A Comparison of Two- and Three-Dimensional Tracer Transport within a
Stratospheric Circulation Model*

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ABSTRACT

Use of the residual circulation for stratospheric tracer transport has been compared to a fully three-dimensional calculation. The wind fields used in this study were obtained from a global, semispectral, primitive equation model, extending from 10 to 100 km in altitude. Comparisons were done with a passive tracer and an ozone-like substance over a two-month period corresponding to a Northern Hemisphere winter. It was found that the use of the residual circulation can lead to errors in the tracer concentrations of about a factor of 2. The error is made up of two components. One is fluctuating with a period of approximately one month and reflects directly the wave transience that occurs on that timescale. The second part is increasing steadily over the integration period and results from an overestimate of the vertical transport by the residual circulation. Furthermore, the equatorward and upward mixing that occurs with transport by the three-dimensional circulation at low latitudes is not well reproduced when the residual circulation is used.

1. Introduction

The distribution of many trace gases in the stratosphere cannot be understood without taking into account transport by stratospheric motions. It is often necessary for practical reasons, however, to simplify the representation of dynamical processes in chemically oriented models, parameterizing in some manner the effects of mean meridional and wave induced transports. This is not an easy task since the transport by planetary waves plays a very important role in the stratosphere. It is about equal in magnitude and opposite in direction to the transport by the mean meridional circulation. Historically, a mass circulation with rising motions at equatorial latitudes and sinking near the pole was first inferred by Brewer (1949) and later by Dobson (1956) in order to explain the observed distributions of water vapor and ozone. Determination of the mean meridional winds from observations (Vincent, 1968) became possible only later and showed a circulation in the opposite direction. This apparent paradox is well understood today in the light of Andrews and McIntyre’s (1976; 1978a,b) Generalized Lagrangian Mean (GLM) theory and Boyd’s (1976) generalization of Eliassen and Palm’s (1960) and Charney and Drazin’s (1961) work. A good review of the evolution of our understanding of these aspects of the stratospheric circulation may be found in Hsu (1980) and Mahlman et al. (1984).

Given the importance of the transport by waves and their intimate relation with the mean meridional circulation, the use of empirical diffusion coefficients to represent transport is very questionable. In addition, such a procedure becomes quite complicated. A portion of the transport by the large scale waves is advective in nature, which means that the diffusion tensor is not strictly symmetric anymore (Strobel, 1981, and references therein). Furthermore, different coefficients should be used for different chemically active species to account for the fact that air parcels describe elliptical orbits around their mean position under the influence of a linear nondissipative wave and that the different constituents experience different net effects from identical displacements by virtue of their differing chemistry (Pyle and Rodgers, 1980).

In recent years, the incorporation of eddy fluxes into the transport calculation has often been made through the use of the diabatic and/or the residual circulation (Dunkerton, 1978; Holton, 1981; Garcia and Salomon, 1982; and others). This approach is based on the non-acceleration/no-transport theorem derived by Andrews and McIntyre (1976, 1978). The theorem states that a steady, conservative wave leaves the Lagrangian mean
flow unchanged and that a transient, nondissipative wave does not induce permanent changes in the Lagrangian mean flow. In a case where only steady and conservative waves are present, the Lagrangian mean flow is driven only by the generalized Lagrangian mean of the net diabatic heating. This diabatic heating is, of course, a result of nonconservative wave processes on various spatial and temporal scales. The resulting accelerations and decelerations are what give rise to the diabatic circulation (cf. Mahlman et al., 1984; and Geller, 1983). No explicit eddy-flux divergences appear in the mean flow equations for the conservative waves, although the effect of the eddies is taken into account by the averaging process. The noninteraction theorem is an exact result, i.e., it holds for finite amplitude disturbances but it involves generalized Lagrangian means as opposed to the conventional Eulerian means.

For this reason, the GLM theory is difficult to apply to practical problems. The Lagrangian mean is an average over “material tubes” which become distorted and change shape according to the evolution of the displacement-vector fields. Once the displacements get large, it becomes difficult to relate the center of mass positions of material tubes to the actual spatial distribution of a tracer at a given time (see Schoeberl, 1981a, for example). As long as the waves in a system are of small amplitude and linear, it can be assumed that the displacements remain small also and that the Lagrangian and Eulerian means of the diabatic heating are equal to a first approximation (Dunkerton, 1978). The meridional circulation obtained in a two-dimensional model by using the Eulerian mean of the diabatic heating (together with some form of dissipation) as the only forcing is usually referred to as the diabatic circulation and is used in a number of stratospheric tracer-models.

The next best approximation to incorporate eddy fluxes into a two-dimensional transport model is to use the residual circulation. Knowledge of the wave fields is necessary to compute the residual circulation.

It was shown by Boyd (1976) that in the absence of critical lines a linear, nondissipative wave induces a mean meridional circulation whose streamfunction is given by

$$\overline{\chi} = \frac{v' \phi_z'}{N^2}, \quad (1.1)$$

where $N^2$ is the static stability, $v'$ is the meridional component of the wave velocity and $\phi_z'$ is the vertical derivative of the geopotential disturbance. The overbar denotes the zonal average. When the conditions of the noninteraction theorem are fulfilled the induced meridional circulation cancels the fluxes associated with the wave itself. The residual circulation is defined by

$$\overline{\nu} = \overline{\boldsymbol{\nu}} - \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 \overline{\chi}) \quad (1.2a)$$

$$\overline{\omega} = \overline{\omega} + \frac{1}{a \cos \theta} \frac{\partial}{\partial \theta} (\overline{\chi} \cos \theta) \quad (1.2b)$$

where $\overline{\nu}$ and $\overline{\omega}$ are the Eulerian zonal averages of the north–south and the vertical velocity components. In the case of a linear, conservative wave the residual circulation velocities $\overline{\nu}$ and $\overline{\omega}$ are just the Eulerian mean circulation components minus the eddy induced Eulerian mean motions, i.e., that part of the mean circulation which is relevant for mass transport.

The residual circulation is always defined by (1.2a) and (1.2b), even when the waves in the system under consideration are not small and conservative. The residual circulation does not, in the general case, agree with the zonally-averaged mass circulation and (1.1) does not represent exactly the eddy induced part of the mean circulation. It has been shown by Holton (1981) that the transport equation for the zonal mean of a quantity $c$, rewritten in terms of the residual velocities, still has the form

$$\partial_t \overline{c} + \overline{\nu} \partial_x \overline{c} + \overline{\omega} \partial \theta \overline{c} = S + \nabla \cdot (K \nabla c) \quad (1.3)$$

with a diffusion tensor $K$ on the right-hand side. This tensor’s symmetric part represents diffusive processes and the tensor’s antisymmetric components account for the fact that the residual circulation is not the true advection velocity in the general case. Expressions for the components of $K$ have been derived in the appendix of Holton (1981).

It is often assumed, however, that the coefficients of the new diffusion tensor $K$ are small and that the bulk of the advective transport is represented by the advection terms on the left-hand side of Eq. (1.3). It is difficult to estimate a priori what errors are induced by neglecting $K$ and using the residual circulation alone for the transport of stratospheric trace constituents, although general circulation models have been used to estimate the values of $K$. (Kida, 1983; Plumb and Mahlman, personal communication, 1984).

Studies have shown that the residual circulation does not represent the actual mass transport in the vicinity of critical lines. Matsuno and Nakamura (1979) showed that a large, jetlike particle dispersion occurs along a horizontal critical line. This study was extended by Schoeberl (1981b) to critical wind lines of arbitrary orientation.

Hsu (1980, 1981) traced a large number of particles released in subtropical and polar latitudes during a simulated major warming. The particles, originally in low latitudes, were mixed rapidly polewards as the warming progressed and the critical wind line descended. Rood and Schoeberl (1983a, b) ran a series of experiments with a $\beta$-plane model in order to isolate the dominant mechanisms of ozone transport. They also concluded that critical line transport was significant and that the final late winter warming determines the timing and magnitude of the polar ozone maximum in spring.

The purpose of this paper is to compare the use of the residual circulation to a fully three-dimensional transport calculation for two idealized models of
Northern Hemisphere winter conditions. Wind fields were obtained by integrating a global stratospheric primitive equation model. The three-dimensional wind fields were used for exact (up to numerical inaccuracies) three-dimensional advection of tracer concentrations. The zonal averages of the mixing ratios as obtained were then compared with those calculated using the residual circulation that was diagnosed from the model output according to Eq. (1.2).

In Section 2, the dynamical model is described briefly. An example of tracer transport during a modeled stratospheric warming is presented in Section 3. This experiment is largely a repetition of one of Hsu’s (1980) studies with the exception that mixing ratios are computed instead of parcel trajectories. In the remaining sections, results from a 60-day “perpetual winter” integration in which a major warming did not occur are discussed. A passive tracer is used in these cases; however, in addition, the evolution of an “ozone-like” tracer is investigated for the perpetual winter situation.

2. Description of the dynamical model

The model used for this study is a semispectral, global primitive equation model, extending from approximately 10 to 100 km in altitude. The spatial grid is the same as in Holton’s (1976) and Holton and Wehrbein’s (1980a,b) model. The boundary conditions are of the same type as they used, i.e., the zonal mean wind is specified at the lower boundary and the zonal mean geopotential is computed to be in geostrophic balance at 10 km. At the top, both zonal mean and wave vertical velocities are required to vanish. Waves are forced by prescribing a geopotential disturbance at the lower boundary.

For the time integration, a version of the split-explicit technique (Madala, 1981, and references therein) was chosen. This integration method makes use of the fact that fast moving gravity waves are essentially linear. Therefore, two different time steps can be used. Terms governing the linear gravity waves are integrated with an explicit time-differencing scheme that requires a small time step. The nonlinear and Coriolis terms are recomputed at larger time-intervals and are kept constant while the linear part of the equation is marched forward. The large time steps are of the same order of magnitude as those that are typically used with a semi-implicit integration method, namely one-half to one hour, depending on the wave activity. The computational efficiencies of these schemes are also comparable. An advantage of the split-explicit technique is that the retardation of gravity waves is not as severe as in implicit schemes.

The split-explicit method was used in combination with a leapfrog scheme for the large time-step integration. A Matsuno step was inserted every two model days. Successive leapfrog steps were averaged (Haltiner and Williams, 1980, pp. 145–148) to prevent decoupling of the physical and computational modes. In addition, a biharmonic diffusion (Holton and Wehrbein, 1980a,b) was used in the latitudinal direction. The diffusion coefficient was relatively small, corresponding to a damping time of about two weeks for $2 \Delta y$ waves, where $\Delta y = a\Delta \theta$ is the grid distance in north–south direction.

The diabatic heating in the model consisted of Strobel’s (1978) parameterization of the absorption of solar UV and Newtonian cooling for which the profile in Schoeberl and Strobel (1978) was used. Mechanical dissipation was parameterized with a Rayleigh friction profile. The frictional damping time in the lower stratosphere was about 60 days. At the top, the coefficients were adjusted to achieve closure of the stratospheric jets, which required a damping time scale of one day. This parameterization is certainly crude and leads to problems in the mesosphere (Holton, 1982, 1983). However, the concentrations in the transport experiments that are discussed in the following are centered at 40 km altitude; therefore, our results are affected very little by the upper part of the model domain. More details of the model formulation can be found in Schneider (1984b).

The model resolution was 6 degrees in latitude. A log($p$) coordinate was used in the vertical with a grid-spacing corresponding to roughly 3.75 km. For the transport calculation, the wind fields were used on a three-dimensional spherical grid with the same resolution as above in the meridional plane and 18 degree grid-spacing in the longitudinal direction. The numerical schemes used for the tracer transport were a two- and a three-dimensional version of the Square Root Scheme (Schneider, 1984a). The only filtering applied to the concentrations was a weak smoother in the polar regions in the 3-D calculations. A time step of 20 minutes was used for the transport calculation; because of the convergence of the meridians and the relatively high velocities associated with wavenumber one in this region, the CFL-parameter became quite large during periods of high wave amplitude. Under these conditions, the Square Root Scheme tends to generate noise (see the discussion in Schneider, 1984a) which is damped out only very slowly by the inherent diffusion in the scheme.

3. Transport of a passive tracer during a linear modeled warming

During a stratospheric warming, all conditions of the noninteraction/no-transport theorem are violated, and it is well known that the residual circulation cannot represent the rapid mixing of air that occurs in the vicinity of a critical wind line (Matsuno and Nakamura, 1979; Schoeberl, 1981a; Rood and Schoeberl, 1983a,b). Hsu (1980, 1981) has computed trajectories of a large number of particles during modeled warmings
and has shown that material which is originally located in subtropical latitudes is mixed rapidly towards the pole. Since it is difficult to translate trajectories into mixing ratios this experiment has been repeated here.

The warming simulation was performed in the standard fashion (Hsu, 1980; Bridger and Stevens, 1982; Lordi et al., 1980). An initial wind profile was chosen, the zonal mean temperatures were computed to achieve thermal wind balance in the initial state, the heating was specified as a relaxation to these initial temperatures with Newtonian cooling coefficients and a wave forcing of 300 gpm was switched on at the lower boundary and kept constant after the switch-on period.\(^1\) The initial wind profile selected here was similar to the one used by Hsu (1980). Since the characteristics of our obtained warming were very similar to that of Hsu’s, we will not describe them again in this paper.

Figure 1 shows the wave-induced mean meridional circulation at model day 15. Figure 1a, b shows the Eulerian meridional and vertical velocity components while Figure 1c, d shows the residual mean velocity components. The magnitudes of the velocities vary over the course of the integration; for most of the time, however, the maximum of the north–south component of the residual velocity is only about 0.5 m s\(^{-1}\) as is the case for this figure. This means that the residual circulation can only effect a northward displacement of tracer material of about two gridpoints during the 30-day period under consideration.

All of the meridional circulation is wave-induced in this case since the initial mean state to which the relaxation is specified contains only a zonal velocity field.

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\(^1\) Although this is the usual way of producing mechanistic models of stratospheric warmings, such models are probably not representative of the manner in which stratospheric warmings take place in the real atmosphere. In such mechanistic models, the initial state, to which the model is continually being relaxed, exists largely due to strong dynamic effects. Section 4 of this paper discusses a more realistic case.
Fig. 2. Evolution of a passive tracer field during the modeled warming. Total column density (arbitrary units) and zonally averaged mixing ratios as obtained from the three-dimensional calculation at day 5, (a and b), (initial condition for the transport calculations), day 15 (c and d) and day 35 (e and f). (g) is the result of the two-dimensional calculation, using the residual circulation. Note that the contour intervals are not the same in all panels.
The meridional velocities of the diabatic circulation, however, are of the same order as the wave-induced mean meridional winds, and therefore, the results would not be changed drastically for this short period integration.

The initial tracer distribution is shown in Fig. 2a, b. Figure 2a is a polar stereographic projection of the total column density and Fig. 2b shows the zonally averaged mixing ratio. Units are arbitrary. The initial distribution is centered at $36^\circ$N and 40 km height. As the wave grows, the velocities associated with it attain values on the order of $30-40 \text{ m s}^{-1}$, and a deformation of the ring can be seen to occur very quickly (Fig. 2c, d). After the warming is completed, at day 35 of the integration, a new maximum in the concentration has been formed right over the pole. The column density there is about 15% higher than the original column maximum of the ring. It can also be seen that southward and upward tracer transport from the equatorward edge of the initial distribution has taken place. By contrast, transport by the residual circulation, starting from the same initial condition, results only in a small northward displacement of the high latitude portion of the ring with the low latitude portion being unchanged (Fig. 2g).

4. Transport during a perpetual winter simulation

While the previously discussed linear warming represents some of the essential dynamics of a major stratospheric warming, it is a very idealized study. Since the diabatic heating was specified as a relaxation to a specified state that is largely maintained through winter dynamics, the radiative drive on the mean flow remained very small (of the order of 2 K day$^{-1}$). Therefore, the restoring of the mean flow after the warming proceeded very slowly. The critical line stayed at the altitude of maximum tracer concentration for a long time so that this case was very unfavorable for comparison with the residual circulation. Also, although major stratospheric warmings are observed to occur every two years on the average, they are hardly representative of typical winter conditions in the stratosphere.

One can ask how well the residual circulation represents the actual mass circulation in a situation where several planetary waves are interacting and have transient components, as is the case in the observed winter stratosphere, but when no major warming is in progress. In the derivation of Eq. (1.1) (Boyd, 1976), the wave is assumed to have a time dependence of the form $\exp(-iut)$. If two linear waves are present, the equations from which (1.1) follows hold only after an additional time average over a time $T = 2\pi/(\omega_1 - \omega_2)$ is performed. In the case where the planetary wave field has a stationary component and a traveling component with a phase speed of $5 \text{ m s}^{-1}$, this averaging time is about two months. The waves in the stratosphere do not remain stationary for such a long time, but significant growth and decay takes place. Therefore, it is not immediately clear if a complete averaging out of the wave motions around some mean position, determined by the residual circulation, can take place over the course of a winter season.

In order to obtain a more realistic simulation of Northern Hemisphere winter conditions in the stratosphere, the model was spun up with diabatic heating and friction as explained in Section 2. Wavenumber 1 and 2 were computed and allowed to interact with each other. The forcing at the lower boundary consisted of a stationary wave one disturbance of 200 m geopotential amplitude centered at $60^\circ$N with a transient 30-day wave of 150 m amplitude superimposed on it. During the spin-up period, only wavenumber 1 was forced at the bottom. Wavenumber 2 was internally forced by the self-interaction of wavenumber 1. Afterwards, wave 2 was also forced at the bottom. The forcing that was chosen is in qualitative agreement with

![Image](https://example.com/image.png)

**Fig. 3.** Time–height cross section of the model-produced mean zonal wind (a), and the model-generated amplitude of $m = 1$ (b) at $60^\circ$N for the perpetual winter simulation. The vertical lines mark 15 day intervals.
the observations of the winter of 1976/77, published by O'Neill and Taylor (1979), and the NMC data set, analysed by Geller et al. (1983, 1984). After 90 days of spinup time, the amplitudes of wavenumber one reached values throughout the model domain that are in the range of observations. The integration was continued from this time onward for another 60 days, and the transport experiments discussed in the following section were made during this period from model day 90 to day 150. Figure 3a shows the time–height cross section of the model-produced mean zonal wind at 60°N. No major warming occurred, but two minor events can be seen to have occurred at around day 105 and day 140. At these times, the mean zonal wind speed diminished significantly. Figure 3b is a time–height cross section of the modeled \( m = 1 \) amplitude at 60°N. Comparing this with the data from four winters analysed by Geller et al. (1983, 1984) that are shown in Fig. 4, it can be seen that both the range of amplitudes of wave 1 and their time dependence are reasonably realistic. The amplitude of \( m = 2 \) (not shown) remained about a factor of 2 too small as compared to observations.

The meridional circulations for two specific days are shown in Figs. 5 and 6. Day 90 is representative for quiet periods. The residual circulation at such times is almost indistinguishable from the diabatic circulation that would be obtained in this model without planetary waves and using the same mean heating and Rayleigh friction. Day 100 is characterized by a large \( m = 1 \) amplitude.

Transport experiments were again done with the same initial tracer concentrations peaking at 40 km height. During the whole course of the integration, there was never a critical line in an area of nonzero mixing ratio. In fact, the only time a critical line occurred was a few days before the first minor warming, and it never descended below 85 km. Therefore, the differences observed between the transport by the residual circulation and the exact, three-dimensional transport are caused only by the effects of large amplitude waves, nonlinearity, transience and dissipation.

a. A passive tracer experiment

The initial tracer concentration for this experiment is shown in Fig. 7a, b; it is the same as the one used in the major warming case, namely a ring centered at 36°N and 40 km height. The evolution of the tracer field as obtained by the three-dimensional calculation is shown in the series of Figs. 7c–m. Polar projections

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**FIG. 4.** Observed time–height cross sections of the \( m = 1 \) amplitude at 60°N during four winters, from four winters of data analysed by Geller et al. (1983, 1984).
of the column density and cross sections of the zonally averaged mixing ratio are shown side by side for several days. A new maximum is formed within the first ten days, and strong poleward transport is apparent. No cross polar flow occurs during the first minor warming event since high wave amplitudes are not maintained for a sufficiently long time. After day 110, the concentration is again drawn out to some extent, and the 1000-line in Fig. 7g has moved considerably southward over most of the hemisphere compared to day 110. During the second surge of $m = 1$ amplitude the sequence of events is the same again, but the changes are not as drastic anymore. This is because the newly formed concentration maximum in the modeled Aleutian High is already there and a large part of the tracer simply circulates about with the position of the vortex changing only slightly.

The residual circulation cannot, of course, represent the strong zonal asymmetries in the tracer distribution. Using it only results in a much more continuous northward displacement of the initial ring and distributions that are less drawn out at the northern and southern end. Figure 8 shows the difference in the zonally averaged mixing ratios obtained by the two methods. Plotted are the two-dimensional mixing ratios minus the zonal averages of the three-dimensional calculation. The differences are of the same order as the mixing ratios themselves. With every surge of wave-amplitude, tracer material is transported towards the pole; simultaneously, some material from the equatorward end of the distribution is mixed southward and upward. As the wave decays, the transport is reversed. These features are not represented by the residual circulation and are manifested as differences of fluctuating magnitude at the high- and low-latitude end of the distributions in the figure. Furthermore, an overestimate of
the downward transport is apparent throughout the integration period. The reason for this seems to be the fact that the tracer distribution is less coherent spatially in the three-dimensional calculation. Therefore, the tracer material is not distributed between regions of upward and downward velocity in such a way that its zonal mean would experience the net displacement given by the residual circulation.

The southward and upward mixing at the southern end of the tracer field is a consequence of the meridional gradients of the concentration and the velocity field associated with the planetary waves. When, for example, wavenumber 1 is dominant, a high and a low are formed 180 degrees apart in middle to high latitudes. Between these vortices, the north–south component of the wave velocity ($v'$) is negative in some interval of longitude and tracer is advected southward in this region. On the opposite side of the globe $v'$ is positive. The tracer concentration, however, falls off sharply towards the equator so that nothing is advected northward in this region. The role of the equatorial critical wind line (McIntyre and Palmer, 1983) for this process is difficult to assess. The same qualitative behavior of the mixing ratios was observed in control runs with a fixed zonal mean wind profile without any critical line. However, since the wave structure depends strongly on the zonal mean wind profile, a quantitative comparison of the strength of the southward/upward mixing in the two cases was not possible.

One might think that most of the differences between the three-dimensional and two-dimensional transports shown in Fig. 8 are due to the rapid deformations of the tracer field at the beginning of the period that are too fast for the residual circulation to follow. Therefore, the two-dimensional calculation has been repeated, starting from day 100, using the zonally averaged mix-
Fig. 7. Evolution of the mixing ratios of a passive tracer during the 2-months winter simulation. Total column amounts (arbitrary units) in the Northern Hemisphere and zonally averaged mixing ratios as obtained from the three-dimensional calculation. Days are indicated on the panels.
FIG. 8. Difference of the mixing ratios of the passive tracer obtained with the residual circulation and the three-dimensional calculation (2-D ratios minus 3-D ratios) at various days. Both calculations start from the same initial condition at day 90.

ing ratio of the three-dimensional run on that day as the initial condition. The differences from the three-dimensional computation at later days are shown in Fig. 9. The errors are smaller now, but still amount to about 40% of the peak mixing ratio that was obtained in the exact calculation. The fluctuations with the time period of the amplitude changes and the overestimate of the downward transport are still apparent.
**Fig. 9.** As in Fig. 6 but starting from day 100, using as initial condition for the two-dimensional calculation the zonal average of the three-dimensional computation at that day.
photochemistry is modeled adequately in this manner; however, it has been shown by Rood (1982) and Rood and Schoeberl (1983a,b) that many features of the observed ozone distributions can be modeled using this simple approach.

The deviations from the initial state at later days as obtained in the three-dimensional calculation are shown in Fig. 11. The formation of a new tracer maximum in the Aleutian high is seen as well as a general transport from low to high latitudes. The depletion of tracer in equatorial regions is much stronger than observed for actual ozone. This problem has also occurred in other model calculations (Rood, 1982; Mahlman et al., 1980).

To compare these results with those obtained using the residual circulation, the differences of the zonally averaged mixing ratios obtained with the two methods have been plotted for various days (see Fig. 12). During the first 20 days, the three-dimensional poleward transport is stronger than given by the residual circulation. Later, however, this picture changes. The use of the residual circulation leads to higher concentrations near the pole than obtained with the three-dimensional calculation. Furthermore, the mixing ratios are also overestimated in the low latitude upper stratosphere. Considering that the deviations of the mixing ratio from the initial state in the polar regions was of the order of 2 units, the error induced by the residual circulation is about 50%. The error is again caused, to a large extent, by the overestimation of the vertical transport by the residual circulation. In low latitudes, tracer is transported upward from the conservative region, while near the pole it is pushed downward out of the source region.

**5. Conclusions**

The use of the residual circulation for stratospheric tracer transport has been compared to a fully three-dimensional calculation for two idealized Northern Hemisphere winter conditions as simulated with a mechanistic model: a major warming and a 60 day winter simulation without a major warming.

As expected, the residual circulation gives an inadequate representation of the actual mass motions during periods of major warmings. This is caused mainly by the descending critical line and the strong particle dispersion along that line (Matsuno and Nakamura, 1979). Since major warmings are observed to occur about once every two years, and chemical models are often integrated over periods of several years, this problem may be significant.

For the perpetual January case without a major

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2 Mahlman et al. (1984) discuss this in terms of the competing effects of the diabatic circulation and the diffusive effect of eddies in determining the equilibrium slope of tracer surfaces.
Fig. 11. Evolution of the ozone tracer as obtained by the three-dimensional calculation. Both column densities and mixing ratios are the deviations from the initial state at the days indicated.
warming, errors of the order of 50% (more for distributions with large latitudinal gradients) were found in the zonally averaged mixing ratios. The error in using the residual circulation consists of a fluctuating component which follows closely the wave transience and an apparent overestimate of the vertical transport (again, see the discussion in Mahlman et al., 1984). This is related to the fact that the tracer material is not moving in and out of regions of upward and downward velocity but tends to spend a longer time in one or the other of them. As mentioned in the Introduction, the residual circulation cannot be expected to reflect short
term fluctuations related to the fact that air parcels describe elliptical paths around their mean positions under the influence of a wave. The time scale of these motions is of the order of a few days to a week, and some researchers have tried to include corrections for this effect in their models (e.g., Garcia and Salomon, 1982). The fluctuations in the tracer concentrations observed are of the time scale of one month and are determined by variations in the wave amplitude. Therefore, tracers with a chemical life time of the order of one month may not be well represented by using the residual circulation for their transport.

The differences found between the two- and three-dimensional calculation in this study are probably an underestimate of the problems one has to expect from most two-dimensional photochemical models using residual or diabatic circulations for their transport calculation. In the present case, the waves have actually been computed and the residual circulation has been diagnosed at every time step from the actual waves present. Usually, much less information about the wave-fields is available. It should also be noted that the zonal asymmetries in the tracer concentrations are significant, even if only one or two large waves are present. Therefore, errors will be induced by only using the zonal mean of the mixing ratios in a two-dimensional photochemical model (Tuck, 1979). These are thought to be rather small, however, on the order of 10–20%.

The errors induced by using the residual circulation for stratospheric tracer transport depend on the dynamical state of the atmosphere, the gradients of the tracer concentration and the initial distribution of the material. A general assessment of the shortcomings of two-dimensional transport is, therefore, difficult, and has not been attempted here. This study can only point to some possible problems one has to expect from the use of the residual circulation.

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