Comments "On the Lateral Boundary Conditions for the Primitive Equations"

EUGENIA KÁLNAY DE RIVAS

Laboratorio de Dinámica de la Atmósfera, Universidad de la República, Montevideo, Uruguay

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In a recent paper, Davies (1973) states that for the barotropic primitive equations, if the lateral boundary conditions are such that \( I_1 + I_2 \leq 0 \), the solution is unique. Here

\[
I_1 = - \oint_C \frac{1}{2} \left[ (\vec{v} \cdot \hat{n}) \frac{1}{2} \vec{v}^2 + g(v' \cdot \hat{n}) h' \right] \, dc,
\]

\[
I_2 = - \oint_C \frac{1}{2} gh' \vec{v} \cdot \hat{n} \, dc,
\]

where \((\vec{h}, \vec{v})\), \((h, v)\) are two initially close solutions, \((h', v')\) is the difference between them, and \(C\) is the boundary of the region of integration. A similar criterion is developed for the baroclinic case.

From the uniqueness criterion the author concludes that the most reasonable set of lateral boundary conditions for the barotropic primitive equations is the normal velocity given everywhere on the boundary and the height and the tangential velocity given at the inflow points.

However, the uniqueness criterion is not useful in determining a correct set of lateral boundary conditions such that the problem is well posed, because it does not avoid overspecification. In particular, if all variables are specified at all boundaries \((h'=0, v'=0\) at \(C\), \(I_1 = I_2 = 0\), the criterion implies a unique solution even though there is an overspecification of the boundary conditions. Even the set of boundary conditions suggested as the most reasonable by the author is overspecified. In the one-dimensional problem (flow through a narrow open channel, for example), this set would correspond to the specification of the velocity at both ends of the channel, and the height at the inflow end. In this case, however, it is easy to show that for a subcritical flow the problem is well posed if one boundary condition on the height, or the velocity, or a relationship between them, is specified at each end.

REFERENCE