A Slowly Varying Model of the Quasi-biennial Oscillation Involving Effects of Transience, Self-acceleration and Saturation of Equatorial Waves

HIROSHI TANAKA AND NOBUYUKI YOSHIZAWA

Water Research Institute, Nagoya University, Chikusa-ku, Nagoya 464, Japan

(Manuscript received 27 May 1986, in final form 19 November 1986)

ABSTRACT

A numerical simulation of the quasi-biennial oscillations (QBO) is presented using a sophisticated one-dimensional model operating under a slowly varying WKB assumption. The roles of wave transience, wave self-acceleration and wave saturation are included. Wave self-acceleration increases the oscillation period and accelerates mean zonal wind, particularly the easterly wind. Introduction of wave saturation into the model leads to a decrease in the oscillation period and a suppression of acceleration of the wave phase velocity. Tuning wave parameters in the model produces more realistic features of the QBO. Two remaining unrealistic results are the transition of mean zonal flow from westerly to easterly regimes occurring more rapidly than the opposite transition and easterly wind maxima appearing at much higher altitudes than westerly wind maxima. Simulation results reveal that the Rossby-gravity wave saturates over a larger altitude range than the Kelvin wave. Techniques for the removal of these inconsistencies are discussed briefly.

1. Introduction

The quasi-biennial oscillation (QBO) model developed by Holton and Lindzen (1972) excludes some important dynamical concepts. Since their study, outstanding progress has been made in understanding wave–mean flow interactions associated with other kinds of waves, particularly Rossby waves and gravity waves (cf. Andrews and McIntyre, 1976a,b; Plumb, 1977; Dunkerton, 1981a,b, 1982a,b; and Coy, 1983). Dunkerton has devoted himself to the development of the theory of wave–mean flow interactions, focusing on gravity waves and equatorial waves. He stresses the role of wave transience, which contributes to mean flow accelerations even if every kind of wave dissipation is excluded.

In spite of Dunkerton’s efforts, the QBO model is still incomplete in some respects. Tanaka and Yoshizawa (1985) incorporated the effect of wave self-acceleration into the model under an approximation of dissipation dominancy. The result demonstrated that the oscillation period increases more than 25% and that wind velocities increase and partly overshoot the original phase velocities of the equatorial waves. At the same time, the model given by Tanaka and Yoshizawa increased an asymmetry of flow patterns unrealistically. This must be corrected by use of a full set of equations without omission provided that wave saturations of Kelvin and Rossby–gravity waves are included.

The importance of wave saturation in limiting wave amplitudes has been reported by Andrews and McIntyre (1976a), Lindzen and Tsay (1975) and Kousky and Koermer (1974). Kelvin–Helmholtz instability or local convective instability may well be important for Kelvin waves (Andrews and McIntyre, 1976a; Kousky and Koermer, 1974). For Rossby–gravity waves it is difficult to determine whether barotropic instability or local convective instability are dominant.

The purpose of this paper is, first, to build a complete one-dimensional quasi-linear model incorporating all basic mechanisms but assuming these mechanisms to be slowly varying and, second, to evaluate how effectively such mechanisms work on the QBO. It is not our objective to compare the present one-dimensional model with the real QBO phenomenon. Some inconsistent results may appear when the one-dimensional model is used. Rapid transition from westerly to easterly regimes is one example. One purpose is to establish whether such a rapid transition can be improved within the framework of a one-dimensional quasi-linear model rather than a two-dimensional model by including horizontal shear in the meridional direction such as employed by Plumb and Bell (1982a,b) and Dunkerton (1985). Incorporation of wave self-acceleration and wave saturation into a two-dimensional model is difficult, though not impossible. Therefore, construction of a very sophisticated one-dimensional QBO model is a necessary process in order to gain better insight into the future development of a more generalized QBO model.

2. Basic equations

A full set of slowly varying wave equations for the equatorial QBO, involving wave transiences, wave self-accelerations and Newtonian-type radiative dampings of
Kelvin and Rossby–gravity waves, is given by Tanaka and Yoshizawa (1985):

$$\frac{\partial \tilde{u}}{\partial t} + \left( \frac{\partial}{\partial z} - \frac{1}{H} \right) (W_{K} \tilde{u} + W_{RG} \tilde{u}_{RG}) = K \frac{\partial^{2} \tilde{u}}{\partial z^{2}},$$  \hspace{1cm} (1)

$$\frac{\partial \tilde{u}_{K}}{\partial t} + \left( \frac{\partial}{\partial z} - \frac{1}{H} \right) W_{K} \tilde{u}_{K} = -a \tilde{u}_{K},$$  \hspace{1cm} (2)

$$\frac{\partial \tilde{u}_{RG}}{\partial t} + \left( \frac{\partial}{\partial z} - \frac{1}{H} \right) W_{RG} \tilde{u}_{RG} = -\beta + k_{RG}^{2} \tilde{u}_{RG},$$  \hspace{1cm} (3)

$$\frac{\partial \tilde{u}_{RG}}{\partial t} + W_{K} \frac{\partial \tilde{u}_{RG}}{\partial z} + W_{RG} \frac{\partial \tilde{u}}{\partial z} = 0,$$  \hspace{1cm} (4)

$$\frac{\partial \tilde{u}_{RG}}{\partial t} + W_{RG} \frac{\partial \tilde{u}_{RG}}{\partial z} + W_{RG} \frac{\partial \tilde{u}}{\partial z} = 0.$$  \hspace{1cm} (5)

The variables and parameters and some of the values used here are listed in appendix A and are otherwise denoted in context. All of the variables are integrated with respect to the meridional direction in the slowly varying framework. The rhs term in Eq. (1) is artificially introduced for subgrid-scale smoothing in order to prevent the time integration breakdown. Here 0.3 m$^{-1}$ s$^{-1}$ is assumed for the eddy diffusion coefficient, $K$. The expressions for the vertical group velocities of Kelvin and Rossby–gravity waves are

$$W_{K} = \frac{k_{K}}{N} \tilde{c}_{K},$$  \hspace{1cm} (6)

$$W_{RG} = -\frac{k_{RG}^{3}}{N} \tilde{c}_{RG}/(2\beta + k_{RG}^{2} \tilde{c}_{RG}).$$  \hspace{1cm} (7)

The intrinsic phase velocity of the Rossby–gravity wave is usually negative and larger than $-2\beta/k_{RG}$ (more exactly $\tilde{c} = -\beta/k_{RG}$), and therefore the vertical group velocity of the Rossby–gravity wave is always positive.

When we incorporate a wave saturation mechanism, saturation conditions are necessary to determine the limits of pseudomomenta:

$$I_{KS} \geq I_{K} > 0,$$  \hspace{1cm} (8)

$$0 > I_{RG} \geq I_{RGs}.$$  \hspace{1cm} (9)

The bounded values of pseudomomentum are given by the characteristic method (cf. Coy, 1983) in a quasilinear system (see appendix B):

$$I_{KS} = \frac{1}{2} \tilde{c}_{K},$$  \hspace{1cm} (10)

$$I_{RGs} = \frac{1}{2} \tilde{c}_{RG} \left( \frac{2\beta + k_{RG}^{2} \tilde{c}_{RG}}{3\beta + k_{RG}^{2} \tilde{c}_{RG}} \right),$$  \hspace{1cm} (11)

with

$$2 > \frac{2\beta + k_{RG}^{2} \tilde{c}_{RG}}{3\beta + k_{RG}^{2} \tilde{c}_{RG}} > \frac{1}{2}$$

for

$$0 > \tilde{c}_{RG} > -\frac{\beta}{k_{RG}}.$$  \hspace{1cm} (12)

The pseudomomentum limit of the Kelvin wave given by (10) is, by chance, equivalent to the local convective instability. On the other hand, the pseudomomentum limit of the Rossby–gravity wave given by (11) is not caused by the local convective instability but instead is the result of an instability associated with the $\beta$-effect (see appendix B for more details). This instability appears well before the local convective instability. In fact, the pseudomomentum of the Rossby–gravity wave is amplified more extensively than that of the Kelvin wave before reaching the instability limit.

The wave saturation limits derived here by the characteristic method do not always reflect real instabilities since the horizontally integrated one-dimensional model is a theoretical product, particularly for the Rossby–gravity wave. The pseudomomentum limits shown in (10) and (11) are, therefore, regarded as theoretical products, which are incorporated to keep the quasi-linear equation system consistent with the slowly varying WKB framework and to prevent breakdown of the system. The relationship of these pseudomomentum limits to realistic wave saturation conditions, if any, must be revealed by the use of multidimensional models, especially for the Rossby–gravity wave.

3. Numerical time integrations

The effect of wave transience is predominant when time changes of wave momenta dominate wave dissipations. In the equatorial stratosphere, however, wave dissipation due to Newtonian cooling seems more effective than wave transience, as shown by Tanaka and Yoshizawa(1985). The Newtonian cooling coefficient used in this paper for the typical equatorial stratosphere is almost the same as Holton and Lindzen (1972):

$$\alpha(z) = \begin{cases} \frac{1}{21} + \left( \frac{1}{7} - \frac{1}{21} \right) \frac{z - 16 \text{ km}}{14 \text{ km}} \text{ day}^{-1} & \text{for } 30 \text{ km} \geq z > 17 \text{ km}, \\ \frac{1}{7} \text{ day}^{-1} & \text{for } 36 \text{ km} > z > 30 \text{ km}. \end{cases}$$  \hspace{1cm} (13)

Wave transience becomes approximately comparable to wave dissipation from a theoretical viewpoint when

$$\alpha_{c} = \frac{k c^{2}}{NH},$$

where $\alpha_{c}$ is the critical value of the Newtonian cooling

---

1 We have already tried a numerical simulation by the use of a Fels' (1982) type Newtonian cooling coefficient, which depends on the vertical wavelength of the equatorial waves, though the result is not shown here. The modification is that the westerly becomes weaker by 2 m s$^{-1}$ and the easterly becomes stronger by only 1 m s$^{-1}$. Saturation of the Rossby–gravity wave vanishes below 20 km, and on the contrary, the Kelvin wave survives only around an 18 km level. On the whole, however, the modification is not significant at QBO altitudes. This type of damping could be important in the upper stratosphere and the mesosphere.
coefficient (Dunkerton, 1981b). By and large, $\alpha(z)$ shown in (13) is larger than $\alpha$, and the wave transience effect becomes relatively small, especially for the Kelvin wave. For the Rossby–gravity wave, Newtonian cooling becomes less effective since the rhs of Eq. (3) is approximately $-0.5\alpha I_{RG}$.

Three cases for the QBO model are considered in this paper:

1) $c_K$ and $c_{RG}$ are constant in time and space, and wave saturations are ignored. Equations (1)–(3) are used;

2) $c_K$ and $c_{RG}$ are allowed to be variable in time and space, but wave saturations are still ignored. Equations (1)–(5) are used;

3) $c_K$ and $c_{RG}$ are allowed to be variable in time and space, and wave saturations are also taken into account. Equations (1)–(5) are used with conditions (8) and (9).

Case 1 is the same as Dunkerton’s (1981b) model. The oscillation period is expected to be increased in Case 2 due to the self-acceleration of waves (cf. Tanaka and Yoshizawa, 1985). Case 3 is the most improved and complete model since all the dynamical effects are incorporated.

The lower and upper boundary conditions of all the dependent variables are specified at 17 and 36 km, respectively. First, for $\bar{u}$:

$$\bar{u}(17\text{ km}, t) = 0,$$
$$\frac{\partial \bar{u}(z, t)}{\partial z} \bigg|_{z=36\text{ km}} = 0,$$  \hspace{1cm} (15)

second, for $I_K$ and $I_{RG}$:

$$I_K(17\text{ km}, t) = 1 \text{ m s}^{-1},$$
$$I_{RG}(17\text{ km}, t) = -1 \text{ m s}^{-1},$$  \hspace{1cm} (16)

and third, for $c_K$ and $c_{RG}$:

$$c_K(17\text{ km}, t) = 30 \text{ m s}^{-1},$$
$$c_{RG}(17\text{ km}, t) = -30 \text{ m s}^{-1}.$$  \hspace{1cm} (17)

Since we apply forward difference schemes to Eqs. (1)–(5) for both time and space domains together with some techniques to stabilize the scheme, upper boundary conditions of the pseudomomenta and phase velocities become unnecessary.\(^2\)

Initial conditions of the variables are specified for $\bar{u}$, $c_K$ and $c_{RG}$:

$$\bar{u}(z, 0) = 30(z - 17 \text{ km})/20 \text{ km m s}^{-1},$$
$$c_K(z, 0) = 30 \text{ m s}^{-1},$$
$$c_{RG}(z, 0) = -30 \text{ m s}^{-1},$$  \hspace{1cm} (18)

and for $I_K$ and $I_{RG}$:

$$I_K(z, 0) = 0,$$
$$I_{RG}(z, 0) = 0,$$  \hspace{1cm} (19)

except for the lower boundary. The initial condition of $\bar{u}$ shown in (18) is the same as employed by Dunkerton (1981b).

Equations (1)–(5) are integrated in time simultaneously with finite difference grids of time, $\Delta t = 864$ s, and space, $\Delta z = 250$ m, together with (6), (7) and (8), (9). Each of the three cases stated above will be compared with one another.

4. Results of the simulations

a. Mean zonal flow

Time–height sections of the mean zonal flows for the three cases stated in section 3 are shown in Figs. 1a, b and c, respectively. Oscillation periods for Cases 1, 2 and 3 are 1.8, 2.6 and 2.1 years, respectively.

Looking at Case 1, the pattern of oscillation is rather similar to Holton and Lindzen (1972), probably because radiative dissipation dominates wave transience. Transition from westerly to easterly regimes is much more rapid than that from easterly to westerly regimes, which seems inconsistent with the observation (cf. Coy, 1979).

Incorporation of the wave self-acceleration (Case 2) leads to an increase of the oscillation period by a factor of 1.5. A large increase in the mean wind velocity is found in both easterly and westerly regimes. The easterly wind, particularly, seems to experience larger acceleration than the westerly wind due to drastic self-acceleration appearing in the Rossby–gravity wave. The maximum value of the easterly wind velocity approaches 60 m s\(^{-1}\) on and near the upper boundary, overshooting the initial phase velocity of the Rossby–gravity wave by a factor of 2. The overshooting of the easterly wind is enormously larger than that of the westerly wind since Kelvin wave self-acceleration is relatively weak. This tendency was also found in the results of Tanaka and Yoshizawa (1985).

Introduction of wave saturation (Case 3) into the model leads to a decrease in the oscillation period of the mean zonal flow, reducing it to a period only slightly greater than the case of constant phase velocity (Case 1). Wave saturation and consequent wavebreaking work to pull the crests of the mean zonal flow downward. Thin layers of wave saturation are always situated just below the crests of the mean zonal flow in both westerly and easterly regimes. Effective but short-time acceleration of wind velocity in the thin layers and subsequent descent of the saturation layers are repeated and, as a result, contribute to a decrease in the oscillation period. It is interesting that Case 3 is more similar to Case 1 than Case 2 as far as the oscillation period of the mean zonal flow is concerned, ex-

\(^2\) If small viscosities are included in Eqs. (2), (3), (4) and (5), upper boundary conditions of wave momentum and phase velocity are necessary.
cept that the easterly wind maxima appear at higher altitudes (around 30 km) than those for the westerly wind maxima (around 23 km). Strong easterly accelerations occur just after the borders of transition and are still larger than westerly accelerations. Easterly wind velocities slightly exceed 30 m s\(^{-1}\) around 30 km altitudes. Rapid transition from westerly to easterly regimes is also found in this case and still remains unsolved. In other words, pseudomomentum flux divergences associated with the Rossby–gravity wave saturation are not effective at smoothing the mean wind shear. This will be discussed later.

Wave saturation of both waves occurs just after the transition of wind direction below the crests of mean zonal flow. The Rossby–gravity wave saturates at 30 km first and the saturation layers move downward, approaching the lower boundary. The Rossby–gravity wave saturation begins at higher altitudes than the Kelvin wave.

b. Phase velocities

Time–height sections of the Rossby–gravity wave phase velocities are shown in Figs. 2a and 2b for Cases 2 and 3, respectively, and those of the Kelvin wave for the same cases are shown in Figs. 2c and 2d. In Case 2, the self-acceleration of Rossby–gravity waves in the easterly regime and the self-deceleration of the wave
FIG. 2. Time-height sections of wave phase velocities. Solid contours are every 10 m s$^{-1}$ and in between dashed contours are every 5 m s$^{-1}$. Regions where absolute phase velocities are larger than 30 m s$^{-1}$ are shaded. Dark belts show wave saturation. (a) Rossby–gravity wave for Case 2, (b) Rossby–gravity wave for Case 3, (c) Kelvin wave for Case 2, and (d) Kelvin wave for Case 3.
in the westerly regime are incredibly large, as shown in Fig. 2a. The phase velocities of the Rossby–gravity wave exceed 60 m s\(^{-1}\) in the easterly regime. In the westerly regime velocities decrease drastically and finally change direction after reaching zero. The self-acceleration of the Kelvin wave is relatively inactive compared with the Rossby–gravity wave in Case 2 (Fig. 2c). Maximum phase velocity of the Kelvin wave is about 35 m s\(^{-1}\) in the westerly regime, and the minimum is 15 m s\(^{-1}\) in the easterly regime. An analogous situation was found by Tanaka and Yoshizawa (1985), under the assumption that \(\partial I_K/\partial t = \partial I_{RG}/\partial t \approx 0\) in Eqs. (2) and (3).

Wave saturation (Case 3) suppresses wave self-acceleration, as shown in Figs. 2b and 2d. Maximum phase velocities of the Rossby–gravity wave are a little more than 40 m s\(^{-1}\) in the easterly regime. Minimum phase velocities are never smaller than several meters per second and never change direction. Furthermore, such acceleration and deceleration occur at very localized altitudes and times. Suppression of Rossby–gravity wave self-acceleration is obviously due to Rossby–gravity wave saturation, as is seen in Fig. 2b.

On the other hand, self-acceleration of the Kelvin wave is less affected by wave saturation. Kelvin wave saturation starts at lower altitudes and is unlikely to
have a serious effect on self-acceleration at higher altitudes. Maximum phase velocity of the Kelvin wave is about 35 m s$^{-1}$, the same as in Case 2. Thus, wave self-acceleration is effectively suppressed when wave saturation occurs at higher altitudes.

c. Pseudomomenta

Time–height sections of the pseudomomenta of the Rossby–gravity waves are shown in Figs. 3a, b and c for cases 1, 2 and 3, respectively. Those of the Kelvin waves are also shown in Figs. 3d, 3e and 3f for Cases 1, 2 and 3, respectively. Pseudomomenta of the Rossby–gravity waves are amplified along the borders from easterly to westerly regimes. Those of the Kelvin waves are amplified along the borders from westerly to easterly regimes. Pseudomomenta of Rossby–gravity and Kelvin waves are accompanied by negative and positive wind shears, respectively, and both move downward alternately.

In Cases 1 and 2, pseudomomenta of the Rossby–gravity wave are much larger than those of the Kelvin wave. The former exceed −20 to −30 m s$^{-1}$, but the latter do not exceed 15 m s$^{-1}$. (See Figs. 3a and 3b for Rossby–gravity wave and Figs. 3d and 3e for Kelvin wave.) On the other hand, when wave saturation is taken into consideration, pseudomomenta of both waves are suppressed and never exceed ±10 m s$^{-1}$. The absolute values of Rossby–gravity wave pseudomomenta are even smaller than Kelvin wave pseudomomenta because the saturation limit of pseudomomentum, denoted by (11), for the Rossby–gravity wave is
smaller than that denoted by (10) for the Kelvin wave when intrinsic phase velocity is common.

5. Fine tuning of wave parameters toward a more realistic model

The model of Case 3, which includes wave transience, wave self-acceleration and wave saturation, is more elaborate in principle than the Holton–Lindzen model. However, some unrealistic and inconsistent results are still left unrectified even in this model. First, easterly acceleration is still stronger than westerly acceleration, and therefore, easterly wind velocity exceeds 30 m s\(^{-1}\) at an altitude of about 30 km. Second, transition from westerly to easterly regimes is still more rapid than the opposite transition. Although very realistic QBO reproduction is not the major objective of this paper, some test cases were computed. These are summarized below.

Figure 4 shows the time–height section of the mean zonal flow when Rossby–gravity wave pseudomomentum enforced at the lower boundary is specified as \(-0.5\) m s\(^{-1}\), half the value employed in Case 3. The result shows that the oscillation period is 2.4 years. Easterly acceleration seems to be reasonably suppressed. Nevertheless, the problem of rapid transition from westerly to easterly regimes is left unsolved. Westerly wind velocity maxima slightly exceed easterly wind velocity maxima. The former appears at much lower levels than the latter, which also seems inconsistent with the updated result of QBO analysis given by Coy (1979). The value of Rossby–gravity wave pseudomomentum enforced at the lower boundary, \(-0.5\) m s\(^{-1}\), seems too small if we look at the asymmetric pattern of the time–height section shown in Fig. 4. The real value may lie between \(-1\) and \(-0.5\) m s\(^{-1}\).

Another way can be found to rectify the inconsistent results obtained by the model simulation. Eddy viscosity coefficients associated with turbulence generated by saturations of the equatorial waves (especially Rossby–gravity waves) would be much greater than 0.3 m\(^2\) s\(^{-1}\). Such large viscosity coefficients would smooth negative wind shears around the saturation levels and create smoother transitions of mean zonal flow from westerly to easterly regimes. We have tried to incorporate the value of 1.5 m\(^2\) s\(^{-1}\) only at the levels of wave saturation, which is five times greater than 0.3 m\(^2\) s\(^{-1}\). This trial results in a more gradual transition from westerly to easterly and a reduction of the easterly maximum by 10 m s\(^{-1}\) without changing the westerly maximum. This modification tends to coincide with the observational structure of the QBO.

6. Conclusions and discussions

Holton and Lindzen’s (1972) model of the QBO, which was parameterized on the basis of linear and stationary equatorial waves, is modified and further developed by incorporating wave transience, wave self-acceleration and wave saturation in a quasi-linear system of equations. The present model is more sophisticated than Dunkerton’s (1981b) and Tanaka and Yoshizawa’s (1985) in virtually all respects. The following results are obtained by the numerical simulation of the model:

(i) Wave transience associated with pseudomomenta is not effective when wave self-acceleration is prohibited, probably because the effect of Newtonian cooling is more predominant in the equatorial stratosphere. Thus, the result of Case 1 resembles Holton and Lindzen’s (1972) result.

(ii) In spite of a relatively large effect by Newtonian cooling, the oscillation period of mean zonal flow increases drastically, by about 50%, when wave self-acceleration is taken into account. In addition, the mean zonal wind velocities easily overshoot the initial phase velocities of the waves. Easterly wind acceleration is enormous due principally to active self-acceleration of the Rossby–gravity wave. This result is analogous to Tanaka and Yoshizawa (1985), but surprisingly much larger.

![Figure 4](image-url)

**Fig. 4.** Time–height sections of mean zonal flow for the case in which only Rossby–gravity wave pseudomomentum is specified as \(-0.5\) m s\(^{-1}\) at the bottom boundary. Solid contours are every 10 m s\(^{-1}\). Dashed contours show \(\pm 25\) m s\(^{-1}\). Westerlies are shaded. Dark belts show wave saturation.
(iii) Wave saturation decreases the oscillation period again to almost the same value stated in (i). Mean zonal wind velocities are also suppressed, although they are still slightly greater in the easterly regime than in the westerly regime.

(iv) Transitions of mean zonal flow from westerly to easterly regimes are much faster than the opposite transitions for all cases performed in this paper.

(v) Rossby–gravity wave pseudomomenta are much larger than Kelvin wave pseudomomenta when wave saturations are ignored. Rossby–gravity wave pseudomomenta are suppressed by incorporation of wave saturation and become comparable with Kelvin wave pseudomomenta.

(vi) The Rossby–gravity wave saturates over larger altitude ranges than the Kelvin wave does when one-dimensional calculations of wave saturation conditions are applied.

(vii) Enforcing a half value of Rossby–gravity wave pseudomomentum, \(-0.5 \text{ m s}^{-1}\), at the lower boundary, we can suppress the easterly acceleration to some degree. However, the incorrect asymmetric pattern of time–height sections between easterly and westerly regimes does not disappear.

One-dimensional models of the QBO have been tested by incorporating almost all of the significant mechanisms. By and large, we obtained consistent results in the present model with the exception of the following two points. First, transition patterns of mean zonal flow from one regime to the other are quite different from the observations (cf. Coy, 1979). Second, westerly wind maxima appear at lower altitudes than easterly wind maxima, which also seems unrealistic (cf. Coy, 1979).

Dunkerton (1982a) reported that incorporation of mechanical dissipation of the Rossby–gravity wave would reduce the rapid transition of zonal flow from westerly to easterly regimes even if a one-dimensional model is used. He used a Rayleigh type of mechanical dissipation but did not specify the origin of the dissipation. Turbulent dissipation originated by Rossby–gravity wave saturation is a powerful candidate for Dunkerton’s (1982a) assumption. Unfortunately, it is very difficult to incorporate the turbulent dissipation mechanism into the present model except as the very simple case described in section 5.

Extension of the model to a meridional plane is another promising approach (e.g., Plumb and Bell, 1982a, b). It implies that the mean meridional circulation is involved in producing the observed asymmetry. Simulation of a realistic transition pattern of the zonal mean flow is an important unsolved problem. It should be noted, however, that such an extension to two-dimensionality must be performed carefully under consideration of wave transience, wave self-acceleration and wave saturation.

The saturation conditions used in this paper are incorporated in order to maintain theoretical consistency of the slowly varying model system. Therefore, these conditions may not correspond to real instabilities appearing in equatorial waves. Such instabilities should be reproduced in a multi-dimensional treatment of the model. Although more detailed implications of the one-dimensional saturation conditions, especially for the Rossby–gravity wave, are beyond the scope of this paper, we believe that some close connection exists between one-dimensional saturation conditions and real instabilities.

Incorporation of the equatorial Rossby waves and inertio–gravity waves into the model is important in simulating the QBO phenomenon more realistically (Andrews and McIntyre, 1976b; Cadet and Teitelbaum, 1979). However, we have to wait until more abundant data can be accumulated for those waves.

Acknowledgments. We thank Larry Coy, Manabu Yamanaka and Arthur Aikin for their useful suggestions on this work. Research funds for the Middle Atmosphere Program (MAP) from the Ministry of Education of Japan have supported this study. We also thank S. Ushimaru for typing the manuscript.

APPENDIX A

List of Symbols

\[\begin{align*}
\bar{u} & \quad \text{mean zonal wind velocity} \\
I_K & \quad \text{pseudomomentum of Kelvin wave} \\
I_{RG} & \quad \text{pseudomomentum of Rossby–gravity wave} \\
I_{KS} & \quad \text{saturated pseudomomentum of Kelvin wave} \\
I_{RGs} & \quad \text{saturated pseudomomentum of Rossby–gravity wave} \\
\bar{c}_K & \quad \text{intrinsic phase velocity of Kelvin Wave} (=c_K - \bar{u}) \\
\bar{c}_{RG} & \quad \text{intrinsic phase velocity of Rossby–gravity wave} (=c_{RG} - \bar{u}) \\
c_K & \quad \text{phase velocity of Kelvin wave} \\
c_{RG} & \quad \text{phase velocity of Rossby–gravity wave} \\
W_K & \quad \text{vertical group velocity of Kelvin wave} \\
W_{RG} & \quad \text{vertical group velocity of Rossby–gravity wave} \\
k_K & \quad \text{horizontal wavenumber of Kelvin wave} (=6.37 \times 10^5 \text{ m}^{-1}) \\
k_{RG} & \quad \text{horizontal wavenumber of Rossby–gravity wave} (=4k_K) \\
H & \quad \text{scale height} (=7 \text{ km}) \\
K & \quad \text{artificial viscosity coefficient} (=0.3 \text{ m}^2 \text{ s}^{-1}) \\
N & \quad \text{Brunt–Väisälä frequency} (=0.02 \text{ s}^{-1}) \\
z & \quad \text{height} \\
t & \quad \text{time} \\
\alpha(z) & \quad \text{Newtonian cooling coefficient} \\
\alpha_c & \quad \text{critical Newtonian cooling coefficient} \\
\beta & \quad \text{beta factor} (=2.29 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1})
\end{align*}\]

APPENDIX B

Quasi-linear Saturation Condition of Rossby–Gravity Waves

The quasi-linear saturation condition of the internal gravity wave was first treated by Dunkerton (1982b).
Coy (1983) derived the same condition under an assumption that the phase velocity is allowed to be variable. Two characteristic curves are obtained from theoretical analysis of quasi-linear equations: one is the leading edge and the other is the trailing edge bordering a wave expansion region. The leading edge ascends with time but never descends. The trailing edge ascends in the early stage of propagation but begins to descend some time later. The turning point corresponds to the beginning of saturation.

Equations describing an interaction between mean zonal flow and the Rossby–gravity wave are described as

$$\frac{\partial I_{RG}}{\partial t} + W_{RG} \frac{\partial I_{RG}}{\partial z} + q^2 W_{RG} \frac{\partial c_{RG}}{\partial z} = \frac{W_{RG} c_{RG}}{H} - \frac{\beta + k_{RG}^2 c_{RG}}{2 \beta + k_{RG}^2} \alpha I_{RG}, \quad (A1)$$

$$\frac{\partial \tilde{u}}{\partial t} + W_{RG} \frac{\partial I_{RG}}{\partial z} + q^2 W_{RG} \frac{\partial c_{RG}}{\partial z} = \frac{W_{RG} c_{RG}}{H}, \quad (A2)$$

$$\frac{\partial c_{RG}}{\partial t} + W_{RG} \frac{\partial c_{RG}}{\partial z} + W_{RG} \frac{\partial \tilde{u}}{\partial z} = 0, \quad (A3)$$

where

$$q^2 = \frac{2 I_{RG} 3 \beta + k_{RG}^2 c_{RG}}{c_{RG} 2 \beta + k_{RG}^2 c_{RG}}. \quad (A4)$$

Note that Eqs. (A1) and (A3) are actually identical to Eqs. (3) and (5), respectively.

Three characteristic curves are obtained by the characteristic method:

$$\frac{dz}{dt} = 0, \quad (A5)$$

$$\frac{dz}{dt} = W_{RG}(1 \pm q), \quad (A6)$$

where (A5) is associated with dissipations and (A6) includes two curves, the plus sign is a leading edge and the minus sign is a trailing edge. The trailing edge stops ascending when $q = 1$; that is, wave saturation begins. This is why the saturated wave momentum (11) can be obtained for the Rossby–gravity wave.

Wave saturation defined here does not always correspond to a realistic instability. This kind of saturation is quite different from local convective instability. Saturated pseudomomentum derived from the local convective instability of the Rossby–gravity wave is given by

$$I_{RGS} = \frac{\tilde{c}_{RG} 2 \beta + k_{RG}^2 c_{RG}}{2 \beta + k_{RG}^2} \frac{k_{RG}^4 \tilde{c}_{RG}^2}{(\beta + k_{RG}^2 c_{RG})^3}. \quad (A7)$$

When $\tilde{c}_{RG} = -\beta/3k_{RG}^2$ ($\approx -20$ m s$^{-1}$ for the typical Rossby–gravity wave), saturated pseudomomenta in both (11) and (A7) become equal; that is, $I_{RGS} = -5\beta/48k_{RG}^3$ ($\approx -6$ m s$^{-1}$). Rossby–gravity wave pseudomomentum easily exceeds $-6$ m s$^{-1}$, as is shown in Figs. 3a and 3b. In other words, the saturation condition of (11) occurs prior to the local convective instability.

In the limit of $\beta \to 0$, saturated pseudomomenta in both (11) and (A7) are degenerated to that of the two-dimensional gravity wave, i.e., $\tilde{c}/3$. In the limit of $\tilde{c} \to 0$, the saturated pseudomomentum in (11) leads to $\tilde{c}/3$. Equatorial Rossby waves and inertia–gravity waves also take the same form of saturated pseudomomentum for $\tilde{c} \to 0$. However, local convective instability would be more significant in such small regions of $\tilde{c}$.

REFERENCES


