The Quasi-biennial Oscillation of Ozone in the Tropical Middle Stratosphere: A One-Dimensional Model

XIU-DE LING* AND JULIUS LONDON

Department of Astrophysical, Planetary and Atmospheric Sciences, University of Colorado, Boulder, CO 80309

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ABSTRACT

A one-dimensional model of the quasi-biennial oscillation (QBO) of ozone in the tropical middle stratosphere is derived based on assumed (observed) zonal wind QBO in a coupled dynamic, radiative/photochemical system. It is found that the derived vertical variation of the ozone QBO amplitude has two maxima, one at 32 km and the other at 22 km, and a minimum at 28 km. These are in qualitative agreement with observations. In the height interval 30–35 km, the ozone QBO is closely related to temperature dependent photochemistry and the ozone and temperature variations are out of phase. Below 28 km, where vertical ozone and thermal transports are important, ozone and temperature oscillations are in phase but both are approximately 270° out of phase with the vertical wind variation.

1. Introduction

Early evidence that there exists a steady but different zonal circulation at different levels in the stratosphere was based on individual restricted sets of observed easterly and westerly winds, (subsequently called the "Krakatoa Easterlies" and "von Berson Westerlies") in the tropical stratosphere (see, for instance, Wexler, 1951; Palmer, 1954). The fluctuating character of this stratospheric wind system was reported by Graystone (1959) and by Ebbon (1960) and a quasi-biennial oscillation (QBO) of the tropical stratospheric zonal wind was clearly identified by Reed et al. (1961) and Veryard and Ebbon (1961). It was later suggested by Ebbon (1963) that the so-called "Krakatoa Easterlies" and "von Berson Westerlies" are, in fact, part of a "tropical wind fluctuation" which has been in existence at least since the early 1880s. A corresponding QBO was first reported for stratospheric temperature by Ebbon (1961), and by Angell and Korshover (1962).

Long period datasets of stratospheric wind observations have been analyzed by Tucker (1979), Coy (1979; 1980), London et al. (1983), and others. The observations show that zonally symmetric alternating westerly and easterly winds occur in the tropical stratosphere with an average period close to 28 months, which, however, has varied during the past 35 years from 21 to 33 months. Successive wind regimes first appear at about 35 km and propagate downward at an average rate of about 1–1.5 km mo−1. The maximum zonal winds, about 20 m s−1, are found over the equator at about 25 km. There is a rapid attenuation of the oscillation below 20 km and it is not detected below about 16 km. A general description of the main features of the stratospheric wind QBO has recently been discussed by Plumb (1984) and Dunkerton and Delisi (1985).

The major characteristics of the tropical zonal wind stratospheric QBO have been successfully explained by Holton and Lindzen (1972) based on the concept that the upward transport of momentum associated with equatorial waves is damped as a result of infrared radiational cooling at these levels. The dissipation of the waves produce an acceleration in the mean flow such that Kelvin waves produce westerly acceleration and mixed Rossby–gravity waves produce easterly acceleration. The theory is able to account for the long term, approximately biennial, periodicity and the downward propagation of the phase of the zonal wind oscillation.

Soon after the discovery of the equatorial stratospheric wind QBO, a 24-month cycle in total ozone was reported for two subtropical stations in the Southern Hemisphere by Funk and Garnham (1962), and shown by Ramanathan (1963) to be associated with the stratospheric wind oscillation. The total ozone QBO has since been verified by ground-based and satellite derived observations (see, for instance, Angell and Korshover, 1964; Tolson, 1981; Hilsenrath and Schlesinger, 1981; Oltmans and London, 1982; Hasebe, 1983). The QBO in total ozone seems to have been present at Shanghai (31°N) at least since 1932 (Angell and Korshover, 1964).

At equatorial latitudes, the amplitude of the total ozone QBO is about the same as that for the annual

* Current affiliation: National Center for Atmospheric Research, Boulder, CO.

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variation (i.e., \(\sim 10 \text{ m-atm-cm}\)) and the maximum occurs near the time of the maximum west wind at about 50 mb (e.g., Oltmans and London, 1982). At midlatitudes, however, where the amplitude of the total ozone QBO appears to be a minimum, it is out of phase with the tropical zonal wind oscillation. The equatorial ozone QBO has the same period as the average equatorial stratospheric wind QBO (\(\sim 27.5\) months). At subpolar latitudes there seems to be a weak biennial oscillation (\(\sim 24\) months) which may or may not be associated with the equatorial QBO.

Variations of total ozone are most closely related to ozone variations in the lower stratosphere (see, for instance, London and Angell, 1982). It is reasonable, therefore, to expect that the ozone QBO should also be present at those levels. Analyses of ozonesonde and Umkehr observations have given some indication of the existence of an ozone QBO in the stratosphere (Angell and Korshover, 1981; Oltmans and London, 1982) but the precision of the analysis of these data is limited by the uneven geographical distributions of observing stations, (there are no long period equatorial ozonesonde or Umkehr observations), the frequent absence of data and the inconsistency often found for the QBO between the two types of observed data. Stratospheric ozone observation from satellites, on the other hand, although covering a much shorter period of time than some Umkehr and ozonesonde measurements, can provide much better space and time density of the ozone data. For nadir viewing systems, however, the height resolution is limited approximately to three layers, each about 5 km in depth. Analysis of stratospheric ozone data derived from Nimbus-4 BUV measurements for the period April 1970 through May 1977 is shown in Fig. 1 (taken from Ling, 1984). The curves in Fig. 1 are the deseasonalized ozone mixing ratios (ppmv) for the 22–27 km layer for 5° latitude belts 30°S to 30°N. The quasi-biennial character of the ozone variation in this layer is quite apparent. The period of the oscillation is 27–28 months and its amplitude is a maximum at the equator and decreases to 30°.

In the lower and middle stratosphere, variations of the ozone concentration are determined by the interaction of dynamics and temperature related photochemistry. The major features of the observed equatorial stratospheric zonal wind QBO have been generally accounted for by Holton and Lindzen (1972) and the resulting temperature QBO by Plumb and Bell (1982b). However, no satisfactory theory has yet been proposed for the observed stratospheric ozone QBO.

In principle, it should be possible to derive the height and latitude variation of the stratospheric ozone QBO from the coupled set of relevant dynamic, thermodynamic, photochemical and ozone continuity equations with suitable assumptions involving, for example, the mechanism through which equatorial waves produce accelerations in the mean zonal flow. On the other hand, if we assume, from previous results (Holton and Lindzen, 1972; Plumb and Bell, 1982a,b), that the stratospheric zonal wind and consequently the temperature QBOs are reasonably well explained, we can use the observed wind (or temperature) oscillation to solve a set of simultaneous equations for the QBO components of other variables; that is, perform a diagnostic study based on an assumed (observed) wind or temperature variation.

In the equatorial stratosphere the amplitude of the zonal wind QBO is of the same order as the zonal wind itself (i.e., 10 m s\(^{-1}\)) while the amplitude of the temperature QBO (\(\sim 1\) K) is more than two orders smaller than the average temperature (\(\sim 225\) K). It is known (see, for instance, Ling, 1984) that random errors in a dataset will cause higher uncertainties in the analyzed harmonics than was present in the original set. A realistic estimate of the random error of observed stratospheric winds and temperatures are 1 m s\(^{-1}\) and 1.0 K (Range Commanders Council, Meteorological Group, 1981). We have, therefore, chosen to derive the temperature, ozone, and vertical wind QBOs from the observed zonal wind QBO.

Here we restrict the study to a one-dimensional system near the equator in order to provide a useful picture of the primary mechanisms responsible for the ozone QBO and its vertical variation of amplitude and phase.
in the height interval 18 to 36 km. We will assume, based on observations, a height and time dependent QBO in the equatorial zonal wind, and use a simplified coupled dynamic, radiative and photochemical system to derive analytic solutions for the resulting QBOs of temperature, vertical and meridional wind, and ozone mixing ratio. The assumed ozone photochemical system involves odd oxygen, odd hydrogen and odd nitrogen. Parameterization of photochemical relaxation and temperature feedback in the ozone system, as well as infrared cooling, will be included in our model. In a subsequent paper we will extend this study in latitude to the subtropics.

2. Model development

The basic system for a two-dimensional (2-D) model which couples dynamics, radiation, and ozone photochemistry in the equatorial and tropical regions of the lower and middle stratosphere, neglecting molecular friction and assuming hydrostatic equilibrium and infrared cooling given by a Newtonian cooling coefficient, can be given by the following set of zonally averaged equations in log-pressure coordinates (see, for instance, Holton, 1979; Hartmann and Garcia, 1979):

\[
\frac{\partial \tilde{u}}{\partial t} + v \frac{\partial \tilde{u}}{\partial y} + w \frac{\partial \tilde{u}}{\partial z} + f \tilde{v} = - \frac{\partial}{\partial y} (\tilde{v} \tilde{u}) - \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w \tilde{u})
\]

(1)

\[
\frac{\partial \tilde{v}}{\partial t} + v \frac{\partial \tilde{v}}{\partial y} + w \frac{\partial \tilde{v}}{\partial z} + f \tilde{u} = - \frac{\partial}{\partial y} (\tilde{u} \tilde{v}) - \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w \tilde{v})
\]

(2)

\[
\tilde{\theta} = \frac{\partial \tilde{\phi}}{\partial z}
\]

(3)

\[
\frac{\partial}{\partial y} (\rho \tilde{v}) + \frac{\partial}{\partial z} (\rho \tilde{w}) = 0
\]

(4)

\[
\frac{\partial \tilde{\theta}}{\partial t} + v \frac{\partial \tilde{\theta}}{\partial y} + w \frac{\partial \tilde{\theta}}{\partial z} + \frac{\kappa}{H} \tilde{\theta} = S \tilde{\gamma} - \mu (\tilde{\theta} - \theta_z)
\]

(5)

\[
\frac{\partial \tilde{\gamma}}{\partial t} + v \frac{\partial \tilde{\gamma}}{\partial y} + w \frac{\partial \tilde{\gamma}}{\partial z} = \tilde{F}_c - \frac{\partial}{\partial y} (\tilde{v} \tilde{\gamma}) - \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w \tilde{\gamma})
\]

(6)

The zonal average of a quantity is defined as

\[
\overline{G} = \frac{1}{2\pi} \int_{0}^{2\pi} G \cdot d\lambda
\]

where the subscripts a and q denote the long-term annual mean and QBO components, respectively.

We assume that the zonally averaged dependent variables have two parts: the first represents a long-term annual average, and the second a quasi-biennial oscillation. The total zonal mean variables are then given by

\[
\overline{G} = \overline{G}_a + \overline{G}_q
\]

where the subscripts a and q denote the long-term annual mean and QBO components, respectively.

As stated earlier, the basic mechanism for the zonal wind QBO in the tropical stratosphere involves the wave-zonal mean flow interaction due to upward propagating equatorial Kelvin and mixed Rossby-gravity waves. Therefore, the momentum equation in the \(x\) direction should include the eddy advection contributions for the different QBO fields. In the present study we will derive the QBOs of the \(O_3\) temperature, and meridional and vertical velocity fields based on the assumption that they are initiated as a result of forcing by the zonal wind QBO. Thus, all eddy flux contributions will be omitted in the equations governing the QBO fields except for the momentum equation in the \(x\) direction.

In addition, in the stratosphere the nontransport theorem approximately holds (see, for instance, Andrews and McIntyre, 1978a,b). The meridional transport of all quantities can be considered as advected by small residual motion. Furthermore, the meridional residual motion can be deduced from diabatic motion as a first approximation (Dunkerton, 1978). Therefore, the eddy terms in Eq. (8) may be dropped provided

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**Table 1. The definition of symbols appearing in Eqs. (1) through (6).**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x, y)</td>
<td>Tangent plane coordinates pointing east and north respectively</td>
</tr>
<tr>
<td>(z)</td>
<td>Vertical coordinate pointing upwards, defined as (z = H \ln(p/p_0)), (p) pressure, (p_0) reference pressure</td>
</tr>
<tr>
<td>(\tilde{\gamma})</td>
<td>3-D velocity vector</td>
</tr>
<tr>
<td>(t)</td>
<td>Time</td>
</tr>
<tr>
<td>(u, v, w)</td>
<td>Velocity components along the (x, y, z) coordinates respectively</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Coriolis parameter ((2\Omega \sin \psi) where (\psi) is latitude)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Geopotential</td>
</tr>
<tr>
<td>(\tilde{H})</td>
<td>Buoyancy acceleration defined as ((R/T) \cdot \nabla (T)) ((T) is temperature). (T) is converted to (\theta) in order to simplify the hydrostatic equation in the log-pressure system.</td>
</tr>
<tr>
<td>(R)</td>
<td>Scale height</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>The gas constant for dry air</td>
</tr>
<tr>
<td>(S)</td>
<td>Volume mixing ratio of ozone</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Newtonian cooling coefficient</td>
</tr>
<tr>
<td>(\theta_z)</td>
<td>The standard vertical distribution of (\theta) used for the Newtonian cooling approximation, i.e., U.S. Standard Atmosphere, 1976</td>
</tr>
<tr>
<td>(C_e)</td>
<td>The infrared cooling rate parameter corresponding to the standard vertical distribution of (\theta),</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Air density ((\rho_0 \exp^{-\rho_\theta}))</td>
</tr>
</tbody>
</table>

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that the meridional circulation ($\bar{v}_q$, $\bar{w}_q$) is derived from
diabatic heating as is the case in the treatment here.

Thus the momentum equation in the $y$ direction, together
with the hydrostatic, mass continuity, thermodynamic
and ozone continuity equations, constitute a
diagnostic system for the wind, temperature and
ozone QBO fields.

We rewrite equations (1) through (6) by subtracting
the annual from the total fields. Note that $\partial \bar{v}_q / \partial t = 0$
and $(\kappa / \eta) \partial \bar{w}_q / \partial t < (\kappa / \eta) \partial \bar{w}_q / \partial t$. Then the set of equations for
the QBO variables are

$$\frac{\partial \bar{u}_q}{\partial t} + \bar{v}_q \frac{\partial \bar{u}_q}{\partial y} + \bar{w}_q \frac{\partial \bar{u}_q}{\partial z} - f \bar{u}_q + \frac{\partial \bar{u}_a}{\partial y} + \frac{\partial \bar{u}_a}{\partial z}$$

$$+ \bar{v}_a \frac{\partial \bar{u}_a}{\partial y} + \bar{w}_a \frac{\partial \bar{u}_a}{\partial z} = \text{FORCING DUE TO EQUATORIAL WAVES}$$

$$\frac{\partial \bar{v}_q}{\partial t} + \bar{v}_q \frac{\partial \bar{v}_q}{\partial y} + \bar{w}_q \frac{\partial \bar{v}_q}{\partial z} + f \bar{u}_q + \frac{\partial \bar{v}_q}{\partial y}$$

$$+ \bar{w}_q \frac{\partial \bar{v}_q}{\partial z} + \frac{\partial \bar{v}_a}{\partial y} + \bar{w}_a \frac{\partial \bar{v}_a}{\partial z} = - \frac{\partial \phi_d}{\partial y}$$

(8)

$$\frac{\partial \bar{u}_q}{\partial t} + \frac{1}{\rho} \frac{\partial (\rho \bar{w}_q)}{\partial z} = 0$$

(9)

$$\frac{\partial \bar{w}_q}{\partial t} + \frac{\partial \bar{w}_q}{\partial y} + \bar{w}_q \frac{\partial \bar{w}_q}{\partial z} + \bar{v}_q \frac{\partial \bar{w}_q}{\partial y} + \frac{\partial \bar{w}_q}{\partial y} + \bar{w}_q \frac{\partial \bar{w}_q}{\partial z}$$

$$+ \bar{w}_q \frac{\partial \bar{w}_q}{\partial z} + \bar{w}_a \frac{\partial \bar{w}_a}{\partial z} = \kappa \gamma_q - \mu \bar{w}_q$$

(10)

$$\frac{\partial \bar{\bar{\gamma}}_q}{\partial t} + \bar{v}_q \frac{\partial \bar{\bar{\gamma}}_q}{\partial y} + \bar{w}_q \frac{\partial \bar{\bar{\gamma}}_q}{\partial z} + \bar{v}_a \frac{\partial \bar{\bar{\gamma}}_q}{\partial y} + \frac{\partial \bar{\bar{\gamma}}_a}{\partial y} + \bar{w}_a \frac{\partial \bar{\bar{\gamma}}_a}{\partial z}$$

$$+ \bar{w}_a \frac{\partial \bar{\bar{\gamma}}_a}{\partial z} = \Gamma \gamma_q + \alpha \bar{\bar{\gamma}}_q$$

(12)

where $\bar{N}^2$ is the square of the Brunt–Väisälä frequency
defined as

$$\bar{N}^2 = \frac{\partial \phi_d}{\partial z} + \frac{\kappa}{\eta}$$

which is equivalent to the traditional definition

$$\bar{N}^2 = \frac{g}{T} \left( \frac{\partial T}{\partial Z} + \frac{g}{c_p} \right)$$

where $Z$ is geometric height.

The $\Gamma$ and $\alpha$ are two photochemical parameters,
which describe the ozone relaxation to equilibrium due
directly to the ozone perturbation itself and indirectly
to the temperature perturbation. The parameter $\alpha$,
therefore, defines a negative feedback effect of tem-
perature on the ozone concentration. Analytic expres-
sions for $\Gamma$ and $\alpha$ are derived in the Appendix.

The set of equations (8) through (12) can be simpli-
ified somewhat by comparison of the different scaled
terms in each equation. The magnitudes of various
quantities, applicable to tropical stratospheric condi-
tions (18–36 km), are listed in Table 2. By using the
zonally averaged indicated magnitudes of relevant
quantities it can be seen that $f \bar{u}_q$ is generally about five
orders of magnitude larger than the other terms on the
left side of Eq. (8). Even if we take the latitudinal scale
$L_y$, as $10^3$ m (i.e., only 1 km away from the equator),
$f \bar{u}_q$ is still two orders larger than the other terms. It has
been shown (see, for instance, Reed, 1964; Ling, 1984)
that the wind and temperature oscillations are consist-
ent with the thermal wind relation, even very close to
the equator. Thus, we can use the geostrophic wind
relationship in the vicinity of the equator. Scaling ap-
proximations applied to Eq. (11) show that $w_q \bar{N}^2$ and
the diabatic terms on the rhs of Eq. (11) are at least
two orders of magnitude larger than the remaining
terms.

In the vicinity of the equator $\bar{w}_q(\partial \gamma_q / \partial z)$ is more
than one order of magnitude larger than the other advec-
tive terms in Eq. (12). The photochemical terms, on
the other hand, vary with height. So that, in the 18–
36 km layer considered in this study, they are an order
larger than $\bar{w}_q(\partial \gamma_q / \partial z)$ in the upper part and somewhat
smaller in the lower part of the layer. We will use
$\bar{w}_q(\partial \gamma_q / \partial z)$ to represent the advective contribution to

<table>
<thead>
<tr>
<th>Variable</th>
<th>Order</th>
<th>Reference or source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_q$</td>
<td>$10 \text{ m s}^{-1}$</td>
<td>Hamilton (1982)</td>
</tr>
<tr>
<td>$u_q$</td>
<td>$10 \text{ m s}^{-1}$</td>
<td>Reed (1965), Tucker (1979), Coy (1979), London et al. (1983)</td>
</tr>
<tr>
<td>$\theta_q$</td>
<td>$10^{-2} \text{ m s}^{-2}$</td>
<td>Reed (1965), London et al. (1983)</td>
</tr>
<tr>
<td>$\gamma_q$</td>
<td>$10^{-7} \text{ ppv}$</td>
<td>BUV data</td>
</tr>
<tr>
<td>$N_q$</td>
<td>$10^{-6} \text{ s}^{-2}$</td>
<td>Holton (1979)</td>
</tr>
<tr>
<td>$\bar{\bar{\gamma}}_q$</td>
<td>$10^{-10} \text{ ppv m}^{-1}$</td>
<td>BUV data</td>
</tr>
<tr>
<td>$\frac{\partial \bar{\bar{\gamma}}_q}{\partial z}$</td>
<td>$10^{-13} \text{ ppv m}^{-1}$</td>
<td>BUV data</td>
</tr>
<tr>
<td>$v_q$</td>
<td>$10^{-2} \text{ m s}^{-1}$</td>
<td>Louis (1974)</td>
</tr>
<tr>
<td>$w_q$</td>
<td>$10^{-3} \text{ m s}^{-1}$</td>
<td>Louis (1974)</td>
</tr>
<tr>
<td>$v_q$</td>
<td>$10^{-2} \text{ m s}^{-1}$</td>
<td>model results of Plumb and Bell (1982b)</td>
</tr>
<tr>
<td>$w_q$</td>
<td>$10^{-4} \text{ m s}^{-1}$</td>
<td>model results of Plumb and Bell (1982b)</td>
</tr>
<tr>
<td>$L_y$</td>
<td>$10^4 \text{ m}$</td>
<td>assumed</td>
</tr>
<tr>
<td>$L_z$</td>
<td>$10^4 \text{ m}$</td>
<td>assumed</td>
</tr>
<tr>
<td>$S$</td>
<td>$10^{-2}$ to $10^{-1} \text{ m s}^{-3}$</td>
<td>height dependent, see Table 3</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$10^{-4} \text{ s}^{-1}$</td>
<td>see Fig. 6</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>$10^{-6}$ to $10^{-3} \text{ s}^{-1}$</td>
<td>height dependent, see Table 3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$10^{-14}$ to $10^{-11} \text{ m}^{-1} \text{s}$</td>
<td>height dependent, see Table 3</td>
</tr>
</tbody>
</table>
the ozone mixing ratio variation in the present 1-D study.

On the basis of the preceding discussion and the assumption of geostrophy even close to the equator, we can simplify Eqs. (8) through (12) to give:

$$\frac{\partial \tilde{\varrho}_q}{\partial y} = -f \frac{\partial \tilde{u}_q}{\partial z}$$  \hspace{1cm} (13)

$$\frac{\partial \tilde{u}_q}{\partial y} + \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \tilde{w}_q) = 0$$  \hspace{1cm} (14)

$$\frac{\partial \tilde{q}_q}{\partial t} + \tilde{w}_q \nabla \tilde{q}_q = S \tilde{\gamma}_q - \bar{\mu} \tilde{q}$$  \hspace{1cm} (15)

$$\frac{\partial \tilde{\gamma}_q}{\partial t} + \tilde{w}_q \frac{\partial \tilde{\gamma}_q}{\partial z} = \Gamma \tilde{\gamma}_q + c \bar{\gamma}_q.$$  \hspace{1cm} (16)

We have eliminated the momentum equation in the $x$ direction since we will assume a known function for the zonal wind QBO derived from observations. We can use the four equations above to solve for the four unknowns: $\tilde{q}_q$, $\tilde{\varrho}_q$, $\tilde{w}_q$ and $\tilde{\gamma}_q$. At stratospheric equatorial and tropical latitudes there are no significant longitudinal variations in the zonal wind or temperature QBOs (e.g., Dunkerton and Dilisi, 1985).

In the present study we assume that the mean zonal wind QBO can be expressed as

$$\tilde{u}_q = u_m F(z)e^{-\alpha y^2}e^{i(\omega t + Kz)}.$$  \hspace{1cm} (17)

where $u_m$ is the maximum value of the zonal wind QBO, $\omega$ and $K$ are the angular frequency and vertical wavenumber and $\omega/K$ is the negative rate of phase displacement with height. In Eq. (17) and subsequently, we will take the real part of the complex expressions analogous to that in (17). It should be noted that the observed equatorial and tropical stratospheric wind QBO shows a relatively rapid change from east to west winds and vice-versa. Also, the wind variation with time is somewhat asymmetrical since the average interval from east to west wind maximum is slightly shorter than west to east wind maximum (see, for instance, Coy, 1980; Quirouz, 1981; Labitzke, 1982). However, the wind variation given by Eq. (17) is a good representation of the wind applicable to our present study.

The value of the parameters used in Eq. (17) were based on the analysis by London et al. (1983) of the observed zonal wind QBO at 15 tropical stations over a period of about 18 years. The best fit to the observed data gives the value of $a$ as $3.5 \times 10^{-13}$ m$^{-2}$. The adopted distribution of $F(z)$ is shown in Fig. 2. Here $F(z)$ has a value of 0.24 at 18 km, a maximum of 1.0 between 25.2 and 28 km and 0.5 at 36 km. For $y = 0$ (equator) the fitted vertical profile of the amplitude $[u_m F(z)]$ with $u_m$ set to 17 m s$^{-1}$ and the height variation of the phase, corresponding to the month of maximum westerly wind, is given in Fig. 3. Also shown for comparison are the observed variations of amplitude and phase. For purposes of later analysis, we have normalized the phase so that month 1 = April 1970. The assumed phase was then set equal to the normalized observed phase at 21 km.

3. Analytic solutions

In order to solve Eqs. (13) to (16) we assume that $\tilde{q}_q$, $\tilde{\varrho}_q$, $\tilde{w}_q$ and $\tilde{\gamma}_q$ have similar $z$ and $t$ dependence as the mean zonal wind QBO [see Eq., (17)]. Thus $\tilde{q}_q$, $\tilde{\varrho}_q$, $\tilde{w}_q$, $\tilde{\gamma}_q = (\tilde{q}, \tilde{\varrho}, \tilde{w}, \tilde{\gamma})e^{i(\omega t + Kz)}$ where $\tilde{q}, \tilde{\varrho}, \tilde{w}$ and $\tilde{\gamma}$ are the complex QBO amplitudes which are functions of $y$ and $z$. Substituting the above expressions into Eqs. (13), (14), (15) and (16) yields

**Fig. 2.** Amplitude profile of the equatorial zonal wind QBO.

**Fig. 3.** Phase variation with height of the equatorial zonal wind QBO.
\begin{align}
\frac{\partial \tilde{u}}{\partial y} &= -\beta y \left( u_m \frac{dF}{dz} + iK u_m F(z) \right) e^{-\alpha y^2} \\
\frac{\partial \tilde{v}}{\partial y} + \left( \frac{\partial}{\partial z} - \frac{1}{H} + iK \right) \tilde{w} &= 0 \\
i \omega \tilde{w} + \tilde{w} N^2 &= S \gamma - \mu \tilde{\theta} \\
i \omega \gamma + \tilde{w} \frac{\partial \gamma}{\partial z} &= \Gamma \gamma + \alpha \tilde{\theta}
\end{align}

where \( \beta = \partial f/\partial y \) and use was made of the \( \beta \)-plane approximation in Eq. (18) (see, for instance, Holton, 1979).

In order to derive a set of solutions to the above equations we need to impose a lateral boundary condition. It has been shown from observed data (see for instance, Reed, 1964; London et al., 1983) that there is a phase reversal of the thermal QBO at a latitude of about 15°. Therefore, in the following calculation, we will set the phase shift of the temperature QBO at 15° latitude, i.e., \( \delta = 0 \) at \( y_t = 1655 \text{ km} \). Integrating (18) gives \( \tilde{\theta}_0 \), in the vicinity of the equator, as

\[ \tilde{\theta}_0 = -A \left( e^{-\alpha y_t^2} - 1 \right) \]

where

\[ A = \frac{\beta \left( (dF/dz) + iKF \right)}{2a} u_0. \]

and \( u_0 = u_m \) at the equator. Then solving (20) and (21) simultaneously yields

\[ \tilde{w}_0 = \frac{(\mu + i \omega)(i \omega - \Gamma) - S \alpha}{N^2(i \omega - \Gamma) + S \lambda} A \left( e^{-\alpha y_t^2} - 1 \right) \]

\[ \tilde{\gamma}_0 = \frac{\alpha N^2 + \lambda (\mu + i \omega)}{N^2(i \omega - \Gamma) + S \lambda} A \left( e^{-\alpha y_t^2} - 1 \right) \]

where \( \lambda \) denotes \( \partial \gamma_a/\partial z \). Equations (22) through (24) are the complete set of solutions for the QBOs of temperature, vertical velocity, and ozone mixing ratio in the vicinity of the equator. At the equator \( \tilde{\theta}_0 = 0 \).

Parameterizations. In order to evaluate the solutions described above, we need to quantify the parameters used in Eqs. (23) and (24), that is, the two photochemical parameters \( \Gamma \) and \( \alpha \); the heating rate parameter \( S \) due to ozone absorption; and the Newtonian cooling coefficient, \( \mu \).

a. Photochemical parameters

Ozone photochemistry in the lower and middle stratosphere involves mainly the odd oxygen (O_3), odd hydrogen (H_2O) and odd nitrogen (NO_x) catalytic cycles. In addition, the destruction of odd oxygen is enhanced by the odd chlorine (ClO_x) cycle. However since the principal influence of the odd chlorine cycle occurs at about 40 km (see, for instance, WMO, 1982; Turco, 1985), it has been omitted in the present study.
Nimbus-4 BUV observations (see Ling, 1984) and the HNO₃ distribution was taken from Fischer et al. (1980) which agrees quite well with the LIMS measurements reported by Gille et al. (1984).

The reference solar spectral irradiance data for the wavelength intervals λλ 175–200 nm, λλ 200–310 nm and λλ 310–735 nm are from WMO (1982). Cross sections for O₂ and O₃ for different wavelengths are taken from Allison et al. (1971), Frederick and Hudson (1979), Shardanan and Prasad Rao (1977) and Griggs (1968). The O₂ cross sections used here for the Herzberg continuum may be somewhat high as suggested by recent laboratory and atmospheric measurements (see, for instance, JPL, 1985). However, use of the lower cross sections would only slightly alter the ozone variation near the upper limit of our model. Cross sections for HNO₃ are taken from Molina and Molina (1981) as recommended by JPL (1983). The wavelength range from 175 to 735 nm was divided into 76 subbands in the computation of the photochemical parameters.

Molecular (Rayleigh) scattering was included in the calculation of the ozone photodissociation rate Jₘ. The scattering cross sections were taken from Penndorf (1957). In the middle and lower stratosphere reflected visible radiation contributes significantly to absorption by ozone (see, for instance, London and Bjarnason, 1982). An albedo of 0.32 for the solar radiation at 12 km was included in the computational scheme.

The long-term annual averaged photodissociation rates were derived in the following way:

For each month, characterized separately by the monthly mean distribution of the Nimbus-4 BUV observed O₃ and mean solar declination, we calculated the monthly average daily mean Jₓ, Jₓ and JₓHNO₃ by using an empirically derived approximation (see Cunnold et al., 1975)

\[
\bar{J} = \frac{\tau}{2.05\pi} [J₁(\sec\theta₁) + J₁(\sec\theta₂)]
\]  

where J₁ denotes the photodissociation rate for Jₓ, Jₓ or JₓHNO₃, 2τ (radians) is the duration of daylight over the day and θ₁ and θ₂ are the solar hour angles corresponding to local time τ/4 and 3τ/4. The J₁ values were averaged over 12 months to get the long-term annual mean dissociation rate.

b. Diabatic heating due to ozone

The heating rate per unit volume due to absorption of solar radiation by ozone can be expressed as

\[
Q₃ = n₃ \int_0^{\infty} F(\lambda)\sigma(\lambda) \exp[-\sigma(\lambda)\chi₃]d\lambda
\]

where Q₃ is a function of time and height, F(λ) is the solar irradiance at wavelengths λ, σ(λ) the ozone absorption cross section for ozone, n₃ is the ozone number density, χ₃ the ozone slant-column density \((=1/\cos\theta)\) and \(\theta\) is the solar zenith angle and Z is geometric height. We define the molecular heating rate as

\[
q₁ = \frac{Q₃}{n₃} \int_0^{\infty} F(\lambda)\sigma(\lambda) \exp[-\sigma(\lambda)\chi₃]d\lambda
\]

The heating rate per unit mass is then

\[
\frac{q₃}{\rho} = m_a \frac{q₃}{\gamma}
\]

where \(m_a\) is the molecular mass of air. Since in the thermodynamic equation we have transformed the variable \(T\) to \(\theta\), the heating rate parameter (molecular heating rate per unit molecular mass) can be written as

\[
S = \frac{q₃}{m_a H C_p} = \frac{q₃}{m_a H} \frac{k}{\lambda + \Delta\lambda}
\]

where \(\Delta\lambda\) is in units of \((m \cdot s^{-2}) \cdot s^{-1}\). Obviously, \(q₃\) is directly related to the photodissociation rate of ozone. By considering a small frequency subband \((\lambda, \lambda + \Delta\lambda)\), we have

\[
q₃(\lambda, \lambda + \Delta\lambda) = h \nu \cdot Jₚ(\lambda, \lambda + \Delta\lambda)
\]

Thus, the scheme described above gives the height distribution of the long-term annual average \(q₃\), hence the heating parameter \(S\). The vertical variation of \(S\) is plotted in Fig. 5. The heating rate due to absorption of solar radiation by ozone, computed by using the values of \(S\) as described above and the mean annual ozone distribution from BUV observations is also shown in Fig. 5 for the equatorial stratosphere up to 36 km. The calculated heating rates increase from about +0.2°K day⁻¹ at 18 km to about 4.1°K day⁻¹ at 36 km, in good agreement with results for the middle stratosphere given by Park and London (1974).

c. Infrared cooling parameterization

Infrared radiative damping approximations, with application to the stratosphere, have been developed and discussed extensively (e.g., Sasamori and London, 1966; Dickinson, 1973; Prinn, 1977; Fels, 1982; Alimandi and Visconti, 1983). We here use a Newtonian coefficient for infrared radiative cooling adopted from Plumb and Bell (1982a) with slight modification:

\[
\mu(Z) = \{1 + 2(Z - 18)/18\} 5.51 \times 10^{-7} \text{s}^{-1}\]

\[
\mu(Z) = \{1 + 2(Z - 18)/18\} 4.76 \times 10^{-2} \text{day}^{-1}
\]

where \(Z\) is geometric height in km. The analytic expression above gives values for the Newtonian cooling parameter of 21 days at 18 km decreasing linearly to 7 days at 36 km. This is close to the radiative damping rate calculated from radiative transfer theory by Alimandi and Visconti (1983), as shown in Fig. 6, for mean conditions corresponding to 0°, January. The ozone QBO could affect the infrared cooling, and thus feed back into the cooling coefficient. However, since the ozone QBO amplitude is two orders of magnitude
wind QBO between 18–36 km has a period of 28 months. The phases and amplitudes of the dependent variable oscillations were computed using the above input quantities.

4. Discussion of results

The calculated results for the vertical variations of amplitude and phase of the ozone QBO in the lower and middle stratosphere near the equator are shown in Figs. 7 and 8. Analogous QBO variations computed from 86 months of BUV ozone observations are also shown for comparison with the theoretical values. Since the number of independent pieces of information which can be retrieved by the BUV observing technique is limited (e.g., Twomey, 1977), only the observed variations for three layers were computed, i.e., 22–27, 27–32 and 32–36 km. For these datasets, the amplitude and phase of the 28-month oscillation was computed after the data were detrended and deseasonalized.

As can be seen in Fig. 7, the derived (theoretical) amplitude of the ozone QBO has a major maximum near 22 km, a secondary maximum near 32 km, and a pronounced minimum near 29 km. The observed mean QBO amplitude values are for 5-km layers and are quite similar to the derived distribution except that the lower maximum has been smoothed considerably. Unfortunately, there are no observations available that can provide better height resolution for the QBO ozone amplitude near the equator. Also, it should be noted

less than its absolute values, the effect of this feedback is small and will be neglected in the present study.

The vertical variation of the parameters $S$, $\Gamma$ and $\alpha$ are listed in Table 3 along with the average values of $\partial y_a/\partial z$ from BUV observations. The following have been specified for use in Eqs. (22) through (24):

$$\beta = 2.28 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$$
$$H = 6700 \text{ m}$$
$$N^2 = 4.667 \times 10^{-4} \text{ s}^{-2}$$
$$K = 1.75 \times 10^{-4} \text{ m}^{-1}$$
$$\omega = 8.657 \times 10^{-8} \text{ s}^{-1}$$

where the value for $\beta$ corresponds to that at the equator; those for $H$ and $N^2$ represent lower and middle stratospheric conditions at the equator (average temperature of 230°C with a lapse rate of +1.2°C km$^{-1}$). The values for $K$ and $\omega$ come from the assumption that the zonal

![Fig. 5. Computed vertical distribution of the heating parameter ($S$) and the long-term heating rate.](image)

![Fig. 6. Variation with height of the Newtonian cooling coefficient (i.e., the radiative damping rate) in units of day$^{-1}$.](image)
Table 3. Calculated and observed values of the parameters used in this study. (Note numbers in parentheses give powers of 10.)

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>$S$ (m s$^{-3}$)</th>
<th>$\Gamma$ (s$^{-1}$)</th>
<th>$\rho$ (m$^{-1}$ s)</th>
<th>$\frac{\partial \gamma_z}{\partial z}$ (ppv m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.0</td>
<td>1.56 ($-1$)</td>
<td>-3.65 ($-6$)</td>
<td>-1.24 ($-11$)</td>
<td>-3.35 ($-10$)</td>
</tr>
<tr>
<td>32.5</td>
<td>1.08 ($-1$)</td>
<td>-1.34 ($-6$)</td>
<td>-6.01 ($-12$)</td>
<td>8.90 ($-11$)</td>
</tr>
<tr>
<td>30.0</td>
<td>8.99 ($-2$)</td>
<td>-5.36 ($-7$)</td>
<td>-2.77 ($-12$)</td>
<td>5.74 ($-10$)</td>
</tr>
<tr>
<td>27.5</td>
<td>8.39 ($-2$)</td>
<td>-2.24 ($-7$)</td>
<td>-1.24 ($-12$)</td>
<td>8.68 ($-10$)</td>
</tr>
<tr>
<td>25.0</td>
<td>8.18 ($-2$)</td>
<td>-9.74 ($-8$)</td>
<td>-4.65 ($-13$)</td>
<td>9.16 ($-10$)</td>
</tr>
<tr>
<td>22.5</td>
<td>8.05 ($-2$)</td>
<td>-4.18 ($-8$)</td>
<td>-1.35 ($-13$)</td>
<td>9.07 ($-10$)</td>
</tr>
<tr>
<td>20.0</td>
<td>8.00 ($-2$)</td>
<td>-1.92 ($-8$)</td>
<td>-4.06 ($-14$)</td>
<td>5.18 ($-10$)</td>
</tr>
</tbody>
</table>

that the information content of BUV type observations decreases below about 25 km. The minimum amplitude near 29 km is a result of the cancellation of the different dominant processes that affect the ozone variation above and below this level. The very large amplitude near 22 km as indicated by the theory, almost 10 percent of the mean value at this level, is probably an overestimate since the 1-D model does not allow for horizontal advection.

The phases of the 28-month oscillation of the observed and derived ozone mixing ratio, assumed zonal wind, and derived vertical wind and temperature are given in Fig. 8 as functions of height. As stated earlier, the time coordinate was normalized to the 50-mb equatorial zonal wind at Singapore ($1^\circ$N) which was subject to a harmonic analysis analogous to that for the BUV ozone data as discussed above. The phase of the input forcing function (the zonal wind QBO) as given in Eq. (17) is shown in Fig. 8. The phase of the calculated ozone oscillation shows reasonable agreement with the observed phase above 32 km and below 27 km but not between 27 and 32 km. It should be noted, however, that there is an abrupt phase shift in the theoretical curve around 29 km which coincides with the amplitude minimum at that height. It is difficult to discuss details of the ozone phase relationship in the layer 27–32 km because of the decreased signal-to-noise ratio and the lack of adequate vertical resolution of the observed data.

The phase difference between the forcing zonal wind QBO and the temperature QBO increases with depth in agreement with observations. At 35 km the derived phase for ozone lags that for the zonal wind by about 4 months whereas the temperature lags the zonal wind QBO by about 17 months and that of the vertical wind QBO by about 14 months. In the height interval 35–20 km, the observed zonal wind QBO experiences a 12-month phase retardation while the derived vertical wind, ozone and temperature oscillations undergo 17 month, 37 month and 22 month retardation, respectively. This height variation of the phase difference can best be understood by examination of the solution for the variation of the temperature QBO. From Eq. (22) we have

$$\bar{\theta}_{0} = \frac{\beta (dF/dz + iKF(z))}{2a} u_{0}(1 - e^{-\alpha z})$$

which shows that the phase difference between the temperature and zonal wind varies with height even though the two are directly coupled. The ozone variation is determined by both the temperature QBO through the temperature dependence of the photochemical system, and the vertical wind QBO through dynamical transport. The time dependent term can be neglected in the thermal energy equation for the whole height range of interest and can also be neglected in the ozone continuity equation above 24 km. Thus, in
the region above 24 km, the response of the ozone QBO to the zonal wind should be considered as instantaneous regardless of the very large phase difference, most of which occurs during the ozone adjustment from an out-of-phase to an in-phase relation with the temperature in the region 27 to 30 km.

Below 24 km where photochemical influences are of secondary importance, the time dependent term plays a significant role in the ozone continuity equation. Thus, the ozone QBO will respond to the forcing zonal wind and therefore the vertical wind QBO with a time delay which is equivalent to the phase difference between the temperature QBO and the ozone QBO as indicated in Fig. 8. As a result, the ozone QBO responds to the local forcing zonal (and vertical) wind QBO almost instantaneously and the phase difference between the zonal wind QBO and the derived QBO of other variables does not reflect the time delay necessary for a local response. The temperature and vertical wind oscillations are approximately out of phase over the entire height interval 20–30 km. However, the calculated ozone and temperature oscillations are out of phase above 30 km and in phase below about 27 km.

The phase relationships described above can be explained by consideration of a time scale for each of the different processes involved, i.e., dynamics, radiative and photochemical. In this case, the relevant time scale is the time taken to bring about a change in a quantity equal to its characteristic change due to a specific mechanism. The characteristic change in a quantity varies from case to case. For example, in the case of the mean ozone variation along a latitude circle, the characteristic change may be taken as its planetary wave amplitude. Such a time scale for horizontal advection of ozone has been defined by Hartmann and Garcia (1979) (see, also, Garcia and Solomon, 1984). For the present 1-D problem, the dynamical (transport) time scale for ozone, \( \tau_{\text{w}}(z) \), may be of the order of months (see, for instance, Brasseur and Solomon, 1985). For radiative photochemical processes, the time scales are equivalent to the reciprocal of \( -\mu(z) \) and \( -\Gamma(z) \) as given by Eqs. (29) and (A39), respectively.

Above 30 km, ozone QBO variations respond more quickly to photochemical processes than to transport while below 30 km, although both mechanisms are important, the relative influence of vertical transport on the ozone QBO increases with depth. For the temperature QBO, radiative processes involve shorter time scales than those related to vertical transport. Thus, upward vertical motion, above 30 km, results in adiabatic cooling which is damped as radiative relaxation tends to return the temperature towards its equilibrium value. However, because of the fast photochemistry at these levels, the negative feedback of temperature on ozone produces an increase in the ozone mixing ratio associated with the residual temperature decrease. Thus, above 30 km, there is a strong tendency for ozone and vertical wind oscillations to be almost in phase (with the ozone QBO lagging slightly) but ozone and temperature to be 180° out-of-phase.

Compensation among the various influences contributing to the ozone QBO in the layer 27–30 km (i.e., ozone photochemistry including temperature feedback, vertical ozone transport) along with phase differences for these competing effects will determine the actual height at which there is a balance for the ozone QBO to reach its minimum amplitude.

In order to understand the phase relationships between the ozone and temperature QBOs below 27 km it is convenient to refer to Eqs. (20) and (21). We can eliminate \( w \) from Eqs. (20) and (21) to give

\[
\tilde{\gamma} = \frac{\left( \frac{\partial \gamma}{\partial z} \right)}{N^2} \left( \mu + \alpha + i \omega \right) S - \Gamma + i \omega \theta.
\]

The frequency terms in Eq. (31) may be neglected above about 25 km since at those levels the ozone relaxation time is \( \ll 100 \) days. The height variation of the remaining terms in Eq. (31) is given in Table 4 where it can be seen that the denominator is always positive. The two terms in the numerator of Eq. (31) represent a positive feedback due to radiative processes and a negative feedback due to temperature related photochemistry. The phase relationship between temperature and ozone is therefore modified by the competition between these two opposite effects. Since radiative influences are larger than photochemical feedback in this region, temperature and ozone oscillations should be nearly in phase at these levels.

Below about 24 km the time constant for ozone transport is less than that for ozone photochemistry. The frequency term now plays a significant role in the mass budget equation for ozone. At these heights both temperature and ozone mixing ratio gradients are positive and vertical transport contributes in the same way to their variations. That is, upward (or downward) motion leads to a decrease (or increase) in temperature and ozone. The ozone mixing ratio should decrease to its minimum value when the vertical wind changes.

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>( \frac{\partial \gamma}{\partial z} ) (( \text{m}^2 \text{s}^{-1} ))</th>
<th>( \frac{\partial \gamma}{\partial z} ) (( \text{m}^2 \text{s}^{-1} ))</th>
<th>( \frac{\partial \gamma}{\partial z} ) (( \text{s}^{-1} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.0</td>
<td>-1.14 (-12)</td>
<td>-1.24 (-11)</td>
<td>-1.12 (-7)</td>
</tr>
<tr>
<td>32.5</td>
<td>2.74 (-13)</td>
<td>-6.01 (-12)</td>
<td>2.06 (-8)</td>
</tr>
<tr>
<td>30.0</td>
<td>1.58 (-12)</td>
<td>-2.77 (-12)</td>
<td>1.11 (-7)</td>
</tr>
<tr>
<td>27.5</td>
<td>2.11 (-12)</td>
<td>-1.24 (-12)</td>
<td>1.56 (-7)</td>
</tr>
<tr>
<td>25.0</td>
<td>1.92 (-12)</td>
<td>-4.65 (-13)</td>
<td>1.61 (-7)</td>
</tr>
<tr>
<td>22.5</td>
<td>1.61 (-12)</td>
<td>-1.35 (-13)</td>
<td>1.56 (-7)</td>
</tr>
<tr>
<td>20.0</td>
<td>7.47 (-13)</td>
<td>-4.06 (-14)</td>
<td>8.88 (-8)</td>
</tr>
</tbody>
</table>
sign from positive to negative. This would cause the ozone QBO to lag that of the vertical wind by 270°. But the effect of photochemistry would modify this phase lag as the ozone relaxes to its equilibrium value. As a result, the phase lag at these heights would be expected to be between 180° and 270°. However, because the thermal relaxation time is relatively small, the temperature oscillation would lag that of the vertical wind by only slightly more than 180° and would lead that of ozone by less than 90°. This is consistent with the derived results given in Fig. 8 where it is shown that below 24 km the phase difference by which the temperature QBO leads that of ozone increases slightly to about 2.5 months with decreasing height to 20 km.

5. Uncertainties in the photochemical parameterization

As pointed out in the Appendix, for the present oxygen–hydrogen–nitrogen atmospheric model the photochemical parameters were determined from the photodissociation rates $J_3$, $J_{\text{H}}$ and $J_{\text{HNO}_3}$, and average vertical distributions of the H$_2$O and NO$_2$ mixing ratios as derived from observed values in the stratosphere. We examine the effect of changes in the assumed distributions of HO$_2$ and NO$_2$ on the derived ozone QBO. Although there are insufficient long-term measurements to give high precision information on the variability of stratospheric H$_2$O and NO$_2$ in the tropics, there are some indications from recent reports that there is a small relative quasi-biennial oscillation of water vapor in the lower stratosphere, up to about 22 km, at midlatitudes of both Northern and Southern hemispheres (Mastenbrook and Oltmans, 1983; Hyson, 1983). The observations indicate that in the lower stratosphere the relative amplitude of the midlatitude water vapor QBO may be of the order of 10 percent of the mixing ratio at that level with about a 10–12 month phase lag relative to the equatorial temperature QBO at 16 km. This would be consistent with the suggestion that a lower stratosphere water vapor QBO over the equator is largely controlled by a freezeout process at and just above the equatorial tropopause. We do not know what the vertical variation of the amplitude and phase of the QBO in H$_2$O would be but there is reason to believe that it might be different from that given by Eq. (17) and it is beyond the scope of the present study to derive such a profile. As far as we know, no QBO has yet been detected for stratospheric NO$_2$. In order to test the sensitivity of the derived amplitude and phase of the ozone QBO to the assumed mean values of the H$_2$O and NO$_2$ vertical distributions, we repeated the calculations as described above for doubled and 50 percent reduced H$_2$O and NO$_2$ mixing ratio amounts. The results of the sensitivity calculations are shown in Fig. 9a, b and Figs. 10a, b for the derived ozone QBO amplitude and phase changes.

In general, there are only small relative changes in the amplitude or phase of the ozone QBO below the level of minimum amplitude and rapid phase transition (approximately 28–30 km), where ozone transport becomes more and more important in determining the ozone QBO. Above the transition region an increase of H$_2$O or NO$_2$ reduces the amplitude of the ozone

Fig. 9. Changes of the derived ozone QBO amplitude with indicated changes of (a) the H$_2$O mixing ratio and (b) the NO$_2$ mixing ratio.
oscillation and vice versa. This result is, of course, consistent with the roles played by the HOx and NOx catalytic chain cycles in the stratospheric ozone photochemical system. As expected at these levels, the ozone variation shows a larger response to changes in NO2 than to analogous changes in H2O. Above 30 km the QBO amplitude can be reduced by as much as 60 percent with doubling of the NO2 mixing ratio but reduced only 20 percent with a similar increase of H2O. In addition increased NO2 results in a slight lifting of the height of the amplitude minimum since the increased NO2 makes the NOx catalytic cycle more efficient, decreases the ozone photochemical relaxation time and thus permits vertical ozone transport to play a more important role in influencing the ozone variation. QBO phase variations are not affected by changes in H2O and only slightly affected by changes in NOx.

The primary purpose of testing the model response to varying H2O and NO2 concentrations was to examine the uncertainties of the model results with respect to the input H2O and NO2 vertical profiles. Although it is not likely that uncertainties in the assumed vertical distributions of H2O and NO2 mixing ratios would qualitatively affect the stratospheric ozone QBO, the model needs to be further tested with realistic evaluation of H2O and NO2 QBO amplitude and phase variations.

Acknowledgments. We would like to thank Mary S. Sable for editorial assistance in the preparation of the manuscript. This study was supported by grant NSF-5153 from the National Aeronautics and Space Administration. We are also thankful for the computing facilities made available to us by the National Center for Atmospheric Research, which is sponsored by the National Science Foundation.

APPENDIX

Parameterizations

For an oxygen–hydrogen–nitrogen atmosphere, the most important reactions involving the production and destruction of ozone for the lower and middle stratosphere (Nicolet, 1975) are listed in Table A1.

The basic mechanism for the existence of odd oxygen particles (O, O1D and O3) in the oxygen–hydrogen–nitrogen stratosphere is described by reactions (A1) through (A5). In addition, odd hydrogen (OH and HO2) (A8), (A9) and (A12), and odd nitrogen (NO and NO2) (A15) and (A16) catalytic reactions contribute to the destruction of the odd oxygen family. The Js in Table A1 represent photodissociation rates (s⁻¹). The second order (cm³ mol⁻¹ s⁻¹) and three body (cm⁶ mol⁻² s⁻¹) reaction rates (k₃) were taken from WMO (1982), except for k₃ which was taken from Huie et al. (1972). The value for k₃ shown in Table A1 is about 6 percent lower than that listed in WMO (1982) for the temperature range found in the tropical stratosphere. Since this difference is less than the uncertainty for k₃ given in WMO (1982), the earlier value, which is given in Arrhenius form, was used as a matter of convenience for computing the photochemical relaxation and temperature feedback parameters as described below.

In the stratosphere the interchange reactions between members of an odd family are usually much faster than reactions involving exchange between different odd families. As a result mutual equilibrium exists for each individual odd family. Thus, we have the following mutual equilibrium relationships:

\[ [O] = [O₃] \frac{J₃}{k₃ [O₂][M]} \]  

(A19)
### Table A1. Photochemical reaction rates used in the present model.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Reaction rate*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_2 + h\nu(\lambda &lt; 242 \text{ nm}) \rightarrow 20$</td>
<td>$J_2$ (A1)</td>
</tr>
<tr>
<td>$O_2 + h\nu + M \rightarrow O_2 + M$</td>
<td>$k_2 = 1.05(-34) \exp(510/T)$ (A2)</td>
</tr>
<tr>
<td>$O_3 + h\nu(\lambda &lt; 1140 \text{ nm}) \rightarrow O_2 + O$</td>
<td>$J_3$ (A3)</td>
</tr>
<tr>
<td>$O_3 + h\nu(\lambda &lt; 310 \text{ nm}) \rightarrow O_2 + O(\cdot D)$</td>
<td>$J_\ddagger$ (A4)</td>
</tr>
<tr>
<td>$O_3 + O \rightarrow 2O_2$</td>
<td>$k_3 = 1.5(-11) \exp\left(-\frac{2218}{T}\right)$ (A5)</td>
</tr>
<tr>
<td>$O(\cdot D) + N_2 \rightarrow O + N_2$</td>
<td>$k_4 = 1.8(-11) \exp(107/T)$ (A6)</td>
</tr>
<tr>
<td>$O(\cdot D) + O_2 \rightarrow O + O_2$</td>
<td>$k_5 = 3.2(-11) \exp\left(\frac{67}{T}\right)$ (A7)</td>
</tr>
<tr>
<td>$O_3 + OH \rightarrow HO_2 + O_2$</td>
<td>$k_6 = 1.6(-12) \exp\left(-\frac{940}{T}\right)$ (A8)</td>
</tr>
<tr>
<td>$O_3 + HO_2 \rightarrow OH + 2O_2$</td>
<td>$k_7 = 1.4(-14) \exp\left(-\frac{580}{T}\right)$ (A9)</td>
</tr>
<tr>
<td>$O(\cdot D) + H_2O \rightarrow 2OH$</td>
<td>$k_8 = 2.3(-10)$ (A10)</td>
</tr>
<tr>
<td>$OH + NO_2 + M \rightarrow HNO_3 + M$</td>
<td>$k_9 = 2.6(-30) \exp\left(-\frac{T}{300}\right)$ (A11)</td>
</tr>
<tr>
<td>$O + HO_2 \rightarrow OH + O_2$</td>
<td>$k_{10} = 3.5(-11)$ (A12)</td>
</tr>
<tr>
<td>$OH + HO_2 \rightarrow H_2O_2 + O_2$</td>
<td>$k_{11} = 8.0(-11)$ (A13)</td>
</tr>
<tr>
<td>$OH + HNO_3 \rightarrow H_2O + NO_3$</td>
<td>$k_{12} = 9.4(-15) \exp\left(\frac{778}{T}\right)$ (A14)</td>
</tr>
<tr>
<td>$O_3 + NO \rightarrow NO_2 + O_2$</td>
<td>$k_{13} = 3.8(-12) \exp\left(-\frac{1580}{T}\right)$ (A15)</td>
</tr>
<tr>
<td>$O + NO_2 + NO + O_2$</td>
<td>$J_{NO_2}$ (A16)</td>
</tr>
<tr>
<td>$NO_2 + h\nu(\lambda &lt; 398 \text{ nm}) \rightarrow NO + O$</td>
<td>$J_{NO_2}$ (A17)</td>
</tr>
<tr>
<td>$NO_2 + h\nu(\lambda &lt; 320 \text{ nm}) \rightarrow OH + NO_2$</td>
<td>$J_{HNO_3}$ (A18)</td>
</tr>
</tbody>
</table>

* Pre-exponential numbers in parenthesis represent powers of ten.

\[
[O(\cdot D)] = \frac{J_\ddagger}{k_4[N_2] + k_5[O_2]} \quad (A20)
\]

\[
[\text{NO}] = \frac{k_{14}[O] + J_{NO_2}}{k_{13}[O_3]} \quad (A21)
\]

\[
[HNO_3] = \frac{k_{8}[OH][NO_2][M]}{k_{12}[OH]} \quad (A22)
\]

\[
[OH] = \frac{k_5[O_2] + k_{10}[O]}{k_6[O_3]} \quad (A23)
\]

Reactions (A10), (A13) and (A14) are considered to be the source and the sinks of HO$_2$. The rates of these three reactions determine the total number of HO$_2$ particles available in the HO$_x$ catalytic cycle for known concentrations of H$_2$O and HNO$_3$. Furthermore, stratospheric HO$_x$ is in photochemical equilibrium since OH and HO$_2$ are both in photochemical equilibrium. Thus the balance between the source reaction (A10) and the sink reactions (A13) and (A14) gives

\[
[OH] = \frac{k_5[H_2O][O(\cdot D)]}{k_{11}[HO_2] + k_{12}[HNO_3]} \quad (A24)
\]

Eliminating [HO$_2$] and [HNO$_3$] from (A22), (A23) and (A24), and using (A19) and (A20) to replace [O] and [O(\cdot D)] by [O$_3$] yields

\[
C_0[OH]^2 + C_1[OH]^2 + C_2[OH] + C_3 = 0 \quad (A25)
\]

where

\[
C_0 = k_2k_4k_1k_2[O_2][M] \quad (A26)
\]

\[
C_1 = k_9k_{12}[NO_2][M](k_2k_1[O_2][M] + k_{10}J_3)
\]

\[
+ k_2k_4J_{HNO_3}[O_2][M] \quad (A27)
\]

\[
C_2 = \frac{k_8k_{12}J_\ddagger(k_2k_7[O_2][M] + k_{10}J_3)[H_2O]}{k_4[N_2] + k_5[O_2]} \quad (A28)
\]

\[
C_3 = \frac{k_8J_{HNO_3}[OH][H_2O][k_2k_7[O_2][M] + k_{10}J_3]}{k_4[N_2] + k_5[O_2]} \quad (A29)
\]

The real solution to the third degree algebraic equation (A25) is (Condon and Odishaw, 1958)

\[
[OH] = \left(-\frac{D_1}{2} + D_2^{1/2}\right)^{1/3} + \left(-\frac{D_1}{2} - D_2^{1/2}\right)^{1/3} \quad (A30)
\]

where

\[
D_1 = \frac{4C_1^3 - 9C_0C_1C_2 + 27C_0^2C_3}{27C_0^3} \quad (A31)
\]

\[
D_2 = \frac{(4C_1^3 - 9C_0C_1C_2 + 27C_0^2C_3)^2}{108C_0^3} + \frac{(3C_0C_2 - C_1^3)^3}{81C_0^2} \quad (A32)
\]

In this study we use the observed distribution of NO$_2$, so the total number of NO$_2$ is given approximately by equations (A21) and (A22).

The rates of change of O, O(\cdot D) and O$_3$ are

\[
\frac{d[O]}{dt} = 2J_2[O_2] + J_3[O_3] + k_4[M][O(\cdot D)][N_2]
\]

\[
+ k_5[O(\cdot D)][O_2] + J_{NO_2}[NO_2] - k_2[O][O_2][M]
\]

\[
- k_3[O][O_3] - k_{14}[OH][NO_2] - k_{16}[OH][HO_2] \quad (A33)
\]

\[
\frac{d[O(\cdot D)]}{dt} = J_\ddagger[O_3] - k_4[M][O(\cdot D)][N_2]
\]

\[
- k_5[O(\cdot D)][O_2] - k_8[O(\cdot D)][H_2O] \quad (A34)
\]

\[
\frac{d[O_3]}{dt} = k_2[O][O_2][M] - (J_3 + J_\ddagger)[O_3] - k_3[O][O_3]
\]

\[
- k_6[O_3][OH] - k_9[O_3][HO_2] - k_{13}[O_3][NO] \quad (A35)
\]

Since O and O(\cdot D) are in photochemical equilibrium, we have
\[ \frac{d[O_3]}{dt} = 2J_2[O_2] + J_{NO_2}[NO_2] - 2k_3[O][O_3] \\
- k_6[O_3][OH] - k_7[O_3][HO_2] - k_{13}[O_3][NO] \\
- k_{14}[O][NO_2] - k_8[O(1D)][H_2O] - k_{10}[O][HO_2]. \]  
(A36)

By using relations (A19) through (A23), and (A30) and changing the [O_3] concentration to the ozone volume mixing ratio \( \gamma \) the above expression may be written as

\[ \frac{d\gamma}{dt} = \frac{2J_2[O_2]}{[M]} - 2 \frac{k_3J_3}{k_2[O_2]} \gamma^2 - E \\
- 2 \frac{k_{14}J_3[NO_2]}{k_2[O_2][M]} \gamma - \frac{k_8J[H_2O]}{k_4[N_2] + k_5[O_2]} \gamma \]  
(A37)

where

\[ E = -2k_4\gamma \left( \left( -\frac{D_1}{2} + D_2^{1/2} \right)^{1/3} + \left( -\frac{D_1}{2} - D_2^{1/2} \right)^{1/3} \right). \]

Note that OH, HO_2, HNO_3 and NO no longer appear explicitly in the equation for \( d\gamma/dt \). In equation (A37), the rhs represents the production and destruction rates of the O_3 volume mixing ratio, i.e.,

\[ P_C = \frac{2J_2[O_2]}{[M]} - 2 \frac{k_3J_3}{k_2[O_2]} \gamma^2 - E \\
- 2 \frac{k_{14}J_3[NO_2]}{k_2[O_2][M]} \gamma - \frac{k_8J[H_2O]}{k_4[N_2] + k_5[O_2]} \gamma. \]  
(A38)

For radiative/photochemical equilibrium, the lhs of (A38) would vanish if the temperature and O_3 mixing ratio involved in the terms appearing on the right hand side of (A38) are replaced by their equilibrium values. Therefore, for an arbitrary state which has a temperature departure \( \Delta T \) (hence \( \Delta \theta \) and an O_3 mixing ratio departure \( \Delta \gamma \) from their equilibrium values, the production/destruction rate of O_3 volume mixing ratio is equal to the total differential of the right-hand side of (A38) evaluated at the radiative/photochemical equilibrium value. Taking the total derivative of (A38) leads to

\[ P_C = \Gamma \Delta \gamma + \alpha \Delta \theta \]

where the two photochemical parameters \( \Gamma \) and \( \alpha \) are given by

\[ \Gamma = -4 \frac{k_3J_3}{k_2[O_2]} \gamma - \frac{\partial E}{\partial \gamma} - 2 \frac{k_{14}J_3[NO_2]}{k_2[O_2][M]} \\
- \frac{k_8J[H_2O]}{k_4[N_2] + k_5[O_2]} \]  
(A39)

\[ \alpha = 2 \frac{k_3J_3(q_2 - q_2)}{k_2[O_2]T^2} \gamma^2 - \frac{\partial E}{\partial T} - 2 \frac{k_{14}J_3[NO_2]q_2}{k_2[O_2][M]T^2} \gamma \]

\[ \frac{k_8J[H_2O]}{(k_4[N_2] + k_5[O_2])^2T^2} \gamma \times k_4[N_2] + q_s + k_6[O_2]q_s \times \frac{H}{R}. \]  
(A40)

The value for \( q_i \) is the numerator in the exponential part of the appropriate reaction rate given in Table A1. That is, \( k_i = A_i \exp[q_i/T] \) where \( A_i \) is the Arrhenius pre-exponential factor. The vertical distributions of the photochemical parameters \( \Gamma \) and \( \alpha \) are given in Fig. A1 based on the assumed height variations of H_2O and NO_2, and the photodissociation rates \( J_3 \), \( J[F] \) and \( J[HNO_3] \), as discussed earlier.

REFERENCES


and —, 1978b: Generalized Eliassen-Palm and Charney-


