The Mesoscale and Microscale Structure and Organization of Clouds and Precipitation in Midlatitude Cyclones. VIII: A Model for the “Seeder-Feeder” Process in Warm-Frontal Rainbands

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ABSTRACT

Previous field studies have indicated that warm-frontal rainbands form when ice particles from a “Seeder” cloud grow as they fall through a lower-level “Feeder” cloud. In this paper we present results from a parameterized numerical model of the growth processes that can lead to the enhancement of precipitation in a “Seeder–Feeder” type situation. The model is applied to two types of warm-frontal rainbands. In the first (Type 1 situation) the vertical air motions are typical of those associated with slow, widespread lifting in the vicinity of warm fronts. In the second (Type 2 situation) the vertical air motions are stronger, and more characteristic of the mesoscale.

The model simulations show that in the Type 1 situations the growth of the “Seed” ice crystals within the feeder zone is due to vapor deposition. The feeder zone in this case is slightly sub-saturated with respect to water due to the presence of the seed crystals. In regions where the feeder zone is not “seeded” from aloft, snow crystals originating in the feeder zone, grow by deposition and riming and produce a precipitation rate of $\sim 1 \text{mm h}^{-1}$, compared to $\sim 2 \text{mm h}^{-1}$ for the combined seeder–feeder cloud system. The presence of seed crystals allows for the efficient removal of condensation produced by the feeder cloud. In the Type 2 situation, the strong mesoscale ascent provides liquid water from which the seed crystals grow primarily by riming.

For both Type 1 and 2 situations the condensation rates, radar reflectivities and rainfall rates predicted by the model are in reasonable agreement with field observations.

1. Introduction

Observational studies of cyclonic storms in the Pacific Northwest have shown that the regions of heaviest precipitation are often organized in the form of rainbands (Houze et al., 1976; Hobbs, 1978; Matejka et al., 1980). Six types of rainbands have been identified: warm-frontal, warm-sector, wide cold-frontal, narrow cold-frontal, prefrontal cold-surge and postfrontal.

The field studies indicate that in the case of the warm-frontal (Hobbs, 1978; Hobbs and Locatelli, 1978; Herzegh and Hobbs, 1980; Matejka et al., 1980; Houze et al., 1981) and the wide cold-frontal (Hobbs, 1978; Hobbs et al., 1980; Matejka et al., 1980) rainbands, precipitation is enhanced through a “Seeder–Feeder” mechanism, similar to that proposed by Bergeron (1950) for the enhancement of precipitation in orographic clouds. In the seeder–feeder process ice particles from a “Seeder” cloud grow as they fall through a lower-level “Feeder” cloud. Cunningham (1951), Wexler (1955) and Plank et al. (1955) identified a seeder–feeder mechanism in warm-frontal and stratiform precipitation in the Northeastern United States, and Marshall (1953) and Gunn et al. (1954) observed it in winter precipitation over Montreal, although these workers did not associate the enhanced precipitation with moving mesoscale features in the form of rainbands.

In this paper we present a diagnostic model of the microphysical processes associated with a seeder–feeder mechanism operating within prescribed steady-state air motion patterns within the feeder zone. When used in conjunction with field observations, obtained in the University of Washington CYCLones (CYCLonic Extratropical Storms) Project, this model provides improved insights into the important precipitation mechanisms operating in warm-frontal rainbands.

The model is applied to two situations that have been observed in warm-frontal rainbands. In the first case, described by Herzegh and Hobbs (1980) and Matejka et al. (1980), the vertical air motions in the feeder cloud are weak (0.1–0.2 m s$^{-1}$) and rather uniform over horizontal distances of $\sim 100$ km or more beneath a warm-frontal surface. Updrafts of this magnitude and on this horizontal scale are typical of warm-frontal lifting. In the second situation, described by Houze et al. (1981), the vertical air motions

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in the feeder zone are greater (~0.7 m s⁻¹), and the horizontal distance over which lifting of this magnitude occurs is ~30 km; such air motions are characterized as mesoscale.

In both cases the model is used to assess the relative contributions of various mechanisms to the growth of precipitable particles in the feeder zone, and model predictions of condensation rates, rainfall rates and radar reflectivities are compared to measurements.

2. Model description

Two basic types of numerical models have been used to simulate the microphysics involved in mixed-phase clouds. The first is the bulk water type model, in which various cloud microphysical processes responsible for transferring the water substance from one phase to another are parameterized; in this type of model, the particles are assumed to be continuously distributed in specified size distributions. Such models have been used in studies of convective precipitation by Ogura and Takahashi (1971), Wisner et al. (1972), Orville and Kopp (1977) and Chang (1977).

In the second type of model, the cloud microphysical processes are dealt with explicitly and distinct size categories of particles are maintained. Cotton (1972) and Scott and Hobbs (1977) have developed cumulus models of this type, and Young (1974) presented an orographic model. More recently, Hall (1980) has discussed an extremely detailed cloud model of the explicit type. Models of the second type allow, in part, for the complex shapes of ice particles, whereas, the parameterized models generally assume that the ice particles are spherical.

In the present study we use a bulk water parameterized model, similar to that described by Lin et al. (1983). This type of model requires the input of various initial fields and numerical values to the parameterized equations; these inputs were available from the CYCLES field data set.

The model is two-dimensional. The model domain is an x-z plane, where x is the horizontal distance perpendicular to the length of the rainband and z the height coordinate. Model variables include: temperature T, and the mixing ratios of water vapor q_v, cloud water q_c, cloud ice q_i, snow q_s and rain q_r. The fields of cloud water and cloud ice advect with the prescribed airflow in both the horizontal and vertical directions, whereas both the rain and snow, while moving with the horizontal wind, fall relative to the updraft. All symbols are defined in Appendix B.

a. Parameterization

The particles comprising the cloud water and cloud ice fields are each assumed to be monodispersed. The sizes of the rain and snow particles, however, are assumed to be distributed continuously, according to an inverse exponential distribution. Houze et al. (1979) showed from airborne observations obtained in the Pacific Northwest that precipitation particles within rainbands follow this type of distribution. This finding provides support for our use of the bulk water type model. For rain, the size distribution given by Marshall and Palmer (1948) is:

\[ N_{DR} = N_{OR} \exp(-\lambda_R D_R) dD_R. \] (1)

Similarly, snow is assumed to follow the size distribution given by Gunn and Marshall (1958):

\[ N_{DS} = N_{OS} \exp(-\lambda_S D_S) dD_S. \] (2)

In this study we use values of \(N_{OS}\) given by Houze et al. (1979).

The slope factors \(\lambda_R\) and \(\lambda_S\) for rain and snow are given by

\[ \lambda_R = \left( \frac{\pi \rho_l N_{OR}}{\rho q_r} \right)^{0.25}, \] (3a)

\[ \lambda_S = \left( \frac{\pi \rho_S N_{OS}}{\rho q_s} \right)^{0.25}. \] (3b)

b. Continuity equations

The continuity equations for the fields of \(q_v\), \(q_c\) and \(q_i\) are of the form

\[ \frac{\partial q}{\partial t} = -u \frac{\partial q}{\partial x} - w \frac{\partial q}{\partial z} + S_0 \frac{q}{\rho}, \] (4)

and for the precipitating fields \(q_s\) and \(q_r\),

\[ \frac{\partial q}{\partial t} = -u \frac{\partial q}{\partial x} - (w + V) \frac{\partial q}{\partial z} - q \frac{\partial (\rho V)}{\partial z} + S_0 \frac{q}{\rho}. \] (5)

In Eq. (4) \(S_0\) is given by Eqs. (A29), (A30) or (A31) and \(S_0\) in Eq. (5) is given by Eq. (A32) or (A33).

The thermodynamic energy equation for \(T\) is

\[ \frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - w \left( \frac{\partial T}{\partial z} + \Gamma_v \right) + S_T \frac{T}{\rho}. \] (6)

c. Cloud processes

Only a brief description of the various processes included in the model will be given here; detailed equations are given in Appendix A.

Fig. 1 depicts the microphysical processes included in the model of the feeder zone. The acronym we use for these processes (contained in parenthesis in Fig. 1) are largely the same as those used by Lin et al. (1983). The initiation of cloud ice (PINT) and subsequent growth by deposition (PDEPI) occurs whenever the air is saturated with respect to ice and \(T < 0^\circ\text{C}\). The concentration of these small ice crystals is assumed to follow the temperature-dependent expression given by Fletcher (1962) for the concentration of ice nuclei in the atmosphere. When the cloud ice mixing ratio reaches a threshold value (cor-
responding to the monodispersed cloud ice particles achieving a diameter large enough to be considered as snow in our model), the excess cloud ice is converted to snow (PSACI). The growth of snow by vapor deposition is given by PSDEP.

Cloud ice evaporates when the air is below ice saturation, or it melts to form cloud water (PSMLTI) when $T \geq 0^\circ$C. An additional cloud ice sink is collection by snow (PSACI). The melting of snow at $T \geq 0^\circ$C (PSMLT) is a source for rain. The quantity of water evaporated from a melting particle (PMLTEV) is computed when the air is below water saturation and $T \geq 0^\circ$C.

Cloud water is formed by condensation (PCOND) when the air is saturated with respect to water and it evaporates when the air is below water saturation. Depletion of cloud water occurs via autoconversion (PRAUT) or accretion (PACW), both of which are sources of rain. The collection of cloud water by snow (PSACW) is a source for snow when $T < 0^\circ$C. The evaporation of rain (PREVP) is calculated when the air is below water saturation.

The seeder cloud is not dealt with in detail in the present model. Instead, the particles from the seeder cloud are represented by a snow mixing ratio $q_{s0}$ entering the top of the feeder zone, and this is held constant during any particular run of the model. The values of $q_{s0}$ used in the model runs to be shown were determined from radar reflectivity measurements using the relation:

$$M_S = aZ^\beta \text{ (g m}^{-3}).$$

The values of $a$ and $b'$ are taken from Herzegh and Hobbs (1980) who compared in situ measurements of ice water contents with simultaneous radar reflectivity measurements for deep stratiform clouds associated with warm fronts in cyclones in the Pacific Northwest.

Our field observations indicate that the role of the seeder cloud is to provide the ice particles that subsequently serve as centers upon which the available water in the feeder zone can collect and be converted into precipitation. Furthermore, the observations indicate that the dominant ice particle types within the feeder zones associated with warm-frontal rainbands are similar to those in the seeder clouds above.

d. Initial condition and numerical procedures

The initial fields of water vapor mixing ratio, temperature and pressure are specified in the model. These values were obtained from sounding data in the vicinity of the warm-frontal rainbands to be modeled. Horizontal winds and vertical air velocities, derived from Doppler radar measurements, are input into the model and held constant during the numerical simulation. Because of this, no feedback mechanisms between microphysical and dynamical processes are simulated in the model.

The continuity equations are integrated in finite difference form, using forward-time extrapolation for local changes in time, and upstream-space differencing for the advection terms. The model is integrated until such time as steady-state values of the various
modeled fields are achieved. The grid increment is 4 km in the horizontal and 200 m in the vertical. The time step is 10 s.

Actual model calculations proceed by first computing dummy values of $T$ and the water continuity variables from horizontal and vertical advection alone. This dummy value of $T$ is then used to compute the saturation mixing ratios with respect to water $q_{sw}$ and ice $q_{iw}$. The saturation vapor pressures over water and ice are computed from equations given by Lowe and Ficke (1974). All production terms for the water continuity variables are then computed as functions of these dummy values. In air saturated with respect to ice and at $T < 0^\circ$C, the deposition growth of the small ice crystals comprising the cloud ice field is calculated first. If the air remains above ice saturation, the deposition growth of snow is computed. Checks are made to ensure that these rates do not exceed the actual amount of water vapor available for growth. Finally, the next value of $T$ and all water continuity values are computed with the inclusion of the source terms. This completes one time step in the integration.

3. Model results and comparisons with observations

In this section the results of several model simulations are presented and comparisons are made with field observations. The model simulations are of two types. Type 1: ice particles from aSeeder cloud interacting with a feeder cloud consisting of uniform updrafts on the frontal scale, as observed by Herzegh and Hobbs (1980) and Matejka et al. (1980). Type 2: ice particles from aSeeder cloud interacting with a feeder cloud forced by a stronger updraft on the mesoscale, as reported by Houze et al. (1981).

a. Type 1: TheSeeder-feeder process with updrafts on the frontal scale

In this section we present the results of model calculations for the case when the feeder cloud is characterized by quite weak vertical motions, similar to those that might be present due to upglide over a warm-frontal surface as documented by Herzegh and Hobbs (1980). This particular simulation utilizes the field data presented in their study.

Shown in Fig. 2 is the model domain for this case; it depicts the Seeder–feeder zones together with the horizontal and vertical wind profiles. The snow mixing ratio fed by the Seeder cloud into the top of the Seeder zone ($q_{s0} = 0.16 \text{ g kg}^{-1}$) is that derived by Herzegh and Hobbs for their 13 January 1975 case study [they applied Eq. (7) above]. The value of $q_{s0}$ was held constant during the model integration.

Clearly, the weak convective cells comprising theSeeder zone do not form, mature and dissipate in unison. However, we assume that the total contribution from the cells involved can be approximated by a steady flux of snow at the base of the Seeder zone. The concentration of Seed particles at the base of the Seeder zone can be computed from the quantity

$N_{0x}/\lambda_S$ where $\lambda_S$ is given by (3b) with $d_s = d_{s0}$. This yields a concentration of 7 L$^{-1}$ for the Seed particles, which is typical of warm-frontal clouds (Matejka, 1980).

The feeder cloud extends over a temperature range of 0 to $-17^\circ$C. The vertical wind profile is also similar to that deduced by Herzegh and Hobbs. A maximum updraft of 0.15 m s$^{-1}$ is located just above the 4 km level. The horizontal wind $u$ shown in Fig. 2 is relative to the motion of the rainband. This prescribed air motion pattern is assumed to extend continuously in the horizontal.

1) PRECIPITATION DEVELOPMENT IN THE FEEDER CLOUD ALONE

To better understand the role of the Seeder clouds, as well as interaction between the Seeder and feeder zones, we first discuss model results for the Seeder cloud alone ($q_{s0} = 0$). Any precipitation in this case is required to form totally within the broad stratiform region of weak ascent resulting from warm-frontal overrunning.

All values of the water continuity fields, their source terms, and other relevant information shown in the following figures for the Type 1 study, are steady state values after 8000 s of simulation time. For the Seeder cloud alone, the simulation is begun by initiating the updraft. Approximately 1 h of integration time was required for the Seeder zone to reach water saturation in the presence of weak updrafts. After this time cloud ice was initiated and grew by deposition to form snow. The simulation continued an additional 4000 s until a steady-state was
reached; this corresponds to the time required for snow to pass through the entire depth of the feeder zone.

Shown in Fig. 3a are the snow $q_s$ and rain $q_r$ mixing ratios for the feeder cloud alone simulation. Values of $q_s$ increase rapidly downward from 0.01 to 0.20 g kg$^{-1}$ prior to melting. Peak values of $q_s$ were 0.25 g kg$^{-1}$. Values of $q_r$ resulting from the melting of snow decrease downward due to evaporation. Shown in Fig. 3b are the corresponding precipitation rates; for snow the rate is the melted equivalent. The snowfall rate is 1.0 mm h$^{-1}$ across nearly the entire width of the feeder zone. Smaller rainfall rates are present below 0°C, again due to particle evaporation. The surface rainfall rate is 0.3 mm h$^{-1}$.

Listed in Table 1 are the values of the various microphysical terms and $q_r$ for simulation of the feeder cloud alone. The formation and growth of snow in the model proceeds as follows. Cloud ice is initiated and grows by deposition (PDEPI), converting cloud ice to snow (PCONV $\sim 10^{-3}$ g kg$^{-1}$ s$^{-1}$). This process is responsible for producing the initial 0.01 g kg$^{-1}$ of snow at the top of the feeder zone [Fig. 3a]. The terms involving the cloud ice field (PINT, PDEPI, PSACI) decreased rapidly with decreasing height due to the strong dependence of the cloud ice (ice nucleus) concentration on temperature. After the snow forms, it grows by riming and deposition.

The important sources of snow mass are deposition (PSDEP) and riming (PSACW) (Fig. 4a and 4b). Even though the cloud liquid water values $q_r$ are small (Fig. 5), riming contributes significantly to the growth of the snow. The precipitation rates are decreased by nearly a factor of two when riming growth is eliminated.

![Fig. 3](image_url)

**Fig. 3.** Cross sections of the snow and rain mixing ratios and the precipitation rates for the feeder cloud alone simulation. (a) Snow mixing ratios $q_s$ (g kg$^{-1}$)—solid lines. Rain water mixing ratios $q_r$ (g kg$^{-1}$)—dashed lines. (b) Snowfall rates—solid lines. Rainfall rates—dashed lines. Both rates are in millimeters of water per hour.
The cloud water shown in Fig. 5 represents condensation produced in the feeder cloud that is stored and not converted to precipitation. In this case, depositional growth was insufficient to remove all condensation (due to the small values of q_s). The ideal situation would be for the snow to consume water vapor (through deposition) at the same rate that it is made available by the feeder zone.

2) SIMULATION OF THE COMBINED SEEDER-FEEDER CLOUD SYSTEM

We turn now to a simulation of the combined seeder-feeder cloud system. The input parameters are the same as the previous simulation, except that now a steady line source of snow q_s into the top of the feeder cloud is added to simulate the input from the seeder cloud.

Again all values of the water continuity fields, their source terms, and other relevant quantities, are steady-state values after 8000 s of simulation time. At the beginning of this simulation the updraft profile and the line source of snow q_s were turned on. During the first hour of integration, the snow mixing ratio decreases downward due to sub-saturated conditions. Once ice saturation is achieved the snow particles increased in size. After a further 4000 s of integration
time, steady-state conditions are achieved. The following figures and discussions apply to the seeded region of the feeder zone.

Shown in Fig. 6a are the snow $q_s$ and rain $q_r$ mixing ratios. The values of $q_s$ and $q_r$ shown earlier in Fig. 3a represent the areas not seeded from aloft, adjacent to the region in Fig. 6a. The snow mixing ratio increases from 0.16 g kg$^{-1}$ to peak values of 0.54 g kg$^{-1}$ prior to melting. Corresponding rain water mixing ratios are lower, due to evaporation as well as decreases in particle concentrations caused by the large differences in the fall speeds of snow and rain.

The corresponding snowfall and rainfall rates are shown in Fig. 6b. The seeding from aloft results in a precipitation rate of 0.5 mm h$^{-1}$ at the top of the feeder zone which increases to peak values of 1.8 mm h$^{-1}$ prior to melting. Surface rainfall rates are 1.2 mm h$^{-1}$. The snowfall rate from the combined seeder-feeder cloud system (1.8 mm h$^{-1}$) is nearly a factor of two larger than the snowfall rate from the surrounding stratiform cloud (i.e., the unseeded feeder zone).

Shown in Table 2 are the peak and average values of various microphysical terms in the seeded portion of the feeder zone. Again, those terms involving the cloud ice field decrease rapidly with decreasing height in the feeder zone. The average values for these terms apply only in the upper 1 km of the feeder zone.

The increase in the precipitation rate from the feeder zone when it is seeded by the seed particle zone is due to the mass acquired by the seed particles as they fall through the feeder zones. The snow produced by the feeder cloud (from conversion, PCONV) is small compared to the input snow value $q_{s0}$. In the unseeded case, the formation of snow particles occurred in the upper few hundred meters of the feeder zone by conversion of cloud ice to snow (PCONV). In the seeded case, the conversion of cloud ice to snow is an order of magnitude smaller due to the presence of the seed crystals. Cloud ice in the feeder zone is collected by the seed particles (PSACT), hence less cloud ice is available for conversion. The combined sources of snow through conversion and collection in the upper region of the feeder zone (where they

Fig. 6. Cross section of the snow and rain mixing ratios and precipitation rates in the seeded region of the feeder zone for the Type 1 case. (a) Snow mixing ratios $q_s$ (g kg$^{-1}$)—solid line—and rain water mixing ratios $q_r$ (g kg$^{-1}$)—dashed line. (b) Snowfall rates—solid line—and rainfall rates—dashed line. Both rates are in millimeter of water per hour.
Table 2. Values of various microphysical terms in the seeded portion of the feeder zone.

<table>
<thead>
<tr>
<th>Source</th>
<th>Peak value</th>
<th>Average value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCONV</td>
<td>$8.0 \times 10^{-7}$</td>
<td>$2.1 \times 10^{-7}$</td>
</tr>
<tr>
<td>PSACI</td>
<td>$2.8 \times 10^{-8}$</td>
<td>$1.8 \times 10^{-8}$</td>
</tr>
<tr>
<td>PSACW</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PSDEP</td>
<td>$2.7 \times 10^{-4}$</td>
<td>$2.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>PSMLT</td>
<td>$-1.7 \times 10^{-3}$</td>
<td>$-1.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>PDEPI</td>
<td>$8.2 \times 10^{-7}$</td>
<td>$3.7 \times 10^{-7}$</td>
</tr>
<tr>
<td>PINT</td>
<td>$2.1 \times 10^{-8}$</td>
<td>$1.8 \times 10^{-8}$</td>
</tr>
<tr>
<td>$q_0$ (g kg$^{-1}$)</td>
<td>0.54</td>
<td>0.40</td>
</tr>
</tbody>
</table>

reach their maximum values) can add, at maximum, only 0.01 g kg$^{-1}$ to the input snow (0.16 g kg$^{-1}$). Therefore, since the bulk of snow mass present at the top of the feeder zone originates in the seeder clouds, subsequent water consumed in the feeder zone should be dominated by the "seed" crystals.

Although collection of cloud ice by seeder snow makes an insignificant contribution to the snow mass, it is important in the following way. The growth rate by vapor deposition $dm/dt$ of plate-like ice crystals is proportional to $m^{1/2}$, where $m$ is the crystal mass at time $t$. For a 100 $\mu$m diameter plate ($m \sim 10^{-11}$ kg) growing in a water-saturated environment at $-12$°C, $dm/dt$ is about $10^{-12}$ kg s$^{-1}$. The growth rate of a 1 mm diameter plate ($m \sim 10^{-9}$ kg) under the same conditions is about $10^{-11}$ kg s$^{-1}$. The effective growth rate of the smaller crystal when attached to the larger crystal is about $10^{-14}$ kg s$^{-1}$, a decrease of two orders of magnitude.

The mass of the larger plate is not significantly increased by collecting numerous cloud ice crystals. However, the collection process prevents the small cloud ice crystals from achieving precipitation sizes on their own. In this way, the seed ice crystals can inhibit the formation of snow within the feeder zone.

The presence of the seed crystals also eliminates the water-saturated conditions in the feeder zone. In the unseeded case, the deposition growth rates were sufficiently low, due to the small amounts of snow, that the feeder cloud remained water saturated and cloud water was present. However, when seed particles are added, this cloud water is suppressed and the growth of snow particles takes place by deposition (PSDEP). The average relative humidity in the feeder was 97% with respect to water and 107% with respect to ice, compared with 100% and 110%, respectively, when seed particles were absent. The presence of cloud water outside the seeded region, and its absence within seeded region, agrees with field observations (Hobbs, 1978; Matejka et al., 1980).

Shown in Fig. 7 is the depositional growth of snow that accreted onto the seed crystals (PSDEP). The maximum rates of deposition occur near the $-12$°C level, where saturation with respect to ice is a maximum.

The importance of the seed crystals can be seen by comparing the total mass removed by snow within the feeder zone in the case of the feeder alone with that removed in the combined seeder-feeder case. The difference between the snow mixing ratios at the bottom $q_{SB}$ and top $q_{ST}$ of the feeder zone is the amount contributed by the feeder cloud. For the feeder alone, $q_{SB} = 0.25$ g kg$^{-1}$ and $q_{ST} \approx 0.0$ g kg$^{-1}$, resulting in an increase of 0.25 g kg$^{-1}$. For the seeder-feeder case, $q_{SB} = 0.54$ g kg$^{-1}$ and $q_{ST} (=q_{00}) = 0.16$ g kg$^{-1}$ for an increase of 0.38 g kg$^{-1}$. Therefore, an additional 0.13 g kg$^{-1}$ is removed from the feeder zone in the seeded case. The additional 0.13 g kg$^{-1}$ of snow present at the base of the feeder cloud in the seeded case is accounted for by the removal of water vapor that was stored as cloud water in the unseeded case (see Fig. 5). The snow produced in the feeder cloud in the absence of seed crystals was not capable of efficiently removing the condensate and, therefore,
cloud water was stored in the feeder and not converted to precipitation.

In the seeded case, larger depositional growth rates, due to the presence of seed crystals, more efficiently removed the condensate in the feeder zone.

As the feeder zone becomes colder, it becomes increasingly capable of supplying sufficient quantities of ice to efficiently remove the condensate, thereby lessening the impact of any seed crystals from above. For a warmer feeder zone, on the other hand, seed crystals supply the required quantities of snow to efficiently remove the condensation. The extreme case of a warm-cloud feeder zone, capable of producing no ice on its own, will be considered later in our discussion of the Type 2 situation.

Shown in Fig. 8 are profiles of the mass of snow per unit volume of air $M_S$ computed from the model and values of this parameter deduced by Herzegh and Hobbs (1980) from radar measurements. The profile derived from the model is a composite based on several vertical profiles taken within the seeded region of the feeder zone. Increases in $M_S$ with decreasing height are due to particle growth, which the model computations showed took place through the deposition of water vapor onto the crystals from the feeder cloud. Using the profiles of $M_S$ shown in Fig. 8, we can estimate the contribution of the feeder zone to the precipitation. The Herzegh–Hobbs profile shows that 80% of the mass of the precipitation present at the 2.5 km level was produced within the feeder zone, compared to 75% predicted by the present model.

Shown in Fig. 9 is the reflectivity profile at $x = 55$ km predicted by the model. The radar reflectivity was computed in the model using the relations

$$\text{dB}(Z_R) = 42.2 + 16.8 \log M_R, \quad (M_R \text{ in g m}^{-3}) \quad (8b)$$

for rain, which is a rearranged version of the $Z-R$ relation for stratiform rain given by Marshall and Palmer (1948).

The effects on radar reflectivity of water-coated ice particles, particle aggregation and the collapse of wet snowflakes into drops, all of which contribute to the radar "bright band," are not included in our parameterization. Therefore, in order to obtain a more realistic radar reflectivity profile near the 0°C level, we include, in a similar manner to that described by Cheng (1981), the "equivalent rain profile" and the "interpolated profile."

The "equivalent rain profile" is computed assuming the radar reflectivity of snow just above the 0°C level is the same as that for rain, with $M_S$ replacing $M_R$ in 8b. This is equivalent to assuming that the snow particles present just above the 0°C level are coated instantaneously with water as they pass through the 0°C level. This process gives the 36 dB(Z) value at 2 km in Fig. 9. Furthermore, strong aggregation of snow particles is likely just above the 0°C isotherm (Hobbs, 1974), the "interpolated profile" is added to simulate the increase in reflectivity due to this process.

It can be seen from Fig. 9 that the largest increase in radar reflectivity with decreasing height (apart from the melting region) occurs in the −16 to −10°C layer where the depositional growth rate (PSDEP) is greatest. Below 2 km, where the air is below saturation with respect to water, the evaporation of melted snow produces a decrease in radar reflectivity with decreasing height.

Herzegh and Hobbs (1980) suggested that in this case study the growth of the seed particles within the
feeder zone took place primarily by vapor deposition. The average precipitation rate at cloud base predicted by the present model (1.2 mm h\(^{-1}\)) agrees well with the values of 1.2–2.0 mm h\(^{-1}\) derived by Herzegh and Hobbs (1980) from field measurements for this case. These measurements represent both the precipitation rates from the combined seeder–feeder system and that from the surrounding stratiform (or feeder) cloud only. Although the combined seeder–feeder cloud system was fairly deep (5 km), the precipitation rate was small due to the slow growth of the seed particles within the feeder zone. The slow growth rate, in turn, was a consequence of the gentle updrafts and low liquid water content.

Observational studies indicate that the seeder–feeder mechanism also operates in wide cold-frontal rainbands (Hobbs, 1978). In the wide cold-frontal rainband documented by Hobbs et al. (1980), depositional growth of seed crystals in a feeder cloud was deduced to have increased the snowfall rate by a factor of four. This is similar to the increase in snowfall rate predicted by the model described in this section.

Several model simulations were conducted in which the concentrations of ice nuclei (and hence cloud ice) were either one, two, or three orders of magnitude above those given by Fletcher’s (1962) relation [Eq. (A13) in Appendix A]. All other input parameters remained unchanged.

When the concentration of cloud ice was increased by one and two orders of magnitude, there were no appreciable changes in any of the model outputs, such as those shown in Figs. 3–7. Depositional growth of the seed snow was the dominant growth mechanism. However, when the concentration of cloud ice was increased by three orders of magnitude above that given by Eq. A13, noticeable changes occurred in the model outputs. Shown in Fig. 10 are the primary cloud microphysical processes in their approximate location within the feeder zone when cloud ice was assumed to be present in concentrations that are three orders of magnitude above that given by Eq. A13. From the base of the seeder zone down to 4 km, the dominant snow source within the seeded portion of the feeder zone is now the conversion of cloud ice (PCONV). Because of the high concentrations of cloud ice in this region (\(\sim 200 \text{ L}^{-1}\)), the depositional growth of these smaller particles (PDEPI \(\sim 10^{-4} \text{ g kg}^{-1} \text{s}^{-1}\)) removes the bulk of the water vapor made available by vertical motions within the feeder zone. Consequently, the depositional growth of the seed snow crystals (PSDEP), which dominated this region in the earlier simulation, is suppressed.

The snowfall rate at the base of the seeded portion of the feeder zone remained nearly the same in all cases. When cloud ice was assumed to be present in concentrations up to two orders of magnitude above those given by Eq. (A13), the model showed the depositional growth of snow (PSDEP) extending throughout the depth of the feeder zone, as shown in Fig. 7, and it was the dominant source of snow mass. However, for high concentrations of cloud ice

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**Fig. 10.** Schematic showing approximate locations of cloud microphysical processes within the feeder zone when the concentrations of cloud ice were increased by three orders of magnitude above those given by Eq. (A13). Growth terms are in units of g kg\(^{-1}\) s\(^{-1}\). Symbols are explained in the text.
particles, conversion of cloud ice to snow (PCONV) and the collection of cloud ice (PSACI) replaced depositional growth (PSDEP) as the main source of snow mass in the upper regions of the feeder zone (Fig. 10). However, the sum of PCONV and PSACI in this case was nearly the same as the value of PSDEP, so they contributed equally to the snow mass.

The presence of high concentrations of cloud ice simply changes the manner in which the condensate produced by the feeder zone is converted into precipitation. In the case of large concentrations of cloud ice particles, vapor is removed by particles produced within the feeder zone; when lower concentrations of ice are present, vapor is removed by particles produced within the seeded zones. Since in both cases sufficient ice particles are present to efficiently remove the condensate, no increase in the precipitation rate is expected.

The only increase in snowfall rates was outside the seeded portion of the feeder zone. Here the mixing ratio of cloud ice increased rapidly by deposition (PDEPI), and there was conversion of cloud ice to snow (PCONV). The conversion of cloud ice to snow in regions outside the seeded portion of the feeder zone took place at a larger rate than within the seeded zone, hence sufficient snow was present to efficiently remove the condensation. When cloud ice was present in low concentrations, the model shows that the condensate in the unseeded feeder zone is stored as cloud water (Fig. 5) and does not add to the precipitation. Since it is unlikely that cloud ice would be present in high concentrations in a weak stratiform cloud, we consider the model results when cloud ice is present in the lower concentrations [where the dominant snow growth mechanism is deposition (PSDEP)] to be more realistic.

b. Type 2: The seeder-feeder process with updrafts on the mesoscale

We turn now to the model simulations for the case where the ice particles from a seeder zone fall into a feeder cloud that is characterized by updrafts on the mesoscale, as documented in the case study described by Houze et al. (1981). The important mechanisms by which precipitation grows in this case are found to be quite different from those deduced in Section 3a.

Shown in Fig. 11 is a schematic of the model domain for this simulation. At the 3.5 km level, individual trails from generating cells are assumed to merge into a continuous pattern in the horizontal. The vertical velocity profile extends continuously within the dashed region enclosing the feeder zone.

Using the radar reflectivity measurements at the 3.5 km level reported by Houze et al. and Eq. (7), we compute the snow mixing ratio at this level. We take this to be the input \( q_{00} \) of the feeder cloud into the top of the feeder cloud. The model is run for two values of \( q_{00} \): 0.86 g kg\(^{-1}\) [corresponding to 25 dB(Z)] and 1.14 g kg\(^{-1}\) [27 dB(Z)]. Ice particle concentrations from the seeder zone, based on these computed values of \( q_{00} \), are 5.7 and 6.2 L\(^{-1}\), respectively. For this Type 2 study, \( N_{05} = 8 \times 10^6 \) m\(^{-4}\). The vertical air velocity in the feeder zone is taken to be a smoothed profile of the measured values given by Houze et al. (their Fig. 4a), which range from 0.1 to

![Fig. 11. Schematic showing the x-z domain used in the model for the seeder-enhanced feeder simulation, the Type 2 situation. Profile of vertical wind \( w \) is from Houze et al. (1981) and extends throughout the dashed region.](Image)
0.6 m s\(^{-1}\). Outside of this region of strong ascent, we assume a vertical velocity of 0.05 m s\(^{-1}\). Results from the model run with \(q_{v0} = 1.14 \text{ g kg}^{-1}\) will be discussed in detail, but various parameters from both model runs will be presented in tabular form for comparative purposes.

All results to be discussed are steady-state values after 3000 s of integration time, considerably less than that required for the Type 1 case. There are two reasons for this. First, in the Type 2 case the feeder zone reaches saturation rapidly due to strong mesoscale updrafts. Second, because of the shallower depth of the Type 2 feeder zones (1.5 km as compared to 3 km in the Type 1 case), less time is required for snow particles from the feeder zone to pass through the feeder zone. Furthermore, since the level at which snow is initiated in the Type 2 case (3.5 km) is closer to the surface than in the Type 1 case (5.6 km), the particles reach the ground sooner.

Shown in Fig. 12 are the mixing ratios of snow and rain water in the \(x-z\) plane for the \(q_{v0} = 1.14 \text{ g kg}^{-1}\) case. The snow mixing ratio increases from its initial value at the top of the feeder cloud (shown as 1.0 g kg\(^{-1}\)) to a peak value of 2.2 g kg\(^{-1}\) just prior to melting. The snow mass is present uniformly in values of 2.0 g kg\(^{-1}\) just above the 2 km level over a horizontal distance of nearly 25 km.

Rapid melting takes place below the 0°C level, as evidenced by the rapid decrease in the snow mixing ratio. Maximum values of the rain-water mixing ratio \(q_r\) (0.4 g kg\(^{-1}\)) are located directly below the maximum values of \(q_v\). The \(q_r\) values decrease to 0.2 g kg\(^{-1}\) outside the region from \(x = 50\) to \(x = 80\) km; these lower values of \(q_r\) are due to snow from the feeder zone melting and falling to the surface without passing through the feeder zone.

Shown in Fig. 13 are the precipitation rates due to snow and rain. The snowfall rate increases uniformly from 4 mm h\(^{-1}\) at 3 km to 6 mm h\(^{-1}\) at 2.5 km, and then more rapidly to \(\geqslant 8\) mm h\(^{-1}\) just prior to melting. The maximum rate obtained in the snow is 8.5 mm h\(^{-1}\).

Fig. 13. Precipitation rates for the seeder-enhanced feeder simulation. Dashed line is precipitation rate of snow (mm hr\(^{-1}\) of water) and the solid line is the precipitation rate of rain (same units).
The rain rates are nearly equivalent to the rates in the snow above. The peak rainfall rate at cloud base (1 km) was 8.6 mm h⁻¹, therefore no significant increase in the precipitation rate occurred between the 0°C level (at 2.0 km) and cloud base. Hence, the bulk of the precipitation growth in the feeder zone took place between the 3.5 km level (top of the feeder zone) and the 0°C level (which was 1.5 km below).

Shown in Fig. 14a is the cloud liquid water content in the feeder cloud. Cloud water is absent in the upper 500 m of the feeder zone, at lower levels, where the vertical velocities as well as the water vapor mixing ratio increase, cloud liquid water appears. Maximum liquid water contents are located just below the 0°C level, and coincide with the maxima in the condensation rate (Fig. 14b). The condensation rate reaches peak values in this region due to strong cooling caused by the melting snow.

Shown in Fig. 15 are the important growth mechanisms within the feeder cloud. In the upper 500 m of this zone, where cloud water is absent, depositional growth of snow (PSDEP) is dominant. The maximum growth rate is $4 \times 10^{-4}$ g kg⁻¹ s⁻¹ (Fig. 15a). As the cloud liquid water begins to increase from 0.05 to 0.3 g kg⁻¹ (Fig. 14a), the snow mass begins to increase due to the accretion of cloud water (PSACW), as shown in Fig. 15b. At 3 km, the accretion rate of cloud water by snow is as large as the maximum values of the depositional growth rate. Hence, in this case, unlike that described in Section 3a, cloud water in the feeder zone is not depressed below the point that growth by riming is significantly hindered. At lower levels in the feeder zone, where larger amounts of cloud water are found, the riming rate increases rapidly to peak values of $14 \times 10^{-4}$ g kg⁻¹ s⁻¹ and it is generally a factor of two to three greater than the depositional growth term. It is within this region that the most rapid increase in the snow mixing ratio (and hence precipitation rate) occurs within the entire feeder zone. Rapid melting of the snow occurs within an average depth of 400 m, as shown in Fig. 15c. This melted snow is a direct source for rain. Some additional growth takes place below the 0°C level where rain collects cloud water, as shown in Fig. 15d. However, this growth is of less importance than the growth of precipitation above the 0°C level (where snow grows rapidly by the accretion of cloud water).

Throughout the entire feeder zone the cloud water content $q_c$ was well below the assumed threshold value of 0.7 g kg⁻¹. Therefore, the model shows no production of precipitation within the feeder cloud itself. Instead, the precipitation was a direct result (and originated on) the particles from the feeder cloud aloft. In reality, some liquid-phase precipitation might
FIG. 15. Cross sections of various sources and sinks of snow and rain for the seeder-enhanced feeder simulation: (a) depositional growth of snow PSDEP, (b) accretion of cloud water by snow PSACW, (c) melting of snow PSMLT and (d) accretion of cloud water by rain PRACW. All in units of $10^{-4}$ g kg$^{-1}$ s$^{-1}$. 
have been produced in the feeder zone by condensation onto large nuclei and chance coalescence, but this was probably a small contribution to the overall precipitation rate.

Using the results shown in Fig. 12 we can estimate the fraction of the precipitation mass content contributed by the feeder zone above the melting level. The snow mixing ratio at the top of the feeder zone is 1.14 g kg$^{-1}$, and just prior to melting it has an average value of 2.1 g kg$^{-1}$. Therefore, 0.96 g kg$^{-1}$ was due to growth within the feeder zone. Hence, approximately 50% of the total precipitation mass present just before melting was acquired by growth of the seed crystals within the feeder cloud.

We now compare these model results with the measurements described by Houze et al. (1981). They concluded that the main process responsible for the enhancement of precipitation in the warm-frontal rainband was the collection of condensate within the mesoscale feeder zone by snow particles from the seeder zone (before, during and after their melting). As we have seen, the model results also show snow particles from the seeder zone first growing by deposition in the feeder zone and then growing rapidly by riming.

Results from the model simulation are compared with those from the field study in Table 3. The average surface rainfall rate of 7.4 mm h$^{-1}$ shown by the model between $x = 50$ and $x = 85$ km compares well with the measured value of 8 mm h$^{-1}$. Reducing the input snow mixing ratio $q_{s0}$ from 1.14 to 0.86 g kg$^{-1}$ reduces the surface rainfall rate to 6.6 mm h$^{-1}$. The average surface rainfall rate predicted by the model for $q_{s0} = 1.14$, without a feeder cloud being present, is 3.7 mm h$^{-1}$. Clearly, both the seeder and feeder clouds are necessary to reproduce the observed rainfall rate at the ground. The total condensation rate (PCOND) in the feeder zone is greater by a factor of two than the condensation rate deduced by Houze et al. from rawinsonde measurements. However, in view of the uncertainties associated with deducing condensation rates from rawinsonde data, this difference is probably not significant.

A further check on the representativeness of the model results is provided by a comparison of the vertical profiles of the radar reflectivities computed from the model and the field measurements for this case (Fig. 16). The measurements were obtained when the rainband passed over Point Brown on the Washington Coast. This occurred ½ h after detailed measurements were obtained of the air motions which were used in the model calculations. However, since the rainfall rate was nearly constant throughout this period (and presumably the air motions near steady-state) this comparison seems justified.

Apart from the layer between 1.5 and 2.3 km, there is excellent agreement between the radar reflectivity profile calculated from the model and the measured reflectivities (Fig. 16). Reflectivity measurements below the 1 km level were not reliable when the radar was pointing vertically.

As in Section 3a, there are significant differences between the initially computed radar reflectivities and those in the vicinity of the “bright-band” (1.5–2.3 km). However, these differences were largely removed by applying the same method as that described in Section 3a to allow for the effects of melting. Thus, assuming that the radar reflectivity of the snow just above the 0°C level is the same as that for rain, we obtain a maximum predicted reflectivity of 47 dB(Z). This is the “equivalent rain profile” in Fig. 16 and it agrees well with the measured value. To simulate the increase in reflectivity due to particle aggregation, the “interpolated curve” is added. This curve, together with the equivalent rain curve and the remainder of the profile computed by the model, pro-
vides model predictions of the radar reflectivity in a vertical profile through the rainband.

4. Summary and conclusions

A parameterized microphysical model has been developed and used to study the growth mechanisms leading to enhanced rainfall rates in two types of warm-frontal rainbands in which a seeder-feeder mechanism operates.

In the first model simulation (Type 1), the updraft velocities in the feeder cloud were typical of those observed in warm-frontal lifting. In the “seeded” region of the feeder zone, growth of the seed crystals by deposition efficiently removed the condensation provided by the updraft in the feeder zone. In regions of the feeder zone where seed crystals were absent, sufficient quantities of ice were not present to utilize the available moisture, hence some condensate was stored as cloud water and did not precipitate. The feeder zone in the seeded region was glaciated. Mass added to seed crystals by deposition in the feeder zone accounted for ~75% of the total mass of precipitation reaching the ground, with the remaining 25% originating in the seeder clouds aloft.

In the second model simulation (Type 2), the feeder cloud was maintained by a strong mesoscale updraft approximately 30 km in width. In this case, the dominant mechanism for the growth of snow input from the seeder cloud was the collection of cloud water supplied by condensation within the updraft. The snow mixing ratio doubled within the feeder zone, which had a depth of 1.5 km. However, more than half of this increase occurred within a shallow layer approximately 500 m in depth, where the riming rates reached peak values.

In both simulations, the importance of the seed crystals was to provide sufficient quantities of snow to efficiently remove condensation products in the feeder zone.

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APPENDIX A

Cloud Physics Parameterization

In the model described in this paper the parameterization of cloud microphysical processes is similar to that described by Lin et al. (1983). In this parameterization, all fields are treated as bulk quantities, as described briefly below. Symbols and units are described in Appendix B.

a. Mass-weighted fallspeeds

All particles in the precipitating fields of rain and snow are assumed to fall at their mass-weighted fallspeeds. For rain this fallspeed is defined as

\[
\nu_R = \frac{\int_0^\infty N_{DR}(D_R)M(D_R)V_R(D_R)dD_R}{\int_0^\infty N_{DR}(D_R)M(D_R)dD_R},
\]

where, \(M(D_R) = \pi/6(\rho_l D_R^3)\). A polynomial fit to the experimental data of Gunn and Kinzer (1949) yields

\[
V_R(D_R) = -0.267 + 51.5D_R - 102.25D_R^2 + 75.5D_R^3,
\]

where the drop diameter \(D_R\) is in cm and \(V_D\) is in m s\(^{-1}\). From (A1), (A2) and Eq. (1) in the text:

\[
\nu_R = (-0.267 + 206\lambda_R^{-1} - 2.045 \\
\times 10^3 \lambda_R^{-2} + 9.06 \times 10^3 \lambda_R^{-3})\left(\frac{p_0}{p}\right)^{0.4},
\]

where \(\lambda_R\) (cm\(^{-1}\)) is given by Eq. (3a).

For snow, we assume \(M(D_S) = \pi/6(\rho_s D_S^3)\) and that the snow crystals consist of aggregates of dendrites for which (Locatelli and Hobbs, 1974):

\[
V_S(D_S) = a^*D_S^b\left(\frac{p_0}{p}\right)^{0.4}.
\]

The factor \((p_0/p)^{0.4}\) allows for the change in fallspeed with air pressure (Foote and DuToit, 1969). Dendrites and aggregates of dendrites were the dominant crystal types observed from aircraft for both case studies discussed in this paper.

From (A1), (A4) and Eq. (2):

\[
\nu_s = a^*\frac{\Gamma(4 + b)}{6} \lambda_s^{-b}\left(\frac{p_0}{p}\right)^{0.4},
\]

where \(\lambda_s\) is given by Eq. (3b).

b. Sources and sinks of the water continuity variables

1) CONDENSATION AND EVAPORATION OF CLOUD WATER (PCOND)

Following Yau and Austin (1979), we express the condensation of water vapor to cloud water (PCOND) as

\[
PCOND = \rho(q_v - q_{sv})\left[\Delta t\left(1 + \frac{L_v}{c_p R_w T^2}\right)^{0.5}\right].
\]

If \(q_v < q_{sv}\), the cloud water evaporates.

The formulation of microphysical rates, such as Eq. (A6), which has \(\Delta t\) in the denominator, may result in the model outputs being sensitive to the chosen time step. To investigate this we used time steps of
2 and 5 s. Over this range of values for $\Delta t$, there were no changes in the various model outputs. However, in applying models of this type to other situations (i.e., strong convective clouds) it would be wise to formulate microphysical conversion rates that are independent of $\Delta t$. These cautionary remarks also apply to Eqs. (A14), (A15), (A19), and (A28) below.

2) AUTOCONVERSION OF CLOUD WATER (PRAUT)

Autoconversion is the process whereby cloud water droplets form raindrops through collisions with each other. Following Kessler (1969), it is parameterized as

$$PRAUT = \alpha \rho (q_c - q_0),$$

where $\alpha$ is a rate coefficient and $q_0$ the mass threshold value for autoconversion. Weinstein (1970) showed from a sensitivity analysis that $q_0$ is the crucial parameter in this parameterization. Order of magnitude variations in $\alpha$ do little to alter the overall microphysics. We use the value of $\alpha$ given by Kessler (1969) and have deduced a value of $q_0$ from airborne observations of cloud water and rain water contents in warm stratiform clouds.

3) COLLECTION OF CLOUD WATER BY RAINWATER (PRACW)

The collection of cloud water is assumed to follow the continuous collection equation:

$$\frac{dM(D_R)}{dt} = \frac{\pi}{4} \rho D_R^2 V_R(D_R) q_c E_{RC},$$

(A8)

Multiplying (A8) by Eq. (1) and integrating yields

$$PRACW = \frac{\pi}{4} \rho q_c E_{RC} N_{OR} (\frac{p_0}{p})^{0.4} \times \left[ \frac{a_0 \Gamma(3)}{\lambda_R^3} + \frac{a_1 \Gamma(4)}{\lambda_R^4} + \frac{a_2 \Gamma(5)}{\lambda_R^5} + \frac{a_3 \Gamma(6)}{\lambda_R^6} \right].$$

(A9)

4) EVAPORATION OF RAINWATER (PREVP)

The evaporation of rainwater (PREVP) is calculated if the air is subsaturated with respect to water and the evaporation of cloud water (PCOND) is insufficient to remove the subsaturation. Similarly, if the air is above water saturation, growth by condensation occurs. From Byers (1965):

$$\frac{dM(D_R)}{dt} = \frac{2\pi D_R(S - 1)F}{A' + B'},$$

(A10)

where

$$A' = \frac{L_v}{K_o T} \left( \frac{L_v M_w}{R^{*} T} - 1 \right)$$

and

$$B' = R^* T / \chi M_w e_{sw}. \text{ We use the value of } A' \text{ given by Pruppacher and Klett (1978). The factor } F, \text{ which allows for ventilation of the drop, is given by (Beard and Pruppacher, 1971):}$$

$$F = 0.78 + 0.31 S_c^{1/3} R_c^{1/2}.$$

(A11)

The total evaporation is then found by substituting (A11) into (A10), multiplying by Eq. (1) and integrating over all drop sizes, to give

$$PREVP = \frac{2\pi N_{OR} (S - 1)}{A' + B'} \times \left[ \frac{0.78}{\lambda_R^2} + 0.31 \left( \frac{a^2 \rho / \mu}{\lambda_R^3} \right) \Gamma(3) \left( \frac{p_0}{p} \right)^{0.2} \right].$$

(A12)

To make the integration more manageable, we write $V_R(D_R) = a' D_R$, which still gives good accuracy.

5) INITIATION OF CLOUD ICE (PINT)

The initiation of cloud ice in our model follows that discussed by Stephens (1979). The cloud ice phase is initiated by assuming the immediate presence of small plate-like ice crystals whenever the air is saturated with respect to ice and $T < 0^\circ$C. These crystals have an initial diameter $D_0$ of 12.9 $\mu$m which corresponds to an approximate mass $M_0$ of $10^{-15}$ kg. The concentration of these crystals is assumed to be given by the concentration of ice nuclei active at temperature $T$ (Fletcher, 1962):

$$n_c = n_0 \exp[\beta(T_0 - T)].$$

(A13)

Following Stephens the rate at which these small ice crystals are initiated is given by

$$PINT = \frac{M_0 n_c}{\Delta t},$$

(A14)

where $\Delta t$ is the time step. At low temperatures, where large concentrations of ice nuclei are active, this rate can exceed the amount of water vapor available for crystal growth. Therefore, this rate is compared with the maximum amount of vapor available for growth and the lesser of these two values is chosen for the initialization term:

$$PINT = \min \left\{ \frac{M_0 n_c}{\Delta t}, \frac{\rho (q_e - q_0)}{\Delta t} \right\},$$

(A15)

where $q_{ni}$ is the saturation mixing ratio with respect to ice.

6) DEPOSITIONAL GROWTH OF CLOUD ICE (PDEPI)

The growth rate by vapor deposition of a small ice crystal is given by

$$\frac{dM}{dt} = \frac{C(S_c - 1)}{e_0} A'' + B'' + B^*,$$

(A16)

where

$$A'' = \frac{L_v}{K_o T} \left( \frac{L_v M_w}{R^{*} T} - 1 \right)$$

and

$$B'' = R^* T / \chi \lambda M_w e_{sw}. \text{ The factor } A'', \text{ which allows for deposition of the drop, is given by (Beard and Pruppacher, 1971):}$$

$$F = 0.78 + 0.31 S_c^{1/3} R_c^{1/2}.$$

(A11)
ice crystals, so \( C = 4D_1 \rho_0 \), where \( D_1 \) is the average diameter of the cloud ice particles. From Hobbs et al. (1972) the diameter \( D_1 \) of a hexagonal plate can be computed from the mass \( M_t \) of the plate:

\[
D_1 = 16.3M_t^{1/2}, \tag{A17}
\]

where \( D_1 \) is in meters and \( M_t \) in kg. We compute \( M_t \) from \( q_i \sigma/n \) and use (A17) to calculate \( D_1 \) since the cloud ice crystals present at a gridpoint are assumed to be monodispersed. Therefore, the growth rate of cloud ice via deposition is

\[
PDEPI = \frac{4D_1(S_t - 1)\eta_c}{A^* + B^*}. \tag{A18}
\]

7) Conversion of cloud ice to snow (PCONV)

The conversion of cloud ice to snow is computed whenever the average cloud ice crystal mass \( M_t \) has exceeded a maximum allowed crystal mass (\( M_{\text{max}} \)), which corresponds to the mass of a 500 \( \mu m \) diameter ice particle. Hence from (A17), \( M_{\text{max}} = 9.4 \times 10^{-10} \) kg. This conversion term transfers the excess cloud ice mass to snow such that the remaining cloud ice mass has a maximum average diameter of 500 \( \mu m \). The formulation is

\[
PCONV = \rho(q_i - q_{i\text{max}})/\Delta t \tag{A19}
\]

where \( q_{i\text{max}} = M_{\text{max}} n_i/\rho \).

8) Collection of cloud ice by snow (PSACI)

The collection of cloud ice by snow (PSACI) is parameterized in the same manner as the collection of cloud water by rain water, namely, by using the continuous collection equation. Hence (A8) becomes

\[
\frac{dM(D_S)}{dt} = \frac{\pi}{4} \rho D_S^2 V_s(D_S) q_i E_{ST}. \tag{A20}
\]

Multiplying (A20) by Eq. (2), using (A4), and integrating over all particle sizes yields

\[
PSACI = \rho \pi a^* q_i E_{ST} N_{05} \left( \frac{p_0}{\mu} \right)^{0.4} \Gamma(b + 3) \frac{1}{\lambda_S^{b+3}}, \tag{A21}
\]

where \( \lambda_S \) is given by Eq. (3b).

9) Collection of cloud water by snow (PSACW)

For \( T < 0^\circ C \), the collection of cloud water by snow (PSACW) leads to growth by riming. However, for \( T \gg 0^\circ C \), PSACW represents the rate at which melting snow accretes cloud droplets, which is a source for rain. The parameterization is similar to (A21):

\[
PSACW = \rho \pi a^* q_i E_{SC} N_{05} \left( \frac{p_0}{\mu} \right)^{0.4} \Gamma(b + 3) \frac{1}{\lambda_S^{b+3}}, \tag{A22}
\]

where the continuous collection equation has been used.

10) Melting of snow (PSMLT)

All snow upon melting is assumed to contribute to rain. The snow melted per unit time is given by (Mason, 1971)

\[
\frac{dM_{\text{melt}}}{dt} = -\frac{2\pi}{L_f} K_a D_S(T - T_0) F', \tag{A23}
\]

where \( F' \) is a ventilation factor which is given by (Thorpe and Mason, 1966)

\[
F' = 0.65 + 0.44S_c^{1/3} R_e^{1/2}, \tag{A24}
\]

where \( R_e \) is now \( V_s(D_S) D_s p / \mu \).

Substituting (A24) into (A23), multiplying by Eq. (2) and integrating over all snow sizes we obtain for the melting of snow (PSMLT):

\[
PSMLT = -\frac{2\pi N_{05}}{L_f} K_a(T - T_0) \times \left[ \frac{0.65}{\lambda_S^5} + 0.44 \left( \frac{a^* p}{\mu} \right)^{1/2} \left( \frac{p_0}{\mu} \right)^{0.2} \frac{1}{\lambda_S^{b+3+5/2}} \right]. \tag{A25}
\]

An additional contributing factor to melting occurs when water vapor condenses onto the surfaces of the snow particles, thereby liberating the latent heat of condensation. This process is not included in the present model; it is included in the model discussed by Wisner et al. (1972).

11) Depositional growth of snow (PSDEP)

When the air is supersaturated with respect to ice, the growth rate of snow by deposition of vapor (PSDEP) is given by (A16) integrated over all sizes of snow particles. The capacitance is again \( 4D_{50} \). Multiplying (A16) by Eq. (2) and integrating we obtain

\[
PSDEP = \frac{4(S_t - 1) N_{05}}{A^* + B^*} \times \left[ \frac{0.65}{\lambda_S^5} + 0.44 \left( \frac{a^* p}{\mu} \right)^{1/2} \left( \frac{p_0}{\mu} \right)^{0.2} \frac{1}{\lambda_S^{b+5+5/2}} \right]. \tag{A26}
\]

12) Evaporation of melting snow (PMLTEV)

This term is identical to (A26) except that the evaporation is from a liquid surface and takes the form

\[
PMLTEV = \frac{4(S_t - 1) N_{05}}{A^* + B^*} \times \left[ \frac{0.65}{\lambda_S^5} + 0.44 \left( \frac{a^* p}{\mu} \right)^{1/2} \left( \frac{p_0}{\mu} \right)^{0.2} \frac{1}{\lambda_S^{b+5+5/2}} \right]. \tag{A27}
\]
13) MELTING OF CLOUD ICE (PSMLTI)

Melted cloud ice is a source for cloud water. This process is assumed to occur instantaneously and is given by

\[ PSMLTI = \rho q_i / \Delta t \]  \hspace{1cm} (A28)

for \( T \geq 0 \degree C \) only.

c. Source terms for the water continuity variables

The source terms for the five water continuity variables are listed below.

For water vapor \( q_v \):

\[ S_v = -[\text{PCOND} + \text{PREVP} + \text{PSDEP} + \text{PMLTEV} (T \geq 0 \degree C) + \text{PDEPI} + \text{PINT}] \]  \hspace{1cm} (A29)

For cloud water \( q_c \):

\[ S_c = \text{PCOND} + \text{PSMLTI} (T \geq 0 \degree C) - \text{PRAUT} - \text{PRACW} - \text{PSACW}. \]  \hspace{1cm} (30)

For cloud ice \( q_i \):

\[ S_i = \text{PDEPI} - \text{PSMLTI} (T \geq 0 \degree C) - \text{PSACI} - \text{PCONV} + \text{PINT}. \]  \hspace{1cm} (A31)

For rain \( q_r \):

\[ S_r = \text{PREVP} + \text{PRAUT} + \text{PRACW} - \text{PSMLT} (T \geq 0 \degree C) + \text{PSACW} (T \geq 0 \degree C). \]  \hspace{1cm} (A32)

For snow \( q_s \):

\[ S_s = \text{PSDEP} + \text{PMLTEV} (T \geq 0 \degree C) + \text{PSACI} + \text{PSMLT} (T \geq 0 \degree C) + \text{PSACW} (T < 0 \degree C) + \text{PCONV}. \]  \hspace{1cm} (A33)

Similarly, the source for \( T \) is:

\[ S_h = \frac{L_v}{c_p} (\text{PCOND} + \text{PREVP} + \text{PMLTEV}) + \frac{L_s}{c_p} (\text{PINT} + \text{PDEPI} + \text{PSDEP}) + \frac{L_l}{c_p} [\text{PSMLT} - \text{PSMLTI} + \text{PSACW} (T < 0 \degree C)]. \]  \hspace{1cm} (A34)

APPENDIX B

List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>SI units</th>
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<tr>
<td>A'</td>
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<td>m s kg⁻¹</td>
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<td>Thermodynamic term in PDEPI</td>
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<td>$R^*$</td>
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REFERENCES


Herzegh, P. H., and P. V. Hobbs, 1980: The mesoscale and mi-


