Severe Downslope Windstorm Calculations in Two and Three Spatial Dimensions Using Anelastic Interactive Grid Nesting: A Possible Mechanism for Gustiness

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ABSTRACT

The Clark nonhydrostatic anelastic code is extended to allow for interactive grid nesting in both two and three spatial dimensions. Tests are presented which investigate the accuracy of three different quadratic interpolation formulae, which are used to derive boundary conditions for the fine mesh model. Application of the conservation condition of Kurihara and others is shown to result in significant improvements in the treatment of interactive nesting. A significant improvement in the solutions for interactive versus parasitic nesting is also shown in the context of forced gravity wave flow. This result, for the anelastic system, is in agreement with the earlier results of Phillips and Shukla, who considered the hydrostatic shallow water system of equations.

The interactive nesting model is applied to the simulation of the severe downslope windstorm of 11 January 1972 in Boulder using both two and three spatial dimensions. The three-dimensional simulation results in a gustiness signature in the surface wind speed. The cause of this gustiness is attributed to the development of turbulent eddies in the convectively unstable region of the topographically forced wave. These eddies are transported to the surface by downdrafts formed in the leading edge of the convectively unstable region. A type of periodicity to the wind gustiness signature is then produced by a competition between the two physical processes of wave build up via forced gravity wave dynamics and wave breakdown via convective instability. The actual source/sink terms for the turbulence are still under investigation. Some preliminary comparisons between the two- and three-dimensional windstorm simulations are also presented.

1. Introduction

This paper presents an interactive grid nesting model which is designed using the nonhydrostatic anelastic equations. There are many problems in atmospheric fluid dynamics in which higher spatial resolution is required in only a limited portion of the computational domain. Grid nesting allows one to focus on such desired regions of the domain and obtain higher spatial resolution with greater computational efficiency. One particular example which is considered in this paper is that of nonlinear wave dynamics where a localized region may be highly nonlinear due to wave breaking, i.e., severe downslope windstorms. Other examples include: critical level calculations where a sharp change in scale occurs near the critical level; boundary layer simulations which are embedded in a larger scale model; convective cloud simulations where fine mesh models may be devoted to better resolving of the cloud dynamics; or even a combination of such phenomena. Here the discussion has been purposefully limited to cases where a nonhydrostatic model would be desired.

One series of experiments is presented using two models in two spatial dimensions where the differences between interactive (two-way) nesting and parasitic (one-way) nesting are demonstrated. These experiments are for topographically-forced gravity wave flow. The improved results of the interactive versus parasitic are presented with a discussion of the earlier experiments by Phillips and Shukla (1973) who performed similar comparison experiments for the hydrostatic shallow water equations. Another series of experiments is presented that demonstrates the importance of the Kurihara et al. (1979) conservation condition. These experiments use a bubble collapse in two spatial dimensions and examine the performance of three different quadratic interpolation formulae. One of the three interpolation formulae is derived using a variational procedure with a constraint such that the conservation condition is obeyed.

None of the experiments presented in this paper consider differences in temporal resolution between models. While grid nesting with only spatial resolution differences is clearly only a temporary consideration, it does allow for the execution of certain higher resolution experiments with reasonable efficiency. For example, the inclusion of variable time resolution in the three-dimensional experiment to be presented would result in only about 25 percent less computational cost. The generalization to nested models with differing temporal resolutions will be considered in future work.

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The severe downslope windstorm of 11 January 1972 reported on by Lilly and Zipser (1972) is simulated in three spatial dimensions using a two-dimensional forcing identical to that employed by Peltier and Clark (1979). This choice of surface forcing restricts the source of the three-dimensional motions in the flow to internal dynamics. Peltier and Clark presented simulations in two-dimensions demonstrating the importance of wave breaking on the severity of the windstorm event. They argued that the wave breaking produced a self-induced resonance which led to the constructive reflection of vertically-propagating wave energy resulting in the severe winds. This theory contrasts with an earlier theory by Klemp and Lilly (1978), who argued that the severe windstorms were due to constructive reflections from the tropopause. The reader is referred to Lilly and Klemp (1980) and Peltier and Clark (1980) for a discussion on the merits of the two theories. Other examples of two-dimensional simulations of the 11 January 1972 windstorm include Klemp and Lilly (1978) and Durran and Klemp (1983). A question which arises with respect to all the experiments performed in two spatial dimensions is their dynamic stability to the introduction of fully three-dimensional motions. Of particular note in the Peltier and Clark and Durran and Klemp solutions is the existence of a strong flow reversal in the core of the main wave. This flow reversal represents a region in which the isentropes are overturned and the flow is convectively unstable. Such super adiabatic regions have been found to be sources of turbulence in Kelvin–Helmholtz (K–H) waves by Klaassen and Peltier (1983b). Similarly, for severe downslope windstorms this region may represent a strong source of turbulence via a release of convective instability. One should keep in mind, though, the rather significant differences in mean wind profiles between the K–H flows and the downslope wind storm profiles. One is typically a hyperbolic tangent while the later is a jet structure. One might expect significant differences in the response to three-dimensional flow perturbations between the K–H phenomena and the downslope wind storm. The introduction of the third spatial dimension into the downslope windstorm simulation allows for a more realistic turbulence simulation of large eddies in which vortex stretching and twisting can occur. The question of whether or not such a source of turbulence can lead to the surface gustiness which is observed for such windstorm events will be examined. Also, some limited comparisons between the two and three dimensional simulations will be presented.

2. The model

The model which has been adapted to the grid-nesting procedure is a considerably modified version of the three-dimensional anelastic code of Clark (1977, subsequently referred to as C77). The C77 code has been employed in three-dimensional simulation studies such as: the deep convective cloud simulations of Clark (1979, 1982); and the simulation of flow over isolated topography of Clark and Gall (1982) and Peltier and Clark (1983). A simplified version of the C77 code has been used in a variety of two-dimensional numerical experiments among which are: the mountain-forced gravity wave experiments of Clark and Peltier (1977) and Peltier and Clark (1979); the Kelvin–Helmholtz simulations of Peltier et al. (1978) and Klaassen and Peltier (1983a); and the cloud simulation studies of Hall (1980). The model is now generalized so that it may be used for the efficient simulation of either two or three dimensional flows. The grid nesting is accomplished by running several models in parallel in an interactive sense. The same code is used for each model by simply “switching” the implied dimensions and other relevant variables at the beginning of a model’s time integration. Computer limitations currently restrict the number of interactive models that may be run efficiently to three.

The basic dynamic code will be described in this paper for only the dry atmospheric equations. All features relevant to the nesting have been incorporated for the cloud physics parameterization, but are not active for the experiments described in this paper. The reader is referred to Clark (1979, 1982) or Clark and Gall (1982) where the current status of the warm rain parameterization of Kessler (1969) is described. As far as the nesting is concerned, the same procedures described for the potential temperature apply to the moisture variables.

a. Basic dry model

The anelastic framework requires that the thermodynamic variables be cast into a perturbation form. The form taken is

\[
\begin{align*}
\theta &= \tilde{\theta}(z) + \theta'(x, t) = \tilde{\theta}(z)(1 + \theta^*) \\
T &= \tilde{T}(z) + \tilde{T}'(x, t) = \tilde{T}(z)(1 + T^*) \\
p &= \tilde{p}(z) + \tilde{p}'(x, t) = \tilde{p}(z)(1 + p^*) \\
\rho &= \tilde{\rho}(z) + \tilde{\rho}'(x, t) = \tilde{\rho}(z)(1 + \rho^*)
\end{align*}
\]

where \( z \) is the Cartesian coordinate in the vertical direction. \( \theta, T, p \) and \( \rho \) are the potential temperature, temperature, pressure and density of the dry air. The terms in (1) with overbars represent the atmospheric conditions for an idealized atmosphere with constant stability S. This is the atmosphere about which a linear perturbation theory is applied. The terms in (1) with the tilde represent the differences between the actual hydrostatically-balanced environmental sounding and the constant stability atmosphere. The \( x \) and \( t \) represent the three-dimensional spatial Cartesian coordinate vector and time. The terms in (1) with the asterisk represent normalized deviations from the chosen idealized constant stability environment. The overbar terms are taken as
$\bar{\theta}(z) = \theta_0 \exp(Sz)$

$\bar{\tilde{T}}(z) = \theta_0 \exp(Sz) \left[ 1 - \frac{g}{C_p\rho_0 S} (1 - \exp(-Sz)) \right]$ 

$\bar{\rho}(z) = p_0 \left[ 1 - \frac{g}{C_p\rho_0 S} (1 - \exp(-Sz)) \right]^{1/\kappa},$ (2)

$\bar{\rho}(z) = \rho_0 \exp(-Sz) \times \left[ 1 - \frac{g}{C_p\rho_0 S} (1 - \exp(-Sz)) \right]^{(1/\kappa)-1}$

where $\theta_0$, $p_0$ and $\rho_0$ represent the environmental potential temperature, pressure and density at $z = 0$ of the model. If we chose $S = 0$ we replace $\theta_0$ with a mean environmental value of potential temperature. In this special case the model reduces to the deep equations of Ogura and Phillips (1962). Here $C_p$ and $C_v$ are the specific heats of the air at constant pressure and volume and $\kappa = R/C_p$ where $R$ is the gas constant. The equations used to derive (2) are the ideal equation of state

$p = \rho RT,$ (3)

the definition of potential temperature,

$\theta = T \left( \frac{p}{p_0} \right)^{-\kappa},$ (4)

and the equation for hydrostatic balance,

$\frac{\partial \rho}{\partial z} = -\rho g.$ (5)

A linearization of (3) and (4) allows one to represent $\rho^*$ as

$\rho^* = -\theta^* + \frac{p^*}{\gamma},$ (6)

where $\gamma = C_p/C_v$. For further details on perturbation equations for anelastic models see Lipps and Hemler (1982) where a detailed scale analysis is presented.

The momentum equation of the model is taken as

$-\frac{D\mathbf{v}}{Dt} + 2\bar{\rho} \mathbf{\Omega} \times \mathbf{v}$

$= -\nabla p' + k \rho g \left( \frac{\theta'}{\theta} - \frac{\rho'}{\gamma \rho} \right) + \frac{\partial \tau_{ij}}{\partial x_j}.$ (7)

Here $k$ is the unit vector in the vertical direction, $\mathbf{\Omega}$ the angular rotation vector of the earth, $\mathbf{v}$ the three-dimensional wind vector and $\tau_{ij}$ is the stress tensor due to subgrid-scale turbulent processes. The full Coriolis effects are considered by the model in its current formulation although, for the experiments to be presented here, they are turned off. Note that in the buoyancy term of (7) only primed terms appear because the terms in (1) with tildes cancel out due to their hydrostatic balance with $\bar{\rho}$.

The stress tensor is parameterized according to the first-order theory of Smagorinsky (1963) and Lilly (1962) as

$\tau_{ij} = \bar{\rho} K_M D_{ij}$ (8)

in which the deformation tensor $D_{ij}$ is defined as

$D_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k},$ (9)

and the total deformation Def is defined as

$\text{Def}^2 = \frac{1}{2} \sum_i \sum_j D_{ij}^2,$ (10)

where $\delta_{ij}$ is the Kronecker delta function. The eddy mixing coefficient is taken as

$K_H = K_M = \begin{cases} \frac{(C\Delta)^2}{\sqrt{2}} |\text{Def}| (1 - \text{Ri})^{-1/2}, & \text{if } \text{Ri} < 0, \\
0, & \text{otherwise.} \end{cases}$ (11)

Here $\text{Ri}$ is the local Richardson number and $\text{Pr} = K_M/K_H$ is the chosen eddy Prandtl number. The values chosen for $c$ and $\Delta$ are given in the tables describing the experimental parameters. The thermal collapse experiments of this paper replace (11) by setting $K_H = K_M = \text{constant}$.

The mass continuity equation is approximated as

$\nabla \cdot \bar{\rho} \mathbf{v} = 0,$ (12)

and the equation governing the first law of thermodynamics is taken as

$\bar{\rho} \frac{D \theta}{Dt} \theta = \nabla \cdot (\bar{\rho} K_H \nabla \theta).$ (13)

The model equations have been described in Cartesian $(x, y, z)$ coordinates where in order to treat irregular surface terrain features the equations are transformed onto the non-orthogonal coordinates $(x, y, \tilde{z})$ with

$\tilde{z} = (z - h)H$ (14)

where $h = h(x, y)$ is the height of the terrain above $z = 0$ and $H$ is the height of the outermost model's fixed "lid."

Except for the boundary conditions the analytical representation of the model equations is complete. The boundary conditions will be described in the sections following the presentation of the numerical form of the equations. In the present anelastic framework, where the strong elliptic nature of the solutions is forced through the pressure solution, it has seemed appropriate to let the form of the numerical equations dictate the boundary conditions for the interior nested model. Of course, the boundary conditions for the outermost model remain problematic and they will be discussed separately.
b. Numerical approximations

The equations are approximated in flux conservative form on the staggered-grid mesh of Harlow and Welch (1965) or the Arakawa C grid. A second-order quadratically conservative scheme of Arakawa (1966) and Lilly (1965) is used for the spatial derivative terms and the second order leapfrog scheme is used for the time tendency terms. As in C77, the equations are represented using the Shuman (1962) type linear operators where

\[ \delta_{\phi} = \frac{1}{2} \left[ \phi(n + \Delta \eta) + \phi(n - \Delta \eta) \right], \]  
\[ \delta_{n \phi} = \left[ \phi(n + \Delta \eta) - \phi(n - \Delta \eta) \right] n \Delta \eta^{-1} \]  

for the arbitrary variable \( \phi \) and dimension \( \eta \). Three tensor transformation terms used to describe the equations in the \( (x, y, z) \) coordinate system are

\[ G^{1/2} = 1 - \frac{h}{H}, \]  
\[ G^{1/2}G^{13} = \left( \frac{z}{H} - 1 \right) \partial_z h, \]  
\[ G^{1/2}G^{23} = \left( \frac{z}{H} - 1 \right) \partial_z h, \]

where the transformed velocity \( \omega \) is taken as

\[ \bar{\rho} \omega G^{1/2} = \bar{\rho} \omega + \frac{G^{1/2}G^{13} \bar{\rho} u}{\bar{\rho} z} + \frac{G^{1/2}G^{23} \bar{\rho} v}{\bar{\rho} y}. \]  

For further details on the transformation approach the reader is referred to C77 or Gal-Chen and Somerville (1975). In (20) and the equations to follow, \( \bar{\rho}(z) \) of (2) multiplied by \( G^{1/2} \); \( \omega \) is the velocity component directed normal to \( z = \) constant surfaces and as such represents a combination of the orthogonal velocity components of the Cartesian system. To allow for a rigorous conservation treatment of the Cartesian momentum components the three momentum equations are taken from C77, as

\[ \delta_{2z} \bar{\rho} \omega u + \left[ \delta_x \left( \bar{\rho} \omega \bar{u} \bar{w} \right) \right] + \delta_y \left( \bar{\rho} \omega \bar{v} \bar{w} \right) + \delta_z \left( \bar{\rho} \omega u \bar{w} \right) - \bar{\rho} \omega \bar{u} \bar{w} \bar{f} + \bar{\rho} \omega \bar{u} \bar{w} \bar{f} \]  
\[ = -\delta_x (G^{1/2} \partial_z \bar{p}) \]  
\[ - \delta_x (G^{1/2}G^{13} \bar{\rho} \omega z) + KX^{-1}, \]  
\[ \delta_{2y} \bar{\rho} \omega v + \left[ \delta_x \left( \bar{\rho} \omega \bar{v} \bar{w} \right) \right] + \delta_y \left( \bar{\rho} \omega \bar{v} \bar{w} \right) + \delta_z \left( \bar{\rho} \omega v \bar{w} \right) \]  
\[ + \bar{\rho} \omega \bar{u} \bar{w} \bar{f} = -\delta_y (G^{1/2} \partial_z \bar{p}) \]  
\[ - \delta_y (G^{1/2}G^{23} \bar{\rho} \omega z) + KY^{-1}, \]  
\[ \delta_{2z} \bar{\rho} \omega w + \left[ \delta_x \left( \bar{\rho} \omega \bar{w} \bar{w} \right) \right] + \delta_y \left( \bar{\rho} \omega \bar{v} \bar{w} \right) + \delta_z \left( \bar{\rho} \omega w \bar{w} \right) \]  
\[ + \delta_y \left( \bar{\rho} \omega \bar{v} \bar{w} \right) - \bar{\rho} \omega \bar{u} \bar{w} \bar{f} \]  
\[ = -\delta_z (G^{1/2} \partial_z \bar{p}) \]  
\[ + g \left( \bar{\rho} \omega \bar{r} \bar{w} \bar{f} \right) \bar{p} \gamma R \bar{T}^{-1} + KZ^{-1}, \]

where the two components of the Coriolis force have been included; \( f \) and \( f^* \) are equal to \( 2 \Omega \sin \lambda \) and \( 2 \Omega \times \cos \lambda \), respectively where \( \lambda \) is the latitude of the model. All operators in \( z \) using (15) and (16) refer to the transformed coordinate \( \tilde{z} \). The time level index \( \tau \) is included in (21) to (23) where \( t = \tau \Delta \xi \). Here \( KX \), \( KY \) and \( KZ \) refer to the subgrid-scale mixing terms which are described in C77. The nonlinear advection terms in (21) conserve the momentum and also conserve the kinetic energy to \( O(\Delta \xi^2) \). A detailed analysis of the conservation characteristics of the model is described in C77.

The first law of thermodynamics is taken as

\[ \delta_{2z} \rho \bar{\theta} + \left[ \delta_x (\bar{\rho} \bar{u} \bar{\theta}) \right] + \delta_y (\bar{\rho} \bar{v} \bar{\theta}) + \delta_z (\bar{\rho} \bar{w} \bar{\theta}) \]  
\[ = KT^{-1}, \]

where again \( KT \) represents the mixing terms. The final equation for the dry model is the anelastic mass continuity equation which takes the simple form

\[ \delta_x (\bar{\rho} \bar{u}) + \delta_y (\bar{\rho} \bar{v}) + \delta_z (\bar{\rho} \bar{w}) = 0. \]

Equations (20) to (25) treat the six dependent variables \( u, v, w, \omega, p \) and \( \theta \). Some additional equations of (1) and (2) are also required to define \( T \) and the relationship between \( \theta, \theta' \), and \( \theta'' \). Given sufficient boundary conditions (21) to (25) are solved by forming a diagnostic pressure equation. The elliptic pressure equation is then solved which allows one to step the model forward in time. In the model code, \( w \) is treated as a temporary variable and does not constitute any overhead with respect to computer memory. The three velocity components \( (u, v, \omega) \) are retained in memory (or disk storage) which allows for easy diagnostic checks on the accuracy with which (25) is satisfied. The pressure solution method is described in C77 and allows for an "exact" direct solution of \( p \) to round-off error when \( h(x, y) = 0 \).

c. Inner model boundary conditions

Here the boundary conditions are described for any model boundary which is removed from the outermost boundary of the computational domain. Consider a boundary at \( x = x_0 \) of an interior model where \( x_0 \) is defined as the most westerly limit of the model, i.e., \( u \) has its first grid point value at this point. Fig. 1 shows a schematic of the grid structure of two models for a two-dimensional cross-section in \( x \) and \( z \). The grid nesting is always chosen as an integer ratio between model resolutions such that the \( y-z \) plane of, say, \( u \) values will be coincident with an outer model \( y-z \) plane of \( u \) values. Similarly, the lowest and highest \( z = \) constant surface levels (\( \omega \) positions) for an inner model are coincident with \( z = \) constant surfaces (\( \omega \) positions) of an outer model. The left-hand side of (21) requires the knowledge of four terms at the \( x = x_0 \) boundary. These are

\[ \bar{\rho} \bar{u} \]  

at \( x = x_0 \), \( t = (\tau + 1) \Delta \xi \).
FIG. 1. Schematic of staggered grid structure for two models with a two-to-one nesting ratio. The circles mark CM positions of $\theta$, $\rho$ and $P$ whereas the crosses mark the velocity component positions. Dot marks the FM grid centered variables and an arrow marks the velocity component positions.

\[
\begin{align*}
\left\{ \frac{\rho v}{\sqrt{\bar{u}^2 + \bar{v}^2}} \right\} & \quad \text{at} \quad x = x_0 - \Delta x/2, \quad t = \tau \Delta t. \quad (27)
\end{align*}
\]

The boundary conditions (26) and (27) are obtained by appropriate extrapolation from the next coarser resolution model that contains the interior domain of the inner model under consideration. Details of the extrapolation schemes employed will be described in Section 3. The remaining five boundaries are treated in an identical fashion where we consider the left-hand sides of (21), (22) or (23). The mixing terms $K_X$, $K_Y$ and $K_Z$ are not treated in the same manner. For an inner model boundary the normal derivative of $K_M$ is taken equal to zero, which is sufficient to allow a complete description of the mixing terms.

The boundary conditions for $\theta$ are also obtained by extrapolation from the next coarser resolution model that contains the entire domain of the model under consideration. At an interior boundary $x = x_0$ the inner model value of $\theta$ at $x = x_0 - \Delta x/2$ and $t = \tau \Delta t$ is replaced by an extrapolated value from the appropriate outer model. Similar considerations are applied to the other five boundaries.

The boundary conditions described in this section are sufficient to allow the formulation and solution of the diagnostic pressure equation. For equal temporal and spatial resolutions these boundary conditions allow for an exact reproduction of the solutions in the overlapping domain providing the mixing terms (and spatial filters) are also matched. Tests were performed in both two and three spatial dimensions for such “perfect nesting” conditions and the model solutions were subtracted from each other to obtain round off level differences after one to two hundred time steps. These tests were performed with and without topographical forcing.

d. Outermost model boundary conditions

The boundary conditions on the velocity components and $\theta$ at the upper and lower boundaries are taken as

\[
\omega = \delta_x \rho v = \delta_x \rho \omega = \delta_x \omega = 0 \quad \text{at} \quad \bar{z} = 0, \ H. \quad (28)
\]

Here we have assumed zero normal velocity gradient conditions. The condition on $\theta$ in (28) is actually applied at $\bar{z} = +\Delta z/2$; $H - \Delta z/2$. (28) is used primarily for the advection terms, Coriolis terms and buoyancy terms of (21) to (24). As $\Delta z \rightarrow 0$, (28) becomes inconsistent with momentum exchange between the ground and the near surface air. But for the present applications (28) is used only to provide a local structure to $\omega$ and $\theta$ at $z = 0$. The surface stress is treated by a drag law formulation such that

\[
\begin{align*}
\tau_{13} &= \frac{1}{2} \rho C_D |V_j|(i \cdot V_i) \quad \text{at} \quad \bar{z} = 0,
\tau_{23} &= \frac{1}{2} \rho C_D |V_i|(j \cdot V_i)
\end{align*}
\]

where $C_D$ is the drag coefficient, $V_i$, the horizontal velocity vector tangent to the surface (which from (28) is equivalent to the velocity at $z = \Delta z/2$) and $i$, $j$ are unit vectors in the $x$ and $y$ directions, respectively. At $\bar{z} = H$ both $\tau_{13}$ and $\tau_{23}$ are put equal to zero. The grid placements and numerical formulations of $\tau_{ij}$ can be found in C77.

The surface sensible heat flux is specified either as zero or as in Clark and Gall (1982) to non-zero values dependent upon the incident solar shortwave flux. In more recent work, which will be reported in a later publication, the surface sensible and latent heat fluxes as well as the surface stresses are determined through a surface energy budget calculation. The sensible heat flux at $\bar{z} = H$ is determined so that there is no vertical divergence/convergence of heat near the model “lid.”

A further condition near the model top is required to absorb vertically-propagating gravity waves. As in C77 a region of Rayleigh friction is employed. The inverse of the Rayleigh friction time constant is allowed to vary linearly with height starting from zero at the bottom of the absorber region. A few grid points of the time constant at the absorber bottom are modified slightly to allow a smooth transition into the linear profile. This absorber is applied to $u'$, $v'$, $w'$ and $\theta'$ such that $u$, $v$, $w$ and $\theta$ are continually relaxed to environmental values. An identical absorber was used by Peltier and Clark (1983). Grid nesting is avoided in regions of the absorber.

The lateral boundary conditions are treated using a combination of specification and extrapolation. At outflow boundaries (those boundaries where the normal velocity component is directed outwards) the normal velocity component is calculated using the extrapolation procedure of Orlanski (1976). All other field values are obtained by taking one-sided “averages”
of the advection equations. At inflow boundaries, the normal velocity is treated by a combination of the Orlandi scheme and time relaxation to environmental values. This is the procedure used in Clark and Gall (1982), Clark (1982), and has been more recently investigated by Kurihara and Bender (1983). All other field values are set equal to their respective environmental values at the inflow boundaries.

e. Model filters

Horizontal spatial filters are applied to $u'$, $v'$, $w$ and $\theta'$ at each time step of the model integration. For each model there are two buffer zones of $\nabla^2_H$ and $\nabla^2_H$ filter structure with the major area of filtering being of order $\nabla^2_H$. These spatial filters are applied along $\bar{z} = \text{constant}$ surfaces. Tests to date have shown no adverse effect of filtering along the $\bar{z}$ surfaces as opposed to, say, filtering along $z = \text{constant}$ surfaces. One test, in particular, which demonstrated the utility of the filters was a simulation of the 11 January 1972 windstorm case with and without the filters for over 200 time steps in two spatial dimensions. The only difference in the runs was the development of some very low amplitude $2\Delta x$ structure in the nonfiltered experiment. The resolved-scale streamline structure and isentrope structure remained unchanged. These filters are applied to the model by calculating tendencies for $u$, $v$, $w$ and $\theta$ which are then added to the advection tendency terms. This a priori procedure does not affect the accuracy of the model's divergence equation, nor in the case of moist flow does it produce anomalous sub-saturated or supersaturated regions within a cloud. For the windstorm experiments the buffer zones of $\nabla^2_H$ and $\nabla^2_H$ were each made only one grid increment wide for the FM model. The time filter of Robert (1966) and Asselin (1972) is applied to $u$, $v$, and $w$ to suppress development of the computational temporal mode.

3. Grid nesting considerations

The inherent elliptic nature of the anelastic system of equations is fundamentally different from the parabolic structure of the hydrostatic framework. The solution of the elliptic pressure equation in the anelastic system allows for the possibility of strong nonlocal effects of both local forcing and boundary conditions as well as local smoothing operations. The design of a two-way nesting framework requires that the interpolation formula used to derive boundary conditions from a coarse-mesh (CM) for use in a fine-mesh (FM) model be consistent with the operators used to average FM data down to the CM resolution. This is the conservation condition referred to by Kurihara et al. (1979). As will be demonstrated in this section, a significant improvement in the nesting performance is obtained when the CM to FM interpolation formula and FM to CM averaging procedures are made reversible. Before discussing the averaging and interpolation formula, some specific differences between the anelastic and hydrostatic frameworks will be discussed.

Most nesting calculations to date have been confined to the hydrostatic framework. Miyakoda and Rosati (1977) discuss nesting of models in the one-way (or parasitic) sense, and Ley and Elsberry (1976) discuss nesting performed in the two-way (or interactive) sense. Kurihara et al. (1979) present a two-way hydrostatic model in which they define the dynamical and mesh interface as being physically separated. Their dynamical interface is the boundary separating a change in time resolution whereas their mesh interface defines the boundary separating a change in spatial resolution. This innovative development proved worthwhile in the hydrostatic framework but is not possible to perform in the anelastic system. Due to the required solution of the elliptic pressure equation, each model must be integrated forward in time using a single time step specification over the entire domain of the particular model under consideration. Thus, the dynamical and mesh interfaces are identical boundaries in the present anelastic model.

An exception to the above hydrostatic cases is the nonhydrostatic interactive nesting model of Blechman (1981) in which vortex generation in thunderstorms was studied. No information on the detail of the numerical approaches used or experimental tests performed on the nesting framework were presented in this paper. Thus it is not possible to intercompare the present results and methods with those used by Blechman.

a. Averaging and interpolation formula

Define $\Phi$ to be a CM variable and $\phi$ to be the equivalent variable in the FM. After the FM model has been integrated in time up to the most recent time value of the CM, the values of $\Phi$ in the entire overlapping domain are replaced using

$$\Phi_i = \sum_{i=1}^{n} \Phi_i \Delta L \Delta x,$$

where $n$ is the integer nesting ratio, $\Delta L$ is the FM grid size and $\Delta L$ is the CM grid size. Eq. (30) is identical to the formula (3.1) used by Kurihara et al. (1979). The $\Delta$ can represent any of $(\Delta x, \Delta y, \Delta z)$ corresponding to the independent dimensions $(x, y, z)$. In the case of a nonstaggered variable such as $\theta$, (30) is applied appropriately in each of the three dimensions. In the case of a staggered variable such as $\omega$, then (30) is applied in only two dimensions $(x, y)$ upon the common $z = \text{constant}$ surface to obtain the CM value of $\omega$. A density weighting is applied, in all cases, when (30) is used. This averaging procedure then insures that the mass continuity equation (25) remains satisfied after averaging provided it was satisfied before averaging.
A similar procedure to that of Kurihara et al. (1979) was followed in the determination of the interpolation formula. They defined a linear formula such that

\[ \sum \phi \Delta l = \Phi_0 \Delta L \]  

(31)

which they referred to as a conservation condition. This is, of course, what was meant earlier in this paper by reversibility. Instead of using a linear interpolation formula, a quadratic interpolation formula was chosen in the present model. It was felt that a quadratic formula would allow a somewhat better fit in regions where the fields showed considerable structure. Any order fit higher than quadratic is likely to lead to oscillations so this was avoided. Comparisons between the linear and quadratic fit were not performed because this requires a substantial change in the model code. The interpolation formula can be written as

\[ \phi = E_- \Phi_- + E_0 \Phi_0 + E_+ \Phi_+ \]  

(32)

where

\[ E_- = \epsilon \left( \epsilon - 1 \right) / 2 + \alpha \]  

\[ E_0 = \left( 1 - \epsilon^2 \right) - 2\alpha \]  

\[ E_+ = \epsilon \left( \epsilon + 1 \right) / 2 + \alpha \]  

(33)

with \( \epsilon = \epsilon_0 \) which corresponds to the position of \( \phi \) and with either

\[ \alpha = 0 \]  

(34)

\[ \alpha = (\Delta l / \Delta L)^2 / 24 \]  

(35)

or

\[ \alpha = [(\Delta l / \Delta L)^2 - 1] / 24. \]  

(36)

In (33), \( \epsilon \) is a normalized distance which varies between \( \pm 1 \). To explain (32) and (33) consider the case of \( \alpha = 0 \). This case corresponds to quadratic collocation. The positions of \( \epsilon = -1, 0, +1 \) represent positions in the FM model that exactly correspond to the CM model positions of \( \Phi_- = \Phi_0 = \Phi_+ \). Now, the actual position of the variable \( \phi \) in (32) will be \( \epsilon = \epsilon_0 \) where \( -1 < \epsilon_0 < +1 \). By modifying (33) with the \( \alpha \) terms it is very easy to convert the mathematical meaning of (32) from quadratic collocation to other potentially useful forms. Considering the case of \( \alpha = 0 \) it is easy to show that this procedure of interpolation does not satisfy the conservation (or reversibility) condition (31). The specification of \( \alpha \) in (35) is obtained by integrating the quadratic approximation of \( \phi \) over the appropriate grid distance \( \Delta L \) of the FM grid. Again this quadratic integration fit does not satisfy (31). The specification of \( \alpha \) in (36) was obtained by solving a least squares fit problem for \( \phi \) and applying (31) as a constraint. If we take \( \phi \) as our final interpolation formula where

\[ \phi = a_0 + a_1 \epsilon + a_2 \epsilon^2 \]  

over the range \( \epsilon = \pm 1/2 \), then a minimization of \( \Lambda \), where

\[ \Lambda = \int_{-1/2}^{1/2} \left[ \phi - \phi(\epsilon) \right]^2 d\epsilon + \lambda \left[ \int_{-1/2}^{1/2} \phi d\epsilon - \Phi_0 \right] \]  

(38)

with respect to \( a_0, a_1, a_2 \), and the Lagrange multiplier \( \lambda \) results in (36) after we integrate (37) over the appropriate \( \Delta l \) grid distance. Note that the range of integration of \(-1/2 \leq \epsilon \leq +1/2 \) is chosen so that the domain of \( \Phi_0 \) is properly spanned. In (38) \( \phi(\epsilon) \) is the structure of \( \phi \) in (32) obtained with variable \( \epsilon \) and with \( \alpha = 0 \). This last variational interpolation formula exactly satisfies (31) and allows one to use some information about the second derivative behavior of \( \Phi \) in the region of interpolation.

In Section 2c the boundary conditions required by the FM model were described. These are all boundary conditions on a two-dimensional plane. Interpolation to the particular two-dimensional plane was performed using (32) and (33) with \( \alpha = 0 \), i.e., (31) was not considered for this direction of interpolation at the time of these tests. Subsequently all interpolations have been converted to using (36). In reducing the two-dimensional plane from the CM to FM resolution (32), (33), and (36) were used. The successive application of (30), (32), (33) and (36) in two or three spatial dimensions preserves the reversibility characteristic.

### b. Interpolation formula tests

To show the difference in performance between the use of (34), (35) and (36) in reducing the two-dimensional arrays from the CM to FM resolutions a series of bubble collapse experiments were performed. The main numerical aspects of the experiments are described in Table 1. These experiments are very similar to those performed by Orlanski (1976). The main idea of these experiments is to initialize the model with an area of 1.2 by 1.2 km of isentropic fluid centered in the computational domain. This region of constant potential temperature is embedded in a still atmosphere having a constant stability of \( S = -d \ln \theta / dz = 10^{-2} \text{ m}^{-1} \). Such an initialization is dynamically unstable and quickly leads to the development of primarily horizontally propagating gravity waves. Thus, these experiments are designed to test the performance of the lateral boundaries, between the FM and CM, to propagating gravity waves. In order to allow for a strong similarity between the control experiment (BC-CON) and the nested simulations the horizontal filters were turned off and the \( K_w \) was set equal to a constant value of 10 m² s⁻¹. The differences between the FM and control experiment are due entirely to differences in the spatial resolution between the FM and CM and due to the treatment of the interpolated boundary conditions.

Figure 2 shows the time evolution of two error diagnostics for the FM simulations. The first of these is the FM domain integrated kinetic energy differences, \( \Delta KE \), where

\[ \Delta KE = \frac{1}{2} \int \rho (\Delta u^2 + \Delta w^2) dx dz, \]  

(39)
and where $\Delta u$ and $\Delta w$ are the differences between the FM and control experiment velocity components. The control experiment was a single model run which used the FM resolution throughout the entire computational domain. The second is the FM domain integrated potential temperature differences squared, $\Delta T$, where

$$\Delta T = \frac{1}{2} \int \int \rho \Delta \theta^2 dx dz.$$  \hfill (40)

Also shown in Fig. 2 for reference purposes are the total FM domain $KE$ and $T$ versus time. These last two variables have been divided by 100 so that they fit conveniently on the same plots. Apparent from Fig. 2 is the dramatic reduction in error level for the experiment BC-FM36 which used (36) resulting in an interpolation formula which is consistent with (31). The maximum error levels were approximately 0.3 and 0.8% at $t = 24$ min for $\Delta KE$ and $\Delta T$ for this experiment. The other two FM experiments obtained error levels in excess of 2 and 3% for $\Delta KE$ and $\Delta T$ at the same time. Fig. 3 shows plots of $\theta$ and $u$ at the concluding time of the experiment for the control experiment. The concluding time levels of $\Delta \theta$, $\Delta u$, and $u$ are also shown for experiments BC-FM34 and BC-FM36. The results of BC-FM35 are similar to BC-FM34. The error plots show that the main improvement is due to intensity levels being reduced by using (36).

An Orlanski (1976) boundary condition was attempted for this series of comparisons but the error levels quickly exceeded the limits on the graphs of Fig. 2. This may be due in part to the nature of these experiments where information is being transferred from the CM to the FM area and vice versa. Any extrapolation scheme which considers information to be going in only one direction will fail in such cases.

c. One-way versus two-way interaction tests

A series of forced gravity wave flow simulations was employed to test the performance of the grid nesting at the upper boundary of the FM domain. Table 2 describes the main features of these experiments. These experiments were performed early in the model development and as a result used (34) for interpolation. The difference between the third experiment and the first two is that the perturbations are defined as departures from a constant stability environment. The first two experiments had used the Ogura and Phillips (1962) deep atmosphere equations. The differences between the results of FM03 and FM02 are very slight and are all attributable to the differences in $\rho(z)$. Thus, the conversion of the model to perturbation about constant stability showed no significant changes when none would be expected. Only the results of the first two experiments will be shown. A Rayleigh friction absorber has been employed in the upper 20 levels of the CM model. The minimum Rayleigh friction time constant of 1000 s at $\tilde{z} = H$ was used.

These experiments are very similar to those presented in Clark and Pettier (1977) where the solutions were compared with linear theory. The surface wave-drag which is defined as

$$D_w(0) = -\int_{-\infty}^{+\infty} h(x) \frac{\partial p}{\partial x} \, dx$$  \hfill (41)

can be found analytically (Miles and Huppert, 1969) as

---

**Table 1. Description of bubble collapse (BC) experiments. All nesting experiments were interactive. ($NX$, $NZ$) refer to the number of grid points in the $x$ and $z$ directions where $\Delta x$, $\Delta y$ and $\Delta t$ are the respective spatial and temporal resolutions.**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$NX$, $NZ$</th>
<th>$\Delta x$, $\Delta z$ (km)</th>
<th>$\Delta t$ (s)</th>
<th>Interpolation formula used</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC-CON</td>
<td>82, 82</td>
<td>0.1, 0.1</td>
<td>6.0</td>
<td>—</td>
<td>Constant stability atmosphere with $S = 10^{-4}$ m$^{-1}$ used in (2)</td>
</tr>
<tr>
<td>BC-CM34</td>
<td>42, 42</td>
<td>0.2, 0.2</td>
<td>6.0</td>
<td>(34)</td>
<td>$h(x, y) = 0$ in (14)</td>
</tr>
<tr>
<td>BC-FM34</td>
<td>42, 42</td>
<td>0.1, 0.1</td>
<td>6.0</td>
<td>(35)</td>
<td>$K_M = 10$ m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>BC-CM35</td>
<td>42, 42</td>
<td>0.2, 0.2</td>
<td>6.0</td>
<td>(36)</td>
<td></td>
</tr>
<tr>
<td>BC-FM36</td>
<td>42, 42</td>
<td>0.1, 0.1</td>
<td>6.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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**Fig. 2.** Error plots of $\Delta KE$ and $\Delta T$ versus $t$ for the bubble collapse experiments. See Table 1 and text for a description of the experiments. $\Delta KE$, $\Delta T$, $KE$ and $T$ are described by (39), (40) and in the text.
\[ D_w(0) = \frac{1}{4} \rho_0 \pi N U h_0^2 \left[ 1 + \frac{7}{16} \left( \frac{N h_0}{U} \right)^2 + \cdots \right] \] (42)

and has a value of \(-1.60 \times 10^{13}\) kg s\(^{-2}\) for these experiments. The inverse Froude number \(N h_0/U\) is 0.495, which is small enough (<0.85) that freely propagating waves are expected, but large enough that nonlinearities due to the surface boundary condition are not insignificant. When 0.85 is exceeded one obtains breaking waves. The vertical wavelength of the vertically propagating waves can be estimated from hydrostatic theory as

\[ \lambda_z = 2\pi U/N = 2.54 \text{ km} \] (43)

and from (43) it is apparent that the CM model has only 6.35 grid intervals per vertical wavelength whereas the FM model has 12.70. Thus, the CM model has only marginal resolution in the vertical whereas the FM model has quite good vertical resolution. In the horizontal direction the mountain half width (\(a = 3\) km) is equal to 3 CM and 6 FM grid intervals.

All forced gravity wave simulations in this paper are initiated with a potential flow type solution. Streamlines are obtained which satisfy mass continuity and which have a vorticity equal to the initial environmental value. The potential temperature field is then initialized as horizontal surfaces. This type of initiation produces results very similar to the earlier (Peltier and Clark, 1979) smooth start-up procedure. Horizontal \(\theta\) surfaces result in a zero initial time tendency on all vorticity components so that the development of gravity waves results from the second time derivative at \(t = 0\).
Phillips and Shukla (1973) showed that allowing two-way interactions between the FM and CM produced improved solutions with respect to those obtained for simple one-way interactions. Their simulations considered the shallow water equations which were assumed to be in hydrostatic equilibrium. Here a similar set of calculations is performed for the anelastic treatment of forced gravity waves over topography.

For the one-way interaction, the FM model has no effect whatsoever on the CM model because the FM model takes only the required boundary conditions from the CM model and feeds back no information. Thus, any differences caused by the different resolutions (spatial in these particular experiments) are allowed to grow with time. On the other hand, when the FM model results are averaged and fed back to the CM model, this brings the CM model phase velocities more in line with the FM model solutions over the FM domain. One would expect then that this second level of interaction will improve the boundary conditions for the FM model and at the same time change the CM solutions. Fig. 4 shows two solutions of the horizontal vorticity component $\eta$ at $t = 240$ min for the one-way (FG-CM01) and two-way (FG-CM02) experiments. The vorticity was chosen for display because this is one of the noisiest fields and is most likely to show any problems. From Fig. 4 there is a slight difference between the one-way and two-way interaction solutions for the CM. The two-way interaction solution is more developed in the upper most levels. The positive region of vorticity near the surface downstream of the mountain is due to the shock wave that is typically produced by the initialization.

Figure 5 shows two time levels each of the three fields, $w$, $\theta$, and $\eta$ for experiment FG-FM01. This is a one-way interaction calculation and there are modes in the solutions which are clearly due to reflections from the models upper interface. The analytical solutions for this case (not shown) produce very smooth contours with only single local extrema in any particular wave section. The irregularities and multiple local
asymptotes to the expected Long's type solution. The wave drag for the CM02 experiment is systematically smaller in magnitude than the FM solution. This result is attributed to the lack of sufficient resolution of the CM. The results of this particular experiment suggest that the interactive nesting may be quite useful for purposes of "foussing" in on the resolved solution.

4. Severe downslope windstorm simulations

Numerical simulations of the severe windstorm of 11 January 1972 (Lilly and Zipser, 1972) are presented in this section. Earlier simulations on this case have been presented by Klemp and Lilly (1978), Peltier and Clark (1979) and by Durran and Klemp (1983). All of these simulations were performed in two-dimensional space. Here simulations very similar to those of Peltier and Clark (1979) will be presented using both two-dimensional and three-dimensional models.

The results of the two-dimensional experiments are very similar to the results of both Peltier and Clark (1979) and Durran and Klemp (1983) for their dry case. The main purpose of the present two-dimensional runs is to test the performance of the interactive grid nesting framework for such a strongly nonlinear fluid flow regime. Also it is necessary to have such runs

extrema are the result of a poor upper boundary condition taken from the CM model. Fig. 6 shows the same time levels for the same fields as in Fig. 5, but for the interactive (two-way) experiment FG-FM02. The introduction of FM to CM feedback has produced a significant improvement in the structure of the FM solutions. Thus the feedback has enabled the CM model to produce a greatly superior boundary condition for the FM. This improvement in the FM solution has resulted in no apparent degradation of the CM solutions as discussed with respect to Fig. 4. The improvement of the solutions by introducing the model interaction is more dramatic than that displayed by Phillips and Shukla (1973), which is attributable to the elliptic nature of the anelastic system. Fig. 7 shows the later time evolution of the $w$, $\theta'$ and $\eta$ fields for FG-FM02. There is no apparent degradation of the solutions with time. The latest time of $t = 480$ min corresponds to 38.4 "launching" time scales $t_L = a/\mu$.

Figure 8 shows the surface wave drag versus time. The FM01, which is one-way, shows a somewhat irregular pattern as compared to FM02 which very nicely

Fig. 5. The time development of $w$, $\theta'$ and $\eta$ for the FG-FM01 experiment. Plates (a) and (b) show $w$ at $t = 120$ and 240 min with a contour interval of 0.05 m s$^{-1}$; (c) and (d) show $\theta'$ at the same times with a contour interval of 0.1 K; (e) and (f) show $\eta$ with a contour interval of $6 \times 10^{-4}$ s$^{-1}$. Solid and dashed contours follow the same format as in Fig. 4.

Fig. 6. As in Fig. 5 except for early time development of FG-FM02. The times shown are $t = 120$ and 240 min.
performed for intercomparison with the fully three-dimensional calculations.

Peltier and Clark discussed the importance of the wave breaking region on the severity of the windstorm near the surface. They argued that the breaking region acted as a self-induced critical region with small (if not negative) Richardson numbers. This region is then viewed as a strong reflector of vertically propagating wave energy due to $\text{Ri} < \frac{1}{4}$ and the mean flow reversal. This type of reflection mechanism has been discussed previously by Davis and Peltier (1976) in the context of parallel flows. There are many questions regarding the influence of the third spatial dimension upon the dynamics of these windstorm events. One question, in particular, which will be addressed in this paper is whether or not the two-dimensional calculations are dynamically stable with respect to perturbations in the cross-stream direction. Of particular concern is the initial influence of the third spatial degree of freedom in the region of wave breaking, where a superadiabatic region (SAR) forms. Relaxing the model constraints by allowing this region to form fully three-dimensional convective rolls in the cross-stream direction may lead to the onset of turbulence. This turbulence can in turn expand from its initial region of generation and possibly influence a much larger domain of the fluid. It is conceivable that such a source of turbulence could eventually lead to the gustiness near the surface which is frequently observed in nature but not obtained with the two-dimensional models. Note that the present study considers turbulence development well past the initial onset so that shear production as well as convective development within the SAR is likely.

It was decided to use a two-dimensional ridge for the three-dimensional simulations, so that one could in principle obtain identical solutions with the two and three-dimensional models, providing the two-dimensional solutions were dynamically stable. The three-dimensional model assumes cyclicity in the cross-stream direction. The width in this direction was chosen to be 10 km which is approximately twice the vertical depth of the convectively unstable region obtained in the two-dimensional simulation. This choice of dimensions allows convective rolls to chose an aspect ration of near unity.

Table 3 describes the main model parameters for all of the experiments performed on the 11 January 1972 windstorm case. Fig. 9 shows the environmental profile of $U$ and $\theta$ which were taken from a Grand Junction sounding. Note that the height coordinate in Fig. 9 is in kilometers above the plains. The same bell-shaped mountain was chosen as in the study of Peltier and Clark (1979). The peak mountain height above the plains of 2.044 km was assumed with a half-width of 10 km.

Figure 10 shows two plots of $u$ at $t = 80$ min (or $600\Delta t$) after initialization. The method of initialization was discussed in section 3c. Shown in Fig. 10(a) is the $u$ field for the control experiment where a single model was employed using the resolution corresponding to the FM model (J11-FM2D). There are some small differences but in general the two solutions agree rather well. One source of difference between these two experiments is that the spatial filters have a buffer zone of $\nabla_u$ and $\nabla_u$ near the two $x$-direction boundaries of

![Fig. 7. As in Fig. 5 except for later time development of FG-FM02. The times shown are $t = 360$ and 480 min.](image1)

![Fig. 8. Surface wave drag versus time for three of the experiments described in Table 2. The expected Long's-type solution which allows for nonlinearities of the lower boundary is also shown.](image2)
Table 3. Description of 11 January 1972 severe windstorm experiments performed in two and three spatial dimensions.

The first experiment J11-CON is the two-dimensional control experiment. See Fig. 9 for environmental sounding.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>NX, NZ</th>
<th>Δx, Δz (km)</th>
<th>Δt (s)</th>
<th>NY</th>
<th>Δy [km]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>J11-CON</td>
<td>242, 82</td>
<td>(1.0, 0.4)</td>
<td>8</td>
<td>N/A</td>
<td>N/A</td>
<td>$S = 2.8 \times 10^{-3}$ m$^{-1}$ in (2).</td>
</tr>
<tr>
<td>J11-CM2D</td>
<td>122, 42</td>
<td>(2.0, 0.8)</td>
<td>8</td>
<td>N/A</td>
<td>N/A</td>
<td>$h(x) = h_0/[1 + (x/a)^2]$ where $h_0 = 2.044$ km $a = 10.0$ km</td>
</tr>
<tr>
<td>J11-FM2D</td>
<td>122, 42</td>
<td>(1.0, 0.4)</td>
<td>8</td>
<td>12</td>
<td>N/A</td>
<td>$C\Delta = 0.19, 0.11$ (km) in (11)</td>
</tr>
<tr>
<td>J11-CM3D</td>
<td>122, 42</td>
<td>(2.0, 0.8)</td>
<td>8 → 5</td>
<td>12</td>
<td>1.0</td>
<td>for two- and three-D runs.</td>
</tr>
<tr>
<td>J11-FM3D</td>
<td>122, 42</td>
<td>(1.0, 0.4)</td>
<td>8 → 5</td>
<td>12</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

The FM model whereas these buffer zones occur far away in the control experiment. In spite of this difference the two solutions are in close agreement. All other fields agreed equally well between the control experiment and the FM2D experiment.

Figure 11 shows the time history of the surface wave drags for the two high resolution two-dimensional experiments and for the single FM three-dimensional experiment. The control and FM2D results are in good agreement up to the time of termination of the control experiment. There is a small departure between the two solutions near $t = 70$ min which does not appear to be growing. The wave drags of the J11-FM2D and J11-FM3D are in close agreement until the effects of wave breaking are felt at the surface such that for $t > 60$ min the two drag histories depart. The time averaged value of the wave drag for $t > 120$ min is approximately 20% larger for the two-dimensional solution than for the three-dimensional solution. All of the maximum amplitudes of wave drag shown in Fig. 11 are somewhat larger than previously obtained by Peltier and Clark (1979). They had obtained a maximum value of $1.6 \times 10^6$ kg s$^{-2}$ which was apparently still growing whereas the present results have obtained values in excess of $3.0 \times 10^6$ kg s$^{-2}$. These differences may be due in part to differences in initialization. The earlier plot of Peltier and Clark (1979) in their Fig. 32

![Fig. 9](image_url)

**Fig. 9.** Environmental sounding used for the 11 January 1972 windstorm experiments. The vertical coordinate is taken as the height above the plains. Shown in the figure are (a) the westerly component of wind and (b) the potential temperature.

![Fig. 10](image_url)

**Fig. 10.** The u-fields at $t = 80$ min for (a) experiment J11-CON and (b) J11-FM2D. Plate (a) shows the control experiment solution for a single model simulation with a dashed outline of the FM region corresponding to experiment J11-FM2D. The contour interval is taken as 8 m s$^{-1}$ with positive values shown as solid contours and negative values shown as dashed.

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appears to be delayed by about 20 min compared to the present results.

The three-dimensional model experiment (J1I-FM3D) was run from \( t = 0 \) to a final time of 155 min. During this time a random forcing of \( \pm 0.005^\circ\text{C} \) on \( \theta' \) was applied to the model buoyancy field throughout the FM domain. This small perturbation allowed for the excitation of cross-stream modes within any portion of the domain. Up to the time of wave breaking these perturbations were stable and resulted in purely random perturbations of equally small amplitude on the three-dimensional components of velocity and vorticity. Subsequent to wave breaking the perturbations began to grow in a SAR associated with wave breaking and from this time onward their amplitudes far exceeded the white noise generator. Fig. 12 shows the evolution of the three components of vorticity and cross-stream velocity component.

The early time profiles of the three components of vorticity, \( (\xi, \eta, \zeta) \), and \( v \) are shown in order to demonstrate that the source regions for the three-dimensional components of fluid flow are at heights corresponding to the wave breaking region. This height range is between 9 to 10 km above the plains. The only vorticity component which occurs in the two-dimensional solutions is the \( y \)-direction vorticity \( \eta \). The mean and standard deviation of \( \eta \) are shown in plates (a) and (b) at the two times of 40 and 93.33 min. There is a significant modification of \( \eta \) over this time period which is due primarily to two-dimensional wave development. In plates (c), (d) and (e) the standard deviations of \( v, \xi \) and \( \zeta \) are shown. The means of these three fields are very near zero and are not shown. At \( t = 40 \) min the abscissa has been multiplied by a factor to show a plot of the inherently three-dimensional dynamical components as they start to appear at the wave breaking height. Cross-sectional plots of these fields clearly show their initial development is confined to the wave breaking region where the SAR occurs. Subsequent evolution of these three fields shows a significant amplification and development outwards from the original source region suggesting that shear production and advection have become important. For example at \( t = 93.33 \) min the extrema of \( v \) have attained values of \( \pm 28 \text{ m s}^{-1} \) which is representative of the subsequent extrema of \( v \). The amplitudes of the standard deviations of \( \xi \) and \( \zeta \) are within a factor of two to three of those on \( \eta \) at the latest time shown in the figure with \( \xi \) being about twice the amplitude of \( \zeta \). One explanation of the increased amplitude of \( \xi \) over \( \zeta \) is that the higher vertical resolution of 0.4 km allows for increased gradients in \( \xi \) as compared to the horizontal gradients of \( \zeta \).

Time series plots of some dynamic field variables were calculated at various points of the FM model for the three-dimensional model experiment. These plots were calculated from 54 equally spaced data sets 100 s apart covering the last 90 min of model simulation time. This spacing corresponds to 20 time steps. Fig. 13 shows time series plots of \( w, \theta \), horizontal wind speed and direction for two elevated locations in the FM domain. The first point was chosen at \( z = 9.53 \) km almost directly overhead of the mountain peak whereas the second point was chosen at \( z = 8.06 \) km about 51.5 km downstream of the mountain peak. The first spatial point shown in plates (a) and (d) shows a rapidly varying structure of all variables throughout this time period. The turbulence had already established itself at the start of this time period in this region. The vertical velocity varies between about \( \pm 15 \text{ m s}^{-1} \) and the horizontal wind speed is rather weak and ranges between 0 to 15 m s\(^{-1}\). The wind direction indicates a wide range of values over about three quarters of the compass, but with a tendency towards easterly flow. The wave breaking region was the only region where such strong changes in horizontal wind direction occurred. The regions outside of this area showed predominantly westerly winds as seen in (h) for the elevated downstream position. There is more than an hour’s delay in the onset of strong transients at the elevated downstream position shown in plates (e) to (h). The wind speeds are considerably stronger in this area where a “gust” has reached nearly 80 m s\(^{-1}\). The three-dimensional motions also penetrate to some surface regions. Fig. 14 shows time series plots of horizontal wind speed at the model surface for six positions. The first position is taken at about 0.50 km downstream of the mountain peak and the remaining positions are equally spaced at 9 km in the easterly direction. The wind directions are not shown but are predominantly westerly with very little variation in direction. The structure of the mountain peak wind speed is quite steady showing only a few meters per second variation. One has to look far enough downstream before any strong transients are observed. This turns out to be about 6 km from the mountain peak. Progressing further downstream in Fig. 14 from the mountain peak, the onset time for gusts increases. At 36.5 km down-
FIG. 12. Vertical profiles of the three components of vorticity ($\zeta, \eta, \psi$) corresponding to the (x, y, z) directions and of $\psi$ for the J11-FM3D experiment. The times of 40, 53.33, 66.66, 80 and 93.33 min are shown. The mean of $\eta$ is shown in (a) whereas all other plots show standard deviations. The horizontal domain used to derive these profiles was the FM domain. Some of the profiles have been amplified and the amplification factor is indicated in parentheses following the time.

stream in plate (e) no gusts appeared at any time during the simulation. These results suggest a localization of the strong gustiness extending from about $\frac{1}{2}$ mountain half-width to approximately three half-widths. The region of strong gustiness from the model seems to be closely related to the region where one obtains strong
Fig. 13. Time series plots of vertical velocity $w$, potential temperature $\theta$, horizontal wind speed and direction. Plates (a)–(d) are for the coordinates $(x, y, z) = (120.5, 0.5, 9.53)$ km in the FM. This corresponds to a position almost directly overhead at the mountain peak in the wave breaking region. Plates (e)–(h) have coordinates $(171.5, 0.5, 8.06)$ km. These plots are derived from data taken every 100 s over the last 90 min of the J11-FM3D experiment.
Fig. 14. Time-series plots similar to those of Fig. 13 but only for horizontal wind speeds. The positions in $x$ for (a)–(f) are 120.5 to 165.5 km with equal spacing of 9 km. All data are at $y = 0.5$ km and $z = 0$.

surface wind predictions from a two-dimensional model simulation.

These results of surface wind speed are in some ways similar to what is observed, in that extreme gusts occur about every 10 to 15 min, but there are also some notable differences. One main difference is that the very high frequency variations on the order of a minute are not reproduced by the model. This could very well be the result of insufficient spatial resolution resulting in a crude representation of only the largest "eddies". Another difference is that the maximum wind speeds simulated are about 15–20 m s$^{-1}$ larger than observed. This result can be attributed to a number of factors. The first factor is that this simulation was run with a dry upstream sounding. Durrant and Klemp (1983) have shown for the two-dimensional case that inclusion of the observed upstream moisture decreases the wave launching efficiency of the mountain which results in reduced wind speeds for this particular case. A second factor is that of insufficient spatial resolution. As noted

in Fig. 11 there is a noticeable decrease in maximum amplitude surface wave drag for the present three-dimensional simulation as compared to the two-dimensional solution. This translates to a decrease in surface wind speeds. A further increase in spatial resolution may make this difference even larger because of the effective reduction in correlations between, say, $w$ and $u'$ and the associated reduction in wave energy transfer. A third factor is that the surface forcing was idealized as a two-dimensional ridge. Introducing three-dimensional variations in the topography may also contribute to a reduction in the maximum surface wind speeds.

Another aspect of these solutions which needs qualification is the choice of cross-stream direction resolution. The original choice of using 10 grid points of 1 km resolution in the $y$-direction was based upon the two-dimensional simulation. The developed SAR had a depth of about 5–10 km in the two-dimensional simulation and assuming that convection tends to choose an aspect ratio of about unity this suggested a transverse domain length of about 8–10 km. Thus 10 km was chosen on this physical basis. It could very well be that a different choice would change some of the present results by, say, increasing or decreasing the period of the gustiness.

Figures 15, 16 and 17 are designed to show the correlation between surface gustiness and the generation of large eddies in the wave breaking region. Fig. 15 displays four $X - Z$ cross sections of $\theta$ at $y = 1$ km for the four times of 131.67, 133.33, 135.0 and 136.67 min. Plates (a) to (c) show a developed SAR break off at about the 7 km height in the core of the wave and then penetrate to near the surface. Plates (b) and (c) show the development of very strong vertical gradients in $\theta$ as a result of this downward penetration. The subsequent relaxation of these vertical gradients is evident in plate (d). Throughout the time period of these plots considerable changes are occurring in the core of the wave whereas outside this region the changes are, by comparison, rather minor.

Figure 16 shows a time sequence of $\zeta$ plots in the same $X - Z$ cross section shown in Fig. 15 for $\theta$. The vertical vorticity is shown because this is a field which relies totally upon the existence of three-dimensional aspects of the fluid flow, i.e. $\zeta = 0$ for the two-dimensional simulations referred to earlier. The contour interval chosen is $10^{-2}$ s$^{-1}$ which is large for this particular model resolution. Scaling $\zeta$ as $|\Delta \zeta|/\Delta x \sim |\Delta w|/\Delta y$ one obtains $|\Delta \zeta| = |\Delta u'| \sim 10$ m s$^{-1}$. Plates (a) to (d) show an eddy with strong anticyclonic vorticity originating in the core of the wave and penetrating towards the surface and then weakening. The pattern of development of the $\theta$-field discussed in Fig. 15 appears to be closely correlated with the large eddy appearing in Fig. 16. Furthermore the eddy reaching the surface in plate (b) of Fig. 16 at $t = 133.33$ min is well correlated with a surface gust in plate (b) of Fig. 14.
The peak of the last of the three large gusts shown in this plate has a speed of approximately 70 m s$^{-1}$ at $x = 129.5$ km, $y = 0.50$ km and $t = 132$ min. Considering the slight difference in $x$, $y$, and $t$, it seems very likely that these two events are related.

Figure 17 shows the very complicated eddy structure which occurs in the $Y-Z$ plane. Wind vectors derived from the $v$ and $w$ wind components are superimposed on $\theta$ plots. The same four times as displayed in Figs. 15 and 16 are shown here. The $x$ position chosen for these plots is 132 km which is one of the positions appropriate for seeing the transient characteristics in the core of the main wave. This figure shows a downward penetration near the surface at $t = 133.33$ min which is localized in the $y$-direction. At $y = 2$ km the strong vertical gradients of $\theta$ are evident in plate (b) which subsequently relax. Cellular type motion is apparent throughout the four plates of this figure with strong updrafts and downdrafts. By comparison with the 20 m s$^{-1}$ reference vector it can be seen that many of the plotted wind vectors exceed 20 m s$^{-1}$.

It is somewhat difficult to describe and/or even visualize the transient characteristics of a three-dimensional field using two-dimensional cross sections. Sometimes movies of a given cross section can aid in the interpretation of results and for this reason such a movie was produced using the cross-sections of Figs. 15 to 17. One interesting feature of the flow field which is apparent from the movie is a large eddy flow pattern in the core of the main wave. The flow gives the appearance of repeatedly building up regions of convective instability with the subsequent breaking down of these instabilities through convective motions. This cycle produces an apparent oscillation of the lee edge of the vertically steep isentropes such that they appear to move back and forth in the $x$-direction with a period commensurate with the period between large surface gusts described in Fig. 14.
The variability of \( w, \theta, u \) and \( v \) from the model presented at approximately the \( z = 11 \) km MSL. This region was chosen for display because it is in the region of initial turbulence generation. Figs. 12–14 of Lilly (1978) show plots of \( w, u, v \) and \( \theta \) at \( z = 6 \) km MSL which correspond to a height at or just below the bottom of the main wave's trough. The observations show \( w \) varying by \( \sim 30 \) m s\(^{-1}\), \( u \) and \( v \) by \( 25 \) m s\(^{-1}\) and potential temperature varying by \( \sim 5^\circ\). Throughout the main wave's trough from the \( z = 11 \) km MSL to \( z = 6 \) km MSL in the model the dynamic variables \( u, v, w \) and \( \theta \) obtain a similar if not slightly larger variability. Corresponding to the observations there is a negative correlation between \( w \) and \( \theta' \) in the model simulations. It should be noted that this negative correlation is between the total components of \( w \) and \( \theta' \) and may differ for mean wind and turbulent components. No attempt has been made to relate spatial or temporal scales between the model and observations because of the differences in data format, i.e., there is a mixture of temporal and spatial variables in the aircraft observations whereas the model is in an Eulerian format.

There are several aspects of analysis which have not been performed yet for this simulated three-dimensional windstorm event. More detailed analysis including a detailed turbulent kinetic energy budget will be described in a future paper.

5. Conclusions

Results presented have demonstrated the utility of the interactive nesting for the anelastic (nonhydrostatic) system of equations. The importance of the Kurihara et al. (1979) conservation condition in deriving interpolation and averaging formula was shown for a bubble collapse experiment. A quadratic conservative set of formulas produced a significant reduction in the errors as compared to the nonreversible quadratic formula.
Simulations of forced gravity wave flow over topography were presented which demonstrate the superiority of results obtained with interactive nesting as compared with the parasitic nesting. The introduction of the interaction between the fine mesh and the coarse mesh produced a dramatic improvement in the fine mesh solutions such that the fields were extremely smooth and produced the correct surface wave drag. These results produced differences more dramatic than those previously presented by Phillips and Shukla (1973). The reason is probably due to the differences between the anelastic and hydrostatic systems.

The interactive grid-nesting model was applied to a simulation of the 11 January 1972 Boulder windstorm event of Lilly and Zipser (1972) in two and three spatial dimensions. The two-dimensional solutions were very similar to results obtained by previous workers whereas the three-dimensional solutions showed some interesting differences from the two-dimensional solutions. A comparison of surface wave drags indicated that the three-dimensional solution attained values somewhat smaller in amplitude. The time averaged values of the last 35 min of wave drag were approximately 20% larger in amplitude for the two-dimensional than for the three-dimensional solution. Earlier unpublished results by this author on a three-dimensional flow over topography where the vertical wavelength was much smaller (2.5 km) showed about a 30% reduction in surface wave drag for the three-dimensional solutions as compared with the two-dimensional solutions.

The main purpose in simulating the severe wind storm event in three-dimensions was to study the stability of the two-dimensional solutions to perturbations in the transverse or cross-stream direction. It was found that the two-dimensional solutions were quite stable to perturbations produced by a white noise generator prior to the wave breaking. Once wave breaking oc-
curred convective rolls resulted in the superadiabatic region of the core of the main forced wave. This convection probably allows for the further production of turbulent eddies. Analysis of the three components of vorticity plus the cross-stream velocity component showed that the three-dimensional motions were initiated in the wave breaking region. Their further development outside this region suggests that shear production and advection of the turbulence have become important.

Plots of time series of winds and potential temperature at fixed points in the model showed a very strong transient nature. The speed and direction of the winds were shown to be extremely variable in the convectively unstable wave breaking region. At the model surface a gustiness signature to the wind was reproduced in a localized region downstream from the mountain peak. This region extended from approximately one-half to three mountain half-widths downstream. At some locations the wind speed varied by as much as 30 m s\(^{-1}\). The maximum gusts were spaced in time by about 10–12 min. The surface winds were found to have very little variability in their direction as compared with their very strong variability in speed.

The preliminary results and analysis of this paper suggest that the convective breakdown of superadiabatic regions in the wave breaking region may be important as an initial source of turbulence for these severe windstorm events. These convective eddies are then likely to interact with the very strong mean flow shears resulting in further intensification through shear production. The present hypothesis of surface gustiness is that vertically-propagating wave energy builds up regions of flow reversal and regions of superadiabatic potential temperature gradients until convection finally results in the production of strong eddies that penetrate to the surface. A periodicity of large gusts is produced by the model. This periodicity may be due to the competition between the surface forcing, which tends to increase the intensity of the superadiabatic gradients, and the turbulence, which tends to eliminate such unstable gradients. This hypothesis relies of course on the premise that the wave launching characteristics of the flow are sufficient to cause wave breaking.

Two particular aspects of the simulations of the severe windstorm event which were not in good agreement with the observations need discussion. The first is that the maximum gusts were too strong by about 15–20 m s\(^{-1}\). The maximum observed gusts were about 65 m s\(^{-1}\) whereas the simulation produced at least one 80 m s\(^{-1}\) gust. Durran and Klemp (1983) have shown that the inclusion of upstream moisture reduces the wave launching efficiency of the mountain, resulting in a weaker response. This consideration would seem to account for possibly half of the observed discrepancy. Another possible reason for overestimating surface wind speeds is the poor horizontal resolution of the model. As noted earlier there is an apparent reduction in intensity between the three and two-dimensional solutions. Increasing the model resolution would produce even finer scale eddies and the introduction of those which destructively correlate with the main wave field may produce a further reduction in intensity for the three-dimensional simulation. Consideration of full three-dimensional topography may also result in a reduction of the surface wind speeds. The second difference is the absence of very high frequency gusts. This is most likely caused by poor spatial resolution. The influence of the three-dimensional topography will also be important but is outside the scope of this paper.

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