°, then decrease poleward.

These features are well documented in the literature. The U. S. Navy Weather Research Facility (1964) and Wallace (1967a) present more detailed descriptions.

Previous diagnostic studies by Reed (1964, 1967) utilized the observed zonal momentum field to deduce the oscillations of the fields of meridional motion, temperature and ozone. He confirmed the geostrophy of the oscillation in zonal wind and showed that the perturbation vertical motion at the equator is downward when westerlies overlie easterlies and upward when easterlies overlie westerlies. He also deduced that only a non-Fickian eddy flux divergence of momentum can satisfy the zonal momentum balance at the equator. However, Reed's analysis was not able to give information on the probable mechanism of downward propagation.

Tucker's (1964) analysis led him to conclude that the downward propagation is of an advective nature. This implies a mean downward motion throughout the tropical stratosphere. Wallace (1967a) argued that the difference in descent rates between easterlies and westerlies suggests a superposition of Tucker's mean downward motion and Reed's perturbation vertical motions at very low latitudes. However, he pointed out that it would be difficult to explain the downward propagation at higher latitudes in terms of vertical advection because of prohibitively high meridional motions which would arise from continuity requirements.

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Previous theoretical studies have stressed radiative forcing mechanisms. Staley (1963) attempted to model the oscillation as a geostrophic response to the vertical diffusion of a temperature perturbation induced by a 26-month periodicity in solar output. Lindzen (1965, 1966) also considered radiative forcing. However, his vertical propagation was due to advection of a basic mean state of westerly shear by the perturbation vertical motion. Neither of these theories is adequate to explain the observed downward propagating oscillation. Diffusion causes a much too rapid attenuation of amplitude with decreasing height, and Lindzen’s mechanism actually gives upward propagation when the observed easterly shear of the mean state is applied to his formulae.

As the above discussion indicates, there is still little agreement on such fundamental questions as whether the oscillation is radiatively or mechanically driven, and whether the downward propagation is of a wavelike, diffusive or advective nature.

In the present study the zonally averaged equations of motion, thermodynamics and continuity are used to formulate a numerical model in which various types and distributions of forcing may be tested. The equations are integrated in time over several cycles of the oscillation and the zonal winds and temperatures computed in the model are compared with the observed fields. In this manner it is possible to test various distributions of the forcing functions in order to obtain some insight into the physical mechanisms which might produce the observed zonal wind and temperature oscillation.

2. Basic equations

Following Leovy (1964) we assume that the zonal wind, pressure and temperature fields are in hydrostatic and geostrophic balance. The zonally averaged equa-
tions of motion, continuity and thermodynamic energy (letting asterisks denote dimensional variables) are:

\[
\begin{align*}
\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} &= -2\Omega (\sin \phi) v^*, \\
\frac{\partial \Phi^*}{\partial t^*} + v^* \frac{\partial \Phi^*}{\partial y^*} + w^* \frac{\partial \Phi^*}{\partial z^*} &= -F^* (y^*, z^*, t^*) - G^* (y^*, z^*, t^*), \\
\frac{\partial \Phi^*}{\partial y^*} &= \frac{\partial \Phi^*}{\partial y^*} + \frac{RT^*}{\partial y^*}. \\
\frac{\partial T^*}{\partial y^*} - \frac{\partial v^*}{\partial z^*} &= \frac{\partial T^*}{\partial y^*} + \frac{RT^*}{\partial z^*} + f, \\
\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} &= -P^* (y^*, z^*, t^*) + Q^* (y^*, z^*, t^*),
\end{align*}
\]  

where \( \theta^* \) is the departure from the basic state temperature. Using this definition we combine (2.2) and (2.3) to eliminate \( \Phi^* \). The result is the thermal wind relationship,

\[
-2\Omega \sin \phi \frac{\partial u^*}{\partial y^*} + \frac{\partial \Phi^*}{\partial z^*} = \frac{R}{H} \frac{\partial \theta^*}{\partial y^*}.
\]  

(2.7)

In addition we may rewrite (2.5) as

\[
\frac{\partial \Theta^*}{\partial t^*} + \frac{\partial \Theta^*}{\partial y^*} + \frac{\partial \Theta^*}{\partial z^*} = -P^* (y^*, z^*, t^*) + Q^* (y^*, z^*, t^*),
\]  

(2.8)

where \( g \) is the gravitational acceleration.

Eqs. (2.1), (2.4), (2.7) and (2.8) are next nondimensionalized by letting

\[
\begin{align*}
\tilde{z} &= \frac{z^*}{H}, \\
\tilde{y} &= \frac{y^*}{L}, \\
\tilde{t} &= \frac{t^*}{\tau}, \\
\tilde{u} &= \frac{u^*}{(2\Omega \sin \phi_0 L)}, \\
\tilde{v} &= \frac{v^*}{(\tau / L)}, \\
\tilde{w} &= \frac{w^*}{(\tau / H)}, \\
\tilde{\theta} &= \frac{R \theta^*}{(2\Omega \sin \phi_0 L)^2},
\end{align*}
\]

Here \( L \) is a horizontal scale length, \( \tau \) the annual period, and \( \phi_0 \) a reference latitude (not the equator). Defining a static stability parameter

\[
\epsilon = \frac{(2\Omega \sin \phi_0 L)^2}{gH^2} \left[ \frac{1}{T_0^*} \left( \frac{RT_e^*}{c_p H} + \frac{dT_e^*}{dz^*} \right) \right]
\]

the nondimensional equations may be written as follows:

\[
\begin{align*}
\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{y}} + \frac{\partial \tilde{u}}{\partial \tilde{z}} &= - \tilde{f} = - \tilde{F} - \tilde{G}, \\
\frac{\partial \tilde{\theta}}{\partial \tilde{y}} &= 0, \\
\frac{\partial \tilde{\theta}}{\partial \tilde{y}} + \frac{\partial \tilde{\theta}}{\partial \tilde{z}} &= - \tilde{P} + \tilde{Q},
\end{align*}
\]  

(2.9)

(2.10)

(2.11)

(2.12)

where

\[
\begin{align*}
\tilde{F} &= \tilde{F}^* (\tau / 2\Omega \sin \phi_0 L), \\
\tilde{G} &= \tilde{G}^* (\tau / 2\Omega \sin \phi_0 L), \\
\tilde{P} &= \tilde{R} \tilde{P}^* / (2\Omega \sin \phi_0 L)^2, \\
\tilde{Q} &= \tilde{R} \tilde{Q}^* / (2\Omega \sin \phi_0 L)^2,
\end{align*}
\]

and

\[
\tilde{f} = \sin \phi_0 / \sin \phi_0.
\]

It is convenient to define a meridional mass transport stream function \( \psi \) according to

\[
\begin{align*}
\frac{\partial \psi}{\partial \tilde{z}} &= -e^{-\tilde{v}} \frac{\partial \psi}{\partial \tilde{y}}, \\
\frac{\partial \psi}{\partial \tilde{y}} &= e^{-\tilde{v}} w.
\end{align*}
\]  

(2.13)
Eq. (2.12) is then satisfied identically. With the aid of (2.10) the time derivatives may be eliminated between (2.9) and (2.11) to obtain a diagnostic equation for the meridional motion. In terms of \( \psi \) this equation is

\[
\int \left( j - \frac{\partial u}{\partial y} \right) \left( \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial z} \right) + 2 f \frac{\partial u}{\partial \psi} + \frac{1 - R/c_p}{2} \frac{\partial \psi}{\partial y} \right) \right] \frac{1 - R/c_p}{\psi} \frac{\partial \psi}{\partial y} \right) e^{-j} + \left( \frac{\partial P}{\partial y} + \frac{\partial Q}{\partial y} \right) e^{-i}. \tag{2.14}
\]

Eq. (2.14) is elliptic in \( \psi \) provided that

\[
f^2 \left( \frac{\partial \psi}{\partial z} \right)^2 - \left( f^2 - j \frac{\partial u}{\partial y} \right) \left( \frac{1 - R/c_p}{\psi} \frac{\partial \psi}{\partial y} \right) < 0. \tag{2.15}
\]

Since \( \epsilon \ll 1 \) and \( (f - \partial u/\partial y) > 0 \) this condition is certainly satisfied in the tropical stratosphere. A similar equation was discussed by Kuo (1956).

If initial and boundary conditions are specified, (2.9), (2.10) and (2.14), together with the definitions of (2.13), constitute a closed prediction system for the zonally averaged momentum and temperature fields provided that the forcing functions on the right hand side in (2.9) and (2.14) are known.

A finite difference analogue to this system is developed in the Appendix.

3. Boundary conditions

For simplicity we assume that the zonal wind and temperature fields are symmetric with respect to the equator. This assumption precludes study of the annual cycle, but observations indicate that the quasi-biennial oscillation approximately satisfies the symmetry condition. Observations also indicate that relatively large wind oscillations occur only equatorward of about 26° latitude, and are confined to the lower and middle stratosphere. Therefore, in this study the model equations will be integrated for the region bounded laterally at \( \phi = 0^\circ \) and \( \phi = 26^\circ \), and bounded vertically at 15 and 50 km.

For integration of (2.9), either \( u \) or its normal derivative must be specified on all the boundaries. Since (2.14) is elliptic in \( \psi \) we must also specify the values of \( \psi \) at all boundaries. The symmetry condition at the equator implies that the meridional velocity must be zero there and that the normal derivations of \( u \), \( u \) and \( \theta \) all vanish. With the aid of (2.13) we then obtain as boundary conditions at \( \phi = 0^\circ \):

\[
\frac{\partial u}{\partial y}, \quad \frac{\partial u}{\partial y} = 0. \tag{3.1}
\]

As conditions at the upper boundary we let

\[
u, \quad v = 0. \tag{3.2}
\]

Although actual conditions at 50 km probably do not fulfill these requirements, the constraint of geostrophic balance guarantees that any numerical errors caused by the oversimplified conditions (3.2) will not propagate away from the boundary.

The remaining two boundaries require more complicated conditions. Observations suggest that for all practical purposes long-period variations in the zonal wind at 26° latitude and at 15 km can be neglected, except for the annual cycle, which is not under consideration here. Accordingly, the long-term mean values as deduced from observations (U. S. Navy Weather Research Facility, 1964) are specified for these quantities.

The stream function distribution at these boundaries (i.e., the mass flux through the boundaries) may then be calculated with the aid of the governing equations (2.9)–(2.11). Considering first the lateral boundary, we solve (2.9) for \( \psi \) and substitute from (2.13) to obtain

\[
\frac{\partial \psi}{\partial z} = \frac{\left( G - F + \frac{\partial u}{\partial z} \right) e^{-i}}{\left( f - \frac{\partial \psi}{\partial y} \right)}, \tag{3.3}
\]

where the partial derivative of \( u \) with respect to time is neglected since \( u \) is specified as a constant on the boundary. Using the condition \( \psi = 0 \) at 50 km, \( (3.3) \) may be integrated with respect to \( z \) to give \( \psi(z) \) at \( \phi = 26^\circ \).

For \( \psi \), and in estimating \( F \), data obtained from the previous time step are used.) To determine \( \psi \) along the lower boundary we note that in the tropical stratosphere where \( \epsilon \ll 1 \), (2.11) may be written

\[
\frac{\partial \psi}{\partial y} = e^{+\epsilon} = \epsilon(Q - P) + O(\epsilon), \tag{3.4}
\]

where \( O(\epsilon) \) indicates small neglected terms. If \( Q \) and \( P \) are parameterized as functions of \( \theta \) (see Section 5) we may integrate (3.4) with respect to \( \gamma \) to obtain \( \psi(\gamma) \) along the lower boundary. The value of \( \theta \) used in specifying \( Q \) at the lower boundary is determined by applying the thermal wind equation to the zonal wind shear between the grid points on the boundary and those immediately above. As explained in the Appendix, it is necessary to adjust \( Q \) by a constant at the lower boundary so that \( \psi(\gamma) \) matches properly to its values at \( \phi = 0^\circ \) and \( \phi = 26^\circ \). This adjustment in no way affects the interior dynamics which depend on \( \partial Q/\partial y \), not \( Q \) itself as is readily seen from (2.14).

Physically, the condition at 26° specifies that the meridional mass flux at each point on the boundary is of the proper size to balance exactly the combined effects of vertical advection, diffusion and momentum.
4. Momentum forcing

One of the most difficult problems in working with the zonally averaged equations is to specify the interaction terms which represent the dynamic coupling between the zonally symmetric circulation and the eddy motions. In general, the eddy momentum fluxes, through which this interaction takes place, have two effects on the zonal momentum budget.

1) We assume that there exist small eddies, whose structure depends largely upon the zonal flow in which they are imbedded. These should produce momentum transports which limit the magnitudes of horizontal and vertical wind shears in the atmosphere. The smoothing effect of these eddies may be parameterized in terms of Fickian diffusion and is represented in (2.9) by the term $F^*$. 

2) Large-scale, organized eddies may have preferred shapes which lead to eddy momentum fluxes that can not be simply parameterized in terms of the zonal momentum distribution. These eddy fluxes, represented by $G^*$ in (2.9), are often counter to the zonal momentum gradient. The effect of small-scale eddies is represented by Fickian diffusion in the form

$$F^*(y^*, z^*, t^*) = K_{zz} \frac{\partial^2 u^*}{\partial z^2} + K_{yy} \frac{\partial^2 u^*}{\partial y^2}. \quad (4.1)$$

The vertical diffusion coefficient $K_{zz}$ was assigned the value $10^6 \text{ cm}^2 \text{ sec}^{-1}$, which is the order of magnitude suggested by tracer diffusion studies.

There is some question as to whether angular or linear momentum should be diffused laterally. Both forms of diffusion were tried in the model and it was found that the former type created an unrealistic concentration of easterly momentum at the equator. Therefore, the results discussed in Section 6 are all for linear momentum diffusion models. The maximum value of $K_{yy}$ used in the model was $2 \times 10^8 \text{ cm}^2 \text{ sec}^{-1}$, which is close to the average value estimated by Hettler (1965) on the basis of the diffusion of radioactive material injected into the atmosphere.

$G^*(y^*, z^*, t^*)$ is specified in accordance with the observed momentum flux distribution. It should include the combined effects of horizontal and vertical fluxes on the zonal wind, thus maintaining the latter at its specified value. By continuity, the vertical mass flux through the lower boundary must equal the mass flux through the lateral boundary, i.e., $\psi(26^\circ, 30 \text{ km}) - \psi(0^\circ, 16 \text{ km}) = 0$. The horizontal distribution of this flux is determined in such a way that the vertical shear of the zonal wind remains geostrophic and the adiabatic heating due to vertical motion just balances the diabatic heating at every point on the lower boundary. Thus, the $\psi$ distribution on these boundaries varies with time to assure that $u(26^\circ, z)$ and $u(\phi, 15 \text{ km})$ remain constant.
by both transient and standing eddies. Because of the very limited station coverage, and the difficulties involved in measuring vertical motions, data are available only for the horizontal transports by transient eddies, and even these are subject to considerable uncertainties.

The observations of the meridional transport of zonal momentum by transient eddies in the Northern Hemisphere indicate that in the long term mean the flux is generally poleward, except perhaps within about 10° of the equator where it is small and the direction is uncertain. The poleward flux increases with latitude, especially north of about 18N. These conditions produce a general flux divergence, except possibly at very low latitudes. The divergence increases markedly north of 18°. The height variation shows a minimum of eddy activity at about 50 mb, with the tropospheric jet stream causing larger fluxes at lower levels, and the polar night vortex contributing to an increase with height at higher levels. Fig. 2a shows the long term mean of the poleward flux of zonal momentum by transient eddies, based on 15 years of data at about 30 stations, and Fig. 2b gives the distribution of the time mean component of $G^*$ which was adapted from it.

Wallace and Newell (1966) have reported variations in eddy activity within the polar night vortex from one winter to the next. This has resulted in large year-to-year variations in momentum fluxes which are detectable above 50 mb as far south as 28N. Still further south there are signs of the same fluctuations above 25 mb but the evidence is less convincing. Reed (1967) reports that spectral analysis of momentum transports at tropical stations show no statistically significant periodicities in the neighborhood of two years.

In the experiments with radiative forcing, $G^*$ was taken as the steady state component (Fig. 2b) alone. The convergence of momentum flux at the equator assumed in this distribution provides the mechanism for producing westerly accelerations at the equator.

In the experiments with dynamical forcing, this was combined with a time varying component of the $G^*$ field, patterned after Wallace and Newell's observations.

The oscillation in momentum fluxes was assumed to decrease linearly from its observed value at 28N to zero at the equator. This produced a latitudinally uniform $G^*$ oscillation, the amplitude of which was assumed to increase smoothly from zero at 25 km and below to a maximum value of $2.5 \times 10^{-5}$ cm sec$^{-2}$ at 35 km. The observed momentum flux appears to be synchronous at all heights and the $G^*$ field in the model was specified accordingly. The important implications of this restriction are discussed in Section 7. The time variations of the $G^*$ field were assumed to be of the form $\sin(t/(2\pi/700))$ where $t$ is in days. This produced alternating positive and negative pulses, simulating the dominance of the winter seasons.

5. Diabatic heating

Diabatic heating was treated in the simplest possible manner (which still preserves its dependence upon temperature) in the form of a Newtonian cooling term plus Fickian diffusion. In dimensional form, this can be expressed as

$$Q^* = K_v (\theta_* - \theta_0) + K_{uv} \frac{\partial \theta_*}{\partial y} + K_{vv} \frac{\partial^2 \theta_*}{\partial y^2},$$

where $K_v$ is an inverse radiative relaxation time, $\theta_*$ is the radiative equilibrium temperature and $K_{uv}$ and $K_{vv}$ are the vertical and horizontal heat diffusion coefficients, respectively. $K_v$ is specified in such a way that radiative relaxation time varies smoothly from 50 days at 100 mb to 10 days at 10 mb, in accordance with the numerical results of Manabe and Strickler (1964).

The $\theta_*$ field was held constant for the dynamically forced experiments, and given a constant value plus a sinusoidal variation in time for the experiments with a radiative drive.

The steady state component of the $\theta_*$ field used in the experiments (with the exception of Experiment 1) is shown in Fig. 3, together with the distribution of heating rates which it implies. We have employed the radiative heat sink at the equator for the sole purpose of...
testing whether downward motion in that region is important in advecting wind regimes downward. We make no attempt to justify this assumption on radiative grounds.

The radiative heating rates assumed in this paper are an order of magnitude larger than the convergences of the meridional transports of heat by transient eddies reported by Wallace and Newell (1966). Accordingly, the $P^*$ term is neglected in the numerical experiments.

6. Numerical results

As explained in the Appendix, the basic equations and boundary conditions were expressed in finite difference form and integrated using the forms of the forcing functions discussed in Sections 4 and 5. The integrations were started from an initial state of rest and continued until cyclic solutions were obtained. In general only two cycles of the oscillation were required for the motion field to converge to an approximately periodic solution.

The four experiments described below were selected from a much larger number which represented many combinations of $G^*$ and $\theta^*$ distributions. It was clear from these experiments that realistic looking downward propagating solutions could be obtained only under the condition of a radiative heat sink at the equator. Hence, our choice of radiative equilibrium temperature distribution shown in Fig. 3 and used in Experiments 2, 3, and 4. Experiment 1 shows a typical solution in the absence of radiative cooling at the equator. The other experiments are presented for the purpose of demonstrating that even with downward motion at the equator it is not possible to simulate with the model all the observed characteristics of the quasi-biennial oscillation. Experiments 2 and 3 explore the possibility of dynamical forcing as the driving mechanism. The final experiment examines the possibility of a radiative drive.

Before discussing the solutions for the biennially forced experiments, it is worthwhile to examine the long-term mean which may be obtained by using the steady state values of momentum flux divergence and radiative heating shown in Figs. 2 and 3 and integrating the equations until the solutions approach a steady state. In Figs. 4a and 4b the steady state zonal wind and meridional stream function distributions are displayed. Figs. 5a and 5b show the corresponding vertical and meridional velocities. In the mean the zonal wind exhibits a small easterly shear with height. The lateral shear is also small, and the core of maximum easterlies is at about 12°. This compares well with the observed distribution. The meridional circulation consists of an
overall upward and northward mass flux except near the equator where our assumption of a radiatively cooled region requires a reverse cell with downward motion between 30 and 20 km, and an equatorward drift between 30 and 25 km to maintain continuity. The steady state momentum balance may be understood physically as follows: At the higher latitudes the eddy flux divergence is balanced primarily by the generation of westerly momentum through the Coriolis force acting on the northward moving air. Near the equator, where the Coriolis force term in the zonal momentum equation becomes small, the steady convergence of momentum by the eddy fluxes is balanced by removal through horizontal diffusion and vertical advection. It should be noted that there exist other combinations of eddy flux and radiative heating distributions which lead to a similar long-term mean zonal wind field. The above arrangement was chosen because it optimized downward propagation in the model. We now describe the results of four experiments with biennially periodic forcing.

a. Experiment 1: Varying eddy flux, mean upward motion at the equator. In order to test whether downward propagation of the wind regimes could be accounted for without vertical advection, an experiment was run in which the radiative equilibrium temperature was specified as a function of height only. The radiatively cooled region at the equator was eliminated and the mean downward current over the equator was replaced by the general upward and poleward motion. Upward velocities over the equator were in the order of 0.02 cm sec\(^{-1}\). Elsewhere, the mean meridional circulation was changed little from the earlier distribution. In this experiment the forcing function \(G^*\) was initially set equal to the distribution shown in Fig. 2b. However, it was found that in the absence of mean downward motion to act as a sink for momentum, strong mean westerlies were generated at the equator.

This experiment indicated (in agreement with Lindzen's theory) that in the presence of strong mean westerly wind shear downward propagation does occur at the equator even in the absence of mean subsidence. In order to eliminate the unrealistic mean westerly wind shear, the steady state component of \(G^*\) was modified by reducing the eddy flux convergence at the equator until a realistic mean wind shear was obtained. The coefficient of horizontal diffusion was set equal to \(2 \times 10^8\) cm\(^2\) sec\(^{-1}\).

Time-height sections of the resulting oscillation are exhibited in Fig. 6. It is readily apparent that there is no tendency for downward propagation and the oscillation...
tion damps rapidly away for the region of direct momentum flux forcing.

b. Experiment 2: Varying eddy flux, mean downward motion at the equator. In Fig. 7 time-height sections of the zonal wind velocity at the equator and $10^\circ$ latitude are exhibited for a case in which $G^e$ was given by the sum of the steady and time-varying eddy flux divergence distributions discussed in Section 4 and $\theta^e$ was specified in Fig. 3. Again $K_{yy} = 2 \times 10^8$ cm$^2$ sec$^{-1}$. A comparison of Figs. 7 and 1 indicates that the amplitude of the computed oscillation at the equator is comparable to the observed amplitude. The observed downward propagation of about 1 km per month is also apparent in the model, primarily as a result of vertical advection. However, the oscillation damps more rapidly with decreasing height below 25 km than observations indicate. At $10^\circ$ the general appearance of the oscillation is also qualitatively correct, but the damping below 25 km is much too rapid. In general, the vertical wind shears are much smaller than observed at both latitudes. It will be shown that this is largely a consequence of horizontal diffusion.

c. Experiment 3: The effect of suppressing horizontal diffusion. To test the relative importance of the meridional motions and horizontal diffusion in producing the downward propagation away from the equator, an experiment was run with conditions identical to Experiment 2 except that $K_{yy}$ was reduced to $2 \times 10^7$ cm$^2$ sec$^{-1}$. In Fig. 8 the time-height sections for this experiment are presented.

At the equator the model resembles the observations more closely than in any other experiment. The wind regimes propagate downwards with very little loss of amplitude. (They propagate downward further than is observed, but this could be remedied by reducing the strength of the heat sink at low levels.) The strong vertical wind shears and the extreme thinness of the easterly regimes at low latitudes in the observations are reproduced well. However, at $10^\circ$ latitude, where the observations show fluctuations very similar to those at the equator, the model shows only a very weak response to the momentum flux oscillation and is unable to simulate the downward propagation.
Detailed analysis of the calculations reveals that any change in the $G^*$ field evokes a response in the mean meridional motion field which produces accelerations in opposition to its own. At all but the lowest latitudes, this reaction effectively compensates for changes in $G^*$, and consequently, the momentum flux oscillation produces little response. Only within a few degrees of the equator, where mean meridional motions are relatively ineffective in controlling the accelerations, do large responses occur. Hence, in Experiment 2, it was chiefly horizontal diffusion of the momentum acquired at the equator which produced the in-phase fluctuations at $10^\circ$.

It is also clear from comparing the results of Experiments 2 and 3 that the appearance of downward propagation at $10^\circ$ does not result from any vertical transport of momentum at $10^\circ$, but rather from the lateral diffusion of the characteristics prevalent at the equator. However, the price we pay for this lateral spreading of the oscillation is the unrealistic attenuation below 25 km at the equator. The dilemma we are faced with would be less serious if the current of mean downward motion at the equator covered a broader latitudinal extent. Then more momentum would be advected downward in successive wind regimes and diffusive losses at low latitudes would be smaller. Tucker (1964) suggested that mean downward motion might occur in the lower stratosphere everywhere between 20N and 20S.

In a series of experiments we attempted to widen the current of descending motion by extending the lateral extent of the heat sink. We found that any substantial changes in this direction produced strong meridional motions toward the equator at 10 mb, and large easterly accelerations. Only by applying a large convergence of momentum in the steady state component of the $G^*$ term could a reasonable mean zonal wind distribution be retained. Since the available eddy flux measurements indicate a divergence of momentum in this region, we did not continue this line of experimentation.

d. Experiment 4: Varying radiative equilibrium temperature, mean downward motion at the equator. The possibility that the oscillation might be driven by a time variation of the radiative heating was investigated in this final experiment. The time varying eddy flux was set equal to zero. A biennially oscillating thermal drive was introduced by multiplying the $G^*$ distribution of Section 5 times the factor $[1 - \sin(2\pi t/700)]$, where $t$ is in days. The effect of this is to introduce a biennial oscillation of $\sim 18^\circ$C in the radiative equilibrium temperature of the lower stratosphere at the equator. This implies an oscillation in the radiative heating rate of $\sim 1.5$C day$^{-1}$ which is more than an order of magnitude greater than the response calculated by Lindzen (1965) for a 10% change of solar ultraviolet radiation. The results of this experiment are shown in Fig. 9. The oscillation has approximately the right amplitude at the equator, and it propagates downward. However, contrary to observation, the downward propagation is more rapid for the leading edge of the easterly regime than for that of the westerly regime.

Aside from the obvious difficulties inherent in accounting for a thermal driving mechanism of sufficient strength to produce the observed amplitude of the wind oscillation, this result demonstrates a serious flaw in theories based on a radiative origin of the oscillation. Observations indicate beyond any doubt that the leading edges of westerly regimes propagate downward more rapidly than those of easterly regimes. But as shown in Experiment 4, a radiatively driven oscillation exhibits just the opposite behavior in propagation speeds. This result is easily comprehensible in terms of the energetics of the oscillation. At the equator, where the horizontal advection of kinetic energy vanishes, the conversion of zonal available potential energy (APE) to zonal kinetic energy (KE) must, in the absence of eddy flux forcing, account for the oscillation in kinetic energy of the zonal wind. In order that APE be con-

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**Fig. 8.** Time-height sections of the zonal wind velocity at the equator (upper) and $10^\circ$ latitude (lower) for Experiment 3. Solid lines are 10 m sec$^{-1}$ isopleths.
verted to KE there must exist a positive correlation between the perturbation vertical velocity and temperature fields i.e., cold air must sink relative to warm air. Thus, the downward motion must be relatively weak when westerlies overlie easterlies (warm equator) and relatively strong when easterlies overlie westerlies (cold equator). Vertical advection will then cause easterlies to propagate downward more rapidly than westerlies. Since the opposite is observed to occur, we must conclude that in fact KE is being converted to APE at the equator and that the oscillation is dynamically driven. Further evidence is given in favor of dynamical forcing in Wallace (1967b).

7. Discussion and conclusions

Our numerical results have shown that using boundary conditions and eddy fluxes based upon observations, it is possible to obtain a time-dependent solution which resembles in some respects the biennial wind oscillation. However, in order to simulate the observed downward propagation, it is necessary to assume the existence of a strong radiative heat sink in the equatorial stratosphere. We know of no physical basis for such an assumption. Aside from this difficulty there are also some notable differences between the model results and the atmosphere, the most serious being the inability to produce downward propagation over a broad range of latitude without rapid attenuation.

In view of the dissimilarities between the model and the real atmosphere, we are forced to conclude that our conception of the dynamical forcing mechanism, reflected in the $C^*$ field in the model, is not wholly correct. There appears to be no physical mechanism that will account for the observed downward propagation unless we resort to an oscillating $C^*$ field which has a phase dependence upon height, i.e., a momentum flux oscillation which propagates downward. This suggests that the year-to-year differences in the poleward momentum fluxes observed by Wallace and Newell (1966) are not adequate to explain the behavior of the zonal wind field. It would appear that an important, if not the major, quasi-periodicity in the momentum transports is still undetected.

It is clear that if the restriction that the $C^*$ oscillation be synchronous at all levels were removed it would be possible to reproduce in the model any desired zonal wind field. In fact, as Deardorff (personal communication) has suggested, the problem could be worked in reverse: Given the observed time dependent zonal wind field, and making the same type of assumptions regarding radiation as in Section 5, it should be possible to solve for the $C^*$ field. However, such a result would be useful only if one had some knowledge of the transport processes responsible for the $C^*$ oscillation; for example, whether they were horizontal or vertical.

The model should be applicable to other situations where zonal symmetry prevails. Two problems of particular interest are the semi-annual oscillation noted by Reed (1965) in the 30-60 km region over the equator, and the polar night jet.

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APPENDIX

Finite Difference Model

The system (2.9)–(2.14) may be expressed in finite difference form most economically by utilizing a staggered grid system. We let the indices $j$ and $k$ denote grid points in $y$ and $z$, respectively, according to

$$
y = (j-1)\Delta y/2 \quad \text{and} \quad z = (k-1)\Delta z/2.
$$

The dependent variables are then assigned grid points as follows: $\psi_{j,k}$ all odd $j$ and odd $k$ points; $u_{j,k}$ all odd
j and even k points; and \( \psi_{j,k} \), all even j and odd k points. Thus, \( \Delta y \) and \( \Delta z \) represent the grid spacing of the variables in \( y \) and \( z \), respectively.

We first evaluate the thermal wind (2.10) at odd \( j \), \( k \) using the formula

\[
f_j(u_{j,k+1} - u_{j,k-1})/\Delta z = - (\psi_{j+1,k} - \psi_{j-1,k})/\Delta y. \tag{A.1}
\]

If we define meridional and vertical mass transport by

\[
v_{j,k} = - (\psi_{j+1,k} - \psi_{j-1,k})/\Delta z
\]
\[
w_{j,k} = + (\psi_{j+1,k} - \psi_{j-1,k})/\Delta y,
\]

respectively, (2.9) may be expressed in flux form for odd \( j \) and even \( k \) as

\[
\varepsilon u_{j,k} = \frac{f_j v_{j,k} - (w_{j+1,k} - w_{j-1,k})}{2\Delta y} = \frac{(w_{j+1,k} - w_{j-1,k})}{\Delta z} e^{-(k-1)\Delta z} G_{j,k}, \tag{A.3}
\]

Similarly, (2.11) may be written for even \( j \) and odd \( k \) as

\[
\varepsilon \psi_{j,k} = - \frac{u_{j,k}}{\varepsilon} e^{(k-1)\Delta z} + K_e (\psi_{j+1,k} - \psi_{j-1,k}), \tag{A.4}
\]

where the small nonlinear advective terms have been neglected. The operator \( \mathcal{L} \) in the above is defined as

\[
\mathcal{L} = - K_{ss} \frac{\partial^2}{\partial s^2} - K_{yy} \frac{\partial^2}{\partial y^2}.
\]

The values of \( u_{j,k} \) and \( w_{j,k} \) at odd \( j \), odd \( k \) grid points, which are required in (A.3) are obtained by linear interpolation from the values at odd \( j \), even \( k \) and even \( j \), odd \( k \), respectively.

We next use (A.1) to eliminate the operator \( \mathcal{L} \) between (A.3) and (A.4) and obtain a diagnostic equation for the stream function, i.e.,

In the integrations the grid system consists of 27 points in the horizontal with separation of 1° latitude, and 36 points in the vertical with 1 km separation. If the values of \( u_{j,k} \) and the forcing functions are known, (A.4) may be solved by relaxation to obtain \( \psi_{j,k} \) at all odd \( j \), \( k \) interior grid points, provided that \( \psi_{j,k} \) is specified along the boundaries. The boundary conditions are determined from the following:

\[
\psi_{j,0} = 0, \quad (1 < k < 35),
\]
\[
\psi_{j,32} = 0, \quad (1 < j < 27),
\]
\[
\psi_{27,k} = \psi_{27,k+2},
\]
\[
\psi_{j,3} = \psi_{j+2,3} + K_e (\psi_{j+1,1} - \psi_{j+1,3}), \quad (27 > j > 1),
\]
\[
\psi_{j,3} = \psi_{j+2,3} + K_e (\psi_{j+1,1} - \psi_{j+1,3}), \quad (35 > k > 1),
\]

where \( j, k \) are odd in all the above formulae. In actual integrations \( \theta \) at the lower boundary must be adjusted so that \( \psi_{1,k} = 0 \). This adjustment does not change the dynamics (see Section 4).

In the numerical experiments (A.3) was integrated forward in time using a two step Lax-Wendroff method. This method proved to be stable for time steps of 10 days and suffered very little truncation error for the forced oscillatory models reported here.

**REFERENCES**


