The Net Generation of Large-Scale Available Potential Energy by Subgrid-Scale Processes

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ABSTRACT

A procedure is introduced which allows estimation of the net influence of subgrid-scale thermodynamic processes on large-scale available potential energy. A numerical example representing the average energetics over North America for March, 1962 suggests that “net” subgrid-scale generation greatly exceeds the average grid-scale generation, with positive contributions occurring in the low and middle troposphere.

1. Introduction

In the mid-1950’s Lorenz (1955a, b) applied Margules’ (1903) fundamental ideas to formulate the theory of available potential energy, providing a framework for a more adequate representation of the baroclinic state of the atmosphere and the role of differential heating in producing that state. In order to assess the role of diabatic heating in generating available potential energy, it is necessary to specify the horizontal and vertical distributions of the various heating components. Such specification is made inherently difficult because in many cases diabatic temperature changes are produced by processes occurring on a scale smaller than the standard data networks can explicitly resolve. One solution of this problem is to model subgrid-scale processes in terms of measurable grid-scale parameters. The purpose of this note is to introduce an alternate means of accounting for the influence of subgrid-scale heating on large-scale available potential energy through the application of large-scale energy budget analyses.

2. Generation of available potential energy

As given by Smith (1969) the Eulerian budget equation representing the contribution of a limited region to the global available potential energy \( A \) is

\[
\frac{\partial A}{\partial t} = G + CA + HFA + VFA + BA, \tag{1}
\]

where:

\( G \) — diabatic generation of \( A \)
\( CA \) — product of individual pressure change \((dp/\partial t)\)
and specific volume, referred to by the author as $A$-conversion.

- $HFA$: horizontal flux divergence of $A$
- $VFA$: vertical flux divergence of $A$
- $BA$: additional term required when the pressure of the lower boundary is allowed to vary.

The generation term is more specifically given by an integral of the product of diabatic heating and the efficiency factor (Dutton and Johnson, 1967). Two methods are available to estimate diabatic heating. The first is to model each heating term individually and sum their contributions to obtain the total heating. While, in principle, the most desirable approach, the difficulties associated with formulating realistic models are formidable. The second, the thermodynamic method, is to estimate the total diabatic heating as the residual quantity required to balance the first law of thermodynamics after first evaluating an appropriate time derivative and horizontal and vertical flux terms. The appeal of the second method is its simplicity, but certain shortcomings must be recognized.

Fundamentally, the thermodynamic method is incapable of resolving contributions from individual heating terms; however, if the researcher is content to diagnose the total generation, this is not a serious deficiency. Of much greater importance is the realization that the spatial and temporal resolutions of standard meteorological data allow only the estimation of large-scale heating by this method. It remains, therefore, to consider whether the influence of subgrid-scale processes can be indirectly considered.

The author proposes the following approach to this problem: Regarding the terms in (1) as being composed of grid- and subgrid-scale processes, it can be separated into the two equations:

$$
\frac{\partial A}{\partial t}_g = G_0 + CA_0 + HFA_g + VFA_g + BA_0 + X_{as},
$$

$$
\frac{\partial A}{\partial t}_s = G_s + CA_s + HFA_s + VFA_s + BA_s - X_{as},
$$

where the subscripts $g$ and $s$ refer to grid and subgrid scales, respectively, and $X_{as}$ represents the exchange between grid and sub-grid scale available potential energy; or more specifically, the role of grid/subgrid scale diffusion of sensible heat in producing changes in available potential energy. All of the terms in (2) except $X_{as}$ can be calculated directly, leaving $X_{as}$ to be determined as the residual required to balance the equation, analogous to the residual determination of dissipation in the kinetic energy budget equation. Considering (3) it seems plausible that certain terms will be negligible when averaged over a large-scale wave system and/or an extended time period. In particular, since the averaging scale would greatly exceed the spatial or temporal scale of the subgrid-scale events, one should expect the net horizontal and vertical flux divergences, the mean temperature change, and the mean surface pressure change to be small. However, heating and temperature and vertical motion and temperature could be significantly correlated; thus, subgrid-scale generation and conversion cannot in general be neglected. Hence, for a large spatial and/or temporal average (3) can be approximated by

$$
G_0 + CA_0 \approx X_{as}.
$$

Both of the left-hand side terms correspond to physical mechanisms for altering the thermal structure within subgrid-scale systems. Through (4) the net effect of these processes is seen to be the establishment of a structure which is capable of exchanging sensible heat with large-scale systems in such a way that the large-scale available potential energy will be changed. Thus, one can regard $X_{as}$ as the "net" generation of large-scale available potential energy by subgrid-scale processes, and the sum of $G_0$ and $X_{as}$ as the total generation.

Before proceeding to a numerical example it would be well to comment on the existence of computational uncertainties. Most evident of these are the random observational errors associated with wind and temperature data. In addition, false grid-scale information may be interpreted as a result of subgrid-scale events occurring at some observation points, a form of aliasing. In both cases the uncertainties could be overwhelming if unfiltered data from only a small sample of stations were considered in the averages obtained. Clearly, these problems can be substantially reduced by application of some data filtering procedure, an operation that may also extract valuable real information. Alternately, one can average over a large sample of stations, realizing that the contribution of random uncertainties to the resulting mean value is reduced in proportion to the square root of the sample size. This is especially applicable to observational errors. In the case of non-representative data it is subject to doubt because correlated subgrid-scale quantities may contribute systematic errors in the large-scale mean value of the product of two quantities. In the author's view this later consideration is still not large for sizeable samples. First, aliasing will not enter at every point, as is the case for observational error. Second, subgrid-scale quantity pairs will, in general, not be perfectly correlated even at points where they do occur. Third, many budget quantities either are themselves or are derived from gradient calculations. Even systematic aliasing errors can influence gradient calculations in a random manner. The author's experience with residual kinetic energy dissipation estimates (Smith, 1973) suggests that 12-hr means over a region as large as North America are reliable as to sign, order of magnitude, and general trends. However, they do possess fluctuations which are at least partly due to computational uncertainties. For 36-hr periods or longer, errors in mean
quantities are reduced to levels adequate to derive reliable conclusions.

3. Numerical example

Because the basic calculations have already been done and are presented in a manner convenient for the present work, the author has chosen to consider the results given by Smith and Horn (1969) with the following modifications. As was acknowledged in their paper, the estimates of CA and VFA are very likely too large because of erroneous vertical motion estimates arising from systematic errors in the computed divergence values. Such errors can be reduced by objective adjustment schemes (O’Brien, 1970; Smith, 1971) which, in general, apply the largest adjustments to levels above 500 mb. In order to approximate the effect of this adjustment on the previous calculations of Smith and Horn, it is first assumed that the vertical motion at the upper boundary (250 mb in this case) is zero. Coupled with the same boundary condition at the lower boundary, this requires that the surface to 250 mb value of VFA be zero. Assuming that the difference between this value and the original total column estimate can be attributed to errors in the 500–250 mb (layer 3) value, the layer 3 estimate is adjusted by this difference. Finally, the layer 3 estimate of CA is adjusted by the same percentage amount as VFA. Note that the surface to 750 mb (layer 1) and 750–500 mb (layer 2) estimates are identical to those of Smith and Horn.

Table 1 contains estimates of the terms in (2) averaged over North America for March 1962. Keep in mind in the ensuing discussion that the budget terms represent the contribution of this region to the global available potential energy budget. No attempt has been made here to define the available potential energy of the region itself, as suggested, for example, by Johnson (1970). No attempt is made to review the complete discussion of budget quantities given by Smith and Horn; rather, attention is focused on the generation term. Also, it should be acknowledged that in Smith and Horn the present author regarded the residual quantity in (2) as an expression of accumulated errors.

Although some errors no doubt still exist, the author’s former interpretation appears to be largely incorrect.

In each layer it is apparent that the subgrid-scale influence dominates the total generation of \( A_n \). This could be accounted for in three ways. First, the small magnitude of \( G_n \) can be attributed to a very limited contribution from zonal differential heating, probably because of the absence of data from tropical and polar latitudes. Second, grid-scale heating and cooling may be alternately correlated with positive and negative efficiency factors as wave systems initiate, grow and decay within the computational region. On the other hand, subgrid-scale heating (e.g., convective latent heat release) would be more likely to exhibit systematic correlations with efficiency factors. Third, subgrid-scale processes may indeed provide the greatest differential heating in the middle latitudes.

The positive values of \( X_{ac} \) in layers 1 and 2 appear to reflect dominance by subgrid-scale sensible and latent heat release in the warmer, higher pressure (positive efficiency factor) parts of the atmosphere. The maximum in layer 2 agrees qualitatively with the maximum mid-tropospheric latent heat release indicated by Krishnamurti (1968) and Bullock and Johnson (1971). The negative value in layer 3 could also result from latent heat release, in this case occurring at high enough levels to place it in the negative efficiency region; or it could be attributed to the systematic occurrence of rising warm air and/or sinking cool air in unsampled subgrid regions. Clearly, however, the rather crude procedure used to correct CA and VFA in the upper layer renders this calculation less certain than those in the lower two layers. For example, if the vertical flux at 250 mb were an order of magnitude less than that at 500 mb, rather than zero, the value of \( X_{ac} \) in layer 3 would be reduced to \(-1.27 \text{ W m}^{-2}\).

In summary, it appears that the approach suggested here is capable of providing an assessment of the role of subgrid-scale processes in large-scale available potential energy budgets independent of the assumptions of any particular parameterization models.

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