Further Evidence of Traveling Planetary Waves

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ABSTRACT

Evidence of regularly propagating, large-scale waves is found in a 73-year record of Northern Hemisphere sea-level pressure data and in a nine-year record of upper air data. Cross-spectrum analyses indicate that south of 50°N, in all seasons, a zonal wavenumber 1 disturbance moves westward around the world in 5 days. In addition, north of 50°N a zonal wavenumber 1 disturbance moves westward around the world in one to three weeks with an average period near 16 days. This disturbance appears to be strongest in winter and spring. The structure of the 16-day wave during winter is studied in detail, and it is shown to be consistent, in many respects, with that of a theoretically predicted free planetary wave, or wave of the second class. A similar conclusion can be made concerning the 5-day wave.

1. Introduction

In recent years there have been many observational studies of large transient waves. In this paper we limit our discussion to only the largest transients—those whose horizontal scale is that of a single wave around a latitude line. We refer to this largest-scale wave as zonal wavenumber 1, or simply wave-1.

Because different investigators studied wave-1 disturbances with data from different years and seasons, and because they used different time filters, one cannot expect them all to report identical propagation speeds and local periods. However, most do report westward movement and local periods averaging near 5 days (Deland, 1964; Eliassen and Machenhauer, 1965, 1969; Wallace and Chang, 1969; Madden and Julian, 1972; Misra, 1972, 1975; Madden and Stokes, 1975; Madden, 1975) or near 16 days (Kuboto and Iida, 1954; Eliassen and Machenhauer, 1965, 1969; Arai, 1970; Hirota, 1968, 1975; Madden, 1975).

Here we present further evidence for westward propagating wave-1 disturbances on these time scales in sea-level pressure and the geopotential height of constant pressure surfaces. Two primary data sets are studied. The first is daily grid-point sea-level pressure data over the Northern Hemisphere from the period 1899–1972. The second set consists of daily Northern Hemisphere geopotential heights for 9 years from 1964–72. In both of these data sets we find evidence for a 5-day wave in all four seasons. This evidence is strongest south of 50°N. Regular westward propagation indicating periods averaging about 16 days is also present, primarily in winter and spring at latitudes north of 50°N.

Details of the two data sets are discussed in Section 2. The methods used to filter the data in space and time are described in Section 3, and the resulting evidence for the existence of westward-propagating disturbances is presented in Section 4. Based on theoretical considerations which are discussed in Section 5, four features characteristic of the vertical and horizontal structures and the frequency of linear free planetary waves, or waves of the second class, in an isothermal atmosphere on a rotating globe, are outlined. Results of additional cross-spectrum analyses and compositing of the data are presented in Section 6 to demonstrate that the period and structure of the 5- and 16-day waves are consistent with these features.

2. Data

a. Sea level pressure

Daily sea level pressure values at 10° latitude intervals from 30–70°N and at 10° longitude intervals from 0–350°E for the period 1899–1972 are studied. These data are based on the Historical Weather Maps (U.S. Weather Bureau, 1899–1939) and the daily maps which succeeded them. Digitized grid-point values were originally obtained from the National Oceanic and Atmospheric Administration, Massachusetts Institute of Technology, the U.S. Navy, the National Meteorological Center and the National Climatic Center. The entire record is now available at the National Center for Atmospheric Research (Jenne, 1975). No grid-point data are available from December 1944–December 1945. Although there is evidence over the 73 years of non-
stationarity in both time averages and spectra computed from these daily data (e.g., Madden, 1976), Madden and Stokes (1975) have shown that the 5-day wave is manifest throughout. Here the data are broken up and studied as separate seasonal segments, so that slow changes (time scales of a year or more) in time-means should not affect a search for transients whose time scales are on the order of days to weeks. Similarly, abrupt changes in time-means, if they do not occur within a single season, should present no problem. Evidence of non-stationarity in the spectra suggests that the largest relative changes have occurred in their high frequency end, that is, at periods shorter than about 4 days (Madden and Stokes, 1975). Despite these changes in the data over the years we conclude that disturbances with periods longer than about 4 days and less than a season should be reasonably well represented throughout the entire 73-year period.

The sea level pressure data were standardized in such a way as to minimize effects of seasonal variations. The standardization procedure is outlined in detail in Madden (1976). It will suffice to say here that at each grid point, long-term means estimated for each day of the year were subtracted from the daily value, and the resulting differences divided by a standard deviation appropriate for the day. The resulting standardized data should be approximately stationary in their mean and variance during any one year.

b. Geopotential height

Geopotential height data taken from daily analyses made by the National Meteorological Center are also studied. The data are from several tropospheric and stratospheric levels and from the nine-year period 1964–72. These data were broken up into nine spring, summer and autumn segments, and eight winter segments as was done with the sea-level pressure data. The data were not, however, standardized as were the sea-level pressure data.

3. Space and time filtering

The space filtering used is simple Fourier analysis along latitude lines. The resulting sine and cosine coefficients of wave-1 are then filtered in time. The time filtering procedure used is similar to the quadrature spectral analysis (QSA) used by Deland (1964) and Eliassen and Machenhauer (1969). Deland plotted sine and cosine coefficients of 500 mb height data for the large-scale waves on polar diagrams. He pointed out that the resulting wave vectors frequently form roughly circular patterns. He argued that since a propagating wave would cause fluctuations in the cosine coefficient which would be one-quarter of a cycle out of phase with those of the sine coefficient, the most direct numerical way of showing such a relationship in a sequence of data was to compute the out-of-phase or quadrature part of the cross spectrum. Deland’s QSA is then a simple and direct way of quantifying the circular patterns in his polar diagrams.

More complicated techniques have been devised to explain the variance of the sine and cosine coefficients of wave vectors in terms of eastward and westward propagating waves as well as fluctuating standing waves (Kao, 1968; Hayashi, 1971; Deland, 1972; Pratt, 1976; Hayashi, 1977). Here we only attempt to establish the dominant character of wave-1 transients. Cross spectra between the sine and cosine coefficients are computed and the coherence squares and phase angles are studied. Because the coherence squares include information on the in-phase part of the cross spectrum (cospectrum) as well as the out-of-phase part (quadrature spectrum), the technique differs from Deland’s QSA method. However, as we shall see, where coherence squares are highest, the sine and cosine coefficient series tend to be out of phase by one-quarter cycle. This means that most of the covariability is in the quadrature spectrum and the technique gives results similar to Deland’s QSA.

Specifically, the grid-point data were expanded into zonal Fourier harmonics at each latitude for each day. The resulting time-dependent cosine and sine coefficients of wave-1 \([C_1(t)\) and \(S_1(t)\)] were then treated as separate time series. Next, cross-spectral analyses were estimated as follows: each \(C_1(t)\) and \(S_1(t)\) time series was broken up into 96-day seasonal segments, or realizations, and the 96-day averages were removed. A fast Fourier transform was used to determine 48 harmonic coefficients (sine and cosine coefficients in time) and the spectra of \(C_1(t)\) and \(S_1(t)\) (\(P_c\) and \(P_s\)), cospectrum (\(K\)) and quadrature spectrum (\(Q\)) were estimated by averaging the appropriate squared coefficients over the number of available segments. The bandwidth of the resulting spectral estimates is approximately \((1/96)\) cycles day\(^{-1}\) and the degrees of freedom (df) are about 2\(q\) where \(q\) is the number of seasonal segments. One can further increase df by averaging \(L\) adjacent spectral estimates. This frequency averaging increases the bandwidth of the analysis to \(L/96\) and the df to \(2qL\).

For sea level pressure, adjacent spectral estimates were not averaged \((L=1)\) since the many seasonal segments available for averaging provide sufficiently stable spectral estimates. The number of available seasonal segments averaged 64 with a minimum of 58 summers at 70°N resulting in a minimum of 116 df for each spectral estimate. The standardization procedure should not reduce the df by more than two.

Eight winter (1964–65 through 1971–72), nine spring, and, because of some missing data, only eight summer and seven autumn (1964–72) upper air segments were
Fig. 1. Coherence squares (solid lines) and phase (dots) between the sine and cosine coefficients of wave-1 sea level pressure (SLP) for the winter (WIN), spring (SPR), summer (SUM) and autumn (AUT) seasons at the latitude indicated on the right. The approximate 99% significance level is indicated by the dashed line. The band width (bw) is indicated on the winter panel.

Similarly averaged. Since the number of available seasonal segments here is small, three adjacent spectral estimates were averaged reducing frequency resolution to (1/32) cycles day\(^{-1}\) and increasing df to approximately 48 for winter and summer (2qL=2.83). The upper air data was not standardized but nine values at the beginning and end of each seasonal segment were tapered with a cosine bell in order to reduce possible leakage of relatively low frequency seasonal variations to higher frequency spectral estimates. This procedure reduces the df of the upper air spectra by approximately 10% (see Julian, 1971).

The coherence squared is given by

\[ \text{coh}^2 = \frac{K^2 + Q^2}{P_e P_s} \]  

(3.1)

and the phase angle \( \phi \) by

\[ \arctan \phi = \frac{Q}{K} \]  

(3.2)

Ambiguities can arise in the use of spectral analysis to isolate traveling waves (e.g., Tsay, 1974; Pratt, 1976). Proper interpretation of the coherence squares
and phase angles computed here is made especially difficult in the presence of large standing oscillations with geographically fixed nodes and antinodes. A measure of the variance associated with such standing oscillations has been given by Pratt (1976) and Hayashi (1977) as

\[ K^2 + \frac{1}{2} (P_e - P_r)^2. \]

(3.3)

Where \( \phi \) is approximately one-quarter of a cycle, \( K \) is small. Furthermore, inspection of \( P_e \) and \( P_r \) indicates they are of comparable magnitude so that we do not expect the variance associated with standing oscillations to be particularly large and their effect on the interpretations of the coherence squares and phase angles presented in the following sections should therefore be minimal. Eq. (3.3) is evaluated and compared with the quadrature spectrum, or "propagating variance" (Pratt, 1976) for a few cross spectra to assure us that this is so.

4. Cross-spectral results

a. Sea level pressure

The coherence squares and phase angles between the \( C_1(t) \)'s and \( S_1(t) \)'s are presented in Fig. 1. Assuming at least 116 df, coherence squares of 0.075 are significantly above an expected zero background at the 99% level based on tabulations published by Amos and Koopmans (1963). Positive phase angles of 90° indicate that \( S_1(t) \) leads \( C_1(t) \) by one-quarter of a cycle which suggests a dominant westward motion.
At 30°N coherence squares are significantly different from zero over a broad frequency range from about 0.05–0.25 cycles day\(^{-1}\) in all seasons. Phase angles in this frequency range are approximately +90° so we conclude that this large coherence reflects westward moving disturbances that propagate with a high degree of regularity. These coherence squares clearly maximize near periods of 5 days. In summer the westward moving 5-day disturbance is apparent as far north as 50°N. In winter coherence squares reflecting regular westward movement in the 5-day period range are small north of 30°N. At 70°N there are significant coherence squares from about 0.15–0.25 cycles day\(^{-1}\), but here \(C_1(t)\) leads \(S_1(t)\). That is, the dominant motion of wave-1 disturbances is eastward at that latitude. Although interesting, this apparent eastward propagation at northern latitudes is not studied further here.

Some additional evidence of westward motion is, however, present at 50 and 60°N in both winter and spring. This evidence is seen in the significant coherence squares in approximately the 0.05–0.10 cycles day\(^{-1}\) frequency range. This frequency range includes the often mentioned local period near 16 days.

At 30°N in summer the ratio of the variance associated with standing oscillations [Eq. (3.3)] to the propagating variance \((Q)\) is 0.15 in the 4.6–5.6 day period range. At 60°N in winter that ratio is 0.72 at a period of 16 days.
b. Geopotential height

Coherence squares and phase angles estimated from the cross spectra of the winter and summer season geopotential height $C_1(t)$'s and $S_1(t)$'s are presented in Fig. 2. Assuming a minimum of 48 df, coherence squares of 0.21 are significantly above an expected zero background at the 99% level. Excepting 850 mb at 40°N in winter, there are significant coherence squares associated with phase angles of $+90°$ near 5-day periods at all levels and in all seasons at both 30 and 40°N. This includes spring and autumn seasons (not shown). In summer at 30°N the 850 and 500 mb $C_1(t)$ and $S_1(t)$ reveal significant coherence squares over a frequency interval from about 0.05–0.25 cycles day$^{-1}$ in a manner similar to the sea-level pressure cross spectra. In other seasons and at some other levels this broad maximum appears to resolve itself into two peaks, one near 5-day periods and one near 10-day periods. This occurs, for example, in winter at 30°N and 500 mb.

At 60 and 70°N there are significant coherence squares at every level in the 0.05–0.10 cycles day$^{-1}$ frequency range in the winter and spring (not shown) seasons. At 80°N in winter (not shown) there are significant coherence squares at every level at a period of 16 days. As with the sea-level pressure results, phase angles indicate that these coherence squares reflect westward propagating disturbances. A difference is that the significant coherence squares associated with sea-level pressure are at 50 and 60°N in these same two seasons rather than at 60 and 70°N. In addition, significant coherence squares are evident at 60 and 70°N at this period at some other levels in summer and autumn (not shown).

At 500 mb and 30°N in summer, the ratio of the variance associated with standing oscillations to the propagating variance is 0.15 in the 4.6–5.6 day period range. At that level at 60°N in winter the same ratio is 0.40 for 16-day periods while at 100 mb and 60°N in winter it is 0.22 for 16-day periods. As with the sea-level pressure data the variance associated with possible standing oscillations is less than the propagating variance.

It was mentioned in the Introduction that several researchers have reported westward propagating wave-1 disturbances whose local periods averaged near 5 and 16 days. From the long sea-level pressure record studied here we see evidence of the 5-day disturbances in all seasons and primarily at latitudes south of 50°N. In addition, evidence for westward propagating disturbances having an average local period on the order of 16 days is most clearly present in the winter and spring seasons north of 50°N. Similar evidence is found in the nine-year record of upper air data. Because the nine years of upper air data overlap the 73 years of sea-level pressure data, the two do not constitute completely independent samples. However, sea-level pressure results averaged to 1963 (i.e., excluding the period of upper air data) yield very nearly identical coherence squares and phase angles as those that are averaged through 1972 and shown in Fig. 1.

5. Theory

a. Introduction

In their analysis, Kubota and Iida (1954) divided disturbances into two parts, namely "... a stationary wave perhaps due to the topography and a progressive one which might be called Rossby waves." Deland (1964) and Eliassen and Machenhauer (1965) proposed a similar two-component model. About the same time Dikly and Golitsyn (1968) solved the Laplace tidal equation and determined phase velocities for certain predicted wave modes that were in excellent agreement with Eliassen and Machenhauer's (1965) observations. In the following, we outline some implications of this last theoretical treatment with regard to the expected periods, and the vertical and horizontal structures of long waves on a sphere. This outline provides us with a theoretical wave model. In Section 6 we then use both spectral analysis and compositing in an effort to better define the structures of the observed 5- and 16-day disturbances, and compare those structures with those of the corresponding wave model.

b. A specific model

The model we consider here is that of linear, free waves in an isothermal atmosphere on a rotating globe. Furthermore we will assume that the basic state of the atmosphere is motionless. The equations of such a model are well known and go back, at least, to Laplace's dynamical theory of tides which he summarized in the Mécannique Céleste published in a series of volumes between 1799–1823. He treated an ocean of uniform depth and extended the results to an isothermal atmosphere in which oscillations took place isothermally. The intervening work has been discussed by many writers (e.g., Chapman and Lindzen, 1970; Holton, 1975).

Solving the problem of free waves in an isothermal atmosphere on a rotating globe involves manipulating the linearized horizontal momentum equations, the thermodynamic equation and the continuity equation, expressed in spherical coordinates to separate vertical and horizontal dependence. Considering the resulting vertical structure equation and suitable boundary conditions, one finds that for free waves in an isothermal atmosphere the separation constant, or equivalent depth $h$, is given by

$$h = \gamma H,$$  \hspace{1cm} (5.1)

where $\gamma$ is the ratio of specific heats and $H$ the scale height. This particular result is independent of the horizontal coordinates. For example Haurwitz (1975) derives (5.1) explicitly in his study of circumpolar
waves on a plane tangent to the earth at the pole, and Lindzen (1967) discusses it with regard to planetary waves on β-planes. An implication following from (5.1) and the vertical structure equation is that free waves in an isothermal atmosphere will be external with constant phase in the vertical, and that their amplitudes \( a \) will vary as

\[
\ln a = \kappa \ln \left( \frac{p_0}{p} \right) + C,
\]

(5.2)

where \( \kappa \) is the gas constant divided by the specific heat of air at constant pressure, \( p \) the pressure, \( p_0 \) some reference pressure and \( C \) an arbitrary constant (e.g., Geisler and Dickinson, 1976).

Hough (1898) showed that the solutions to the horizontal structure or Laplace tidal equation could be obtained in the form of a series of associated Legendre functions. These eigenfunctions are referred to as “Hough functions.” For a given equivalent depth they give the latitudinal structure of various wave modes and their frequencies are eigenvalues of the Laplace tidal equation. For equivalent depths in the range of interest the solutions fall into two classes—gravity waves (waves of the first class) and planetary waves (waves of the second class). It is the waves of the second class that concern us here.

With knowledge of the equivalent depth \( h \) of the atmosphere, one can refer to Longuet-Higgins (1968) for the frequency and horizontal structure of free wave modes of the second class. For an atmosphere with a constant temperature of 244 K, \( h \) from (5.1) is equal to 10 km. The observed 5- and 16-day periods suggest that the relevant wave modes for the current discussion are those with \( m = 1 \) and \( n = 2, 3, 4 \) or 5 (\( m \) is the zonal wavenumber and \( n \) relates to the latitudinal scale) since from Longuet-Higgins’ Fig. 2 we find that periods of these four modes are approximately 5.0, 8.3, 12.5 and 17.9 days, respectively. For these wave modes and \( h = 8.29 \) km, Haurwitz (1937) reports periods a fraction of a day longer than these, as did Hough (1898) in his consideration of the \( m = 1, n = 3 \) mode with \( h = 8.84 \) km.

The Hough function solutions for \( H_2^2, H_3^2 \) and \( H_4^2 \) (\( H_n^m \) represents a given Hough function) are depicted in Fig. 10 of Longuet-Higgins for \( h = 8.84 \) km. Kasahara (1976) has presented these same solutions for \( h = 10 \) km and the differences are very small. Kasahara also presents the Hough function for the \( H_3^2 \) mode.

The wave model that we have chosen for comparison with the observations is simplified in many respects. For that reason we have included in an appendix some likely consequences of more realistic atmospheric conditions.

c. Summary

We now have four features that the observed disturbances should have in order that they be consistent

\[ \text{with the model of free waves of the second class:} \]

- The disturbances should have constant phase with height, that is, there should be no vertical slope.
- The amplitudes of the disturbances should vary in the vertical approximately as indicated by (5.2).
- The disturbances should have a frequency that is similar to one of the model wave modes.
- The disturbance should have a latitudinal structure that is similar to one of the model wave modes.

The observed 5-day period is consistent with that expected for the \( H_2^2 \) mode. It has been argued that this wave does not slope with height (Madden and Julian, 1972; Misra, 1975), and that its latitudinal structure is similar to the \( H_2^2 \) mode between 50°S and 50°N (Madden and Julian, 1972, 1973). Figs. 1 and 2 indicate that the disturbance we refer to as the 16-day wave is a broad-band one, with periods ranging from about 1–3 weeks. In the presence of background winds the \( H_3^2 \) mode may have a period exceeding 30 days, but the expected periods of the \( H_3^2 \) and \( H_4^2 \) modes fall within this 1–3 week range (see Appendix). In Section 6 we establish further similarities between the vertical and horizontal structures of the observed 16-day wave and those predicted for these latter two modes. Some additional estimates of the structure of the 5-day wave are also made. We then conclude that the 5- and 16-day waves are very likely atmospheric counterparts of the theoretically predicted free waves of the second class. As such, they may be excited by random forcing.

6. Observed structure of the 5- and 16-day wave-1 disturbances

Cross-spectral analyses and compositing are used in order to establish details of the vertical and horizontal structures of the 5- and 16-day wave-1 disturbances. The cross-spectral analyses are carried out as discussed in Section 3, and they give evidence regarding the vertical and latitudinal slope of the disturbances. The compositing method is described in the following and the composite waves give a measure of the amplitude variation with height and latitude.

a. Vertical structure of the 16-day wave in winter

While we make specific reference to the 16-day wave, we again point out that Figs. 1 and 2 indicate that the disturbance is a broad-band one, with periods ranging from about 1–3 weeks. From Figs. 1 and 2 we see that the coherence associated with the 16-day wave is larger in winter than in summer, so we estimate details of its structure based on winter season data. To begin, \( S_{1}(t) \) at 60°N and 850 mb is considered one time series, and the \( C_{1}(t) \)'s at 60°N and other pressure levels are considered separate second time series. The coherence and phase of resulting cross spectra are shown in Fig. 3. The

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dashed vertical lines represent 11- and 21-day periods. Coherence squares are significantly different from zero in this range (0.21 is the approximate 99% level), and the 850 mb $S_1(t)$ series leads the $C_1(t)$ series at other levels by very nearly one quarter of a cycle. From Fig. 3 we conclude that the 16-day, wave-1 disturbance is coherent in the vertical and has little slope with height.\footnote{The phase angle between the 850 mb $S_1(t)$ series and that of the 30 mb $C_1(t)$ series is 65° at a period of 16 days. This may indicate a small, but possibly real, westward tilt between those levels since the 95% confidence limits for the phase is about ±12° for 48 df and coh² = 0.49 (Jenkins and Watts, 1968, p. 381). Deland and Johnson (1968) found a small westward slope in the stratosphere for the westward traveling waves that they studied.}

Composite waves were constructed by first passing the $S_1(t)$ and $C_1(t)$ at 60°N through a 16-day band-pass filter. The amplitude response (AR) of the filter was designed to match the maxima in the coherence squares of Figs. 1–3, and it is shown in Fig. 4. The maximum response of the filter is at 16 days and half-power points are near 21 and 11 days.

Sixteen-day filtered $S_1(t)$ and $C_1(t)$ at 60°N and 100 mb during January, February and March of 1970 are shown in Fig. 5. This is typical of the filtered data during all of the eight winters studied. Regular westward propagation is not always present (16–21 January); but, once established, several consecutive circuits are not uncommon. Indeed, there were two and one-half more circuits following the last point plotted here before the regular propagation broke down in mid-April 1970. Polar diagrams similar to Fig. 5 were examined for all of the eight winters. When regular westward propagation was evident dates were compiled into one of eight groups or categories according to phase. For example, 22 January and 6, 7 and 24 February 1970 are compiled into category one (0–45°W) and 23, 24 January and, 8, 9, 25 and 26 February 1970 into category two (45–90°W), etc. $S_1(t)$'s and $C_1(t)$'s were averaged in each category \cite[based on dates determined from filtered 60°N, 100 mb $C_1(t)$ and $S_1(t)$ at each level and each latitude to produce a composite wave. The resulting composite 16-day wave at 60°N is shown in Fig. 6. Each point is the average of more than 40 dates during the eight winters studied. The circular movement at each level reflects westward propagation,}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig3}
\caption{Coherence squares (solid line) and phase (dots) between the sine coefficient of wave-1 geopotential height at 850 mb and the corresponding cosine coefficients at indicated levels at 60°N for eight winter seasons. The approximate 99% significance level is 0.21. The dashed vertical lines are at 11- and 21-day periods.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig4}
\caption{Amplitude responses of 16-day band-pass filter (solid curve) and 5-day filter (dashed curve). Dashed vertical lines are at 4.6, 5.4, 11- and 21-day periods.}
\end{figure}
Fig. 5. Polar diagram of band-pass filtered wave-1, sine coefficient (ordinate) and cosine coefficient (abscissa) of geopotential height field at 60°N and 100 mb. Units are meters. Indicated categories are explained in the text.

and the amplitude and phase of the time-mean or standing wave determines the center of the path. From Fig. 6, the standing wave has its ridge about 60°E at 850 mb and it slopes westward to near 130°W at 30 mb. This is consistent with the standing wave-1 at 60°N in January reported by van Loon et al. (1973). The standing wave is presumably forced by heat sources and mountains.

We can estimate an average amplitude for the westward propagating 16-day wave at each pressure level from Fig. 6. These average amplitudes are plotted versus pressure, or height, in Fig. 7 as open circles. To assure that the apparent increase of amplitude with height is not simply due to the fact that compositing dates were selected based on 100 mb data, the entire compositing procedure was repeated by selecting category dates based on filtered values of 850 mb $S_1(t)$ and $C_1(t)$ at 60°N. The amplitudes of this new composited 16-day wave are indicated in Fig. 7 by the dots. They too grow with height. Taking the average of these two amplitudes as the best estimate at each level, one can fit a straight line to the six average values by linear regression. The resulting line has a slope of 0.27. This is in good agreement with (5.2) which, for the theoretical free wave model, requires a slope equal to $\kappa$ (0.285).

b. Horizontal structure of the 16-day wave in winter

Here $S_1(t)$ at 100 mb and 60°N is considered as one time series and the $C_1(t)$’s at 100 mb and other latitudes are considered separate, second time series. The coherence and phase of the resulting cross spectra are shown in Fig. 8. In the 11–21 day period range the coherence is significant at the 99% level between 60 and 70°N and $S_1(t)$ at 60°N leads $C_1(t)$ at 70°N by approximately 70°. Significant coherence is indicated across a wide frequency range between 60 and 50°N.

Fig. 6. Polar diagram of 16-day composite winter season wave at 850, 500, 300, 200, 100 and 30 mb levels at 60°N. Typical time between points at any level is about 2 days. Each point represents the average of over 40 individual 100 mb category dates as described in the text. Composite sine coefficients of wave-1 in the geopotential are plotted along the ordinate and corresponding composite cosine coefficients along the abscissa. Units are meters.

In the 11–21 day period range $S_1(t)$ at 60°N leads $C_1(t)$ at 50°N by approximately 60°. The coherence between 60 and 40°N is not significant. At a period of 16 days the coherence between $S_1(t)$ at 60°N and $C_1(t)$ at 30°N is significant at the 99% level, but here $C_1(t)$ leads $S_1(t)$ by 80°. The coherence between $S_1(t)$ at 60°N and $C_1(t)$

Fig. 7. Amplitudes $a$ of 16-day composite winter season wave at 60°N based on 100 mb category dates (open circles determined from Fig. 6) and 850 mb category dates (dots determined from a composite wave not shown). Regression line determined from $a$’s is the solid line. Dashed line is the theoretical expectation derived from (5.2) (constant is arbitrary).
The amplitudes of the composite 16-day wave [based on 100 mb filtered \( S_1(t) \) and \( C_1(t) \) as discussed above] at different latitudes at 500 mb are shown in Fig. 9. Inspection of the composite waves at each latitude indicated that they are very nearly in phase at 50, 60, 70 and 80°N. The wave at 40°N is about one-quarter of a cycle out of phase with these higher latitudes, and the wave at 30°N is clearly out of phase with the waves at higher latitudes. As a result the 30°N amplitude is plotted as a negative value. The phase at 40°N is indicated as uncertain.

To assure that the relative maximum amplitude at 60°N shown in Fig. 9 is not simply a result of the fact that the composite wave is based on dates selected from filtered \( S_1(t) \) and \( C_1(t) \) at 60°N, an attempt was made to make a second composite using filtered \( S_1(t) \) and \( C_1(t) \) from 100 mb and 30°N. Resulting polar diagrams did not indicate enough regularly propagating disturbances to form an adequate composite. The fact that coherence squares between \( S_1(t) \) and \( C_1(t) \) are significant in the 16-day range only at northern latitudes (Fig. 1 and 2) does support the implication that the wave has maximum amplitude there, however.

c. Vertical and horizontal structure of the 5-day wave in summer

The coherence associated with the 5-day wave is largest at 30°N in summer (Figs. 1 and 2). For that reason we investigated the structure of the 5-day wave in a manner similar to that just described for the 16-day wave except 30°N was chosen as a reference
latitude instead of 60°N, and summer seasons were studied instead of winter seasons.

The summer season cross spectra between $S(t)$ at 850 mb and the $C(t)$’s at other levels at 30°N is shown in Fig. 10. The 5-day wave is seen to be coherent in the vertical with little slope with height.

A composite 5-day wave was constructed in an identical way to that described for the 16-day wave. The AR of the filter used is shown in Fig. 4. Maximum response of the filter is at 5 days and half-power points are near 5.4 and 4.6 days. Each point of the resulting composite 5-day wave, similar to that shown in Fig. 6 for the 16-day wave, is the average of at least 50 dates. The resulting vertical structure at 30°N is presented in Fig. 11. The amplitude of the composite 5-day wave increases with height below 100 mb, consistent with theoretical expectation determined by (5.2).

Cross spectra between $S(t)$ at 30°N and $C(t)$ at other latitudes (not shown) suggest that the 5-day wave is coherent to 70°N and in phase at all latitudes. This is consistent with the latitudinal structure of the $H_2$ mode. However, the composite 5-day wave at 500 mb shows regular propagation only at 30 and 40°N. The estimated amplitudes at these latitudes are 4.4 and 6.2 m, respectively. Presumably the amplitude of the 5-day wave is large enough so that cross spectra averaged over several summer seasons can detect it at 50°N (Fig. 1), for example, but too small to be isolated by the compositing procedures.

d. Summary

Cross spectra between levels indicate that both the 16-day and the 5-day waves have very little tilt in the vertical. In addition, their estimated amplitudes increase with height (only below 100 mb in the case of the 5-day wave) in a manner that is consistent with that predicted by the theory for free waves of the second class. The one to three week periods of what we have in summer from Fig. 1, we conclude that the signal associated with what we call the 5-day wave is most easily identified in this period range.
referred to as the 16-day wave are comparable to those expected for the \(H_1^1\) and \(H_1^1\) modes when the Doppler effect of zonal westerlies is allowed for. Similarly the \(H_1^2\) mode should have a period near the 5 days that is observed.

The theoretically predicted wave modes are global and extend from Northern to Southern Hemispheres. The \(H_1^1\) and \(H_1^1\) modes are symmetric while \(H_1^2\) is antisymmetric across the equator. In an effort to detect possible coherence and symmetry or antisymmetry across the equator, cross spectra between \(S_1(t)\) from 60°N and \(C_1(t)\) from 60°S were computed for several years in the 1950’s when Southern Hemisphere data were available. No statistically significant coherence was found near 16 days. However, Madden and Julian (1973) did find coherence and near-zero phase between the 30–50°N zone and that at 30–50°S in the 4–6 day period range.

Based on the period and the latitudinal structure suggested by Figs. 8 and 9, our tentative conclusion is that the 16-day wave is most similar to the \(H_1^1\) mode, but it must be recognized that lacking confirmation that it is of global scale other interpretations are possible. For example, the 16-day wave may indeed be a free wave of the second class, but, because of horizontal wind shears or other complicating factors, it could be confined to one hemisphere. Available theoretical evidence supports the idea that horizontal wind shears can form wave guides. However, near the equator the 16-day wave is moving at 29 m s\(^{-1}\), and would therefore require an east wind of 29 m s\(^{-1}\) to allow for the zero wind line absorption discussed by Dickinson (1968). Since mean zonal winds in the troposphere rarely reach this speed, we can offer no quantitative evidence suggesting that such absorption might trap the 16-day wave in one or the other hemisphere.

We find no new evidence to dispute earlier claims that the observed 5-day wave is very similar to the \(H_1^2\) mode. Here we have shown that below 100 mb its amplitude grows with height as that of an external wave and that the evidence for the existence of the wave is clearest south of 50°N.

7. Discussion

Cross-spectrum analyses of two, essentially independent, data sets (73-year sea-level pressure record and 9-year upper air record), indicate that south of 50°N, in all seasons, a zonal wavenumber 1 disturbance moves westward around the world in about 5 days. Earlier work had shown that this 5-day wave is similar in many respects to the \(H_1^2\) wave, a theoretically predicted free wave of the second class. Here these similarities are reconfirmed, and in addition evidence presented indicates that below 100 mb the amplitude of the 5-day wave grows with height as that of the external mode. Also, north of 50°N, primarily in winter and spring seasons, a zonal wavenumber 1 disturbance moving westward around the world in 1–3 weeks with an average period of 16 days is indicated. Characteristics of this 16-day wave are most similar to the \(H_1^1\) free wave of the second class. While such free waves do not, for example, transport heat, Madden (1975, 1978) has argued that they can contribute significantly to time variations in horizontal heat transport and atmospheric energetics through alternately constructive and destructive interference with the quasi-stationary forced waves. In this regard, interference between free and forced waves may play an important role in the sudden warmings of the stratosphere and in time variations of the general circulation. We speculate that this sort of wave interference provides a mechanism to explain some of the variations in atmospheric energetics that have been observed to occur on an approximate two-week time scale (e.g., Muench, 1965; Hirota and Sato, 1968; Miller, 1974; Webster and Keller, 1975). A thorough description of this interference phenomenon requires further study. In addition, even though the amplitude of the 5-day wave is small it may modulate precipitation near the equator (Burpee, 1977).

While the evidence presented here indicates the 16-day wave is most similar to the \(H_1^1\) mode, analysis of Southern Hemisphere data is needed to confirm this possibility. Because the amplitude of the 16-day wave increases with height there must be a small accompanying temperature wave. A study of satellite radiometer data, similar to the one that Rodgers (1976) presented for the 5-day wave, might reveal if the 16-day wave does indeed extend across the hemispheres. If it does, and if it is symmetric about the equator, then the \(H_1^1\) mode would likely be the correct choice.

Finally, although the discussion of traveling planetary waves presented here is limited to zonal wavenumber 1 disturbances, many of the papers referenced in the Introduction point to the existence of westward traveling waves on other scales. Evidence that at least one of these other waves, wavenumber 2, has little phase tilt with height has recently been presented by Pratt and Wallace (1976) as well.

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APPENDIX

Comparing the Model to the Real Atmosphere

Any analytical model that we choose for comparison with the observations will necessarily be oversimplified in many respects. The model considered here assumes that the basic state of the atmosphere is isothermal and
motionless, and it neglects diabatic effects. Here we consider, in a somewhat qualitative way, likely consequences of more realistic atmospheric conditions.

It is important that our estimate of the equivalent depth \( h \) be appropriate for the real atmosphere since the vertical and horizontal structures, and the frequency of the model waves, depend on it. Based on (5.1), we have assumed that it is near 10 km. Lindzen (1968) derives an equation similar to (5.1) that includes the effect of a constant zonal flow. It indicates that in an atmosphere with a nonzero basic flow the value of \( h \) appropriate for free waves is dependent on its horizontal scale. Deland (1970) discussed Lindzen’s equation further and shows that for a wave-1 disturbance of large latitudinal scale, a basic flow of 10 m s\(^{-1}\) in an isothermal atmosphere increases \( h \) from \( 6H \) by only 10%. Similarly, the equivalent depths of model atmospheres with varying temperature profiles typically differ from 10 km by less than 20% (e.g., Siebert, 1961). Taylor (1929) estimated \( h \) to be about 10 km based on the propagation speed of a gravity wave produced by the Krakatoa explosion of 1883. He compared this empirical finding with the equivalent depths of several models ranging from one with an adiabatic lapse rate from the surface to 13 km with an isothermal layer above 13 km to one with an isothermal condition throughout. The ground temperature in these models was varied too, and the range of equivalent depths was from 8.3–11.8 km. Using more realistic temperature profiles Kasahara (1976), Lindzen and Blake (1972) and others have solved the vertical structure equation numerically and also find \( h \) to be near 10 km.

Therefore, despite the fact that (5.1) is derived for an isothermal atmosphere at rest, we can consider that \( h = 10 \) km with some assurance that this is a reasonable estimate for the real atmosphere. Even with this assurance, conclusions regarding the external nature of free waves and the resulting vertical structure indicated by (5.2) can still be questioned, since the vertical structure equation on which they are based assumes an isothermal atmosphere. In addition, Newtonian cooling might damp the waves so that the amplitude would not increase with height as rapidly as indicated by (5.2). This would be especially true in the stratosphere where the relaxation time associated with Newtonian cooling is about one week, which is short relative to that in the troposphere (Dickinson, 1969). We can, however, offer the results of a numerical study by Geisler and Dickinson (1976) to support the conclusion that in the presence of a more realistic atmosphere the vertical structure of at least one free wave mode is consistent with (5.2). They considered time-dependent solutions of linearized equations for a wave-1 disturbance on a spherical earth in the presence of realistic zonally averaged temperatures and zonal winds derived from the thermal wind equation. The wave-1 disturbance is forced by a vertical velocity near the ground whose period is near 5 days. When winds alone were included, the amplitude of the wave increased with height in the troposphere and lower stratosphere at a rate somewhat faster than that indicated by (5.2), but with both winds and Newtonian cooling this rate of increase was nearly the same as that of (5.2). In addition, the horizontal structure of the excited wave agreed with that of the \( H_2 \) mode. This result was obtained even when the forcing had a broad Gaussian latitudinal dependence.

We must also make allowances for the tendency of the westerly winds of the atmosphere to increase the period of westward propagating waves of the second class. Considering an incompressible and homogeneous fluid of 8 km depth, Chiu (1952) showed that the effect of a zonal wind of constant angular velocity equal to \( \Omega/33 \) (9 m s\(^{-1}\) at 50° latitude) was to increase the period of the \( H_2 \) and \( H_4 \) mode from 9.2 and 18.5 days of the no-wind case to 10.7 and 31.3 days, respectively. For a similar wind equal to \( 2\Omega/33 \) the periods were 12.8 and 129.4 days. Including more realistic vertical and horizontal wind shears makes it impossible to separate the basic equations into horizontally and vertically dependent parts. For this reason no analytical treatment of the full effects of realistic zonal winds on the periods of theoretically predicted wave modes is available. For the present we again consider effects indicated by numerical experiments.

In Geisler and Dickinson’s study (1976), zonal winds had practically no effect on the period of the 5-day wave. In another numerical study, the effect of zonal winds on the eigenfrequencies and eigenfunctions of a two-layer ocean model on a rotating sphere are considered by Dickinson and Williamson (1972). The wind they use is westerly at 19 m s\(^{-1}\) at 55°N and easterly at 10 m s\(^{-1}\) at the equator (see Fig. 3 of Dickinson and Williamson, 1972). The resulting eigenfrequencies associated with a 10-km depth for the \( H_2 \), \( H_3 \), \( H_4 \) and \( H_5 \) modes are 0.198, 0.093, 0.057 and 0.031 cycles day\(^{-1}\), respectively, or periods of 5.1, 10.8, 17.5 and 32.3 days (their Table 9). Here the winds have little effect on the period of the \( H_2 \) mode but tend to lengthen the period of the \( H_3 \), \( H_4 \) and \( H_5 \) modes. Dickinson and Williamson also show that the latitudinal structure of the \( H_2 \) is not substantially changed by the background wind.

Although the model waves with which we compare the observational results are not completely realistic, the studies discussed here do not indicate the comparisons are inconsistent. In particular, inclusion of zonal winds in these models does not alter the period of the \( H_2 \) mode. It does make the periods of the \( H_2 \), \( H_3 \) and \( H_4 \) modes longer than those expected with no background wind. Neither the Geisler and Dickinson study nor the Dickinson and Williamson study finds large changes in the latitudinal structure of the wave modes due to zonal winds. Also, the vertical structure of the 5-day wave has the characteristics of an external wave event when zonal winds and Newtonian cooling are included. These conclusions are limited to the troposphere and lower stratosphere in Geisler and Dickinson (1976).
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