Lagrangian Mean Circulations in the Stratosphere

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(Manuscript received 30 June 1986, in final form 17 February 1987)

ABSTRACT

Three-dimensional winds derived from LIMS satellite observations for the 1978/79 winter are used to compute the mean Lagrangian motion in the winter stratosphere. Material tubes of air parcels are initialized every 4 days and followed for periods of 10 days each. The initial positions of the tubes are chosen so that they lie along contours of constant geopotential and potential temperature. Maps of air parcel distributions give a qualitative picture of the degree of deformation of the material tubes with time. In addition, quantitative measures of the Lagrangian mean velocity and dispersion are computed.

During quiet periods, when the zonal wind is strong and the vortex is nearly axisymmetric, the air parcel tubes tend to remain coherent for the full 10 days of the integration. When the wave amplitudes are large, many of the tubes break and the parcels disperse. During the observed minor sudden warming, those tubes closest to the vortex center remained coherent with little distortion. In contrast, during the major sudden warming every material tube in the stratosphere was broken, and there was extensive mixing between air parcels from low and high latitudes.

Lagrangian mean vertical motion tended to be smaller than the motion in the transformed Eulerian coordinate system, which is sometimes used to represent the mean Lagrangian flow. The horizontal velocities determined from the Lagrangian parcel trajectories do not in general correspond well with the transformed Eulerian velocities.

1. Introduction

The traditional Eulerian mean framework has, for the last several decades, been used extensively in studying the processes controlling middle atmospheric circulation and transport. The transport of a conservative chemical or dynamical tracer is viewed in this framework as the sum of the transport by the zonally averaged mean meridional circulation (MMC) plus the transport due to wave flux convergences. However, this partitioning of the transport between mean and eddy contributions can be misleading because the Eulerian MMC is itself partially driven by the eddies. Evidence of the nonindependent nature of the two transport components was given in the early model transport study of Hunt and Manabe (1968). They showed that the transport of a conservative tracer by the Eulerian MMC and by eddy flux convergences tended to cancel each other at high latitudes in the lower stratosphere, resulting in only a small net residual transport.

The advection of trace species is closely related to the acceleration of the mean wind due to advection of angular momentum. The nonacceleration theorem (Eliassen and Palm, 1961; Charney and Drazin, 1961; Andrews and McIntyre, 1976; Boyd, 1976) states that steady, conservative waves of small amplitude do not force changes in the zonal-mean wind and temperature fields. When these conditions are satisfied, air parcels move through the waves but there is no net meridional or vertical displacement, and no acceleration of the mean wind. Holton (1980) showed that under these conditions the net transport of a conservative trace species by the waves and wave-induced mean flow is also zero.

Analysis of middle atmosphere observations confirms that eddy-mean flow cancellation occurs. Observations show that the Eulerian MMC in the upper stratosphere and lower mesosphere consists of a flow from the summer to winter pole, while the lower and middle stratospheric circulation has a two-cell struc-
ture, with ascending motion over the tropics and polar regions and descent in midlatitudes (e.g., Vincent, 1968; Hartmann, 1976). The high-latitude wintertime cell is thermally indirect, and therefore must be dynamically forced. Also present in the winter hemisphere are large-amplitude planetary waves that propagate from the troposphere. The adiabatic cooling associated with the upward branch of this indirect cell in the polar regions acts to offset the effect of positive heat flux convergence associated with the waves. This is, of course, just what the nonacceleration theorem predicts. For waves satisfying the nonacceleration condition, the eddy heat flux convergence that would otherwise decrease the latitudinal temperature gradient is exactly cancelled by adiabatic cooling in the upward branch of the indirect cell. Although the nonacceleration conditions are never met exactly for real waves, there is often a high degree of cancellation between the mean and eddy transports in the stratosphere.

The Generalized Lagrangian-Mean (GLM) framework of Andrews and McIntyre (1978b) eliminates this artificial distinction between transports by the Eulerian MMC and by the eddies, and therefore provides a more physical framework for viewing stratospheric circulations and trace constituent transport. The GLM system of equations is a hybrid description which gives the net Lagrangian displacements in an Eulerian form. Unlike the Eulerian averages along latitude circles, the Lagrangian averages are taken on wavy material tubes that would lie on latitude circles if waves were not present. Small-amplitude waves will drive the GLM circulation only if they violate the nonacceleration conditions. The GLM circulation is also forced by non-conservative zonal mean processes such as diabatic heating. (The radiative cooling is, however, directly related to the temperature distribution, which is maintained in part by mixing due to waves.) In the Lagrangian mean formulation, there is no circulation corresponding to the eddy flux convergence; ideally, a single two-dimensional mean circulation can represent the total mean (in a GLM sense) transport in the meridional plane.

The Lagrangian and Eulerian mean circulations are related by the Stokes velocity, with the Lagrangian velocity equal to the sum of the Eulerian and Stokes velocities (McIntyre, 1980a). The Stokes velocity is partly a consequence of variations in wave amplitude with latitude or height. Depending upon the wave structure, parcels will experience a rectified motion while traveling through the wave. For example, if the air parcel rises in a region of small wave amplitude and then sinks in a region of large wave amplitude, there will be a net sinking as it cycles through the wave. Under non-acceleration conditions, this drift, i.e., the Stokes velocity, would be balanced by the wave-induced mean velocity, and the net result would be no transport (assuming no mean nonconservative forcing was present). In general, the Lagrangian velocity will depend on the relative sizes of the Stokes and mean Eulerian velocities. The Stokes velocity can be the dominant term which can result in the Eulerian and Lagrangian circulations being oppositely directed.

The two-dimensional Lagrangian mean circulation is physically more useful for interpreting transport processes than the Eulerian MMC alone. Unfortunately, calculations of a Lagrangian mean are difficult and present a number of problems not present in the Eulerian framework. An improvement over the Eulerian MMC is the transformed Eulerian MMC, also known as the residual circulation, introduced by Andrews and McIntyre (1976; 1978a), which should be qualitatively more akin to the fundamental Lagrangian-mean description. The residual circulation, roughly speaking, is that portion of the Eulerian MMC that remains after the effect of steady conservative waves has been subtracted out. In general, the transformed Eulerian mean circulation, which is a strictly Eulerian diagnostic, is not the same as the Lagrangian mean flow. They are identical in the absence of waves, however, since the Stokes velocity is then zero. The residual MMC more closely resembles the actual air parcel motions than the Eulerian MMC, and is easier to calculate than the Lagrangian MMC, and has therefore been proven useful in chemical transport studies (e.g., Holton, 1981; Garcia and Solomon, 1983).

Several studies of the Lagrangian-mean circulation in the stratosphere have been made with numerical models. Matsuno and Nakamura (1979) derived the Lagrangian mean circulation indirectly, based on results from an Eulerian model of a stratospheric sudden warming. They found, as expected, that the Eulerian and Lagrangian mean circulations were in opposite directions in high latitudes. A similar conclusion, based on observations, was obtained by Mahlman (1969). Another direct calculation was that of Dunkerton (1978), who approximated the Lagrangian circulation from the mean diabatic forcing for the winter. His analysis stemmed from the assumption that all net parcel displacements averaged over a long (seasonal) time scale are due to a nonconservative process, in particular to the zonal mean diabatic forcing. Because of the nature of the analyses, both of these studies considered only the mean Lagrangian displacements, and not the dispersion of air parcels around the mean.

The model of Hsu (1980), which was discussed in detail by Dunkerton et al. (1981), computed the Lagrangian circulation directly by tracing a number of air parcels during a stratospheric sudden warming. Her results showed that there were strong poleward and downward displacements of air parcels in high latitudes, associated with the adiabatic increase in temperature. Even though the model was limited to zonal mean and wave 2, the motion, especially in low latitudes, became so complex that the parcels did not remain in a continuous line, but appeared to become disconnected. In another model simulation (Kida, 1977), parcels were
followed over the course of a season to determine the mean time for transport between low and high latitudes, and between the troposphere and stratosphere in the presence of a wave. The models of both Hsu and Kida confirmed that the mean motion was poleward and downward in the stratosphere, in opposition to the wintertime Eulerian mean circulation. Also, both models found that there is a large amount of dispersion in addition to the net displacements, and that the dispersion is not uniform for all regions of the atmosphere.

Although experiments with models are very useful, they necessarily represent a simplified picture of what is occurring in the actual atmosphere. The best understanding can be obtained by a balance of model studies, in which fields are exactly known, and observations, which have greater uncertainties, but which must be used to evaluate the accuracy of model results. In this paper we present results of a study in which the net displacements and Lagrangian mean velocities were estimated for the 1978–79 winter from middle atmosphere observations. To do this, we selected material tubes (ensembles of parcels) for a number of days throughout the winter, traced them following the three-dimensional wind (derived from LIMS satellite data), and then averaged their net displacements from the initial tube positions.

These calculations depend on the specification of a three-dimensional, time-dependent wind field at all points. The conclusions of this study are therefore limited by the quality of the wind information that goes into the calculations. While we have used what we believe to be the highest quality global stratospheric observations in existence, there are still uncertainties associated with them. Sources of possible error include instrument noise and systematic error, errors in the inversion of radiance to temperature, limited temporal and spatial resolution of the observations, and approximations used to calculate winds from geopotential and temperature. Of these, the most serious are probably the poor temporal and zonal resolution of the data and the geostrophic approximation used to compute horizontal winds. (Smith and Bailey, 1985, showed that the horizontal winds calculated from LIMS agree quite well both in magnitude and in temporal variability with those from rocket measurements. However, the cumulative error from trajectory calculations can be large even if the point-by-point errors are small.) Because of these potential errors, the actual magnitudes of the circulation components calculated should not be given too much confidence. In our discussion of the results we have tried to emphasize the relative importance of various components of the circulation, such as a comparison of residual and Lagrangian mean motions (computed for identical wind fields) and an analysis of the spatial and temporal variations of dispersion.

Section 2 contains a discussion of how the material tubes were initialized, how the integrations were carried out and finally how we calculated the Lagrangian mean velocities of the tubes. Section 3 will present results showing the evolution of material tubes during a quiet period, an active period and a major sudden warming. Section 4 will compare the derived Lagrangian mean circulation with the corresponding results for the residual MMC. Since the transformed Eulerian circulation is often used because of its resemblance to the Lagrangian mean, several comparisons were made to determine how well the residual circulation approximates the GLM circulation and under what conditions this approximation breaks down. Conclusions are summarized in section 5.

2. Calculations

Lagrangian-mean velocities are not unique and can only be calculated relative to a suitable initialization. Therefore, the first step of the calculations was determining the initial positions of material tubes that were to be used later in calculating GLM velocities. When the tubes are properly initialized, the GLM velocity of a tube should be zero if the waves satisfy the nonacceleration conditions, and if no mean nonconservative forcings are present. The tube should also be initialized so that it could have evolved by the action of conservative waves from a zonally symmetric state to its current configuration (McIntyre, 1980a).

Integrations in numerical models such as that of Hsu (1980) begin from a state of zonal symmetry and in this case the natural initialization is to place the tubes on latitude circles. However, in the atmosphere, where such a state does not in general exist, an initialization along latitude circles would lead to distortions of the tube and apparent wave-induced Lagrangian velocities even if the waves present were steady and conservative (Dunkerton, 1980).

One method of initialization satisfying these requirements is that of the modified Lagrangian mean, proposed by McIntyre (1980a,b) and Dunkerton (1980). They delimited the sides of their tubes by surfaces of constant potential vorticity and potential temperature, the tube in this case having some finite cross-sectional area. We could not use this method because the contours of potential vorticity derived from data contained too much noise in the high wavenumbers. Also, we were restricted in practice to a finite number of air parcels rather than a continuous surface.

Our approach was to view the tube as a space curve having infinitesimal cross-sectional area. The tubes were chosen to coincide with isolines of constant height on an isentropic surface. The height contours approximate the potential vorticity contours to first order and are far less noisy. Only those tubes that could have evolved from a state of zonal symmetry are used. This restriction means that height contours surrounding a closed anticyclone were not used. The initialization is similar to the modified Lagrangian mean (McIntyre, 1980a; Dunkerton et al., 1981), but the integration then fol-
allows individual fluid parcels as in an unmodified Lagrangian mean. Tests showed that even during periods of strong change, some of the tubes maintained their coherence for periods of more than a week. (Examples are shown in the discussion of results.) We believe that this provides justification for the assumption that this initialization places parcels approximately in material tubes.

The numbers of parcels placed on a tube was proportional to the arc length of the tube, and varied from a minimum of 16 for the shortest tubes to a maximum of 128 for the longest. The initial spacing between parcels on a tube was in general nonuniform and was on the order of three to four hundred kilometers. The need for such nonuniform spacing can be understood by considering a tube originally zonally symmetric about the North Pole and having uniform spacing between the parcels. Once the tube becomes distorted from this state, portions of the tube will inevitably become stretched and the tube's mass per unit length in these sections will be less than other parts of the tube. These density variations along the tube require that the parcels be placed on the tubes so that the spacing between two parcels is proportional to the horizontal wind speed between those two points (Matsuno, 1980). This method of spacing the parcels will result in the same number of parcels passing a certain point of the tube per unit time.

Once the initialization of the tube positions was completed, the next step of the calculations was determining the future positions of the tubes using the three-dimensional wind field. The wind field was calculated once daily and was derived using temperature data obtained from the LIMS satellite instrument (Gille et al., 1984). The LIMS temperatures, which were Fourier analyzed out to wave 6, were used to compute thicknesses. Then, using 50-mb FGGE heights (50-mb NMC heights for November 1978) as the tie-on height, these thicknesses were integrated to obtain isobaric heights. Values at the pole were estimated by extrapolation from 80°N. The mean and eddy components of the zonal wind and the eddy component of the meridional wind were calculated from the geostrophic wind equation. The vertical and mean meridional wind fields were determined from the thermodynamic and continuity equations in the same manner as in Smith and Lyjak (1985). The net diabatic heating rates used in the thermodynamic equation were calculated as described in Gille and Lyjak (1986).

Determining the future positions of the tubes required knowing the future locations of the parcels on the tube. This was achieved by integrating forward in time using:

\[ \mathbf{X}(t_2) = \mathbf{X}(t_1) + \mathbf{V}[\mathbf{X}(t_1), t_1] \Delta t \]

where \( \Delta t \) is the time step, \( \mathbf{X}(t) \) is the three-dimensional position vector of the parcel at time \( t \), and \( \mathbf{V}[\mathbf{X}(t_1), t_1] \) is the velocity vector at position \( \mathbf{X} \) and time \( t \). Here \( \mathbf{V} \) was determined by linearly interpolating in space and time from once daily grids. The time step used was 30 minutes. Tests were conducted to determine the sensitivity of our results to the time step chosen. These tests indicated that individual parcel positions changed slightly when the time step was reduced to 1 min, but the overall tube position remained unchanged.

The GLM calculations were performed beginning every fourth day between 1 November 1978 and 9 May 1979. Each integration was run for 10 days or until some of the parcels left the data range (100–0.1 mb; Northern Hemisphere). For each initialization, there are 10 levels ranging from 50 mb to 0.4 mb. The number of material tubes varies, but is generally in the range of 5–8 per level.

The calculation of the Lagrangian-mean velocity of a GLM tube involves differencing the tube's mean position at an initial and a final time and dividing by the elapsed time. A major problem in this calculation is determining the tube's mean position. Andrews and McIntyre (1978b) defined the center of mass of the tube to be the mean latitude and height of the parcels constituting the tube. This method worked well for computing the tube's mean vertical position and was the method used in this study. However, for computing the mean horizontal position of the tube, this method is suitable only when planetary wave amplitudes are relatively small, in which case the tube lies near a latitude circle. This was not true for some of the large wave amplitude cases that were considered here. In some of these cases, amplitudes were so large that the geopotential contours used to initialize the tubes were not circumpolar. Therefore, when calculating the GLM horizontal mean tube position and velocity a different definition was used. Rather than using the mean latitude, which is a measure of distance from the pole, we picked a point that was in the "center" of the ring of parcels, and determined the mean distance from this point. The center was defined to be a point whose horizontal position was such that the average distance from each individual point to the center, measured along the surface of the sphere, was minimized. This point's horizontal location, along with the previously defined mean vertical position, will be referred to as the center of mass of the material tube. This center of mass definition works reasonably well for large wave 1 and for quiet conditions. However, when a large wave 2 disturbance is present there are two sets of tubes, one set on each side of the pole, and interpretation of results is then much more difficult. As will be discussed further below, the horizontal GLM velocities resulting from both Andrews and McIntyre's definition and from ours compare poorly in space and time with the transformed-Eulerian mean horizontal velocity.

The horizontal GLM velocity is defined to be the average rate that parcels approach their center of mass. As long as the center stays in roughly the same place
through the integration, the GLM velocity is defined by subtracting the final mean distance, and dividing by the elapsed time. The horizontal velocity defined at some point along the tube is directed toward or away from the center of mass, and in general contains both zonal and meridional components. There is only one horizontal speed for a particular tube, but its zonal and meridional components vary depending on the location along the tube. During a few very active periods, the center of mass also moved substantially, and in these cases the horizontal velocity is directed towards the new center of mass.

Our method of initializing tubes and computing their mean motion is based on an estimate of how material tubes would deform beginning from an undisturbed state. While ours is certainly not the only possible method of computing the GLM motion, the results show that it is a useful one. In general, the method works best, and is most similar to the definition of Andrews and McIntyre (1978), when only small amplitude waves are present. During more disturbed conditions, these GLM calculations give the motion with respect to the location around the polar vortex, rather than with respect to a latitude circle. However, when the vortex itself is rapidly deforming, the mean component of the motion is difficult to isolate. As the results presented in section 4 will show, the calculated mean circulation can be erratic and difficult to envision intuitively.

3. Air parcel distributions

In this section we show maps of the distribution of air parcels that were originally all on the same potential temperature surface. Three cases, which illustrate three different types of behavior, are discussed. The levels and the number of days of integration shown vary; in each case we have examined a number of maps and selected those that best illustrate the overall behavior during that particular period. Vertical displacements and mean velocities are discussed in section 4.

The behavior of the air parcel tubes was first studied for the relatively quiet time period of 16–23 November 1978. On 16 November wave amplitudes were small and the polar vortex was deep and nearly zonally symmetric. Figure 1a shows a horizontal projection of the initial tube positions for an isentropic surface near the 2-mb level. (The potential temperature surfaces used for initialization are close to the standard pressure levels of the data, and will be referred to by the pressure level.) Each parcel whose path was followed is indicated by a letter, and there is a different letter for each material tube. Figure 1b shows the resulting tube positions on 23 November after an integration of 7 days. The tubes have largely remained intact and coherent, with only slight distortions of the outermost tubes F and G.

Maps of the material tubes originally near 10 mb for the same days are shown in Fig. 2. In this region the distortion of the outer rings is much greater than in the upper stratosphere. Although kinks develop in the outer rings, the order does not change; for example, the E ring is still enclosed within the F ring. The zonal wind for 16 November (Fig. 3) shows that the winds are quite strong in the midlatitude upper stratosphere, but are weak (or even easterly) in the lower stratosphere, particularly south of 40°N.
January 1979. The large wave 1 disturbance has shifted the polar vortex well off the pole.

Figure 4b shows the tube positions on 26 January after an integration of 5 days. Despite the very dramatic dynamical changes that have taken place during these 5 days, the innermost three tubes have remained intact around the vortex. The outer tubes (D–F) have broken and are coiling around the Aleutian anticyclone as well as the vortex, but it still is possible to tell that the points form a chain rather than a random mass. Parcels from the tube which was originally outermost are now within the anticyclone. Results for this period at other levels resemble those at the 1.5-mb level shown.

Figure 5a shows the material tube positions at 2 mb on 17 February, just before the wave 2 major sudden warming (Palmer, 1981). After three days (Fig. 5b), all of the tubes are strongly distorted, and some have broken. After one week of integration (Fig. 5c) only short segments of the original tubes are identifiable. In some cases, parcels that were once part of a coherent tube appear to be clustered in a particular area, although it is not possible from this map to identify which parcels were originally adjacent. For the most part, fluid parcels from different tubes appear to be extensively mixed. In fact, near the pole there appear to be more parcels from the outer tubes (E, F and G) than from those originally innermost (A–D). The word “mixing” is used here to mean that the majority of the parcels originally constituting a tube can no longer be identified with the tube. Sections of the tube may still be intact but most of the tube has been torn apart. The tubes cross each other and are unable to return to their original predisturbance configuration. Because the parcels within each tube are originally spaced at horizontal distances of up to several hundred kilometers, the amount of mixing occurring on scales smaller than this is unknown. Therefore, it is unclear from these calculations whether

During late January 1979, there was an extremely large wave 1 in the stratosphere. The dynamical events during this period included a dramatic minor sudden warming (Palmer and Hsu, 1983) and a striking incident of planetary wave breaking (McIntyre and Palmer, 1983; 1984). Descriptions of the evolution of potential vorticity fields during this event are given by Schoeberl and Smith (1986) and Butchart and Rensberg (1986). The behavior of GLM tubes during this very active period is quite different from that in November. Figure 4a shows the initial tube positions for 1.5 mb on 21

![Figure 2. Initial positions (a) and positions after 7 days (b) of air parcels that were near the 10-mb level on 16 November.](image1)

![Figure 3. Zonal mean geostrophic wind in the stratosphere on 20 November.](image2)
mixing of trace species would extend to scales small enough to affect chemical reactions.

Behavior of the material tubes at levels below 2 mb is similar in basic appearance to Fig. 5. However, near the stratopause (around 0.4 mb) the material tubes underwent only minor distortions during this period, except for the region near the pole where horizontal dispersion was large (Fig. 6). The reversal of the mean wind at that level (Fig. 7) was confined to the region poleward of about 60°N.

The three cases shown can be distinguished based on the appearance of the material tubes after an inte-

gration of about one week. For the November case, all of the tubes remain intact and do not cross. The distortion is greatest for the lower-latitude tubes, especially those in the region of weak zonal winds. During the January warming, the tubes closest to the vortex center stay intact with little distortion. The outer tubes break and wrap around and into the anticyclone, but still remain coherent after seven days. The January case is similar in some respects to the limited results of Dunkerton and Delisi (1986) for early February, in which parcels from one material tube were followed. Their results appear to be relevant to a material tube whose behavior is similar to our D–F tubes in Fig. 4. During the major warming in February the mixing in the stratosphere is so strong that it is not possible to trace fully the connection between the parcels from any of the material tubes. At the stratopause level this event had a less-disruptive effect, and the destruction of material tubes was confined to high latitudes. For all three cases described, plots (not shown) of initial and final GLM tube positions projected onto the latitude–height plane show that vertical dispersion is much smaller than horizontal dispersion.

The change in behavior of the material tubes is consistent with the idea of planetary wave breaking proposed by McIntyre and Palmer (1983, 1984) based on observations for the same winter. During early winter, there is a continual erosion of the vortex by breaking waves in low latitudes where the mean flow is weak. The distortion of the outer material tubes during November is symptomatic of this nonlinearity. Because the initialization was based on regularly spaced contours of geopotential, we were not able to examine material tubes near the zero wind line, where the nonlinearity is expected to be greatest.

During the January warming, potential vorticity maps shown by McIntyre and Palmer suggest that large pieces of the distorted vortex are being broken off and mixed with the lower potential vorticity air originally in low latitudes. The center of the vortex was not, however, destroyed at this time. The fluid parcel distributions (Fig. 4) show that material tubes from the outer three tubes have broken and are mixed, while the innermost tubes (A–C) remain intact. As long as the tubes remain connected, it may be possible for them to return to the predisturbance state of near-zonal symmetry. Fluid parcels in the tubes D–F cannot regain their former state by conservative action; in this sense, the air has been irreversibly mixed. During the February warming, the analysis of McIntyre and Palmer suggested that at 10 mb the entire vortex was broken apart, and that mixing was even more extensive. This erosion of the vortex was quantified by Butchart and Remsberg (1986). The parcel trajectories at stratospheric levels also support this idea. There are no material tubes that remain connected, and, except for a few short segments, the fluid parcels are no longer indentifiable with a particular tube.

Fig. 4. Initial positions (a) and positions after 5 days (b) of air parcels that were near the 1.5-mb level on 21 January.
4. Comparison of Lagrangian and transformed Eulerian velocities

This section presents a quantitative comparison between the GLM and residual mean circulations. There are several fundamental differences between these two descriptions of the circulation, which should be borne in mind when comparing them. The residual circulation is the net mean component of the Eulerian twodimensional transport, but is not independent of the dispersion. Dispersive mixing by waves is, to a large extent, responsible for maintaining the warm polar temperature which in turn lead to the strong diabatic forcing of the mean circulation. Therefore, a mean circulation based on the observed diabatic forcing reflects the presence of dispersion by waves. Under circumstances of small and spatially uniform dispersion, the GLM winds can bear a strong resemblance to the residual circulation. However, the GLM motion is not constrained to be nondivergent in the two-dimensional plane, and does not in general give a coherent transport pattern. Regions or periods of agreement between the two descriptions of the circulation correspond to cases during which dispersion is relatively small. Plumb and Mahlman (1987) describe experiments determining the two-dimensional advective and dispersive transport components from a three-dimensional model, and stress that it is the spatial inhomogeneity of the eddy
diffusion which is responsible for the difference between
residual and Lagrangian mean descriptions.

A latitude–height cross section of the direction of
the GLM circulation for the period 16–23 November
is shown in Fig. 8a. (See section 2 for a description of
how the velocities were computed.) Here one normal-
ized vector is plotted for each material tube. The base
of the arrow is the original latitude and height of the
point on each tube which is closest to 0° longitude,
although for this case the point selected has only a mi-
nor effect on the plotted results. Figure 8a shows that
the flow is generally coherent, with mean descent oc-
curring for almost all tubes. The meridional component
is generally northward above 10 mb with some south-
ward motion below 10 mb and north of 60°N. Mag-
nitudes (not shown) of the vertical velocities generally
increase with height, while horizontal velocity mag-
nitudes generally increase towards the equator. The
regions where the meridional velocities are most erratic
correspond well to the region where the zonal flow (Fig.
3) is weak.

Figure 8b shows normalized vectors of the trans-
fomed Eulerian MMC for the period 16–23 Novem-
ber. The transformed Eulerian MMC was calculated
directly from the transformed thermodynamic equa-
tion using the same mean and eddy winds as were used
for the Lagrangian integrations. A more detailed de-
scription of the residual circulation is given in Gille et
al. (1987). Since identical information went into the
GLM and residual mean calculations, all differences,
except for minor ones due to numerical calculations,
are due to the fundamental differences in the way the
circulations are determined. The most conspicuous dif-
ference between the two velocity fields is in the degree
of smoothness. The GLM field shows a large degree
of scatter, especially in the meridional wind direction.
This may be due in part to the limited number of tra-
jectories used to determine the Lagrangian mean cir-
culation. The correspondence between the directions
of the GLM and transformed Eulerian circulations is
best at 0.4 mb and poorest below 10 mb, where vectors
northward of 60°N show that the meridional compo-
ents are in opposite directions. Magnitudes of the two
circulations for the entire winter are compared below.

The GLM normalized velocity cross section for the
January warming and the corresponding transformed
Eulerian MMC are shown in Fig. 9. Unlike during the
quiet case, the GLM circulation is now rather disor-
ganized and incoherent. (Although the meridional
component of the mean circulation varies depend-

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**Fig. 6.** Initial positions (a) and positions after 7 days (b) of air parcels that were near the 0.4 mb level on 17 February.

**Fig. 7.** Zonal mean geostrophic wind in the stratosphere on 25 February.
difficulty in assigning an average latitude to the tube when large-amplitude planetary waves are present.

Considering all the approximations involved in comparing mean velocities computed in two different coordinate frames, the agreement is remarkably good. During late fall and spring \( \bar{w}^L \) and \( \bar{w}^* \) vary slowly with time in the lower stratosphere (Fig. 10a). The variation is small in the lower stratosphere because there is relatively little wave transience, and most of the forcing at these times is due to comparatively steady diabatic processes. During the winter season both \( \bar{w}^* \) and \( \bar{w}^L \) vary rapidly with time, but generally the curves compare well at 10 mb. Note the large negative values that occur in the stratosphere during the sudden warmings in January and February. The slight difference in the timing of the large negative values may be due to the difficulty in assigning a latitude to a particular tube for plotting purposes. At the stratopause (Fig. 10b) both \( \bar{w}^* \) and \( \bar{w}^L \) indicate strong sinking, but \( \bar{w}^* \) is consistently larger (more negative) by about 2 mm s\(^{-1}\), or about 20\%–50\%. Overall results for all levels indicate

FIG. 8. Latitude–height cross section of the normalized directions of the mean Lagrangian (a) and transformed Eulerian (b) circulations for 16–23 November.

on what location along the ring is used to assign a position, there are no cases that result in a smooth mean meridional Lagrangian flow.) The vertical directions of the two circulations again compare well, while the horizontal component for the GLM circulation is very chaotic. This is in contrast to the transformed Eulerian MMC where the flow is generally northward everywhere. The GLM velocities for the February warming (not shown) also show consistent downward motion and irregular meridional motion.

Quantitative comparisons between the two circulations were achieved by plotting their horizontal and vertical components both as functions of time and of latitude. Time series of the Lagrangian (\( \bar{w}^* \)) and transformed Eulerian (\( \bar{w}^* \)) vertical velocities for 1 November 1978 through 9 May 1979 at 10 and 0.4 mb are shown in Fig. 10. The \( \bar{w}^* \) values plotted are averages for latitudes 60 to 72°N, while the \( \bar{w}^L \) values are calculated using mid- to high-latitude material tubes. This procedure was followed because of the dif-

FIG. 9. As in Fig. 8 but for 21–26 January.
Because they are averages, neither the Lagrangian nor residual velocity is capable of fully characterizing the downward motion of air parcels. The zonal mean thermodynamic equation can be written

\[ \frac{\partial \bar{\theta}}{\partial t} + \bar{v} \frac{\partial \bar{\theta}}{\partial y} + \bar{w} \frac{\partial \bar{\theta}}{\partial z} = \bar{Q} - F_H - D_H \]

where \( \bar{\theta} \) is potential temperature. The forces maintaining the thermal distribution following the two-dimensional circulation are diabatic heating (\( \bar{Q} \)), an eddy heat transport term that vanishes for small amplitude, steady conservative waves (\( F_H \)), and diffusion (\( D_H \)). See Gille et al. (1987) for the form of the eddy heat flux term \( F_H \). Diffusion has been neglected in the calculation of the residual circulation because of the difficulty of determining it from the observations. A time mean value of temperature change due to diffusion that is significant relative to that from mean diabatic forcing would result in an error in the calculated residual circulation. Note that horizontal diffusion could therefore be responsible for an overestimate of the mean vertical velocity. This is the most likely explanation for the consistent discrepancies between the calculated residual and Lagrangian vertical velocities. Given the importance of the diffusion process, it is encouraging that the results shown in Fig. 10–11 suggest that \( \bar{w} \) is not a bad approximation to \( \bar{w}^L \), even during active periods in the winter stratosphere.

Several experiments have been performed which compare transport by the residual circulation with that by the Lagrangian mean or full three-dimensional circulation in numerical models. Rood and Schoeberl (1983) derive expressions relating the Lagrangian and

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**Fig. 10.** Lagrangian (solid) and residual (dashed) mean vertical velocities as a function of time at 10 mb (a) and 0.4 mb (b) for high latitudes (roughly 60°–72°N). The Lagrangian means are based on integrations of 7 days each.

that, where the two circulations did not agree the transformed \( \bar{w} \) was an overestimate of \( \bar{w}^L \) by as much as 50% at middle and high latitudes.

Figure 11 shows \( \bar{w}^L \) and \( \bar{w} \) plotted as functions of latitude on 13 November 1978 (a quiet period) at the 1-mb level. The variable \( \bar{w} \) overestimates \( \bar{w}^L \) at all latitudes by about 1 mm s\(^{-1}\). The latitude location of the parcel in each ring which was closest to 0° longitude is used to assign a latitude to the tube. Such an assignment is arbitrary but in this relatively quiet case would only result in a small translation of the GLM curve. However, the longitude must be chosen carefully when the vortex is displaced off the pole.

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**Fig. 11.** Lagrangian (solid) and residual (dashed) mean vertical velocities vs latitude at 1 mb. The Lagrangian means are based on integrations of 9 days beginning 13 November.
residual circulations in a simple model with small-amplitude waves. Their results indicate that the residual circulation is similar in structure to the Lagrangian, but stronger by about 30%. However, they only considered the case in which the wave amplitudes and Lagrangian mean velocities were small. When the wave amplitudes are large, no simple relationship between the two forms of mean circulation can be derived.

A numerical experiment which compares the three-dimensional advection of a passive tracer with the two-dimensional advection by the residual circulation in the presence of large-amplitude waves was performed by Schneider and Geller (1985). They integrated the model for 60 days under wintertime conditions. The transport calculation was initialized with a zonal mean tracer distribution in a wind field which had zonal asymmetries. This unrealistic initial field results in very large advection during the first few days of integration, and makes it difficult to interpret the results. However, a comparison beginning 10 days into the integration, at which time the tracer distribution had developed asymmetries similar to those of the flow, still shows that the residual circulation gave downward transports larger by about 40% than the three-dimensional flow. The results of Schneider and Geller, like those of Rood and Schoeberl, support our findings that the residual circulation overestimates the downward motion in the high-latitude winter stratosphere.

Our results were also used to compare the mean meridional velocities as a function of time and of latitude. Figure 12 shows a time series of the meridional velocities derived from a high latitude tube near 2 mb. The GLM meridional velocity is much more noisy, and the agreement with the residual velocity is poor, especially during very active periods characterized by breaking planetary waves. Although different quantitative results were obtained when the meridional GLM velocity definition of Andrews and McIntyre (1978b) was used, the agreement between the residual and GLM velocities remained poor.

While the meridional and vertical residual velocities are constrained to satisfy a zonal mean continuity equation, no such relationship exists in general for the mean Lagrangian velocities. The extreme values of $\bar{v}^L$ are not necessarily inconsistent with the more regular vertical motion fields since the Lagrangian mean flow can be divergent even for an incompressible fluid.

Dispersion due to wave motion is expected to be largest when there are large breaking planetary waves, which is when the agreement between $\bar{v}^L$ and $\bar{v}^*$ is poorest. As a quantitative measure of dispersion, we use the horizontal distance after ten days between air parcels which were adjacent at initialization, divided by the initial distance. If the tube is slightly distorted but remains coherent, the length ratio was found to be in the range of 1–2 and in a few cases less than 1. Time series of this measure of dispersion are shown in Fig. 13 for two cases: a high-latitude tube in the upper stratosphere and an outer tube (middle to low latitude) in the lower stratosphere. For high latitudes, the length measure is about 1–1.5 during almost all periods except between mid-January and late February, where the tube lengths increase by a factor of 5–8 during the 10-day integrations.

The dispersion in low latitudes is larger in the time mean than in high latitudes, with dispersion values ranging from 2 to 5. This is consistent with the theory of McIntyre and Palmer (1983), which assumes that wave breaking (and therefore large dispersion) is always occurring around the edge of the vortex. Only during sudden warmings does the wave-breaking region penetrate into high latitudes.

Comparing Fig. 12 with Fig. 13a confirms that the dispersion is relatively small during periods when the $\bar{v}^L$ and $\bar{v}^*$ agreement is good and is large when the $\bar{v}^L$ and $\bar{v}^*$ agreement is poor. This suggests that a large component of the horizontal transport in the stratosphere is in motion which is not included in the transformed Eulerian meridional velocity.

Two-dimensional stratospheric transport models using the Eulerian MMC have had to use a strong eddy diffusivity in the manner suggested by Reed and German (1965) in order to adequately simulate the tracer distribution. The eddy diffusivity works largely to cancel transport by the MMC, and as such represents transport rather than actual dispersion. Particular values which work during one period or in one region of the atmosphere cannot, in general, be used in another situation. More recent eddy diffusion parameterizations derived by Clark and Rogers (1978), Plumb (1979), and Matsuno (1980) are directly linked to the planetary wave structure. Holton (1981) derived an eddy diffu-
sivity specifically for use with the transformed Eulerian circulation. A similar term is used by Garcia and Solomon (1983) in their two-dimensional chemical dynamical model. With these eddy diffusion parameterizations, the eddy advection of a nonconservative tracer contains an additional stirring term that depends on the mean distribution of the tracer in question.

Plumb and Mahlman (1987) separated the (three-dimensional) monthly mean transport in a general circulation model (GCM) into two components: the mean two-dimensional advection and the dispersion around the mean. The latter was parameterized as eddy diffusivities which varied in space and time. Largest diffusion values were found in the tropical upper troposphere and in the subtropical stratosphere. The maximum diffusion rate occurred in the region of the stratospheric surf zone during January. Since the GCM did not simulate a sudden warming, the dispersion in high latitudes in winter is likely to have been underestimated. Despite this, the magnitude of the diffusion coefficients computed by Plumb and Mahlman are substantially larger than those in several recent two-dimensional models. These eddy diffusion parameters were tested in a two-dimensional model and gave fairly good representation of the full three-dimensional transports.

Our results also suggest that there is a substantial amount of dispersive motion, in addition to advective motion, associated with planetary waves in the stratosphere. The dispersion is not random, but instead is closely tied to the relative strength of eddy and mean zonal winds (or the degree of nonlinearity). When wave amplitudes are large, both the advective and the dispersive action of the waves is increased. To create a two-dimensional transport model that adequately simulates the stratospheric circulation, inhomogeneous time-varying eddy diffusivities such as those derived by Plumb and Mahlman should be adapted to the particular circulation patterns of the model.

5. Conclusions

The winter of 1978/79 has been one of the best-observed and most studied periods for the stratospheric circulation. The minor and major sudden warmings of this winter have been the subject of several studies (e.g., Palmer, 1981; Butchart et al.; 1982; Palmer and Hsu, 1983; Gille and Lyjak, 1984; Smith, 1985). Also, the events form the observational basis for McIntyre and Palmer's (1983, 1984) theory of planetary wave breaking. The Lagrangian mean analysis presented in this paper provides a different way of looking at these events.

Maps of horizontal air parcel distributions show features that support the wave breaking hypothesis of McIntyre and Palmer. In our study, parcels are grouped into material tubes that remain in the same position unless the mean state is changing with time. During early winter, when the waves are weak and the zonal wind is strong, the material tubes in low latitudes deform more than those around the vortex. Over the 10-day integrations, the material surfaces do not cross each other; air initially within the vortex does not mix with that initially outside. McIntyre and Palmer call the region of mixing (or irreversible deformation of material surfaces) near the low-latitude critical line the surf zone. Because geopotential contours were used to initialize the material tubes, we were not able to examine the behavior of tubes at the zero wind line where nonlinearity is expected to be most important.

During the January minor sudden warming the surf zone moved into high latitudes and the associated
mixing caused substantial erosion of the vortex. The Lagrangian parcel distributions show that, while the material tubes closest to the vortex remain intact, those further out were broken and fluid parcels were irreversibly mixed. Air parcels from the outer part of the vortex penetrated into the anticyclone.

During the major warming in February, there was a breakdown of the entire vortex and strong horizontal mixing of fluid parcels from all latitudes. Since the distance between adjacent air parcels on a ring is roughly 300–400 km, nothing can be said about mixing on scales smaller than this. On this scale, however, there appears to have been a fair amount of stirring up of parcels from different rings. In the stratosphere none of the material tubes remained coherent over an integration period of one week. At the stratopause, however, only those material tubes in the inner vortex were pulled apart. The air parcels in midlatitudes, where the zonal wind continued to be strong through the warming, remained as coherent material tubes.

While giving general support for the wave-breaking interpretation of McIntyre and Palmer (1983, 1984), the air parcel distributions also show features not evident in potential vorticity maps. Because there are up to 128 parcels per material tube, features can be resolved from these calculations that would be difficult to see directly from satellite data. It is evident, for example, that a coherent tube of air parcels cycles into the anticyclone during the minor warming. Even with this resolution, some of the material tubes are so strongly deformed during the major sudden warming that it is not possible to find the connection between them after an integration of one week. Features also evident in the parcel distributions are kinks that appear in single tubes without a rearrangement of the order of the tubes from their original latitudinal order. Such small distortions are indicative of horizontal dispersion, but might not be evident on maps of potential vorticity because the scale is smaller than can be accurately observed at the present time.

The Lagrangian mean velocities computed from parcel trajectories are compared with the transformed Eulerian circulation (Andrews and McIntyre, 1976). In general, the vertical velocities compare well in structure, but the residual circulation overestimates the magnitude by up to 50%. This result is consistent with studies comparing the transport by the residual circulation to the actual three-dimensional (Schneider and Geller, 1985) or the Lagrangian (Rood and Schoeberl, 1983) transports.

The horizontal velocities compare very poorly in both structure and magnitude with the residual meridional velocities. The Lagrangian mean circulation, unlike the residual circulation, is not constrained to satisfy a two-dimensional continuity equation. It appears that a large component of the horizontal motion is dispersive rather than in a coherent mean flow. Therefore, the use of the residual circulation to represent the mean Lagrangian displacements may have some problems in simulating the horizontal transports. An additional, primarily horizontal, mixing term will be needed. The analysis in this paper indicates that there is a steady dispersion in low latitudes which persists through the winter. This mixing process may be an important component of the transport on seasonal time scales. A similar conclusion was reached by Plumb and Mahlman (1987) based on transport in a general circulation model.

As a further complication, both the Lagrangian parcel distributions and the quantitative dispersion measure indicate that the horizontal mixing varies strongly with space and time. Even with a strong horizontal mixing, it will be difficult to simulate the rearrangement of material tubes, and, for example, to get air from low latitudes to the pole as occurs during sudden warmings.

The good agreement between the vertical motion from the GLM and transformed Eulerian coordinate frames is probably a result of the very strong vertical stratification in the stratosphere. It takes a nonconservative forcing, such as diabatic heating, to move fluid parcels across potential temperature surfaces. On the other hand, motion along potential temperature surfaces, which are closer to pressure surfaces, occurs much more readily.

Acknowledgments. We thank John Gille for providing us with a special analysis of the LIMS temperature dataset. This work was supported in part by the National Aeronautics and Space Administration through grants W15439 and W16215 and by the National Science Foundation through Grant ATM-8513637.

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