Part II: Mesoscale Model

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ABSTRACT

A 20-level, three-dimensional, primitive equation model with 20 km horizontal resolution is used to predict the development of convectively driven mesoscale pressure systems. Systems produced by the model have life histories and structural characteristics similar to observed convectively driven mesosystems. Cooling by (parameterized) convective-scale moist downdrafts is largely responsible for mesohigh formation, while warming by compensating subsidence strongly correlates with mesocyclogenesis.

An hypothesis for mesocyclogenesis associated with deep convective complexes is presented. The hypothesis recognizes that certain configurations of convective activity may produce focused areas of forced subsidence warming aloft. The warming in turn causes a thickness increase aloft which creates a hydrostatic circulation favorable for evacuating mass from the subsidence column. Consequently, pressure falls beneath the layer of high-level warming. Model results supporting this hypothesis are presented.

1. Introduction

Mesoscale weather systems have been observed for over 30 years (see, e.g., Brunk, 1949; Williams, 1948). Yet, perhaps because of the lack of routinely available mesoscale data, or the expanded role of synoptic-scale numerical modeling, only a few mesometeorological concepts have found their way into operational forecasting (e.g., Magor, 1959). Within the past five years, however, research interest in mesometeorology has risen sharply and it appears that continuing developments in remote sensing and numerical modeling will further stimulate research in this area (Kreitzberg, 1976).

Of particular interest are mesoscale systems found in conjunction with deep, moist convection which is organized into squall lines or other multicelled structures. Fig. 1 shows examples of such mesosystems. Fujita (1959) showed that evaporational cooling in moist downdrafts is the predominant mechanism responsible for the production of cold air domes and their associated pressure rises (mesohighs). However, a hypothesis for mesocyclogenesis was not put forth until recently when Hoxit et al. (1976) postulated that convectively forced subsidence warming produces mesolows.

The purpose of this paper is threefold: to demonstrate that mesoscale numerical models are capable of simulating convectively driven mesohighs and mesolows, to enhance understanding of how mesocyclogenesis occurs, and to augment current concepts and hypotheses on the interaction of deep convection with its environment. The following section discusses the processes and circulations instrumental in the formation of mesoscale pressure systems similar to those shown in Fig. 1 (i.e., with diameters on the order of several hundred kilometers). Section 3 presents results of a numerical simulation of mesoscale pressure systems driven by deep convection, and Section 4 briefly discusses the interaction of deep convection with its environment.

2. Formation of mesoscale pressure systems

Under certain conditions, the atmosphere generates a form of moist convection which, through interaction with its environment and because of the characteristics of the convection itself, generates intermediate-scale features in the wind and pressure fields. Some examples are mesoscale high and low pressure systems, squall lines, or simply additional vertical circulations not evident on the larger synoptic scale. Growth or development of mesoscale pressure systems is not necessarily controlled by environmental conditions, but may arise from the interactions among various scale perturbations. Furthermore, certain configurations of the perturbations appear to be more favorable for development than others. We refer to this tendency for mesoscale features to form and amplify as a consequence of convective activity as mesoscale instability. Mesoscale instability as considered here is not to be confused with its larger scale counterpart, baroclinic instability, where horizontal motions dominate and Coriolis forces play a major role. Instead, mesoscale
instability is largely an enhancement of vertical circulation and is manifested primarily in three forms:

1) **Cloud amplification**

Rather than a simple overturning through gravitational instability, the circulation in and around an *individual cumulonimbus* organizes into a mutually beneficial inflow-outflow configuration where updraft and downdraft interact without interference. The outflow causes more boundary layer air to reach its level of free convection and feed the *existing updraft*, thus amplifying the overturning of the atmosphere which in turn produces more outflow, overturning, etc. (e.g., supercell).

2) **Cloud multiplication**

Outflow from an individual cumulonimbus may cause sufficient lifting to induce *new convection* to form away from the original cumulonimbus. Thus, convective overturning is enhanced by increasing the number (area) of active cumulonimbi, and outflow from the new cumulonimbi may cause formation of still more moist convection, etc. (e.g., arc line).

3) **Dynamic enhancement**

Under certain conditions, cumulonimbus lines or systems induce mesoscale highs and lows in areas immediately beneath and adjacent to the active convection. These mesoscale pressure systems may increase low-level horizontal convergence and thereby supply additional mass and moisture to the convective cells. This, in turn, may amplify the convection which intensifies the mesoscale pressure areas, produces more convergence, etc. (e.g., see Fig. 1).

It is recognized that many of the factors which determine whether or not mesoscale development will occur are not yet identified or understood. However, for convective systems large enough for the hydrostatic approximation to be valid (as shown in Fig. 1), certain conclusions about what produces the mesoscale pressure changes can be inferred. Specifically, the only way for pressure to change at a level is if there is a net change in mass above the level. Thus, the problem of hydrostatic pressure change is really a problem of how to move mass into or out of a column. *This implies circulation.* In the case of the thunderstorm generated mesohigh at the surface, the thunderstorm circulation replaces relatively warm, moist, boundary-layer air with colder, drier (more dense) moist downdraft air. Fujita (1959) calculated the change in mass due to the introduction of precipitation-cooled downdraft air into the boundary layer and found that it is proportional to the surface pressure change. While it can be argued that a portion of the pressure rise is produced by non-hydrostatic circulations, the magnitude of such a pressure rise is small [on the order of a millibar or less for a downdraft of order 15 m s⁻¹ (see Schaeffer, 1947; Newton and Newton, 1959)] relative to the hydrostatic pressure change. Furthermore, the non-hydrostatic pressure rise occurs over a small fraction of the total area of the mesosystem.

A more difficult problem is to explain the existence of the mesolows which typically appear along the periphery of mesohighs. Again assuming hydrostatic conditions, there must be a circulation which removes mass from the column. Further, this circulation must be focused, extend over a deep enough layer, and continue long enough to generate the concentration of pressure falls necessary for mesolow formation. Since mesolows form and dissipate on the order of minutes to as long as a few hours, and since they move at speeds up to 30 m s⁻¹, it is difficult to argue that radiation can induce such a circulation. Likewise, horizontal temperature advection is not sufficiently large, nor is a focusing mechanism readily apparent to produce the concentrated pressure fall necessary for mesocyclogenesis. Hoxit *et al.* (1976) attributed mesolow formation to middle- and high-level subsidence. They pointed out that surface pressure falls of 2–4 mb h⁻¹ can be produced hydrostatically by sinking of the order of tens of centimeters per second in the 100–500 mb layer. Their argument assumes that a focused area of subsidence locally warms a high-level portion of a column of air (thereby inducing a circulation to remove mass). More specifically, the following steps describe the hypothesized sequence of events responsible for mesocyclogenesis:

1) Presuppose an existing mesoscale subsidence circulation *forced* by a deep convective complex. Initially, the circulation is in mass balance (i.e., net mass inflow at the top equals net mass outflow at lower levels). Fig. 2a is a schematic of the initial circulation.

2) With time, the circulation generates compressional warming aloft which increases the thickness. This also results in a pressure increase at constant height levels in the vicinity of the tropopause. At lower levels, pressure remains the same (see Fig. 2b).

3) Elevated pressure surfaces aloft accelerate air outward and a net evacuation of mass from the column develops. Pressure falls beneath the layer of mass evacuation and low-level inward acceleration develops (see Fig. 2c).

4) Forced convergence and subsidence fields aloft weaken. Middle- and low-level inflow begins.

Clearly, a low pressure system produced by this type of process must be highly unbalanced and is very likely a transient phenomenon. Indeed, mesolows which form in association with convective com-
plexes usually appear late in the growing phase and through the mature phase of the convective system (Pedgley, 1962) when the meso-β scale subsidence circulation is very likely the strongest and most well organized. This period is relatively brief, typically lasting only a few hours. On occasion, however, rapidly moving mesolows have been documented to travel hundreds of kilometers and persist for over 10 h (Magor, 1958).

Although Hoxit et al. (1976) considered mesolow formation downwind of deep convection, they did not discuss mesocyclogenesis in the rear quadrants of the environment surrounding active convection (i.e., the “wake” low) (see Brunk, 1949; Williams, 1953; Pedgley, 1962). Moreover, mesolows are frequently observed to form on one side or the other of deep convection (Fujita et al., 1956), and, of course, many times are not observed. In this regard, Feteris (1978) observed that the compensating subsidence has a tendency to occur where lapse rates are nearly dry adiabatic and restoring forces are weakest. Since warming cannot occur from dry adiabatic compression in an air mass with a dry adiabatic lapse rate, mesocyclogenesis also cannot occur under these conditions.

Assuming that in some instances subsidence warming is the primary mechanism for hydrostatic generation of mesolows, the timing, location and intensity of mesocyclogenesis should depend on the variables listed in Table 1. The first four variables follow from the first and second laws of thermodynamics if the hydrostatic assumption is invoked and horizontal advection is neglected. Specifically, warming by subsidence compression at a point may be expressed as

$$\frac{\partial T}{\partial t} = -w(\Gamma - \gamma), \quad (1)$$

where $\Gamma$ is the dry adiabatic lapse rate, $\gamma$ the environmental lapse rate and $w$ the environmental vertical motion (see Appendix for list of symbols). In Eq. (1), the term in parentheses is a measure of the static stability.

The remaining four variables in Table 1, although equally important, are more difficult to quantify. For example, the ventilation (item 5) is a measure of the vector difference between the horizontal environmental wind velocity in the subsidence layer and the horizontal propagation velocity of the convection (Gray, 1979). If the convection and the environmental wind move with the same velocity, the ventilation is zero. The weaker the ventilation, the longer the same parcel of environmental air can subside as it moves past the convection, thus maximizing the warming.

In some instances, compensating subsidence may
occur above the region being covered by cold outflow from moist downdrafts. In this situation, the warming aloft and cooling in the boundary layer tend to offset one another and significant surface pressure changes cannot develop. In other situations, a mesohigh may have already formed and, as the convection moves on, subsidence warming in the rear of the convection moves over the mesohigh. In this instance, surface pressure falls rapidly but any mesolow which forms within the cold outflow air is typically very weak.

The last variable in Table 1 is the vertical shear of the horizontal wind. The exact role of vertical shear is not clear although it appears to organize the subsidence into preferred zones (see Hoxit et al., 1976).

Several of the items in Table 1 are strongly influenced by the particular configuration taken by individual thunderstorms. For example, Fig. 3 shows four different configurations of three thunderstorms. Each thunderstorm updraft region is indicated by a circled plus sign and subsidence areas are shaded. We can explore the effects of cell configuration on subsidence fields if the following assumptions are made:

(i) All thunderstorms are the same size and transport equal amounts of mass and moisture.
(ii) All thunderstorms produce the same amount of compensating subsidence.
(iii) Subsidence is symmetrically distributed around each thunderstorm and is strongest nearest the updraft.
(iv) Static stability is horizontally homogeneous throughout the domain.
(v) The initial surface pressure pattern is uniform.
(vi) The horizontal wind velocity in the subsidence layer and the velocity of the thunderstorms are everywhere the same.

In Fig. 3a, none of the subsidence areas overlap and subsidence warming is therefore widely distributed and relatively weak. In Fig. 3b, some overlap is apparent on the outer edges of the subsidence zones. Since the strength of the subsidence diminishes rapidly with distance from the updraft (Lilly, 1960; Fritsch, 1975), subsidence warming, even in the overlap region, would still be rather weak. The strong overlap area in Fig. 3c, however, is providing a more significant center of subsidence while the triple overlap in Fig. 3d provides a well-focused, concentrated area of subsidence warming. Under the above assumption, an identical amount of convective overturning may or may not produce a mesolow, depending on how the individual thunderstorms are positioned and interact with each other and their environment. The same argument applies for convective cloud complexes which may act as individual units and interact with one another.

Considering the variables in Table 1 and the discussion above, it appears that subsidence-generated mesolows can form in any location with respect to the center of convective activity and, therefore, prediction of mesocyclogenesis requires a prediction of the structure and evolution of the convective complex. Clearly, with today’s observational capability, it would be extremely difficult to subjectively forecast where mesocyclogenesis will occur. By using an appropriate convective parameterization procedure in a fine mesh numerical model, however, it may be possible to predict the development of mesoscale pressure systems.

3. Numerical model governing system

Many considerations are involved in the design of a numerical model. While some features of a model are easily selected from prior modeling efforts, others are more difficult to specify and require, to some extent, arbitrary choices. For the numerical simulation of convectively driven mesoscale pressure systems, it is desirable to use a simple model so that the response of the governing system to the convection will not be obscured by other forcing mechanisms such as local terrain or land-sea effects.

a. The governing system

The primitive equations are formulated in an $x$, $y$, $z$ coordinate system with the spherical shape of the
earth approximately included through the Coriolis force and its north-south variation. The model system is essentially the same as the NCAR general circulation model (Oliger et al., 1970) which permits horizontally propagating acoustic waves. The momentum equations in flux form are

\[
\frac{\partial \textbf{v}}{\partial t} = -\nabla \cdot \rho \mathbf{v} - \frac{\partial}{\partial z} (\rho \mathbf{v} u) - \frac{\partial}{\partial x} (\rho \mathbf{v} v) - \frac{\partial}{\partial y} (\rho \mathbf{v} w),
\]

(2)

\[
\frac{\partial \rho v}{\partial t} = -\nabla _z \cdot \rho \mathbf{v} \mathbf{v} _z - \frac{\partial}{\partial z} (\rho vv) - \frac{\partial}{\partial x} (\rho vv) - \frac{\partial}{\partial y} (\rho vv),
\]

(3)

where \( F_u \) and \( F_v \) are the frictional effects applied to the lowest layer. The pressure tendency equation (Haltiner and Martin, 1957) is

\[
\frac{\partial \rho}{\partial t} = B + \rho g w - g \int_{z_0}^{z} \nabla z \cdot \rho \mathbf{v} \mathbf{v} dz,
\]

(4)

where \( B \) is the pressure tendency at the top \( z_j \) of the model atmosphere (50 mb level) and is obtained from the vertical motion equation by applying the upper boundary condition \( \mathbf{w} = 0 \), i.e.,

\[
B = \frac{w_0 + \int_{z_0}^{z} Q}{\nabla \chi \cdot \rho \mathbf{v} \mathbf{v} dz}.
\]

(5)

In Eq. (5), \( w_0 \) is the vertical motion at level \( z_0 \) and \( Q \) is the sum of all heating rates \( = Q_c + Q_s + Q_r \), where \( Q_c \) is the heating by moist convection, \( Q_s \) by stable precipitation and \( Q_r \) by radiation. The radiative contribution only includes surface effects and is defined in Section 3b; \( Q_c \) and \( Q_s \) are given by

\[
Q_c = c_p \frac{(T - T_c)}{T_c},
\]

(6)

where \( T \) is the grid-point temperature adjusted for the effect of convection (see Fritsch and Chappell, 1980), \( T_c \) is the period of convection; and

\[
Q_s = L \frac{\Delta r}{\Delta t}.
\]

The quantity \( J \) is defined by

\[
J = \nabla _z \cdot \rho \mathbf{v} \mathbf{v} - g \int_{z_0}^{z} \nabla _z \cdot \rho \mathbf{v} \mathbf{v} dz.
\]

(7)

Vertical motion is evaluated by using Richardson’s equation, or

\[
w = w_0 + \int_{z_0}^{z} \frac{Q}{c_p T} - \nabla _z \cdot \mathbf{v} - \frac{B + J}{\eta p} dz.
\]

(8)

By making the hydrostatic assumption, temperature is diagnosed from the pressure field using

\[
T = -\frac{g}{\frac{\partial}{\partial z} \ln \rho},
\]

(9)

and density is calculated from the gas law

\[
\rho = \frac{p}{RT}.
\]

(10)

Finally, moisture is conserved according to

\[
\frac{\partial r}{\partial t} = -\nabla _z \cdot \mathbf{v} r - w \frac{\partial r}{\partial z} + \frac{\partial r}{\partial t} - \frac{\partial}{\partial t} \left| \frac{\partial r}{\partial t} \right| - \frac{\partial}{\partial t} \left| \frac{\partial r}{\partial t} \right|,
\]

(11)

where \( \frac{\partial r}{\partial t} \) and \( \frac{\partial r}{\partial t} \) are changes due to convective and stable precipitation, respectively. Surface moisture fluxes are not included.

b. Boundary-layer formulation

1) Diurnal heating

Boundary-layer warming and cooling are specified through a heating function \( H \) rather than by the usual surface energy balance formulation:

\[
\frac{\partial r}{\partial t} = H.
\]

(12)

Using an arbitrarily specified diurnal temperature curve, the surface heating required to simulate the diurnal temperature change is determined in the manner described below.

Diurnal changes in heating rates are approximated by four linear segments shown in Fig. 4. By specifying the times of zero, maximum and minimum heating, as well as the diurnal temperature change, the amplitude of the warming \( (H_w) \) and cooling \( (H_c) \) rates can be determined. If, for example, the diurnal temperature change is 10°C and the net 24 h temperature change is zero, then

\[
c_p^{-1} \int_{0}^{24} H dt = 0°C
\]

\[
= c_p^{-1} \left[ \int_{0}^{t_1} H_1 dt + \int_{t_1}^{t_2} H_2 dt + \int_{t_2}^{t_3} H_3 dt + \int_{t_3}^{t_4} H_4 dt \right],
\]

(13)

where

\[
H_1 = H_w t, \quad 0 \leq t < t_1
\]

\[
H_2 = H_w - \frac{H_c}{\Delta t_1} [t - t_1], \quad t_1 \leq t < t_2
\]

\[
H_3 = \frac{H_c}{\Delta t_2} [t - t_2], \quad t_2 \leq t < t_3
\]

\[
H_4 = H_c - \frac{H_c}{\Delta t_3} [t - t_3], \quad t_3 \leq t \leq t_4
\]

(14)
or, after integrating,
\[ \frac{H_w}{2c_p} t_2 = 10^\circ C. \]  
(16)

Similarly, the cooling is described by
\[ c_p^{-1} \int_{t_2}^{t_4} H_c dt + c_p^{-1} \int_{t_4}^{t_1} H_c dt = -10^\circ C. \]  
(17)

The maximum heating and cooling rates are then
\[ H_w = c_p \frac{20^\circ C}{t_2}, \]  
(18)
\[ H_c = - \frac{c_p \frac{20^\circ C}{t_4 - t_2}}{t_4 - t_2}. \]  
(19)

In view of this simple formulation for \( H \), it is relatively easy to introduce time and magnitude variations in boundary-layer thermal forcing which is an essential degree of freedom for studying the meso-

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**Fig. 5. Outline of iterative sequence in convective parameterization and mesoscale numerical model.**
scale sensitivity of the convective parameterization technique.

2) **Surface friction**

A surface drag formulation is incorporated at the lowest two levels to account for surface friction. In flux form, the two components of frictional force are

\[
F_u = c_D \rho u |V|, \quad F_v = c_D \rho v |V|,
\]

where

\[
c_D = \begin{cases} 
3.42 \times 10^{-6} \text{ m}^{-1} & \text{at level 1}, \\
1.75 \times 10^{-6} \text{ m}^{-1} & \text{at level 2}, \\
F_u = F_v = 0 & \text{at level 3 (z = 988 m) and above}.
\end{cases}
\]

(21)

For the case results presented in Section 5, the drag coefficient was set to zero.

c. **Lateral boundary conditions**

The lateral boundary conditions for limited-area models require special formulations to insure physical and numerical consistency between the large-scale (external) forcing and processes internal to the limited area. Following Perkey and Kreitzberg (1976), a lateral boundary condition which allows large-scale waves to enter the limited-area domain but does not allow exiting waves to be reflected with sufficient amplitude to destroy the integration was selected. Basically, the boundary formulation blends the large-scale tendencies and the model tendencies near and along the boundaries. This is done in a manner so that the tendencies at the boundary itself are completely specified by the large scale (ls), while a short distance inside the boundary they are completely specified by the model scale (ms). In between, the tendencies are a weighted combination of both scales. Thus, the tendency of any variable \(\alpha\) can be determined from

\[
\frac{\partial \alpha}{\partial t} \bigg|_j = F_j \frac{\partial \alpha_{ms}}{\partial t} \bigg|_j + (1 - F_j) \frac{\partial \alpha_{ls}}{\partial t} \bigg|_j,
\]

(23)

where \(F_j\) is a weighting term defined by

\[
F_j = \begin{cases} 
0.0 & \text{for } j = \text{the boundary grid point}, \\
0.4 & \text{for } j = \text{the boundary -1 grid point}, \\
0.7 & \text{for } j = \text{the boundary -2 grid points}, \\
0.9 & \text{for } j = \text{the boundary -3 grid points}, \\
1.0 & \text{for } j = \text{all other interior grid points}.
\end{cases}
\]

(24)

\(\frac{\partial \alpha_{ms}}{\partial t}\) is the mesoscale model tendency,

\(\frac{\partial \alpha_{ls}}{\partial t}\) is the large-scale model tendency.

For the analytical initialization (Fritsch et al., 1980)

\[
\frac{\partial \alpha}{\partial t} \bigg|_j = F_j \frac{\partial \alpha_{ms}}{\partial t} \bigg|_j,
\]

(25)

where \(F_j\) is defined by (24). East and west boundaries are periodic.

d. **Finite-difference formulations**

The finite differencing of the governing system described in Section 3a is centered in space except for the vertical differencing, and centered in time except for the forward differencing used in the convective parameterization

1) **Space differencing**

All horizontal centered differencing follows the formulation

\[
\frac{\partial A}{\partial x} \bigg|_{i,j} = \frac{A_{i+1,j} - A_{i-1,j}}{2\Delta x},
\]

(26)

\[
\frac{\partial A}{\partial y} \bigg|_{i,j} = \frac{A_{i,j+1} - A_{i,j-1}}{2\Delta y}.
\]

(27)

Since the model has uneven spacing in the vertical, a Taylor series expansion is used to approximate the vertical derivatives at each interior level. At the upper and lower boundaries, the vertical derivatives are assumed equal to the derivatives across the upper and lower layers, respectively. Thus, for an interior level

\[
\frac{\partial A}{\partial z} \bigg|_k = (A_k - A_{k-1}) \frac{\Delta z(k)}{\Delta z(k - 1)} + (A_{k+1} - A_k) \frac{\Delta z(k - 1)}{\Delta z(k)\Delta z(k - 1)},
\]

(28)

at the lower level

\[
\frac{\partial A}{\partial z} \bigg|_{k-1} = \frac{A_2 - A_1}{z_2 - z_1},
\]

(29)

and at the upper level

\[
\frac{\partial A}{\partial z} \bigg|_{k=K} = \frac{A_{K+1} - A_k}{z_{K+1} - z_K}.
\]

(30)

The vertical differences, \(\Delta z\) and \(\delta z\) are defined by

\[
\Delta z(k) = z_{k+1} - z_k,
\]

(31)

\[
\delta z(k) = z_{k+2} - z_k.
\]

(32)
2) Time differencing

The centered or leapfrog time algorithm was selected for the time differencing in the prognostic equations. The algorithm is defined by

\[ A^{r+1} = A^{r-1} + \left( \frac{\partial A^r}{\partial t} \right) 2\Delta t, \]  

(33)

where \( \tau \) is the time level \( t \), \( \tau + 1 \) corresponds to \( t + \Delta t \), and \( \tau - 1 \) to \( t - \Delta t \). During application of this algorithm, the friction terms in the momentum equations and the diabatic terms in the pressure tendency and vertical motion equations must be evaluated at time level \( \tau - 1 \) to maintain numerical stability (see Oliger et al., 1970). Advective and forcing terms are calculated at time level \( \tau \). In order to avoid separation of solutions (a consequence of the leapfrog algorithm), the following time filter (see Asselin, 1972) has been included:

\[ A^{*+1} = A^{*+1} + 2\Delta t \frac{\partial A^r}{\partial t}, \]  

(34)

\[ A_i^* = A_i^r + \epsilon[A_i^{r-1} - 2A_i^r + A_i^{r+1}], \]  

(35)

\[ \epsilon = 0.05. \]  

(36)

Finally, the changes in temperature from compensating vertical motions in the environment are calculated from (1). In finite-difference form, (1) is written

\[ T_k^{r+1} = T_k^r - w_k^r \Delta t \left[ \Gamma + (T_k^{r+1} - T_k^{r+1}) \frac{\Delta z(k)}{\Delta z(k - 1) \delta z(k - 1)} \right. \]
\[ \left. + (T_k^{r+1} - T_k^{r+1}) \frac{\Delta z(k - 1)}{\Delta z(k) \delta z(k - 1)} \right]. \]  

(37)

Because forward integration of this equation becomes numerically unstable, an alternate integration procedure from Richtmyer and Morton (1967) is applied. Eq. (37) is rewritten in the form

\[ -A_k T_k^{r+1} + B_k T_k^{r+1} = C_k T_k^{r+1} = D_k, \]  

(38)

where

\[ A_k = -w_k^r \Delta t \frac{\Delta z(k - 1)}{\Delta z(k) \delta z(k - 1)}, \]  

(39)

\[ B_k = 1 + A_k + w_k^r \Delta t \frac{\Delta z(k)}{\Delta z(k - 1) \delta z(k - 1)}, \]  

(40)

\[ C_k = w_k^r \Delta t \frac{\Delta z(k)}{\Delta z(k - 1) \delta z(k - 1)}, \]  

(41)

and

\[ D_k = T_k^r - w_k^r \Delta t \Gamma. \]  

(42)

Eq. (38) has the set of solutions

\[ T_k^{r+1} = E_k T_k^{r+1} + F_k, \]  

(43)

where

\[ E_k = \frac{A_k}{B_k - C_k E_{k-1}}, \]  

(44)

\[ F_k = \frac{D_k + C_k F_{k-1}}{B_k - C_k E_{k-1}}, \]  

(45)

The changes in \( u, v \), and mixing ratio are calculated similarly.

3) Short-wave filter

During the course of the integration high-frequency "noise" may develop, which, if left uncontrolled, may amplify and destroy the integration. Consequently, a five-point short-wave filter, in the form of a smoother-desmoother (see Shapiro, 1970), is applied to all the prognostic variables once each hour or every 30 time steps after correction begins. For any variable \( A \), the filter is defined by

\[ A_{p,i,j} = A_{i,j} + S[A_{i+1,j} + A_{i-1,j} + A_{i,j+1} \]
\[ + A_{i,j-1} - 4A_{i,j}], \]  

(46)

\[ A_{p,i,j} = A_{p,i,j} + D[A_{p,i+1,j} + A_{p,i-1,j} + A_{p,i,j+1} \]
\[ + A_{p,i,j-1} - 4A_{p,i,j}], \]  

(47)

where \( S = 0.125 \) and \( D = -0.135 \). \( A^p \) is calculated during the first pass through the array and \( A^p \) is the desmoothed array which results from the second pass. This cycle is repeated 10 times [see Perkey and Kreitzberg (1976) for an analysis of the effects of the smoother-desmoother].

e. Summary of the governing system and its interaction with the convective parameterization

The governing system has the following characteristics:

1) The conservation of momentum equations are cast in the primitive equation flux form.

2) Height is the vertical coordinate with 20 constant height levels defined at the climatological mean heights of significant pressure levels (i.e., every 50 mb). See Table 2. All variables are defined at all levels.

3) The horizontal domain is 400 km on a side with a 20 km grid mesh.

4) Surface topography is not included.

5) Lateral boundaries are periodic in east-west and constant at the north and south. A "sponge" condition is applied to the prognostic tendencies along the north and south boundaries.

6) The upper boundary condition requires the vertical motion to be zero, i.e., \( d z / d t = w = 0 \).

7) The lower boundary condition includes a surface drag with linear drag decrease through the transition layer; also, \( w = 0 \).

8) The finite-difference formulations are centered
TABLE 2. Constant height levels of the numerical model. The heights are taken from the U.S. Standard Atmosphere pressure-height profile.

<table>
<thead>
<tr>
<th>Level</th>
<th>Height (m)</th>
<th>Pressure (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>111</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>540</td>
<td>950</td>
</tr>
<tr>
<td>3</td>
<td>988</td>
<td>900</td>
</tr>
<tr>
<td>4</td>
<td>1457</td>
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in time and space and a time filter is included to avoid "separation of solutions." Time steps are 30 s.

9) A horizontal diffusion filter, applied as a smoother-desmoother, is used to help control numerical or boundary generated noise.

10) Virtual temperature and density adjustments have been neglected except in the moist convection calculations.

11) The governing system is hydrostatic with non-hydrostatic motions included through the convective parameterization. Pressure, $u$, $v$ and mixing ratio are the prognostic variables while temperature is diagnosed hydrostatically and density is derived through the gas law. Vertical motion is diagnosed using Richardson's equation.

12) Effects of convective processes on the mesoscale are gradually fed into the governing system of equations. Changes are linearly introduced over the estimated period of time that convection is active in a grid element. Temperature changes are incorporated into the governing system through the heating terms which appear in the pressure tendency and vertical motion equations. See Fritsch and Chappell (1980) for a detailed description of the parameterization.

Fig. 5 shows the sequence of calculations whereby the convective forcing is introduced into the mesoscale model. At each time step, every grid point is checked for convective changes still being introduced as a result of convection which began at a prior time step. If yes, then the previously defined convective tendencies $(Q_c, \frac{\partial r}{\partial t}|_c, \frac{\partial u}{\partial t}|_c, \frac{\partial v}{\partial t}|_c)$ are left unchanged. If no, the sounding at the grid point is tested for convective instability. For an unstable situation, the convective contributions to the mesoscale are computed by the cloud model. Otherwise, the convective terms are set to zero. Following the disposition of the convective tendencies, a time step in the governing mesoscale system is computed beginning with the diagnosis of vertical motion and ending with the prediction of mixing ratio.

4. Initial conditions

Using the mathematical formulation described by Fritsch et al. (1980) analytical initial conditions representative of a portion of the warm sector of an extratropical cyclone were constructed. In the center of the domain, these initial conditions are designed to simulate an idealized squall-line environment where a “short” (meso-α scale) wave is propagating through a slower moving synoptic-scale wave (see Fig. 6). The area of upward motion traveling with the short wave encounters a narrow moist tongue in the synoptic-scale warm sector and, along with the boundary-layer warming, initiates deep convection. Along the northern and southern boundaries, horizontal wind speeds and vertical shear of the horizontal wind are very small. With very moist and unstable low-level air, these initial conditions are a severe test of the model’s ability to predict slow-moving, heavy-rain events.

Figs. 7–10 show soundings and various horizontal sections of the initial atmospheric structure. Note in Fig. 7 that the surface pressure difference across the domain is only ~1.5 mb and that the correspond-
ing temperature change is just a little over 2°C. A loft, the meso-α wave shows up primarily in the thermal field and as a jet maximum near the southern end of the weak pressure trough.

For a point along the west side of the axis of the moist tongue (indicated by × in Fig. 7c), the initial potential buoyant energy is shown in Fig. 10. The updraft parcel path was constructed by assuming that the lowest 100 mb is the layer that supports the clouds. At this initial time, the boundary-layer lapse rate is fairly typical of midday conditions and further heating and lifting are required to bring the boundary-layer air to its level of free convection (LFC). As boundary layer heating continues, unstable conditions develop when surface temperatures approach 29°C. This occurs after ~3 h of integration.

5. Results

Evolution of the flow fields during the hours prior to the first appearance of convection is characterized by eastward progression of the meso-α scale wave and warming of the boundary layer. Figs. 11–24 show the evolution of the dynamic and thermo-
grid elements in which convection develops (point × in Fig. 7c). Note the strong warming in the vicinity of the tropopause (∼12 km) and the moist downdraft cooling in the boundary layer. Because the thin layer of anvil cooling cannot always be resolved by the ∼50 mb vertical spacing in the model, it is introduced in the model layer immediately above the highest model level within the convective cloud. In Fig. 11a this cooling appears at 13.6 km. Fig. 11b shows the vertical distribution of wind speed in the same grid element. With this type of wind profile the warming and cooling aloft will be quickly advected out of the grid element, while effects of cooling near the surface dynamic structure of the atmosphere from 5 min before deep convection begins to 2 h following the onset of convection.

a. Characteristics of initial convection

In this model, most warming of the atmosphere by convection is realized through adiabatic compression by compensating subsidence in the convective-cloud environment. Fig. 11a shows the vertical distribution of convective heating in one of the first

![Fig. 8. Initial conditions at z = 5.57 km (∼500 mb level): (a) pressure (mb) and temperature (°C); (b) wind direction (arrows) and isotachs (m s⁻¹). Dark lines in (a) are pressure.](image)

![Fig. 9. As in Fig. 8 except at z = 9.16 km (∼300 mb level).](image)
will, relative to high levels, remain for a longer time. Furthermore, the meso-β scale upward vertical circulation which develops in response to the convective warming (see Fig. 12) will compensate for some of the warming. Consequently, surface pressure should begin to rise soon after convection begins.

b. Sequence of convective development and heating

The sequential development of deep convection is shown by the successive locations of convective cooling at the surface (Fig. 13). As convective heating is introduced at upper levels, the atmosphere immediately begins to compensate through lifting and adiabatic cooling so that large rises in temperature never appear. Similarly, the cooling being introduced at cloud top produces compensating subsidence so that large temperature decreases are never realized. Because the effects of convection are gradually fed into the meso-β scale model governing system, the compensation necessarily must lag the warming (or cooling) so that mesoscale changes which appear in the results reflect a combination of convective-scale effects and meso-β scale responses.

c. Meso-scale changes at the surface

The meso-β scale response to the convection is rapid and quite strong. Fig. 14 shows the history of the surface pressure following the start of convection. After 30 min of convection, the pressure tendencies have reversed in the region where convection first formed and pressure rises now dominate. New convection has formed on the eastern edges of the original convective areas and the meso-β scale vertical circulation now dominates the western half of the domain (see Fig. 15). Surface winds are beginning to respond to the rising pressure and the formation of a meso-β (bubble) high is rapidly proceeding.

Forty-five minutes following the initial convective outbreak, the convection has organized into a well-defined “squall line” with an associated mesohigh (Figs. 13e and 14e). Note that the mesohigh centers on the region which experienced the earliest and
front nearly double the maximum speeds prior to convection. Strong low-level convergence accompanies the leading edge of the outflow and upward motion quickly increases around the 900 mb level (see Fig. 19g).

In addition to the low-level cooling produced by the moist downdrafts, considerable drying is also evident. Fig. 20 shows the evolution of surface mixing ratios as the convection moves through the moist tongue. Typical surface changes are $\sim 2 \text{ g kg}^{-1}$ with a maximum change of $\sim 3 \text{ g kg}^{-1}$.

Coincident with the development of the mesohigh, a mesolow also forms during the period from 15 to 30 min following the convective outbreak. The first mesolow (labeled $L_1$ in Figs. 13, 14, 15, 16 and 20) seems to be closely correlated with a center of upper level subsidence warming (see Figs. 15c and 15d and 21e and 21f). It is important to recognize that the high-level warming, and subsequent formation of a mesolow, must lag in time the appearance of the subsidence center; i.e., the subsidence warming must be focused, continue for a long enough period of time and extend over a deep enough layer, to hydrostatically generate the concentration of pressure falls necessary for mesolow formation (see Section 2). Several other mesolows (labeled $L_2$ to $L_5$) also form in conjunction with upper level subsidence centers (see Figs. 14 and 15) and associated warming (see Fig. 21). Although strong subsidence occurs along the north and south boundaries, the constant boundary condition along with the sponge formulation for the first few interior points, suppress meso-

![Fig. 11. Vertical profiles of convective heating and horizontal wind speed over point $\times$ at 30 min following the onset of convection. Layer of convective warming is shaded.](image)

strongest low-level cooling (Figs. 16c and 16d). Note also that at the 2 km level, above the layer of moist downdraft cooling, the mesohigh is overlain by a weak trough with virtually no temperature changes from before convection developed (see Fig. 17). During the next 30 min the convection moves rapidly eastward as the mesohigh continues to strengthen and expand (see Figs. 13f and 13g and 14f and 14g). By this time the surface winds have had sufficient time to respond to the changing surface pressure and to the intrusion of mid-level momentum into the low levels. Figs. 18f and 18g show a well-developed outflow from the mesohigh with peak winds in the gust

![Fig. 12. Mesoscale vertical motion pattern (cm s$^{-1}$) in an east-west vertical section through the north-south center of the convective complex 45 min following the onset of convection. Shading indicates subsidence $> 5 \text{ m s}^{-1}$.](image)
Fig. 13. Successive surface locations of the moist downdraft cooling (°C h⁻¹). Contours also indicate where active convection is occurring. L's indicate positions of surface mesolows; H indicates mesohigh.
Fig. 14. Evolution of surface pressure pattern (mb). Time sequence begins 15 min before the start of convection and continues for 2 h.
Fig. 15. Evolution of vertical motion pattern (cm s⁻¹) at z = 9.16 km (~300 mb level). Time sequence begins 15 min before the start of convection and continues for 2 h. L’s indicate position of surface mesolows 15 min following each vertical motion pattern; H indicates mesohigh.
Fig. 16. Evolution of surface temperature pattern (°C). Time sequence begins 15 min before the start of convection and continues for 2 h. L's indicate positions of surface mesolows; H indicates mesohigh.
low formation so that only an areally distributed drop in pressure can materialize. There is, however, an additional center of upper level subsidence in the interior of the grid; this center is evident in Fig. 15f at about 100 km east and 200 km north. In this case, the subsidence forms over the dome of cold downdraft air and only a very weak east-west trough appears on the west side of the mesohigh (see Fig. 14g).

d. Mesoscale vertical circulations

Along with the meso-β circulations that form at the surface, strong vertical circulations develop through a deep layer of the atmosphere. Fig. 22 shows the time history of vertical motion in an east-west vertical plane taken at the north-south midpoint of the domain. Notice in Fig. 22a, before convection begins, that the maximum up and down motions are about 1 cm s\(^{-1}\). Thirty minutes following the onset of convection, upward motion has increased to over 0.5 m s\(^{-1}\) and subsidence to over 10 cm s\(^{-1}\). As the convective complex moves eastward, the meso-β vertical circulation amplifies and reaches its peak \(\sim 1\) h after the complex first formed. Forty-five minutes later (Fig. 22h) convection is dissipating and the circulation aloft quickly weakens. Subsidence continues in the mesohigh, however, as the cold air deposited in the boundary spreads out.

A meso-β vertical circulation similar to the one shown in Fig. 22 has been diagnosed by Fankhauser (1969, 1974) and Ogura and Chen (1977) for squall lines that passed through the National Severe Storm Lab (NSSL) upper air mesonetwork in Oklahoma. Fig. 23 compares the mature vertical circulation found by Ogura and Chen to the circulation predicted by the model. Note that in both the model and diagnosed circulations the maximum in upward motion developed around 300 mb and subsidence centers formed about 60 to 80 km on either side of the axis of upward motion (in Fig. 23a, 40 cm s\(^{-1}\) at 300 mb corresponds to \(17 \times 10^{-2}\) mb s\(^{-1}\), while 5 cm s\(^{-1}\) at 800 mb is \(5 \times 10^{-2}\) mb s\(^{-1}\)). Although there are differences in the details of the two circulations, the general structures are remarkably similar. This type of circulation also appears in the mesoscale numerical simulations of Brown (1979) and Kreitzberg and Perkey (1977). In fact, in the Kreitzberg-Perkey results, it is the secondary meso-β scale vertical circulation forming in response to the convection, that produces the bulk of the precipitation in nonsevere storm situations. Other indications of a region of mean mesoscale ascent are the areas of steady rain (persisting for several hours) which develop following the maturation of convective complexes. See, for example, the steady rain reports in Fig. 1c; see also, Pedgley (1962), Houze (1977), Zipser (1977) and Leary and Houze (1979). The exact role of such a circulation in the development of mesoscale

![Fig. 17. Temperature (°C) and pressure (mb) patterns at z = 1.95 km (~800 mb level). Dark lines are pressure. (a) Prior to convective outbreak; (b) 45 min following onset of convection.](image)

weather systems is unclear at this time. However, this type of circulation may be a mechanism by which convection interacts "up-scale" with its environment to organize and enhance future convective growth. Note, however, that to some extent, the dimensions of the circulation may be influenced by the density of observations, or in the model case, by the horizontal grid spacing.

e. Convective precipitation

Because of the large differences in thunderstorm environment across the western half of the domain,
Fig. 18. Evolution of surface wind field. Time sequence begins 15 min before the start of convection and continues for 2 h. Arrows show wind direction and solid lines are isolasts (m s⁻¹).
Fig. 19. Evolution of vertical motion pattern (cm s$^{-1}$) at $z = 0.988$ km ($\sim 900$ mb level). Time sequence begins 15 min before the start of convection and continues for 2 h.
Fig. 20. Evolution of surface mixing ratio pattern (g kg\(^{-1}\)). Time sequence begins 15 min before start of convection and continues for 2 h.
Fig. 21. Evolution of temperature pattern (°C) at z = 9.16 km (300 mb). Time sequence begins 15 min before the start of convection and continues for 2 h. L’s and H’s as in Fig. 14.
Fig. 22. Evolution of mesoscale vertical motion patterns (cm s⁻¹) in an east-west vertical section through the center of the convective complex.

Time sequence begins 15 min before the start of convection and continues for 2 h.
convective precipitation (Fig. 24) varied substantially. In the center of the convective complex, where the vertical wind shear was strongest and the convection moved the fastest, amounts were relatively light—approximately 2 cm. The heavy-rain predictive potential of the model was tested along the northern and southern edges of the mesosystem where horizontal wind speeds and vertical wind shear were exceptionally weak and storms were almost stationary. The heaviest rainfall, about 12 cm in 2 h, occurred along the southwest flank of the mesohigh where low-level moisture (16 g kg$^{-1}$, a dewpoint of $\sim$21°C) was continuously flowing into the system and thunderstorm activity persisted.

6. Discussion and conclusions

The model results indicate that, at least for certain initial conditions, convective parameterization can be used to simulate the development of convectively driven meso-$\beta$ scale weather systems. Furthermore, magnitudes of the mesoscale changes produced by the model are similar to those observed in nature. Of particular significance is the model prediction of a strong meso-$\beta$ scale vertical circulation. The subsidence portion of this circulation is strongly correlated with the production of meso-$\beta$ low pressure systems. This supports the contention of Hoxit et al. (1976) and is one of the major findings.

Along with the prediction of the mesohighs and mesolows, the model results also provide some insight into the role of mesocyclogenesis in dynamically enhancing the environment for convective growth. Although the mesolow appears to be related to the subsidence warming aloft, the warming (on the order of 1–3°C over a layer about 200–400 mb thick) usually does not appreciably reduce the potential buoyant energy. Further, significant warming rarely penetrates below $\sim$500 mb so that the development of new convection is not thermodynamically suppressed. In some cases, though, it is observed that mesoscale subsidence ahead of an active convective system acts to dissipate small growing cumulus clouds (Hoxit et al., 1976). As the area of active convection arrives, however, the suppressed condition is reversed and boundary-layer air is strongly forced into the primary convective circulation. In this manner (first by suppression and then forcing), the original mesoscale convective system may organize and perpetuate itself as it propagates.

Fig. 23. Comparison of observed mesoscale vertical circulations (b) to model predicted circulation (a). Observed circulation was diagnosed by Ogura and Chen (1977); units in (b) are $10^{-3}$ mb s$^{-1}$.
through a region of unstable air created by much larger synoptic-scale forcing. Depending on its strength and duration, the mesolow may enhance the low-level convergence of mass and moisture into the existing convective system. It also may play a role in the development of the cyclonic circulation that frequently accompanies tornado-genesis. Much further study is needed to evaluate these two possibilities.

As pointed out in Section 2, the mechanism for producing mesolows is applicable not only to large convective clouds but to cloud complexes or cloud clusters. This suggests that tropical cyclogenesis may occur in clear areas within, adjacent to, or between cloud clusters. The fact that tropical cyclogenesis frequently occurs in regions of little wind shear and weak ventilation (Gray, 1979) also supports this hypothesis.

In view of some of the restrictions and assumptions that were applied in developing the parameterization, and considering the spatial resolution and dimensions of the numerical model, caution is necessary when interpreting some of the results. For example, mechanisms that propagate the model convection are not entirely the same as those that propagate actual convective systems. Individual clouds are not resolved and, therefore, neither is cloud-scale forcing. Initial and subsequent locations of convection are purely a response to surface heating in conjunction with meso-α and meso-β scale forcing. Thus, even though several important processes are included, the direct, cloud-scale mechanical lifting by moist downdrafts cannot be properly incorporated. Note, however, that substantial compensation for cloud-scale forcing is realized through the lifting provided by the meso-β high-pressure outflow, and the downward transfer of momentum by the cumulus cloud in the convective parameterization procedure.

It should also be noted that the shape and evolution of the mesoscale features were likely influenced by the rigid walls at the north-south boundaries and the periodic conditions in the east-west. The sponge condition applied at the northern and southern boundaries acted to diminish the response of the wind to the hydrostatically generated meso-β pressure systems so that low-level convergence and development were retarded. In the opposite sense, these rigid boundaries may enhance mesolow development since more of the subsidence warming is likely to occur near the active convection than if outflow was permitted through the boundaries. Still another effect is produced by the constant pressure and zero heating conditions along the north-south borders. These two restrictions, combined with the hydrostatic assumption, cause the temperature to remain constant along the boundaries and artificially limit the north-south extent of the convection.

This is not considered too severe a restriction since in actual situations natural environmental features also impose similar limits.

An additional cause for concern stems from the requirement that the cloud-scale compensating subsidence occur within the same grid element in which it was generated. The validity of this assumption is strongly dependent on grid size and the particular characteristics of the clouds and environment. In some situations compensating subsidence may occur in an area extending 50 or even 100 km downwind of the active updrafts (Fritsch, 1975; Hoxit et al., 1976). It is not clear at this time whether subsidence is in response to the meso-β scale up motion in the cloud region or is a direct response to cloud updrafts. More than likely it is a combination of both. In either case, the atmosphere tends to accomplish the same end result, focused warming. What is different is the timing and magnitude of the warming. This ultimately affects surface pressure, the new growth of deep convection, and the structure and movement of the entire convective complex. Until we understand how to distribute the compensating subsidence around the active updrafts, we cannot be certain that convective parameterizations are correctly generating the physical circulations responsible for producing mesoscale pressure systems. In the meantime, we can only compare prediction to reality and so far, these preliminary results are very encouraging.

Finally, several other integrations were performed on initial conditions which were slightly different from those described in Section 4. These integrations produced meso-β scale features very similar to the results shown above. In each case, mesoscale surface pressure systems formed in conjunction with the convection. However, depending on when and where the convection began, the position and intensity of the mesoscale features changed considerably. Furthermore, the horizontal distribution and amount of convective rainfall also varied substantially in response to the changes in location of the initial convection. These results suggest that the evolution of meso-β scale convective systems may be highly sensitive to small changes in the initial location of convective development. They also indicate that in order for a model to accurately predict the timing, location and intensity of convectively driven mesoscale systems, resolution of mesoscale forcing along with an accurate boundary-layer prediction are essential.

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APPENDIX

List of Symbols

\( B \) pressure tendency at the top of the model
\( c_p \) specific heat at constant pressure
\( c_v \) specific heat at constant volume
\( c_p \) surface drag coefficient
\( \text{ETL} \) equilibrium temperature level for convective updraft
\( f \) Coriolis parameter
\( F_j \) weighting function for lateral boundary tendencies
\( F_u \) \( x \) component of surface friction
\( F_v \) \( y \) component of surface friction
\( g \) gravity
\( H \) boundary-layer heating function
\( H_e \) maximum cooling rate in boundary layer
\( H_w \) maximum warming rate in boundary layer
\( L \) latent heat of condensation or sublimation
\( \text{LCL} \) lifting condensation level
\( \text{LFC} \) level of free convection
\( p \) pressure
\( Q \) total diabatic heating \([=Q_c + Q_s + Q_f]\)
\( Q_c \) convective condensation heating
\( Q_r \) radiational heating
\( Q_s \) condensation heating from stable precipitation
\( r \) mixing ratio
\( R \) dry air gas constant
\( T \) temperature
\( u \) \( x \) component of horizontal wind
\( v \) \( y \) component of horizontal wind
\( V_z \) vector horizontal wind
\( w \) vertical component of wind
\( z_t \) height of the top of the model
\( \alpha \) any variable
\( \gamma \) environmental lapse rate
\( \Gamma \) dry adiabatic lapse rate
\( \eta \) \( c_p/c_v \)
\( \rho \) density
\( \tau \) time level
\( \tau_c \) period of time convection is active in a grid element
\( \Delta \) denotes the finite difference approximation to the partial derivative
\( \nabla \) horizontal del operator in Cartesian coordinates

Coordinates

\( t \) time
\( x \) Cartesian coordinate, increases toward the east
\( y \) Cartesian coordinate, increases toward the north
\( z \) Cartesian coordinate, increases with height

Subscripts

\( c \) indicates a convective process
\( i \) coordinate of a grid point in the \( x \) direction (numbered from the west)
\( j \) coordinate of a grid point in the \( y \) direction (numbered from the south)
\( k \) coordinate of a grid point in the \( z \) direction (numbered from the surface)
\( 0 \) indicates lowest level in the model
\( r \) indicates a radiative process
\( s \) indicates a stable (nonconvective) process
\( \zeta \) intermediate step in time filter

REFERENCES


