Horizontal Energy Propagation in a Barotropic Atmosphere with Meridional and Zonal Structure

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ABSTRACT

To investigate how the propagation of energy away from steady sources may be influenced by the horizontal structure of the time-averaged flow in the troposphere, solutions to a barotropic model are displayed and interpreted. The model is steady and linearized about a basic state which varies in latitude and longitude. Emphasis is placed on cases where the longitudinal variations are gradual.

Many of the results can be analyzed by locally applying tools developed for zonally symmetric basic states, e.g., ray tracing and refractive indices. Simple background flows are constructed which have longitudinally confined critical lines, reflective surfaces, wave guides, and regions conducive to propagation. An example is given in which longitudinal derivatives of the basic state are significant. The theory of ray tracing in a two-dimensional wind must be employed as an interpretive tool in this case. It is concluded that zonal variations tend to enhance poleward (equatorward) propagation in large-scale troughs (ridges).

Examples are also shown of propagation through observed January 300 mb mean flow. These experiments suggest that the southern flank of the Aslatic jet may act as a partial reflector, increasing the likelihood of resonance in the midlatitudes. The central tropical Pacific appears to have potential as a corridor for interhemispheric transmission, though during typical January conditions wavetrains emanating from the mid-latitudes are absorbed there.

1. Introduction

The subject of this paper is the propagation of wave energy away from steady sources in a barotropic, non-divergent atmosphere. Earlier work by numerous researchers, most of whom employed linear models with longitudinally invariant basic states, has indicated that the path taken by energy is greatly affected by the meridional structure of the mean flow through which it propagates. Here we will investigate the situation when there are also longitudinal variations in the background flow; note to what extent the behavior found in the earlier studies still exists in this more complicated setting, and gain some insight into how the zonal variations change the propagation path of the energy.

The notion that wave energy may be forced in one region of the atmosphere and then propagate to and influence other regions is often used to explain various aspects of atmospheric motion. Perhaps the most intense application of this concept has been in the study of the vertical propagation of planetary waves from the troposphere into the stratosphere. Over the years, as the study of vertical propagation has progressed, the structure of the assumed mean atmosphere has become more complicated. Charney and Drazin (1961) assumed the mean wind was a function of height only; Dickinson (1968) added meridional variations of a certain analytical form; numerical experiments (e.g., Matsuno, 1970) have allowed arbitrary meridional structure.

Research on horizontal propagation has undergone a similar development. As the subject has developed, the assumed structure of the background flow has become ever more complex, thus rendering the theory applicable to more and more realistic situations. Rossby's (1945) first work on energy dispersion assumed a homogeneous wind. He hoped that a study of energy propagation in atmospheric wave trains might suggest mechanisms for the generation of new waves and for changes in the amplitude of existing waves. To investigate the possible influence that mid-latitude disturbances could have on tropical flow, Bennett and Young (1971) considered cases with constant latitudinal shear. Studying the same problem, Charney (1969) assumed basic state winds which were an arbitrary but slowly varying function of latitude. By utilizing numerical techniques, Hoskins et al. (1977) were able to quantitatively investigate energy propagation through zonally symmetric flows which were based on observed time mean atmospheric wind profiles. This basic technique has since seen many applications, including Gross and Hoskins' (1979) investigation of the remote response of the atmosphere to steady large-scale mountain forcing, Opsteegh and yan den Dool's (1980) study of the at-
mospheric response to sea surface temperature anomalies as a function of season, and Webster's (1981) look at the dependence of the atmosphere's response to sea surface temperature anomalies on the latitude of the anomaly.

These and other investigations which utilized linear models with symmetric background flows have revealed that wave packets moving away from a source can display an interesting assortment of behavior which depends on the structure of the basic state. For example, studies concerning critical lines, that is surfaces at which a wave's phase velocity equals the basic state velocity (e.g., Charney, 1969, and Bennett and Young, 1971) typically show wave energy absorption at the critical line. Geisler and Dickinson (1974) produced reflections in their quasi-linear model when the meridional gradient of potential vorticity of the background flow vanished. Another phenomenon sometimes found in propagation experiments is that which Dickinson and Clare (1973) referred to as overreflection; under certain circumstances, the reflected branch of a wavetrain can actually have greater amplitude than the incident branch. A common attribute of wavetrains excited on the sphere is the arch-like routes which they follow; Longuet-Higgins (1964) showed that for a solid body rotation basic state, the path taken by a steady wavetrain is exactly a great circle.

All of the studies mentioned to this point employed basic states which were zonally symmetric. This greatly facilitated the interpretation of results because the model equation was then separable and so the application of a wide range of theory which has been developed for the one-dimensional case could be easily applied. However, as shown in the plot of January-mean 300 mb streamfunction in Fig. 1a, the environment through which a locally excited wavetrain must propagate in the atmosphere is definitely not zonally symmetric. Thus the applicability of the research with symmetric basic states to atmospheric conditions is in some doubt. Recently, investigations by Simmons (1982), Webster and Holton (1982), Karoly (1983) and the present author, as reported on in this paper, have been undertaken in order to determine the impact of longitudinal basic state variations on atmospheric energy propagation.

How do east–west variations affect wave propagation? It is possible that the symmetric component of the observed flow is strong enough that it will control the dispersion process so that the longitudinal waviness can be ignored. Another possibility is that the wavy variations can be viewed as a series of essentially zonal regions in each of which the zonally symmetric theory holds true. A third possibility is that propagation through a wavy basic state may have little resemblance to propagation through a zonally symmetric state. In the work reported on here these possibilities have been investigated by experimenting with a linear model in which the basic state can vary in both latitude and longitude. Section 2 describes this model, which is then used in Section 3 to examine propagation through various analytically specified basic states which are longitudinally invariant within regions whose longitudinal scale is large compared to the wavelength of the perturbation waves. The experiments suggest that within an invariant region waves propagate in the same manner as they would if the local basic state conditions held at all longitudes. Thus, many of the phenomena found in the earlier studies which utilized zonally symmetric states, like critical lines absorption, reflection, trapping and overreflection, can be identified in the more complex setting. Section 4 examines how the wavetrains are altered when longitudinal variations in the basic state are large enough to affect the local propagation prop-
properties of the model fluid. Examples of propagation through a longitudinal wavenumber-one basic state are presented and interpreted in terms of ray tracing concepts. The observed mean flow of Fig. 1 is also used as a basic state. Much of the behavior found in the simple analytic basic states is found to carry over to the much more complicated observed flow. The model solutions suggest that the mean wintertime pattern over the northern Indian Ocean tends to reflect wavetrains back into the Northern Hemisphere. They indicate that much of the equatorial region is a barrier to meridionally propagating steady wavetrains, but occasionally the central Pacific may act as a corridor between the hemispheres. Also, the exit region of the Asian jet may overreflect waves incident upon it. These findings are discussed and summarized in Section 5.

2. Model

The model used for this study of horizontal energy propagation is the steady barotropic vorticity equation on the sphere, linearized about a basic state which varies longitudinally as well as latitudinally. Because it has this combination of attributes, the model is easy to work with, it produces solutions which can often be interpreted in terms of linear theory and it contains the dynamical processes which earlier studies have shown to be important to the propagation of large-scale atmospheric waves. In particular, the work of Dickinson (1968) and more recently Hoskins et al. (1977) has indicated the importance of including the full spherical geometry in energy dispersion experiments. On the other hand, recent contributions by Hoskins and Karoly (1981) and Phillips (1982), who reported on experiments with baroclinic models, have suggested that wavetrains produced by isolated sources are primarily barotropic away from the forced region. Thus the use of a multi-level model in the present study of propagation does not seem necessary. Since we are finding only steady solutions, the fact that the model supports only Rossby waves should not affect the solutions under most circumstances. Possible consequences of omitting Kelvin and Rossby-gravity modes will be mentioned in Section 3.

In deriving the exact form of the equation to be used in the study, it is useful to begin with the steady, forced barotropic vorticity equation

\[ J(\psi, \nabla^2 \psi + f) + \alpha \nabla^2 \psi + K \nabla^4 (\nabla^2 \psi) = R, \tag{1} \]

where \( \psi \) is the streamfunction, \( f = 2 \Omega \sin \phi \), \( \Omega \) is the earth's rotation rate, \( R \) a forcing function, and

\[ J(A, B) = \frac{1}{r^2 \cos \phi} \left( \frac{\partial A}{\partial \lambda} \frac{\partial B}{\partial \phi} - \frac{\partial A}{\partial \phi} \frac{\partial B}{\partial \lambda} \right) \]

for longitude \( \lambda \), latitude \( \phi \) and earth's radius \( r \). Here \( \alpha \), the Rayleigh coefficient, has been given the value \( 1.57 \times 10^{-6} \text{ s}^{-1} [\text{about (7 days)}^{-1}] \) and is based upon the damping effect Lau (1979) estimated transient eddies have on the stationary flow. Also \( K = 2.34 \times 10^{16} \text{ m}^4 \text{ s}^{-1} \), the same value that Grose and Hoskins (1979) used to control the singularity present at critical lines. Notice that the so-called free surface or deformation term has been omitted; it has no effect on the steady solution.

Now if we separate the streamfunction into a part which contains most of the amplitude \( \tilde{\psi} \) and the remainder \( \psi' \), both of which are functions of latitude and longitude, so that

\[ \psi = \tilde{\psi} + \psi', \]

and if we let

\[ R' = R - J(\tilde{\psi}, \nabla^2 \tilde{\psi} + f) - \alpha \nabla^2 \tilde{\psi} - K \nabla^4 (\nabla^2 \tilde{\psi}), \]

and if products of primed terms are ignored, then (1) becomes

\[ J(\tilde{\psi}, \nabla^2 \tilde{\psi}') + J(\psi', \nabla^2 \tilde{\psi} + f) + \alpha \nabla^2 \tilde{\psi}' + K \nabla^4 \nabla^2 \tilde{\psi}' = R'. \tag{2} \]

Note that the basic state or background field \( \tilde{\psi} \) is not necessarily a free solution to (1), but instead can be thought of as a solution forced by

\[ J(\tilde{\psi}, \nabla^2 \tilde{\psi} + f) + \alpha \nabla^2 \tilde{\psi} + K \nabla^4 \nabla^2 \tilde{\psi}, \]

which represents a time mean climatological source of vorticity.

The method used to solve (2) is described in Appendix A. As part of the solution technique, fields are represented as series of spherical harmonics which in most cases are truncated triangularly at 20. Any deviation from this is noted at the appropriate place in the text. Part of the solution process requires representing fields on a grid made up of 128 longitudes by 64 Gaussian latitudes. All plots displayed in this paper are made from data on this grid.

3. Locally zonal basic states

a. Refractive index analysis

In this section, we will investigate the propagation of energy through a basic state which contains large regions with essentially no zonal variation connected by zones of gradual change. In analyzing the behavior of waves propagating through zonally invariant background flows, one customarily assumes that meridionally the basic state varies slowly so that the concept of a local wavenumber and frequency is applicable. Here this assumption will also be satisfied by longitudinal variations in the basic state. Where the background is locally independent of longitude, this condition is trivially satisfied. One might call these cases very slowly varying in longitude.

In regions where the basic state is zonally invariant, (2) retains only those terms present in a model lin-
earized about a symmetric basic state. Thus, as long as the longitudinal extent of a region of zonal invariance is large in comparison to the scale of perturbation quantities, it is reasonable to speculate that many of the propagation properties found in earlier studies which dealt with zonal basic states can be applied to these regions. A convenient way to classify some of these properties is to cast (2) in the form of a wave equation, as, e.g., Charney and Drazin (1961), Dickinson (1968) and Opsteegh and van den Dool (1980) did in their analyses of propagating waves. Thus, if we assume that all longitudinal derivates of the basic state vanish, then the homogeneous solution of (2) satisfies

$$\frac{\omega}{r^2 \cos \phi} \frac{\partial^2 \psi'}{\partial \lambda^2} + \frac{1}{r^2 \cos \phi} \frac{\partial \psi'}{\partial \phi} + \alpha \nabla^2 \psi' = 0,$$

where the angular velocity is

$$\tilde{\omega} = -\frac{1}{r^2 \cos \phi} \frac{\partial \psi}{\partial \phi},$$

the potential vorticity is

$$\tilde{q} = \frac{1}{r^2 \cos \phi} \frac{\partial}{\partial \phi} \left( \cos \phi \frac{\partial \psi}{\partial \phi} \right) + f,$$  \quad (3)

and the diffusion term has been omitted, for simplicity. Assuming a plane wave solution in the longitudinal direction so that

$$\psi = \Psi(\phi) e^{im\lambda},$$

and changing variables by letting

$$s = \ln \left( \frac{1 + \sin \phi}{\cos \phi} \right)$$

(which is equivalent to using a Mercator projection), we have

$$\frac{d^2 \Psi}{ds^2} + \rho^2 \Psi = 0,$$  \quad (4a)

where

$$\rho^2 = \frac{\cos \phi}{\omega - i \alpha m^{-1}} \frac{\partial \tilde{q}}{\partial \phi} - m^2.$$  \quad (4b)

Thus, $\rho$ can be thought of as a refractive index for meridional propagation. The solutions will be propagating where $\rho$ is real, and evanescent where it is imaginary. Even when dissipation is negligible, evanescent will occur where $\omega$ is very large, where $\tilde{q}$ and $\tilde{\omega}$ are of opposite sign, near the poles, and for large zonal wavenumbers. A particularly interesting special case is where $\tilde{\omega}$ vanishes, a so-called critical latitude, which usually separates a region of evanescence from a region of propagation. Dissipation cannot be ignored near a critical latitude. This can be seen by expanding one of the terms from Eq. (4b):

$$\frac{1}{\omega - i \alpha m^{-1}} = \frac{\omega}{\omega^2 + \alpha^2 m^{-2}} + i \frac{\alpha m^{-1}}{\omega^2 + \alpha^2 m^{-2}}.$$

When $\tilde{\omega}$ is small, the imaginary term in this expansion will be important, so the refractive index will have a large imaginary component and the solution to (4a) will be damped. Experiments by Hoskins et al. (1977), Grose and Hoskins (1979), Opsteegh and van den Dool (1980) and others, using models with zonally symmetric basic states, have all produced results which are consistent with this simple refractive index analysis.

Strictly speaking, the quantity $q$ defined by (3) is the potential or absolute vorticity for a zonally symmetric basic state. In a longitudinally varying basic state, the definition of potential vorticity normally includes the additional term,

$$\frac{1}{r^2 \cos \phi} \frac{\partial^2 \psi}{\partial \lambda^2}.$$

However, a hypothesis which will be tested at a number of points in this paper is that theories derived for zonally symmetric basic states can be applied locally to zonally asymmetric states whose longitudinal variations are gradual. For this reason the phrase "potential vorticity" will refer to the quantity defined in (3) whenever it appears in this paper.

b. Solid body rotation

As a basis for comparing more complicated experiments, it is useful to see how the refractive index analysis applies to a standard case where the background is longitudinally invariant solid body rotation. This basic state has been employed by many other researchers, so we can use their work as well as the refractive index concept to interpret this case.

In our standard case the solid body rotation background has a speed of about 32 m s$^{-1}$ at the equator. Its profile is illustrated in Fig. 2 where it is referred to as SBR. (Solid body rotation is often referred to as superrotation in the literature and in this paper.) The forcing for this case is centered at (50°N, 90°W) where it has a value of $1 \times 10^{-10}$ s$^{-2}$ (or roughly 1 day$^{-2}$). It is prescribed as a linear function of distance from the central point, decreasing to zero in about 1600 km. In our experiments, the zonal mean component is removed from this distribution.

In his study of planetary waves on the sphere, Longuet-Higgins (1964) found that Rossby wave packets propagating through solid body rotation follow great circle routes. This characteristic is evident in the solution of our standard case which is shown in Fig. 3. In their application of ray tracing techniques to propagation on the sphere Hoskins and Karoly (1981) noted that different rays emanating from a source will follow different great circles depending on the wavenumber of the packet. The slope of the ray will be proportional to meridional wavenumber and inversely proportional to the zonal wavenumber. Thus,
rays whose slopes have large meridional components near the vorticity source represent components of the disturbance which have small zonal wavenumber and by (4b) can be expected to propagate freely. On the other hand, rays leaving the source along nearly zonal paths correspond to large zonal wavenumber components and are thus evanescent. In our standard case, this dependence of propagation characteristics on wavenumber is manifested by a split in the energy as it disperses from the source. The void between the two wavetrains to the east of the source is the region crossed by the evanescent rays. The train to the north of the source contains only very large scales because, as (4b) indicates, only waves of very small wavenumber can propagate near the poles. The southern train contains smaller scales and thus cannot propagate as near to the pole as the northern train. The amplitude of each train tends to decrease with distance from the source because of the drag term in (2), though there is a tendency for amplification in regions where $\rho^2$ is small, a result explained by Hoskins and Karoly in terms of conservation of wave action.

\hspace{1cm} \textit{c. Critical lines}

The zonal mean troposphere has a band of easterlies which straddle the equator and play an important part in horizontal propagation studies because, as (4) suggests, they inhibit propagation. Simply stated, where the potential vorticity gradient is positive, Rossby waves are westward propagating relative to the basic current, so stationary Rossby waves can only exist when superimposed on an eastward moving flow. Furthermore, the refractive index analysis presented above suggests that a steady wavetrain in a dissipative system should be damped even in a region of weak westerlies. Given the zonal mean state of the atmosphere in the tropics, this has important consequences for the theory of the general circulation because it implies that interhemispheric interaction and tropical/midlatitude communication on long time scales are hampered, if not totally cut off. Research by Dickinson (1968) and Bennett and Young (1971), among others, which utilized models linearized about zonally symmetric states, supported this idea that there is a barrier to propagation at some “critical latitude” in the tropics.

This zonally symmetric picture may be misleading, however, because as Fig. 1b indicates, there is a longitudinal zone in the central Pacific in which normally there are westerlies during January. To test whether energy might be able to travel through such a zone of local westerlies, a basic state is constructed consisting of westerlies at all latitudes between 180 and 90°W and westerlies decreasing to weak easterlies at the equator in the band between 0 and 90°E. At other longitudes, these two extremes are linearly interpolated. The 180 to 90°W profile is labeled SBR in Fig. 2 and is the same one used in the solid body rotation experiment of Fig. 3; the 0 to 90°E profile is marked C in Fig. 2. Solving for the streamfunction representation of this basic state, as described in Appendix A, gives the streamfunction shown in Fig. 4a. The rapid change in the wind profile is difficult for the spectral model to resolve so in experiments with this background the total wavenumber truncation $N$
When a vorticity source is placed at (50°N, 120°E) as in Fig. 4b, there is a large Southern Hemisphere response because the wavetrain is in strong westerlies as it crosses the tropics. By contrast, Fig. 4d indicates that a source at (50°N, 60°W) produces an enhanced subtropical Northern Hemisphere response, but almost no response in the Southern Hemisphere. Between these extremes, a source at (50°N, 120°W) gives a moderate Southern Hemisphere response, as depicted in Fig. 4c. The wavetrain in this intermediate case propagates through a region where the basic state is not longitudinally invariant, but to a first approximation its characteristics are midway between the extremes of the cases in Figs. 4b and 4d. To quantify the behavior in these three cases, the ratio \( \Gamma \) of the mean square perturbation vorticity south of 30°S to the mean square perturbation vorticity north of 30°N was calculated for each case. For the experiments in Figs. 4b, c, and d, \( \Gamma \) has the values 0.127, 0.053, and 0.023 respectively.

Equation (4) merely suggests that a solution to (2) will decay in a region of easterlies or sufficiently light westerlies. Hoskins and Karoly's (1981) application of ray tracing theory to the zonally symmetric system indicates additional behavior which should be expected as a wavetrain nears a critical line. First, the analysis suggests individual elements of the train will become elongated in the east–west direction; second, the wavetrain path should tend to become meridional; and third, since the group velocity is proportional to the local background wind velocity, the group velocity becomes small, and damping reduces wave amplitude. It is interesting that this same behavior persists in the cases shown in Fig. 4 even though the zonally symmetric condition which the Hoskins and Karoly analysis was based on has been relaxed. As another example of this correspondence, \( \Gamma \) for the purely solid body rotation case of Fig. 3 is 0.307. Thus, propagation has been hampered even in the case of Fig. 4b, apparently because the basic state velocity at, say, (30°N, 150°W) is about two thirds of its value at this location in the solid body rotation background.

It is interesting to note the similarity between the absorption at a local critical line in Fig. 4 and a steady solution to a nonlinear, divergent barotropic model reported by Webster and Holton (1982) recently. They too found that a wavetrain forced by a mid-latitude source was absorbed when it encountered a region of equatorial easterlies, even if the easterlies are of limited longitudinal extent. Similarly, cross equatorial propagation was possible through a zone of westerlies, provided the westerlies were strong enough and the zone wide enough. The experiments illustrated in Fig. 4 confirm that neither nonlinearity nor divergent modes are required to explain such examples of local absorption and propagation.

With the basic state depicted in Fig. 4a, the possible
impact of local conditions in the tropics on transmission of energy from the equatorial region to the midlatitudes can also be examined. In Fig. 5a, the solution to (2) is shown when the forcing is centered at \((10^\circ N, 135^\circ W)\). Energy easily reaches the midlatitudes because the wavetrains are in westerlies. However, note that the northern ray, after being turned southward by the pole, is prevented from reaching the Southern Hemisphere by the local critical line east of the Greenwich meridian. In this case \(\Gamma\) is 0.640. It would be 1.00 if the source were on the equator, so a shift of 10 degrees in latitude can have a large impact on the amplitude of the midlatitude response. A similar sensitivity was found by Webster (1982) with a model linearized about a zonally symmetric state. This effect is even stronger when the source is shifted 180 degrees to \((10^\circ N, 45^\circ E)\) as in Fig. 5b. Again, propagation to the north is strong because much of the source resides in strong westerlies. However, the wavetrain immediately to the south of the source damps quickly, because there the background winds are very weak. In this case, \(\Gamma\) is 0.494, so there is an even greater difference between the northern and southern responses than in the previous example. But given the large difference in amplitudes of the wavetrains northeast and southeast of the source, \(\Gamma\) is larger than one might expect. The reason for the unexpectedly large perturbation variance in the Southern Hemisphere is that the wavetrain emanating northeastward from the forced zone follows its usual great circle route which leads it to the band of tropical westerlies. The westerlies allow it to continue southward so that this branch becomes a major energy source for the Southern Hemisphere.

(Though for the most part this paper is not concerned with the character of the local response to a source, it is worth noting that in these last two experiments the amplitude over the eastern tropical forcing is about three times that over the western forcing. This would seem to be because in the eastern case, both the basic state wind and meridional potential vorticity gradient are much smaller and so a stronger perturbation response is required to balance a given forcing. Taking this difference in the local response into account, propagation into the northern midlatitudes is actually more strongly damped in the easterly case (Fig. 5b) than in the case where westerlies pass over the forcing (Fig. 5a). In the former case, the first downstream high is about one quarter of the response over the source region, while in the latter it has retained about one half of its magnitude.)

As mentioned earlier, in order to simplify the model, it was made nondivergent. One would expect that if this simplification is of any consequence then it would be experiments dealing with equatorial response which would be most affected since Kelvin and Rossby–gravity modes have been left out of the model. It has been noted that there is little difference between the interhemispheric propagation depicted in Fig. 4 and that found by Webster and Holton (1982) with a shallow water model. On the other hand, their experiment with an equatorial source in a region of westerlies showed little midlatitude response, in sharp contrast to Fig. 5a. Tests (not shown) indicate essentially no difference between the response to equatorial forcing of a shallow water model linearized about the SBR profile and using an equivalent depth of 2000 m (just as Webster and Holton did) and the response of the nondivergent model linearized about the same state. Thus it seems unlikely that either the lack of Kelvin and Rossby–gravity modes or the slight distortion of the very long Rossby modes caused by the nondivergent assumption produced the discrepancy between our results and those of Holton and Webster. Rather, the difference in the midlatitude response to equatorial forcing in the two sets of experiments is likely a combination of the field forced (vorticity in our experiments, mass in Webster and Holton’s experiments), the variable displayed (vorticity versus zonal wind), the strength of the westerlies \((32 \text{ m s}^{-1} \text{ versus } 10 \text{ m s}^{-1})\), the meridional scale of the forced region \((1000 \text{ km versus } 500 \text{ km})\) and the close proximity of the source to a region of easterlies in the Webster and Holton experiments. Further evidence that energy can escape the tropics even when divergence is included in the calculation has been provided by Chang and Lim (1983) who found extensive midlatitude response to an equatorial source in their shallow water model on an equatorial \(\beta\)-plane.

**Fig. 5.** Experiments with the same basic state as Fig. 4a and tropical sources. (a) Perturbation vorticity when source is centered at \((10^\circ N, 135^\circ W)\). (b) Perturbation vorticity when source is centered at \((10^\circ N, 45^\circ E)\).
when it was linearized about a 10 m s\(^{-1}\) state, as long as the equivalent depth was at least 1500 m.

d. Reflecting regions

Held (1982) has pointed out how important reflecting surfaces in the tropics may be. Not only might they alter the path energy takes, but without them, wavetrains emanating from midlatitude sources either are absorbed in easterlies, or as we have seen above, propagate into the Southern Hemisphere. Neither type of behavior is conducive to the resonant responses one finds in more confining geometries. Critical lines are one flow configuration which may act as reflecting surfaces but there is much disagreement over this possibility (see Tung, 1979, for a discussion of some of the points related to this controversy). As we have seen in the previous subsection, they act as absorbers in the model used in this study, just as they do in similar models with zonally symmetric background flows (Dickinson, 1968; Bennett and Young, 1971). McIntyre (1982) has noted that regions of vanishing potential vorticity gradient could be effective means of enhancing resonant responses because they may act as reflectors. The simple refractive index argument above indicates that one would not expect propagation into a region of small potential vorticity gradient. In fact, carrying the refractive index analogy a step further, one would expect the wavetrain to bend away from such a region (not into it, as with a critical line). The ray tracing analysis of Hoskins and Karoly (1981) predicts this same tendency. For these reasons, experiments with background states which contain regions of zero potential vorticity gradient are included in this study.

For the reflection condition,

\[ \tilde{u} - \frac{\partial \tilde{u}}{\partial \phi} \leq 0, \]

to be sustained over an extensive latitude band, the basic state wind must undergo large latitudinal variations within that band. Thus, if one requires wind magnitudes to be limited to tropospheric-like values, the wind speed must be small somewhere in the band. Conversely, a region in which there are easterlies flanked by westerlies necessarily has points at which the meridional curvature of the zonal wind profile makes a negative contribution to the potential vorticity gradient. Hence, simply from geometrical considerations, it is apparent that critical lines and the reflection condition tend to go together. In addition, Geisler and Dickinson's (1974) investigation of Rossby waves interacting with a critical layer, suggests that there may be dynamical reasons for this common juxtaposition, in that in their experiments the interaction between critical layer and eddies tended to modify the flow in such a way that the potential vorticity gradient became negative. Comparison of the zonal component of the wind and the meridional gradient of potential vorticity for mean January 300 mb flow in Figures 1b and 1c, especially on the equatorial side of the Asian jet, supports the contention that critical lines and reflecting regions tend to occur together. But, as will be demonstrated in the next section, wave propagation in a region where both the reflection condition is satisfied and a critical line is present, can be quite different from propagation when only one of these conditions is satisfied. The critical line basic state in the previous subsection was prescribed in such a way that the reflection condition was not satisfied. At this point a background field will be constructed which does satisfy the reflection condition but in which there is no critical line.

Such a background can be created by assuming the basic state has no meridional velocity and the refractive index of (4b) vanishes for all scales. This combined with (3) and a requirement of equatorial symmetry gives

\[ \frac{\partial \tilde{u}}{\partial \phi} = 2 \Omega \tan \phi, \]

where

\[ \tilde{u} = \frac{1}{r} \frac{\partial \tilde{v}}{\partial \phi} \]

is the basic state zonal wind component. Integration of this with respect to latitude yields

\[ \tilde{u} = \Omega r \sin \phi \tan \phi + \tilde{u}_e \sec \phi, \]

where \( \tilde{u}_e \) is the prescribed equatorial wind speed. By setting \( \tilde{u}_e \) equal to 8.5 m s\(^{-1}\), the profile marked R in Fig. 2 is produced. Joining the tropical portion of this with the solid body rotation profile used in earlier experiments at 12.5°N and 12.5°S, where they intersect, produces a zonally symmetric background which should allow propagation in midlatitudes but which should cause reflection near the equator. This can be tested by using this basic state in (2) and placing a vorticity source at (50°N, 90°W). The solution, shown in Fig. 6a, when compared with Fig. 3, has much less perturbation vorticity variance in the Southern Hemisphere. In fact, \( \Gamma \) is 0.077 in this case while it was 0.307 for the superrotation case. In addition, the Northern Hemisphere midlatitude response has now acquired a longitudinal wavenumber 3 character. In fact, the variance north of 30°N has increased by about as much as the variance south of 30°S has decreased. These observations support the notion that a partial reflection has indeed occurred in this case; subtracting the solid body rotation response from this solution confirms it. This difference, displayed in Fig. 6b, indicates that the reflecting region has caused much of the southern wavetrain to be replaced by a mirror image in the Northern Hemisphere. The reflected wave is about 50% stronger than the decrease in the southern branch. Though rather modest, this increase does support Held's (1982) idea.
that reflection in the tropics could lead to a more resonant midlatitude response. One other characteristic of this response worth noting is the string of perturbations centered at 12.5°N. The infinitely large refractive index of the analytical basic state at 12.5°N is represented by our truncated model as a very large refractive index in the region near 12.5°N. This region tends to trap components of the perturbation solution with large zonal wavenumbers.

Next, constructing a basic state which satisfies the reflecting condition in only an interval of longitude, we can look at a case which is closer to actual atmospheric conditions. This background consists of the usual solid body rotation winds between 180 and 60°W, the same wind profile used in the zonally symmetric basic state reflecting experiment between 0 and 120°E, and linearly interpolated values elsewhere. A nondivergent background field based upon this distribution of zonal wind is calculated in the manner explained in Appendix A. This basic state is depicted in Fig. 7a. The model response to a source at (50°N, 150°E) shown in Fig. 7b resembles that of the pure superrotation case of Fig. 3. Here Γ has the value 0.284, much larger than the corresponding critical line experiment, presumably because the band of westerlies is wider here and because the westerlies are stronger since the model spherical harmonics can represent this background field more accurately. By moving the source to (50°N, 30°W), as in Fig. 7c, it becomes obvious that the local reflecting region acts much like the zonally symmetric one of Fig. 6a. Once again, the southern ray’s amplitude is decreased and the change in the predominant midlatitude zonal wavenumber is also apparent. The difference between this experiment and the solid body rotation result, shown in Fig. 7d, retains the mirror image character of the symmetric case, and the northern branch has more amplitude than the southern one it replaced.
The presence in part of the tropics of a region where propagation is possible does have an important consequence. Even though the decrease in the amplitude of the southern branches in Figs. 6b and 7d are essentially the same, $\Gamma$ now has a value of 0.120. Thus there is almost double the variance south of 30°S that there was in the zonally symmetric reflecting case. This increase of variance is because the reflected branch skirts the reflecting zone and then propagates through the westerlies into the Southern Hemisphere. For the level of damping used in this experiment, the reflecting surface has made no increase in the amount of energy which can travel around the globe and re-enter the forced region. Thus a reflecting surface in the equatorial region which only extends part way around the globe does not seem to be sufficient to form a resonant cavity in one hemisphere. Figure 1c suggests that this is the situation which exists under normal Northern Hemisphere wintertime conditions.

### Table 1. Mean January 300 mb values of zonal wind $u$ (m s$^{-1}$), meridional potential vorticity gradient $\partial u / \partial \phi$ (10$^{-8}$ s$^{-1}$) and smallest zonal wavenumber for reflected waves $m_n = (\omega^{-1} \cos \phi / \partial u / \partial \phi)^{1/2}$, at latitude $\phi$ (°N) and 60°E, $i = (-1)^i$.

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### e. Waveguides

In his work on wave propagation in the meridional plane, Matsuno (1970) found that vertically propagating waves tended to be channeled into the polar night jet. He attributed this to the negative latitudinal and vertical curvature of the mean winds in the region between the tropospheric and stratospheric jets, which leads to a relative minimum in the squared refractive index there. We might expect propagation in the vicinity of jets in the horizontal plane to exhibit similar behavior. Due to the curvature of the zonal wind profiles in zonally oriented jets, their axes are zones of relatively large potential vorticity gradients while their flanks have small potential vorticity gradients. (Figure 1c shows that the distribution of potential vorticity in the Asiatic subtropical jet is like this.) Thus, by (4), provided the strong winds in the jet core do not completely counteract the contribution of the potential vorticity gradient to the refractive index, a jet could act as a waveguide for all waves of sufficiently large zonal scale. Table 1 indicates that along a meridian passing through the Asian jet, the profiles of zonal wind and potential vorticity are such that it might act as a channel for propagating waves. South of the jet axis, the profile becomes reflective for all waves at 9°N, while to the north it reflects all scales at about 40°N. The center of the jet allows all but the very shortest waves to propagate. Profiles through other parts of this jet are similar except the latitudes of reflection tend to be farther north to the east and farther south toward the west. Table 1 also supports the contention that the reflecting condition and weak westerlies or easterlies tend to occur together, and the arguments of Dickinson and Clare (1973) suggest that this can have important consequences on energy propagation because it can lead to overreflection. The set of experiments described in this subsection demonstrates the waveguide characteristics of jet-like structures and indicates the possible importance of critical lines which are parallel to a jet.

As before, a simple background field is constructed which contains the properties of interest. The jet profile used in these experiments is marked J15 in Fig. 2. It consists of a wind maximum of 40 m s$^{-1}$ at 30°N which decreases linearly in both directions to a value of 20 cos(37.5°) m s$^{-1}$ at 37.5°N and 20 cos(22.5°) m s$^{-1}$ at 22.5°N. To the north and south of this are zones, each 25 degrees wide, in which the refractive index, (4b), is imaginary for all $m$. Profiles for these zones are derived from (4b) in the same way that (5) was except constants of integration are now determined by demanding continuity with the jet just described and with the solid body rotation which comprises the remainder of the profile. Unlike all other superrotations used in this paper, the one used in this basic state has a speed of 20 m s$^{-1}$ at the equator. The two-dimensional background consists of the J15 profile between 120°W and 0, solid body rotation winds between 60°E and 180°E, and linearly interpolated values elsewhere. After transforming this into a streamfunction representation, the basic state has the form pictured in Fig. 8a.

To verify that in fact this jet does act like a waveguide, a source of vorticity is placed at (30°N, 90°W), near the west end of the jet, and the steady solution to the model is calculated. Figure 8b, a depiction of this solution, shows how, in contrast to earlier experiments, the response is confined to a narrow band centered about the jet. Energy moves along the jet until it reaches the east end and then propagates both to the north and south once it reaches a longitude at which the reflecting condition is no longer satisfied.

A striking feature of this case is the large amplitude of the response. (Note that the contour interval in Figs. 8b and c is twice the standard interval.) To a
certain degree, the large amplitude occurs because the output from the source is confined to a relatively small area. A second possible reason is that the conditions for overreflection described by Dickinson and Clare (1973) and Lindzen and Tung (1978) are satisfied north of the jet. An overreflected wave extracts energy from the basic state; thus, overreflection could be the mechanism which leads to the large amplitude of our perturbation solution. Dickinson and Clare and Lindzen and Tung considered zonally symmetric basic states, but as we have noted before, results of such analyses often prove useful in interpreting our simulations with basic states which vary slowly in the zonal direction. Both of these investigations found that overreflection occurs at a critical line which is embedded in a region where $\phi$ is negative provided there is a source of wave activity in an adjacent region where propagation is possible. From Table 2, we see that these conditions hold in our waveguide jet because the source was placed on the jet axis where wave propagation can take place, the meridional gradient of potential vorticity is negative just north of the jet, and there is a zero wind line inside this reflecting region. Another requirement for overreflection is that in the region across the critical line from the wave source, the zonal mean meridional flux of zonal momentum must be small. In our example, the polar reflecting effect mentioned earlier satisfies this requirement. The southern side of the jet is also a reflecting region but probably not overreflecting. There is no reason to expect the wave momentum flux to be small south of this reflecting region. However, a wavetrain partially reflecting off the southern side of the waveguide and overreflecting from the northern side is a possible explanation for the large amplitudes in this case.

That the waveguide properties of the jet basic state are retained even if the forcing is external to the jet can be seen in Fig. 8c. In this experiment the vorticity source has been placed at (30°F, 150°W) so that it is upstream of the jet entrance and in the solid body rotation part of the background. The usual northern and southern wavetrains develop but in addition some of the wave energy enters the jet and excites the waveguide mode present in the earlier experiment. As before, this branch is channeled to about 60°F where it splits on exit from the guide. (The contour interval used in Fig. 8c prevents display of this split.) In this case, growth of the wave in the jet toward the east is evident, supporting the notion of overreflection.

A final experiment with waveguides is presented in order to indicate the sensitivity of the response to modest changes in the background. The basic state used is like the previous waveguide case except that the jet is widened so that it decreases linearly from 40 m s$^{-1}$ at 30°F to 20 cos(42.5°F) m s$^{-1}$ at 42.5°F and 20 cos(17.5°F) m s$^{-1}$ at 17.5°F. The reflecting regions are still 25 degrees wide and the same solid
body rotation used in the previous waveguide experiments fills out the field. Using this background, the model response to a source at (30°N, 90°W) is shown in Fig. 9. When compared with Fig. 8b, the result of widening the jet is striking. Most of the disturbance is still within the confines of the latitudes where the jet is, but the magnitude of the response is now much less, even smaller than when the source was placed upstream of the narrower jet. In addition, the amplitudes no longer increase to the east inside the waveguide. The parameters listed in Table 3 may help explain the smaller amplitudes in this experiment. In particular, the criteria for overreflection are no longer met because on both sides of the jet the reflection condition and the critical lines occur at about the same latitudes. Another characteristic of this response is the fact that the wavetrain continues to propagate zonally, even after the wavetrain has exited from the waveguide. The guide seems to act like a filter, removing those components which would normally propagate meridionally. This filtering may be because the region where propagation can occur in the interior of the jet is bordered by absorbing critical lines so that any waves in the response with meridional components to their trajectories will be absorbed.

4. Slow zonal variations

a. Idealized basic state

To this point in our effort to understand how energy can be expected to propagate in the presence of a zonally asymmetric basic state, we have concentrated on propagation in regions where the background field is locally longitudinally invariant. However, in Fig. 1a it is obvious that there are zones in observed flows where the curvature of the streamlines is great enough that zonal variations in the mean flow are comparable to the meridional variations. In this section, we will consider propagation through regions where streamline curvature is significant. Of course, the model behavior under these conditions is more difficult to interpret than it was when local zonal in-

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Table 3. Same quantities as those listed in Table 1 but for a cross-section at 65°W through the 25° wide jet profile described in the text.

\[
\psi' = \Psi \exp[\ii(kx + ly - \sigma t)],
\]

where \( \psi' \) is the basic state streamfunction, a slowly varying function of space. Following Lighthill's (1978) analysis of energy propagation in the presence of a wind which he based on Whitham's (1960) work on group velocity, we assume that locally \( \psi' \) can be represented by the plane wave solution

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\psi' = \Psi \exp[\ii(kx + ly - \sigma t)],
\]

where \( \Psi, k, l \) and \( \sigma \) are local amplitude, zonal wavenumber, meridional wavenumber and frequency, respectively. Then, as shown in Appendix B, the slope
of a ray as it crosses the axis of a symmetric, meridionally oriented trough or ridge\(^2\) is

\[
\frac{v_y}{u_x} = \frac{l}{k},
\]

(8)

where \(u_x\) and \(v_y\) are the two components of local group velocity. As is further shown in Appendix B, the change in the local wavenumber of a wave packet as it travels along such a ray is

\[
\frac{d_k}{dt} = \left( -l \frac{\partial^2 \psi}{\partial x^2} + l \frac{\partial^3 \bar{u}}{\partial x \partial y} (k^2 + l^2)^{-1} - l \frac{\partial \bar{u}}{\partial x} \right),
\]

(9a)

\[
\frac{d_k}{dt} = \left( \left( \frac{\partial^2 \psi}{\partial y^2} + k \frac{\partial^2 \psi}{\partial y^2} \right) (k^2 + l^2)^{-1} - k \frac{\partial \bar{u}}{\partial y} \right),
\]

(9b)

(where \(d_k/dt\) is the total derivative following the group velocity). Now (8) and (9b) are identical to the expressions which apply to zonally symmetric basic states. However, even with the simplifications assumed here, the expression for changes in the zonal wavenumber has been altered; in the zonally symmetric case \(k\) is constant along rays.

To help see the consequence of (9a), consider a basic state streamfunction

\[
\psi = \Psi_1 \sin ax \cos by - \Psi_2 \sin y,
\]

(10)

which satisfies the conditions of orientation and symmetry of troughs and ridges used in the derivation of (8) and (9). We must assume that

\[
a \ll k, \quad b \ll l,
\]

so that our analysis, which required that the background is slowly varying when compared to the perturbation scale, can be used. Inserting (10) into (9a) yields

\[
\frac{d_k}{dt} = a^2 l \Psi_1 (1 - \epsilon) \sin ax \cos by,
\]

(11)

where

\[
\epsilon = \frac{a^2 + b^2}{k^2 + l^2} < 1.
\]

Though (9), and thus (11), is only strictly valid on trough and ridge axes, it will be a useful indicator within some finite distance from an axis, just as (8) is. Thus, where \(\cos(by)\) is positive (negative), changes in the zonal wavenumber along a ray will have the same (opposite) sign as the meridional wavenumber in ridges and the opposite (same) sign in troughs. In summary, we can expect that near the axes of ridges and troughs the expressions for the slope of a ray (8) and the variation in meridional wavenumber along a ray (9) are unchanged from the zonally symmetric case, but the longitudinal wavenumber is no longer constant along a ray and its tendency can be determined.

In order to illustrate the effects of a wavy basic state upon the dispersion of energy away from an isolated source, we will show results from a set of experiments which utilize the state defined by (10) except that \(x\) has been replaced by \(a\) and \(y\) by \(b\). Here \(\Psi_1\) is chosen so that the maximum basic state meridional velocity is 10 m s\(^{-1}\), \(\Psi_2\) is given a value so that the average equatorial wind is about 32 m s\(^{-1}\), and \(a\) and \(b\) are set to 1. This background field is displayed in Fig. 10a.

In the first experiment (Fig. 10b), a vorticity source is placed on the equator at 90\(^\circ\)W. The centers of high and low for the solution when \(\Psi_1 = 0\), i.e. solid body rotation, are marked by circled plus and minus signs, respectively, on the same figure. Of course, the obvious difference between the two experiments is the fact that the response is no longer symmetric about the equator. Near the source, the northern branch is now more meridionally oriented while the southern branch has become more zonal. This is exactly the behavior the group velocity analysis suggests. In this case, the cosine term in (11) is positive everywhere, so one expects \(d_k/dt\) to take the sign of \(l\) in ridges and the opposite sign in troughs. From (8), this implies the northern branch (where \(l > 0\)) will become steeper than in the solid body rotation case, while the southern branch (where \(l < 0\)) will have a shallower slope. Note that because of the symmetries in this basic state, a source at 90\(^\circ\)E would produce a southern branch with a strong meridional component and a northern branch which rapidly bends eastward.

Further verification that propagation through this basic state cannot be understood simply by applying zonally symmetric theory locally can be made by repeating this experiment with (2) modified so that

\[
\begin{align*}
\frac{1}{r^2 \cos \phi} \frac{\partial \psi}{\partial \phi} &= 0, \\
\frac{1}{r^2 \cos \phi} \frac{\partial \psi}{\partial r} &= 0
\end{align*}
\]

The model is then formally the same as a model linearized about a symmetric state, but \(\bar{u}\) and \(\bar{q}_a\) are still functions of longitude. When this was done, in an experiment not shown here, the rays were found to be essentially the same as when the \(\Psi_1 = 0\) basic state is used, confirming the importance of the zonal gradients in this background field.

Equations (8) and (9) are only valid within some neighborhood of trough/ridge axes. Also, though (9b) is identical to the symmetric basic state result, the meridional wavenumber will be modified along a ray path relative to the symmetric case because it is a function of \(k\). In order to test the validity of the ten-

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\(^2\) For convenience, "trough" will refer to a relative minimum in streamfunction and "ridge" to a relative maximum, regardless of the sign of \(y\).
periment; the curvature of the streamlines still seems to be affecting the paths taken by the energy even though they are far from the axis. Beyond the central meridian there is some evidence of the basic state ridge in the Eastern Hemisphere influencing the pattern as both branches have bent into the Southern Hemisphere, a tendency which (11) supports. Finally, moving the source to (0, 0) produces the solution displayed in Fig. 10d. Eastward of 60°E the paths have the accentuated southern components expected, but near the source they are actually north of their solid body rotation trajectories. Equation (B2) indicates there should be a bias in ray paths toward the direction of the background flow. Near the source of vorticity in this experiment \( \vec{\psi} \) has its strongest northward value and the ridge/trough effect is weakest; this may explain the behavior near the prime meridian.

The basic state used here to demonstrate the usefulness of the ray tracing concept in an environment with zonal variation is rather unrealistic in that it contains "troughs" and "ridges" which extend from pole to pole. Another simple streamfunction which does not have this shortcoming is

\[ \vec{\psi} = \Psi_1 \sin \chi \sin 2\gamma - \Psi_2 \sin \gamma. \]

Keeping with the terminology of this paper, this flow pattern has troughs and ridges which are confined to one or the other hemisphere. An analysis of propagation through this basic state, using the same procedure outlined above, leads to the conclusion that there is preferential propagation poleward in low pressure troughs and equatorward in high pressure ridges. This tendency, which our simplified ray tracing analysis suggests and which our experiments confirm, can also be noted in the rays which Karoly (1983) calculated for a zonally varying midlatitude jet.

b. Observed basic states

Each of the background fields used in experiments described in the above narrative has been concocted so as to elucidate the effects of one special property of the observed mean midtropospheric state with respect to wave propagation. In this section, we shall use the observed flow itself as the basic state and see to what extent the model solution can be explained in terms of the earlier idealized results. Though meridional variations in the basic state have sometimes been large, all of the synthetic backgrounds have had only slow zonal variations in order to be consistent with the accompanying analysis. In order to retain this consistency, the observed patterns used here are filtered so that only zonal waves 0, 1 and 2 are retained. As illustrated in Fig. 11, many of the important features of the unfiltered flow, shown in Fig. 1a, remain in the truncated representation. In particular, the East Asian jet and the large-scale longitudinal variations in equatorial flow are present.
Grose and Hoskins (1979) found that their simulation of the atmosphere’s mean wintertime state by mountain forcing in a linear, steady barotropic model was best when 300 mb zonal mean winds were used as the basic state. This suggests that this level will give a good indication of the horizontal propagation characteristics of the troposphere. We have already noted that it has regions where absorption, reflection and trapping might take place. Some of this behavior can be seen in an experiment (Fig. 11b) in which a source is placed at (40°N, 30°E) in a basic state consisting of filtered January 300 mb mean flow. The response in the vorticity field is rather noisy, but the northern branch of the pattern can be picked out as it passes over Siberia and on to Alaska. The southern branch is quite different from, say, the solid body rotation case of Fig. 3. Part of it does head towards Australia, but because it must pass through easterlies, its amplitude decreases rapidly. Most of the southern branch is reflected back toward Asia in the vicinity of (10°N, 80°E) along a line where the refractive index is imaginary for all scales. The reflected ray then appears to be trapped in the jet until it reaches the central Pacific. At this point the reflecting condition no longer holds and the energy once again moves southward into a region of weak westerlies in the tropics, where it is absorbed. This response has so many small-scale features it is useful to also look at the perturbation streamfunction (Fig. 11c). Here the reflection in the northern Indian Ocean and the trapping in the Asian jet are more distinguishable.

The possibility of interhemispheric or tropical/midlatitude transmission through the westerlies which exist in the equatorial midtropospheric Pacific during the Northern Hemisphere’s winter was raised by Hoskins et al. (1977). Our experiments with analytically defined basic states have demonstrated that indeed propagation through local westerlies is possible. To test this theory further a vorticity source is placed at (30°S, 150°W) in the filtered mean January 300 mb background. From Fig. 12a we see that almost all of the energy is trapped in the Southern Hemisphere. Moving the source to other longitudes does not increase the northern response. Of course, the farther north the forced region is, the more energy reaches the Northern Hemisphere, but clearly westerlies in the central Pacific are normally not strong enough to allow unrestricted interhemispheric interaction.
These results do not preclude the possibility that from time to time westerlies strong enough to allow propagation between the hemispheres exist or that the sparse data in this region leads to an underestimate of meridional propagation there. These possibilities have been tested by finding solutions to (2) with a vorticity source at (30°S, 150°W) and using the filtered monthly mean 300 mb flow patterns observed during the First GARP Global Experiment (FGGE) for December 1978 and January–March 1979 for backgrounds. In three of the months there was no larger Northern Hemisphere response than in Fig. 12a. But for January 1979 the mean square variance in the Northern Hemisphere extratropical perturbation vorticity increased by an order of magnitude (Fig. 12b). The enhanced propagation through the tropics is the result of a deepening of the low latitude Northern Hemisphere trough centered at 120°W. This deepening amplifies the strength of the equatorial westerlies thus increasing the group velocity according to the Hoskins and Karoly (1981) analysis and intensifying the northward component of a ray crossing the equator by the trough/ridge effect described in the previous subsection.

5. Discussion

The results of the experiments described here are meant to suggest how local steady forcing of the atmosphere could be expected to affect remote regions, but certain limitations in the approach should be kept in mind. First, the model used in this study is representative of the workings of the earth’s atmosphere only to the extent that the atmosphere acts like a barotropic, potential vorticity conserving fluid. Of course, other processes are important, but this work is based on the supposition that the conservation law used here is a strong constraint on atmospheric dynamics, so the tendencies demonstrated here must be included in any more comprehensive theory of mean atmospheric motion.

Second, it should be emphasized that all results are for steady solutions to the model equation. They are meant to portray flows averaged over a period of weeks or longer. In some cases, transients can exhibit markedly different behavior. For example, Charney (1969) showed that on the β-plane easterlies are not barriers to certain westward propagating Rossby waves. Hoskins et al. (1977) gave an example of meridional propagation of transients through easterlies on the sphere and Karoly (1983) found that the rays for very large scale, low frequency waves passed through regions of easterlies. Another consequence of investigating steady solutions is that barotropically unstable modes are automatically filtered from the solutions. But the reflecting condition employed above is also a necessary condition for this instability, so an important element of the dynamics has been removed in some cases. Simmons et al. (1983) have shown that our model does have many unstable normal modes when linearized about January-mean 300 mb flow.

Third, restricting ourselves to linear solutions can have important consequences. But nonlinear solutions also contain regions of propagation, absorption and reflection which act in much the same manner described here. For example, Webster and Holton (1982) found regions of local westerlies to be conducive to propagation in experiments with a nonlinear barotropic model. In fact, a good understanding of linear examples can help one to anticipate nonlinear results to the extent that the nonlinear feedbacks can be approximated by a series of linear processes.

Fourth, recall that all examples here were constructed so that zonal asymmetries were slowly varying. While this condition was legitimate for the behavior being demonstrated, the atmosphere is not slowly varying in some places. As an indication of the importance of the slowly varying assumption, the experiment in Fig. 11 is repeated but without the zonal filtering of the basic state. As can be seen in Fig. 13, the southerly wave train is still reflected away from the tropics and virtually nothing penetrates to the Southern Hemisphere. However, instead of turning southward east of the dateline and dying as in the slowly varying case, it continues northward and amplifies dramatically at (45°N, 150°W). Simmons (1982) and Simmons et al. (1983) also found a large response at this location in experiments with a barotropic model linearized about mean January flow. In those experiments, sources were placed in the tropics in the Eastern Hemisphere but the response was similar to Fig. 13 in that a wavetrain was created which was trapped in the East Asian jet and then amplified just to the east of the jet exit region. One possible explanation for the large amplification in the Pacific which occurs in experiments with an unfiltered observed basic state is suggested by the waveguide experiments described in Section 3. In the present study, the only experiments with a prescribed background which showed a wavetrain suddenly increasing in amplitude at a location remote from the source were some of the waveguide experiments. It was noted that this behavior might be because the waveguide basic state locally satisfied the conditions for overreflection. Examination of the dateline profiles of zonal wind and refractive index for the January 300 mb basic states indicates that the filtered basic state does not meet the conditions for overreflection while the unfiltered background is overreflective. Thus, it may be that the added longitudinal detail in the unfiltered January basic state results in latitudinal vorticity gradients near the dateline which are conducive to overreflection. This contention is further supported by experiments (not shown) with zonally symmetric basic states constructed from the January stream-
have in a similar manner in a linear model with a basic state which varies very slowly in the zonal direction. A group velocity calculated from the local zonal wind component gives a good indication of the path that energy takes away from a local source; critical lines, regions of decay and reflecting surfaces can all exist locally. A consequence of this is that the tropics need not be viewed as a barrier to interhemispheric interaction. The region of westerlies in the western Pacific may be a corridor through which the hemispheres can communicate during the Northern Hemisphere winter. The propagation properties of this region may change from month-to-month and year-to-year so in some cases there is free interaction while in others there is little. Similarly, the experiments support the idea that changes in forcing of the tropical atmosphere could have major impacts on the midlatitudes. This could happen in a direct manner: tropical forcing might be in or near a region in which propagation to the midlatitudes is possible; or indirectly: tropical forcing might change the intensity of the westerlies which in turn would lead to more interhemispheric propagation. This does not rule out midlatitude initiation of some of the processes. Dynamics internal to the midlatitudes might, for example, shift the westerlies farther south than normal so that energy from a tropical source could more readily reach midlatitudes.

Held (1982) and McIntyre (1982) have pointed out that the existence of reflecting surfaces in the atmosphere could have important consequences; in particular, it would increase the potential for resonance. The experiments reported here have provided evidence both for and against this theory. On the one hand our experiments have shown that critical lines are not necessary for reflection and that there is a region south of Asia which may act as a partial reflector. This zone tends to bend rays emanating from the north back into the Northern Hemisphere so that their energy is not lost to the tropics or the Southern Hemisphere. However, the experiments also suggested that this reflecting region only delays the loss of energy to the opposite hemisphere or to absorption in easterlies, so that enhanced resonance may not occur after all. The issue is clouded even further when the unfiltered January 300-mb basic state is used as a basic state. Under these conditions, the response to forcing in the Northern Hemisphere is strong throughout the hemisphere. To be sure, there is very little loss of energy to the Southern Hemisphere in this case, but to say that is the sole reason for the large Northern Hemispheric response would be a oversimplification. Given the general decay in energy as it propagates away from the source in other experiments, it would appear that in this case the perturbations must be gaining energy from the basic state. It is possible that this is an example of overrefraction.
Another notion suggested by this work is that jets in the midtroposphere may act as waveguides. They tend to be regions highly conducive to horizontal propagation flanked by bands which inhibit propagation. The east Asian jet is a particularly good example. The steady response to forcing in jets appears to be very sensitive to details in the structure of the waveguide. Depending on the detailed structure of the jet it may act as a resonant cavity or a filter.

Large-scale troughs and ridges may also be instrumental in determining the route taken by energy moving away from a source. Experiments indicate and group velocity arguments support the idea that rays tend to be directed more toward the pole in large scale troughs and toward the equator in large scale ridges than would be expected under zonal flow conditions.

In summary, in a variety of ways relatively small changes in regional flow can have a marked influence on the path taken by energy dispersing from fixed sources. Modification of the horizontal shear can change an absorbing region into a partially reflecting one. Increasing 5 m s\(^{-1}\) westerlies to 10 m s\(^{-1}\) westerlies can greatly enhance propagation. Atmospheric variability can be affected as much by these factors as by shifts in forcing.

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APPENDIX A

Method of Solution of Eq. (2)

Eq. (2) is solved by discretizing \( \psi' \) in terms of a series of spherical harmonics:

\[
\psi'(\lambda, \phi) = \sum_{m=-M}^{M} \sum_{n=|m|}^{N} a_n^m Y_n^m(\lambda, \phi),
\]

(A1)

where \( Y_n^m(\lambda, \phi) = P_n^m(\sin \phi)e^{im\phi} \) and \( P_n^m \) is an associated Legendre function. Inserting (A1) into (2), multiplying by a complex conjugate spherical harmonic, \( Y_n^{-m} \), and integrating over the globe gives

\[
\sum_{m=-M}^{M} \sum_{n=|m|}^{N} L_{nm}^m \psi_n^m = R_n^{m'},
\]

(A2)

where

\[
R_n^{m'} = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \int_{0}^{2\pi} R \psi_n^{m*} \cos \phi d\phi d\lambda,
\]

and

\[
L_{nm}^m = \frac{n(n+1)}{2\pi r^4} \int_{-\pi/2}^{\pi/2} \int_{0}^{2\pi} \left( \frac{\partial \psi}{\partial \phi} \frac{\partial Y_n^m}{\partial \lambda} - \frac{\partial \psi}{\partial \lambda} \frac{\partial Y_n^m}{\partial \phi} \right) \times Y_n^{m*} d\lambda d\phi
\]

\[
\times \left( \frac{\partial}{\partial \phi} \left( \nabla^2 \psi + f \right) \frac{\partial Y_n^m}{\partial \lambda} - \frac{\partial}{\partial \lambda} \left( \nabla^2 \psi \right) \frac{\partial Y_n^m}{\partial \phi} \right) Y_n^{m*} d\lambda d\phi.
\]

The derivatives of \( \psi \) are taken by representing \( \psi \) as a series with the same form as (A1) and analytically differentiating the spherical harmonics. The integrals are calculated with Gaussian quadrature and Fourier transforms using the same technique Eliassen et al. (1970) introduced for spectral forecast models. The series of \((M+1) \times (2N+2-M)/2\) equations represented by (A2) is then solved using a standard Gaussian elimination algorithm.

For most cases in this paper \( M = N = 20 \) (what is commonly referred to as triangular 20 truncation), but in a few \( M = 18 \) and \( N = 24 \). Though mathematically, fewer gridpoints would suffice, in order to give a smooth appearance to the figures, the grid calculations are done on 128 longitudes by 64 Gaussian latitudes.

Where noted, some basic states are constructed by specifying \( \vec{u} \) the zonal component of the basic flow on the transform grid and then calculating a streamfunction \( \psi \) which represents that wind in the following way. Here \( \vec{u} \) and \( \psi \) are related by

\[
\frac{\partial \psi}{\partial \phi} = -r \vec{u}.
\]

This can be integrated on the sphere by assuming \( a_n^m \) are the coefficients which result in the best fit for

\[
\frac{\partial \psi}{\partial \phi} = \sum_{m=-M}^{M} \sum_{n=|m|}^{N} a_n^m \frac{\partial \psi_n^m}{\partial \phi}
\]

in a least squares sense. The least squares constraint leads to a series of equations

\[
\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \int_{0}^{2\pi} \frac{\partial}{\partial \phi} e^{-im\phi} \frac{\partial P_n^m}{\partial \phi} \cos \phi d\phi d\lambda
\]

\[
= \sum_{n=|m|}^{N} a_n^m \int_{-\pi/2}^{\pi/2} \int_{0}^{2\pi} \frac{\partial P_n^m}{\partial \phi} \frac{\partial P_n^{m'}}{\partial \phi} \cos \phi d\phi
\]

for \( n' = 0, \ldots, M \) and \( n' = n', \ldots, N \), which can be solved. But by (A3), the \( a_n^m \) are spectral coefficients for \( \psi \).
APPENDIX B

Ray Tracing in Large-Scale Troughs and Ridges

A ray tracing analysis of (6), which is the Cartesian analog of our basic model equation, can be performed as follows. Inserting (7), the assumed plane wave solution, into (6) produces the dispersion relationship

$$\sigma = -\left( \beta k + \frac{\partial}{\partial y} \nabla^2 \psi k - \frac{\partial}{\partial x} \nabla^2 \psi l \right) \times (k^2 + l^2)^{-1} + \tilde{u}k + \tilde{v}l. \quad (B1)$$

The two components, $u_g$ and $v_g$, of the local group velocity, the velocity at which wave energy propagates, thus have the values

$$u_g = \frac{\partial \sigma}{\partial k} = -\left( \beta - \frac{\partial}{\partial y} \nabla^2 \psi + 2k^2 \tilde{u} + 2\tilde{v} kl \right) \times (k^2 + l^2)^{-1} + \tilde{u}, \quad (B2a)$$

$$v_g = \frac{\partial \sigma}{\partial l} = \left( \frac{\partial}{\partial x} \nabla^2 \psi + 2kl \tilde{u} + 2k\tilde{v} \right) \times (k^2 + l^2)^{-1} + \tilde{v}, \quad (B2b)$$

in the steady case. To simplify the analysis, we look at the group velocity along the axes of troughs and ridges, where $\tilde{v}$ vanishes. If in addition the axes are meridionally oriented, then $\tilde{v}_{xy}$ vanishes, and if the troughs and ridges are symmetric about their axes within some neighborhood of the axes, then $\tilde{v}_{xx}$ also vanishes. With these assumptions the slope of a ray as it crosses a trough or ridge is

$$v_g = \frac{2k\tilde{u}}{-\beta - \frac{\partial}{\partial y} \nabla^2 \psi + 2k^2 \tilde{u} + \tilde{u}(k^2 + l^2)} \frac{l}{k}. \quad (B3)$$

This is the same expression Hoskins and Karoly (1981) derived for a zonally symmetric basic state.

For (B3) to be useful in determining the path taken by a wavetrain the local wavenumber must also be determined. Information about this can be gained from the refraction of wave energy equation

$$\frac{d_x k}{dt} = -\frac{\partial \sigma}{\partial x}, \quad \frac{d_x l}{dt} = -\frac{\partial \sigma}{\partial y} \quad (B3)$$

(where $d_x/dt$ is the total derivative following the group velocity), which can be derived from the principle of conservation of wave crests (Whitham, 1960). By differentiating (B1), these can be shown to be

$$\frac{d_x k}{dt} = -\left( l \frac{\partial^2 \tilde{v}}{\partial x^2} + l^2 \frac{\partial^2 \tilde{u}}{\partial x \partial y} \right) (k^2 + l^2)^{-1} - l \frac{\partial \tilde{v}}{\partial x}, \quad (B4a)$$

$$\frac{d_x l}{dt} = \left( k \frac{\partial \tilde{b}}{\partial y} + k^2 \frac{\partial^2 \tilde{u}}{\partial y^2} \right) (k^2 + l^2)^{-1} - k \frac{\partial \tilde{u}}{\partial y}. \quad (B4b)$$

in a meridionally oriented symmetric trough or ridge. Again, the second expression is identical to the expression found by Hoskins and Karoly for zonally symmetric basic states. However, (B4a) is a significant departure from the zonally symmetric case; under symmetric conditions $k$ is constant along rays.

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