Vacillations Induced by Interference of Stationary and Traveling Planetary Waves

MURRY L. SALBY*

Department of Astrophysical, Planetary, and Atmospheric Sciences, University of Colorado, Boulder, CO 80309

ROLANDO R. GARCIA*

National Center for Atmospheric Research,** Boulder, CO 80307

(Manuscript received 6 October 1986, in final form 8 March 1987)

ABSTRACT

We explore the interference pattern produced when a traveling planetary wave propagates over a stationary forced wave. The interference signature is examined in a variety of diagnostics, ranging from instantaneous local wave amplitudes to cross sections of Eliassen–Palm flux and synoptic maps of Ertel potential vorticity. Results capture the salient characteristics of quasi-periodic disturbances in the stratosphere reported by Madden and others.

The interference process results in a modulation of all the transport properties of the stationary wave, even if the traveling component is purely barotropic as is typical of transient planetary waves in the troposphere and lower stratosphere. Locally, the Eliassen–Palm flux involves a transient vector which orbits about the time-mean component, causing the instantaneous flux of wave activity to vary in both magnitude and direction. For stationary and traveling waves of comparable amplitude, the EP flux vector F can readily be driven through zero, completely altering its strength and direction. This temporal fluctuation actually arises out of the spatial modulation of the wave field and the migration of the resulting pattern with time. In this manner, the steady uniform stream of wave activity associated with the stationary wave is organized into a series of capsules or wave packets which propagate upward and equatorward. Consequently, the signature at a particular location emerges as a series of bursts in wave activity.

Because of rising values of F at the leading edge of each of these wave packets and opposite behavior at the trailing edge, the mean flow experiences an alternating succession of eddy forcing. This gives rise to a fluctuating response in the zonal-mean which, under typical amplitudes, may be considerable. Greatest influence is exerted in the polar stratosphere due to the veering of F to and away from the pole. Convergence of meridians at high latitudes leads to repeated focusing and spreading of wave activity and thereby a magnified response in the mean flow. For amplitudes representative of January 1979, a mean wind reversal occurs in the polar stratosphere and mesosphere, attended by substantial warming over the polar cap and out-of-phase behavior in the tropics and mesosphere.

The synoptic signature of wavenumber 1 interference consists of two basic elements: (i) displacement and wobbling of the vortex about the pole, as a ridge builds in from below; (ii) distortion of the vortex into a comma-like shape, its axis spiraling anticlockingly and equatorward about the ridge. Both features are widely documented in observations. They are introduced here by the eddy field as a capsule of wave activity propagates through a given level. Potential vorticity on isentropic surfaces exhibits similar but exaggerated behavior. The results suggest an alternate, perhaps complementary, interpretation to planetary wave breaking of the evolution of potential vorticity during January 1979.

1. Introduction

Madden (1975) has suggested that the interference pattern established when a traveling planetary wave propagates over a stationary forced wave may be responsible for the sizable fluctuation of eddy heat and momentum fluxes evident at stratospheric levels. A significant fraction of this fluctuating eddy variance is quasi-periodic, associated with the 16-day wave (van Loon et al., 1975). The interference mechanism has also been invoked in connection with the upward migration of wave amplitude vacillations (Madden, 1977). These features have led to speculation that this process might be important in large-scale stratospheric disturbances.

The idea that variations of wave activity and eddy transport in the stratosphere are associated with traveling waves is not altogether new, dating back to suggestions in early observational reports (e.g., Muench, 1965; Finger et al., 1966; Hirota and Sato, 1969). However, the ability of barotropic waves which dominate the transient planetary wave field in the troposphere (Eliassen and Machenhauer, 1965, 1969;
Ahlquist, 1982; Lindzen et al., 1984), to lead to such disturbances is not widely recognized. This is due chiefly to the fact that these \textit{column responses} of the atmosphere have little or no phase tilt and therefore contribute nothing in themselves to the vertical Eliassen–Palm flux or flux of wave activity.

Nevertheless, support for this mechanism has emerged in a simple baroclinic calculation (Garcia and Geisler, 1981), which demonstrated a correlation between fluctuating eddy heat fluxes, arising in just this manner, and the zonal-mean temperature gradient. Exchanges between instantaneous eddy and zonal-mean fields, induced through interference, has also been advanced as an explanation for the tropospheric zonal index cycle (Lindzen et al., 1982). Madden (1983) has demonstrated a parallel between observed oscillations in eddy heat and momentum fluxes and those derived from interference of the observed stationary wave with a simple normal mode structure. In both cases the phase of the oscillation migrates upwards out of the troposphere into the lower stratosphere. However, the crucial eddy quantity, insofar as the mean flow is concerned is neither the heat nor the momentum flux, but rather the Eliassen–Palm flux (Andrews and McIntyre, 1976; Boyd, 1976), whose divergence represents the essential wave driving of the mean flow. It is on this central diagnostic that we shall base the following analysis.

Of all the traveling waves possibly relevant, atmospheric normal modes have received much of the attention. Part of the reason is that they are preferred in the response to random or broadband forcing (Salby, 1984b), and capture a significant fraction of the unsteady planetary-scale variance (e.g., Eliassen and Machenhauer, 1965, 1969; Lindzen et al., 1984). In the troposphere they are of global extent, having negligible phase tilt in both latitude and height (Salby, 1981b). However, in the stratosphere their structures are strongly modified by the influence of wind shear and damping, slower modes penetrating during solstice only into the winter hemisphere where the flow is westerly. A westward phase tilt at upper levels is also introduced. With the advent of global satellite monitoring, many of these features have been verified (Hirooka and Hirooka, 1984; Hirooka and Hirota, 1985). See Madden (1979) and Salby (1984a) for reviews.

Of those normal modes documented, the 16-day wave or second symmetric mode of zonal wavenumber 1 has most captured the imagination of stratospheric investigators, chiefly for its large amplitude and recurrent appearance. It is a significant climatological feature of Northern Hemisphere winter statistics (Madden, 1978, 1983; Rinne and Sarkenan, 1985), its variance being concentrated between 12 and 21 days. Because it attains amplitudes comparable to the stationary wave, it can significantly alter the time-mean disturbance field, making it important in wave-mean flow considerations. Indeed Hirooka and Hirota (1985) have noted the appearance of this and other normal modes prior to the onset of major stratospheric disturbances. In transient episodes such as that of January 1979, the 16-day wave appears capable of dwarfing the stationary forced wave over the entire troposphere and lower stratosphere (Madden and Labitzke, 1981).

The January 1979 transient wave episode has been widely documented, because the traveling wave was anomalously strong, so much so that it could be seen to retrogress even in the \textit{total} wave field over much of the lower atmosphere. This is evident in the polar diagram for wavenumber 1 amplitude (Fig. 1). At each level below 10 mb, a transient component of the wave amplitude orbits systematically about the time-mean value (squares), corresponding to regular retrogression and a concentration of variance about 18 days. Up through the lower stratosphere the transient component dominates the total wave, driving the instantaneous amplitude about the origin, so that the total wave's phase systematically retrogresses.

Above 10 mb, the amplitude of the stationary wave exceeds that of the traveling wave, the crossover altitude being marked by amplitudes of order 1000 gpm. As a result, the phase at the current wave no longer exhibits pure retrogression over the entire episode, even though the transient component moves systematically westward. Rather the propagation of the combined wave is first prograde and then retrograde. Nevertheless, the modulation is considerable, driving the instantaneous wave amplitude at 10 mb between zero and twice its time-mean value.

The rather substantial instantaneous wave field which results corresponds to one of the most disturbed upper level configurations on record, although under WMO classification it is characterized as only a minor stratospheric warming. Synoptically, the picture during constructive interference (20–28 January) is one of an intensifying Aleutian ridge which displaces the (nominally) circumpolar vortex completely off the pole (Fig. 2), resulting in a strong cross-polar circulation. In addition to the back and forth wobbling of the vortex during destructive and constructive interference, there is a rotation of the ridge–trough pair about the pole. Below 10 mb the motion is purely retrograde, consistent with the phase behavior in Fig. 1, but at 10 mb and above propagation alternates direction during the cycle. During destructive interference (16 January), when wavenumber 1 collapses, the vortex is restored to polar symmetry, and a wavenumber 2 anomaly appears. In contrast, when transient and stationary waves interfere constructively, the vortex experiences its greatest displacement, and both ridge and trough be-

\footnote{Over brief intervals such as the episode during January 1979, it is not possible to focus on a single mode, because the short-term response will involve many components which require time to disperse. Individual modes may be isolated only through spatial and temporal filtering.}
come distorted, appearing to spiral about one another. Such behavior is not unusual. Much of the preceding winter is in fact marked by similar, if less pronounced behavior, with a more or less continual displacement and wobbling of the vortex about the pole (e.g., Leovy et al., 1985). Accompanying this movement is a noticeable distortion of the vortex, at times attaining the shape of a comma which tends to wrap about the Aleutian ridge (Fig. 2).

January 1979 is also a celebrated period, because it is widely regarded as the hallmark of planetary wave breaking in the stratosphere. The same scenario, when viewed in terms of Ertel’s potential vorticity, leads to a signature suggestive of nonlinear advection at upper levels, with material being drawn out of the body of the vortex (McIntyre and Palmer, 1983, 1984). This is hinted at by the comma-like distortion of the vortex, seen previously in the geopotential behavior. Adverted material appears subsequently to spiral about the Aleutian ridge and ultimately mix with subtropical air at low latitudes. The aforementioned process has been suggested as playing a principal role in the erosion of the polar night vortex. Its signature, in either geopotential or Ertel potential vorticity, is now well recognized as being intrinsic to disturbed conditions in the stratosphere, having been documented in a number of winters.

While not diametrically opposed, the interference and wave breaking descriptions of January 1979 are conceptually different. Interference is an intrinsically linear process, with nonlinearity restricted for the most part to quasi-linear interaction between the wave field and the mean flow. Wave breaking, on the other hand, presumably involves strong nonlinearity in the form of wave–wave interactions which result in a cascade of enstrophy to smaller scales (although it should be noted that interference, through local amplification, may ultimately lead to wave breaking). It is the appearance of smaller scales accompanying the complex distortion of the vortex that has led more than one investigator

---

**FIG. 1.** Complex amplitude polar of wavenumber 1 during January 1979 at 60°N for selected levels. Instantaneous amplitude given by radial distance to the orbit, phase by the corresponding angle, as labeled. (Adapted from Madden and Labitzke, 1981; Smith, 1985)
to question the justification of wave-mean flow decomposition. The latter is always formally valid, couched in the same theoretical foundation as any Fourier decomposition. Its usefulness in describing complex distortions of the vortex, however, is another issue.

We examine here in detail the mechanics of the interference process within the framework of central dynamical quantities such as Eliassen-Palm flux and potential vorticity. The signature of the process is explored in the wave field, in the mean flow, and in combined synoptic fields. In particular, the response of the mean flow to vacillations in eddy fluxes created by the interference pattern is derived. We examine the behavior starting from rudimentary considerations such as local wave amplitude, progressing to quantities of ever higher dimensionality and order. We conclude by deriving the signature of the interference process in synoptic fields of geopotential and Ertel potential vorticity. Our results suggest an alternate, though perhaps complementary, interpretation (viz., to wave breaking) of the potential vorticity behavior observed during January 1979.

In section 2 we formulate the necessary equations and derive the interference behavior in general form. To elucidate the process, the wave field is then restricted to a diatonic spectrum consisting of two components: a single stationary wave and a single monochromatic traveling wave. In section 3 a simple barotropic normal mode propagating over a simple stationary plane wave is considered, and closed form solutions are obtained. In section 4 the wave fields are restricted spatially, providing more realistic structures without sacrificing the advantages of an analytical solution. Finally, in section 5 both stationary and traveling wave fields are calculated numerically with the linearized Primitive Equations in a realistic basic state. The mean flow reaction to the fluctuating eddy forcing which results from interference is derived in section 6. In section 7 synoptic geopotential behavior corresponding to the combined wave and mean flow fields is presented, and in section 8 the synoptic signature in potential vorticity on isentropic surfaces is examined. Conclusions are drawn in section 9.

2. Formulation

We consider an arbitrary unsteady streamfield represented as

\[
\Psi(\lambda, \phi, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\sigma e^{-i\omega t} \sum_{m=-\infty}^{\infty} e^{im\lambda} \Psi_m(\phi, z)
\]  

(1)

where \( \lambda, \phi, z \) and \( t \) denote longitude, latitude, height and time, respectively, and \( \Psi_m \) is the complex amplitude for wavenumber \( m \) and angular frequency \( \omega \). Moreover, this representation will be abbreviated by introducing the space-time transform and its inverse:

\[
S[\Psi] = \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} d\sigma e^{-i\omega t} \sum_{m=-\infty}^{\infty} e^{im\lambda} \Psi_m[\phi, z]
\]

(2a)

\[
S^{-1}[\Psi] = \int_{-\infty}^{\infty} d\sigma e^{-i\omega t} \sum_{m=-\infty}^{\infty} e^{im\lambda} [\Psi_m[\phi, z]].
\]

(2b)

We examine now the flux of wave activity or Eliassen-Palm flux, \( \mathbf{F} \), resulting from (1). Because the instantaneous streamfield is evolving, so too will be the instantaneous EP flux field \( \mathbf{F}(\phi, z, t) \).

\[ a. \text{Quasi-geostrophic disturbances} \]

For wave fields satisfying the conditions of quasi-geostrophy, the latitudinal and vertical components of \( \mathbf{F} \) are

\[
F_\theta = -\bar{\rho}(a \cos \phi) \frac{\partial \mathbf{V}}{\partial \theta}
\]

(3a)

\[
F_z = \bar{\rho}(a \cos \phi) \frac{\partial \mathbf{V}}{\partial z}
\]

(3b)

where

\[
\bar{\rho} = \rho_0 e^{-z/H},
\]

(4)

\( z \) is log pressure height, \( \rho_0 \) and \( H \) are constant reference density and scale height, respectively; \( u, v \) and \( \theta \) represent zonal and meridional velocities and potential temperature, respectively, and the overbar denotes zonal average. Additional symbols are in standard notation. With the relations

\[
u' = -\frac{1}{a} \frac{\partial \Psi'}{\partial \phi}
\]

(5a)

\[
u' = \frac{1}{a \cos \phi} \frac{\partial \Psi'}{\partial \lambda}
\]

(5b)

\[
\frac{\partial \Psi'}{\partial \theta} = \frac{g}{f} \frac{\partial \theta}{\partial \phi}
\]

(6)

\[
N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z}
\]

(7)

Equations (3a, b) become

\[
F_\theta = \bar{\rho} \frac{\partial \Psi'}{\partial \phi} \frac{\partial \Psi'}{\partial \lambda}
\]

(8a)

\[
F_z = \bar{\rho} \left( \frac{f^2}{N^2} \right) \frac{\partial \Psi'}{\partial \phi} \frac{\partial \Psi'}{\partial \lambda}
\]

(8b)

Substitution of the spectral representation (1) into (8a, b) results in

\[
F_\theta(\phi, z, t) = \frac{\bar{\rho}}{a (2\pi)^3} \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\phi' \int_{-\infty}^{\infty} d\lambda' \sum_{m=0}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ \frac{\partial \Psi_m'}{\partial \phi} e^{-i\sigma t} \Psi_m^* e^{ia \phi'} \right\} e^{i(m-m')\lambda}
\]

(9a)
The instantaneous EP flux vector may be recognized as a convolution of frequency components of the space–time spectrum; $F_{\phi}^{m\sigma'}$ represents the contribution from wavenumber $m$ at frequencies $\sigma$ and $\sigma'$ and oscillates as $e^{-i(\theta-\theta')y}$, i.e., beating at the difference in frequencies between the components. In particular, steady or time-mean contributions are derived from spectral components of the same frequency, while fluctuating contributions arise from cross terms involving different frequencies.

b. Disturbances to the Primitive Equations

The same procedure can be carried out for disturbances satisfying the linearized Primitive Equations, in which case

$$ F_{\phi} = \bar{\rho} a \cos \phi \left[ \frac{\bar{v} \bar{v}'}{\bar{\theta} z} - \bar{v} W' \right] $$

(15a)

$$ F_{z} = \bar{\rho} a \cos \phi \left[ \frac{f - (\bar{u} \cos \phi)}{a \cos \phi} \frac{\bar{v} \bar{v}'}{\bar{\theta} z} - \bar{w} W' \right] $$

(15b)

Then letting

$$ x(\lambda, \phi, z, t) = \begin{bmatrix} u' \\ v' \\ w' \\ \theta' \end{bmatrix} $$

(16a)

$$ X^{m}_{\phi}(\phi, z) = S[x'] = \begin{bmatrix} U^{m}_{\phi} \\ V^{m}_{\phi} \\ W^{m}_{\phi} \\ \Theta^{m}_{\phi} \end{bmatrix} $$

(16b)

we again obtain for the fluctuating EP flux field

$$ F(\phi, z, t) = \sum_{m=0}^{\infty} \frac{1}{(2\pi)^{2}} \int_{-\infty}^{\infty} \sigma' \int_{-\infty}^{\infty} d\sigma' F_{\phi}^{m\sigma'}(\phi, z, t) $$

(17a)

but now with

$$ F_{\phi}^{m\sigma'}(\phi, z, t) = 2\bar{\rho} a \cos \phi \left[ \text{Re} \left\{ \frac{\bar{u} \bar{v}^{m}_{\phi} \Theta^{m}_{\phi}}{\bar{\theta} z} - \bar{v}^{m}_{\phi} U^{m}_{\phi} \right\} e^{-i(\theta-\theta')y} \right] $$

(17b)

c. Reaction of the basic stream

The EP flux represents the essential wave driving of the mean flow. Specifically, it’s divergence constitutes the sole wave forcing of the (quasi-geostrophic) transformed Eulerian equations governing the zonal-mean state:
We will focus on the transient eddy driving embodied in the first term, specifically that introduced by interference as represented in the cross frequency terms of (14) and (17). To this end, we presume the zonal-mean friction and heating are in balance with the time-mean zonal flow and with the time-mean EP flux. Letting

$$\bar{\psi}^* = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\sigma e^{-i\sigma t} \bar{\psi}_g$$

and frequency transforming (24), we obtain after some manipulation

$$L[\bar{\psi}_g] = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} d\sigma' \int_{-\infty}^{\infty} f N^2 a \cos \phi$$

$$\times \frac{1}{\rho} \frac{\partial}{\partial z} \left[ \frac{1}{N^2 a} \frac{\partial \bar{\psi}^*}{\partial \phi} \right] + \frac{f}{N^2 a} \frac{\partial \bar{F}_g}{\partial z} + \left( \frac{S}{N^2 a} \right) \frac{\partial \bar{Q}_g}{\partial \phi}$$

which holds for each frequency component $\sigma$.

In general, the convolution (11a) involves a continuum of frequencies. However, it is of interest to examine situations where the variance is concentrated about particular frequencies, for instance, a red spectrum about zero frequency associated with the quasistationary wave and one or more discrete peaks associated with normal modes. It should be understood that the ideas are about to develop carry through regardless of the spectral make-up of the traveling wave.

### Diatomic spectrum

We consider the simple two component space-time spectrum

$$\Psi_m = \frac{1}{2} \Psi_{m,1} \delta_{nm} \delta(\sigma' - \sigma_1) + \frac{1}{2} \Psi_{m,2} \delta_{nm} \delta(\sigma' - \sigma_2)$$

representing the combination of a single stationary wave ($\sigma_1 = 0$) and a monochromatic westward traveling wave ($\sigma_2 < 0$), both of wavenumber $m$. In addition to being of considerable advantage in elucidating the process, this idealization is also of practical merit because of the concentration of variance associated with the stationary and 16-day waves. The frequency convolution, either (14) or (17), reduces to

$$F = F_{m,1}^o + F_{m,2}^o + F_{m,1}^f + F_{m,2}^f.$$
the steady component derived from matching frequencies and the fluctuating component from cross terms. The fluctuating vector field $F'$ may be thought of in terms of amplitude and phase, both of which vary spatially according to the structures of the two wave components. We examine now in detail the behavior for quasi-geostrophic disturbances.

$$F' (\phi, z, t) = \begin{bmatrix} \frac{\bar{m}}{2} \text{Im} \left[ \frac{\partial \Psi_m^e}{\partial \phi} \Psi_m^e \sin(\mu_m^e - \gamma_m^e) + \frac{\partial \Psi_m^e}{\partial \phi} \Psi_m^e \sin(\nu_m^e - \gamma_m^e) \right] \\ \frac{\bar{m}}{2} \left( \frac{f^2}{N^2} \right) \text{Im} \left[ \frac{\partial \Psi_m^e}{\partial z} \Psi_m^e \sin(\mu_m^e - \gamma_m^e) + \frac{\partial \Psi_m^e}{\partial z} \Psi_m^e \sin(\nu_m^e - \gamma_m^e) \right] \end{bmatrix}$$

(30a)

(30b)

By representing the complex wave fields as

$$\Psi_m^e = |\Psi_m^e| e^{i\gamma_m^e}$$

(31a)

$$\frac{\partial \Psi_m^e}{\partial \phi} = \frac{|\Psi_m^e|}{\partial \phi} e^{i\mu_m^e}$$

(31b)

For the quasi-geostrophic case, (29a-c) reduces to

$$F_0(\phi, z) = \begin{bmatrix} \frac{\bar{m}}{2} \text{Im} \left[ \frac{\partial \Psi_m^e}{\partial \phi} \Psi_m^e \sin(\mu_m^e - \gamma_m^e) \right] \\ \frac{\bar{m}}{2} \left( \frac{f^2}{N^2} \right) \text{Im} \left[ \frac{\partial \Psi_m^e}{\partial z} \Psi_m^e \sin(\mu_m^e - \gamma_m^e) \right] \end{bmatrix}$$

(32a)

where the phases $\gamma_m^e$, $\mu_m^e$ and $\nu_m^e$ are functions of space, and after some manipulation, the steady and fluctuating components of the vector field $F$ can be reduced to

$$F(\phi, z, t) = \begin{bmatrix} A_x \cos(\Delta \phi - \delta_x) \\ A_z \cos(\Delta \phi - \delta_z) \end{bmatrix}$$

(32b)

$$A_x = \frac{\bar{m}}{2a} \left( a_x^2 + b_x^2 \right)^{1/2} \cdot \text{sign}(a_x)$$

(33a)

$$\delta_x = \tan^{-1} \left( \frac{b_x}{a_x} \right)$$

(33b)

$$a_x = \frac{|\Psi_m^e|}{\partial \phi} \sin(\mu_m^e - \gamma_m^e) + \frac{|\Psi_m^e|}{\partial \phi} \sin(\nu_m^e - \gamma_m^e)$$

(33c)

$$b_x = \frac{|\Psi_m^e|}{\partial \phi} \cos(\mu_m^e - \gamma_m^e)$$

(33d)

$$A_z = \frac{\bar{m}}{2} \left( \frac{f^2}{N^2} \right) \left( a_x^2 + b_x^2 \right)^{1/2} \cdot \text{sign}(a_z)$$

(34a)

$$\delta_z = \tan^{-1} \left( \frac{b_z}{a_z} \right)$$

(34b)

The steady component $F_0$ is just the sum of the individual time-mean contributions due to components 1 and 2. If the traveling wave is barotropic, without phase variation, its time-mean contribution vanishes, and $F_0$ reduces to the contribution derived from the stationary wave alone. The fluctuating component $F'$ is new in that it exists only when both waves are simultaneously present. It exists even when the time-mean contribution to $F$ of the traveling wave vanishes. The latitudinal and vertical components of $F'$ oscillate in time and, it will be seen shortly, in space as well due to the phase terms $\delta_x$ and $\delta_z$. At a particular latitude and height, one may imagine a transient EP flux vector $F'(t)$ added to the local time-mean vector $F_0$. Dependent upon the relative phase of the oscillation between the components...
of $\mathbf{F}'$, i.e., $\delta_\phi$ and $\delta_z$, the transient vector's evolution, will describe an orbit about the tip of the time-mean vector which is either circularly polarized, linearly polarized, or in general somewhere between. The tip of the instantaneous EP flux vector moves along this orbit, giving $\mathbf{F}$ the appearance of vacillating in both magnitude and direction.

3. Stationary plane wave and barotropic Lamb mode

Consider now the case of a simple stationary plane wave and a westward propagating Lamb normal mode

$$\Psi_m^{(1)} = A_m^{(1)} e^{i\theta (\ell \phi + k \frac{z}{\mathcal{H}})}$$

(35a)

$$\Psi_m^{(2)} = A_m^{(2)} e^{i\theta (H \ell \phi + f(t))}$$

(35b)

each presumed to behave quasi-geostrophically in the region of interest. The traveling wave has the barotropic Lamb vertical structure, and its horizontal behavior is that of the Hough mode $H_m^{(2)}(\phi)$, of meridional index $n$ and frequency $\omega_2$. Because of this normal mode structure (specifically the absence of phase tilt), the traveling wave in isolation transports nothing in either latitude or height. We consider the second symmetric mode of wavenumber 1, $H_m^{(2)}(\phi)$, associated with the 16-day wave.

The steady and fluctuating components of the vector EP flux field reduce to

$$\mathbf{F}_0(\phi, z) = \begin{bmatrix} \frac{\rho m}{2a} k (A_m^{(1)})^2 \\ \frac{\rho m}{2} \left( \frac{f^2}{N^2} \right) \left( k \mathcal{H} \right) (A_m^{(2)})^2 \end{bmatrix}$$

(36)

$$\mathbf{F}'(\phi, z, t) = \begin{bmatrix} A_\phi \cos (\sigma_2 t - \delta_\phi) \\ A_z \cos (\sigma_2 t - \delta_z) \end{bmatrix}$$

(37)

with

$$A_\phi = \frac{\rho m}{2a} A_m^{(1)} A_m^{(2)} e^{i(\ell \phi + k \frac{z}{\mathcal{H}})} (a_\phi^2 + b_\phi^2)^{1/2} \cdot \text{sign}(a_\phi)$$

(38a)

$$\delta_\phi = -\tan^{-1} \left[ \frac{b_\phi}{a_\phi} \right]$$

(38b)

$$a_\phi = l H_m^{(2)}(\phi) \cos \left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right) - \frac{\partial H_m^{(2)}}{\partial \phi} \sin \left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right)$$

(38c)

$$b_\phi = l H_m^{(2)}(\phi) \sin \left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right) + \frac{\partial H_m^{(2)}}{\partial \phi} \cos \left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right)$$

(38d)

and

$$A_z = \frac{\rho m}{2H} \left( \frac{f^2}{N^2} \right) A_m^{(1)} A_m^{(2)} e^{i(\ell \phi + k \frac{z}{\mathcal{H}})} H_m^{(2)}(\phi) \times \left( a_z^2 + b_z^2 \right)^{1/2} \cdot \text{sign}(a_z)$$

(39a)

$$\delta_z = \tan^{-1} \left[ \frac{b_z}{a_z} \right]$$

(39b)

$$a_z = \left( \frac{1}{2} - \kappa \right) \sin \left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right) + k \cos \left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right)$$

(39c)

$$b_z = \left( \frac{1}{2} - \kappa \right) \cos \left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right) - k \sin \left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right).$$

(39d)

It is illuminating to consider the behavior in the vicinity of the peak value of the Hough mode, where $\partial H_m^{(2)} / \partial \phi$ is small. Then

$$A_\phi = \frac{\rho m}{2a} A_m^{(1)} A_m^{(2)} e^{i(\ell \phi + k \frac{z}{\mathcal{H}})} H_m^{(2)}(\phi)$$

(40a)

$$\delta_\phi = -\left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right)$$

(40b)

and in the limit of large vertical wavenumber

$$A_z \sim \frac{\rho m}{H} \left( \frac{f^2}{N^2} \right) A_m^{(1)} A_m^{(2)} e^{i(\ell \phi + k \frac{z}{\mathcal{H}})} H_m^{(2)}(\phi),$$

(41a)

$$\delta_z \sim -\left( \frac{\ell \phi + k \frac{z}{\mathcal{H}}}{\mathcal{H}} \right), \quad k \to \infty.$$  

(41b)

So for $k \to \infty$,

$$\mathbf{F}'(\phi, z, t) \sim \frac{\rho m}{2} A_m^{(1)} A_m^{(2)} e^{i(\ell \phi + k \frac{z}{\mathcal{H}})} H_m^{(2)}(\phi) \times \left[ \frac{l}{a} \cos \left( \sigma_2 t + \phi + k \frac{z}{\mathcal{H}} \right) \right]$$

(42)

Thus each component of $\mathbf{F}'$ oscillates in time at frequency $\sigma_2$, and also oscillates spatially at the scales of the stationary wave. In the vertical, the amplitude of this oscillation decays exponentially as $e^{i(\ell \phi + k \frac{z}{\mathcal{H}})}$. Fluctuations in the total EP flux field will be greatest at the lowest levels and monotonically wane to zero with increasing altitude. Such behavior is characteristic of this idealized case because of the smaller vertical growth rate of the Lamb mode relative to that of the vertically propagating stationary wave.

The signature just described results from the constructive and destructive interference between the two wave fields (Fig. 3). This interference pattern modulates all of the transport properties of the stationary wave, even though the traveling wave alone transports nothing in either latitude or height. Because of the differing phase structures, specifically the westward phase tilt of the stationary wave and the vertical phase alignment of the traveling wave (Fig. 3a), this modulation does not proceed synchronously at all latitudes and heights. Rather, there exists a phase lag between points which
is manifested in the instantaneous wave field by a modulation in space. As time elapses, this interference pattern propagates vertically and equatorward in a manner similar to the phase surfaces of the stationary wave (42).

In terms of wave activity or Eliassen–Palm flux, the process modulates each of the components of F, both spatially and temporally. By itself, the undamped stationary plane wave transmits a steady and uniform stream of wave activity in the vertical. However, through interference with the traveling wave, this uniform stream of eddy activity is transformed into a series of peaks and troughs (Fig. 3b), all moving upward. For stationary and traveling waves of comparable amplitude, wave activity and its flux are completely localized into a succession of capsules which migrate upwards. In time (Fig. 3c), the local wave flux at some position appears as a sequence of pulses superposed on the time-mean value. This fluctuation in F results from the migration of the spatial interference pattern and is analogous to the passage of a series of wavefronts.

To illustrate these features we assign the values:

\[
\begin{align*}
A_{01}^m &= 12 \text{ m} \\
\ell &= -3.0 \\
kH &= 3x = 0.858 \\
A_{00}^m &= 60 \text{ m}
\end{align*}
\]

for \( m = 1 \), where \( H = 7.3 \text{ km} \), \( \sigma_1 = 0 \), and \( \sigma_2 \) corresponds to westward propagation at a period of 16 days. The wave structures have been normalized so that \( A_{n0}^m \) and \( A_{0n}^m \) denote the surface geopotential amplitudes at 60\(^\circ\), and the stationary wave has latitudinal (deg) and vertical wavelengths of 120\(^\circ\) and 54 km. Both stationary and traveling waves grow continually in the vertical, with the traveling wave dominating below about 7 scale heights (50 km) and the stationary wave prevailing above.

Complex amplitude polars for the instantaneous wave at 60\(^\circ\) for selected heights are shown in Fig. 4. Signatures are marked at all levels by a transient component \( Z(t) \) which orbits about the tip of the time-mean amplitude vector \( Z_0 \). The latter is seen to rotate with height as a result of the westward tilt of the stationary wave. At the lowest levels the transient component dominates, driving the instantaneous amplitude completely about the origin. Thus the phase of the combined wave increases continually with time, corresponding to systematic retrogression. At upper levels, however, the stationary wave prevails, with the result that the combined wave no longer moves systematically in one direction, but rather is prograde over half of the cycle and retrograde over the other half. Near the crossover altitude, roughly 8 scale heights, the combined wave is modulated between zero and twice its time-mean amplitude, giving the appearance of a burst in wave activity followed by complete collapse.

Figure 5 shows the instantaneous EP flux at the same locations as Fig. 4. At all levels the time-mean EP flux is upward and equatorward. Again the transient component, this time \( F' \), modulates both the magnitude and direction of the time-mean EP flux vector. It can be shown that the trajectory given by \( F'(t) \) describes an ellipse. Below 8 scale heights modulation by the transient component drives the instantaneous EP flux vector completely about the origin, so that both components of \( F \) change sign somewhere during the cycle. The Eliassen Palm flux appears to pulsate during the cycle and switch direction from equatorward to poleward. Such behavior is attended at high latitudes by a focusing of wave activity, due to the poleward convergence of meridians, and ultimately by an increased EP flux divergence (O'Neill and Youngblut, 1982). At sufficient altitude the time-mean EP flux eventually prevails, with the crossover level being characterized by a modulation of \( F \) between zero and twice its time-mean value, again assuming the appearance of a rapid growth and collapse of wave activity.

The arrows shown on the transient orbits in Fig. 5 indicate a reference phase of the vacillation. A lag between levels can be seen. As is evident in Fig. 6, which shows a time–height section of \( F \) at 60\(^\circ\), this is simply a signature of the upward migration of packets of wave activity. Both amplitude and direction of \( F \) propagate upwards. However, because the amplitude of the transient component decays with altitude, the EP flux
Fig. 4. Instantaneous wave amplitude at 60° for selected levels (units of gpm), altitude z in scale heights. Progressive propagation: clockwise; retrogressive propagation: counterclockwise. Traveling wave dominates at lower levels, resulting in continual retrogression of the combined wave. Stationary wave prevails at upper levels, leading to a reversal in propagation during the interference cycle.
Fig. 5. Instantaneous EP flux at 60° for selected levels (SI units). Transient component $F'$ orbits about time-mean component $F_0$. At lower levels the transient component drives $F$ completely about the origin, altering both its strength and direction. Time-mean component eventually prevails at upper levels, the crossover point marked by modulation of $F$ between zero and twice its time-mean strength. Wherever the transient contribution is large, there is a considerable shift in the direction of $F$, e.g., equatorward to poleward.
eventually reduces, at upper levels, to the steady equatorward component $F_0$.

4. Localized stationary wave and barotropic traveling wave

In the previous example both stationary and traveling waves extended indefinitely in the vertical and equatorward. Due to refractive and absorptive effects this does not occur in the atmosphere. The stationary wave is confined to the winter westerlies during solstice, and vertical growth rarely persists beyond the stratosphere because of radiative absorption and refraction by strong westerlies. The horizontal extent of traveling waves, e.g., a normal mode, although dependent upon frequency, is greater than that of the stationary wave. However, slower modes such as the 16-day wave are also excluded from the summer stratosphere by strong easterlies (Salby, 1981b). As for their vertical structure, slower modes rarely continue to grow above the stratosphere.

In this section we explore the interference signature with more realistic wave structures by restricting the simple wave fields used in section 3 spatially as

$$
\Psi_m^s(\phi, z) = w_m(\phi)w_z(z) \cdot \hat{\Psi}_m^s(\phi, z)
$$

(44a)

$$
\Psi_m^t(\phi, z) = w_m(z) \cdot \hat{\Psi}_m^t(\phi, z)
$$

(44b)

with $\hat{\Psi}_m^s$ and $\hat{\Psi}_m^t$ representing the simple structures in sec. 3 and

$$
w_m(\phi) = e^{-(\phi - \phi_0)/\nu_m}^2
$$

(45)

$$
w_z(z) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{z - z_0}{H \sigma_z} \right) \right]
$$

(46)

The latter describe a Gaussian envelope in latitude and a step dropping to zero amplitude above a prescribed level.

Fig. 7. Amplitude (solid) and phase (dashed) structures for (a) stationary and (b) traveling waves. (See text.)

We assign the values:

$$
\begin{align*}
A_m^s &= 25 \text{ m} \\
\ell &= -3.0 \\
\kappa H &= .858 \\
A_m^t &= 150 \text{ m} \\
\phi_0 &= 70^\circ \\
\sigma_z &= 10^\circ \\
z_0 &= 6.0 \\
\sigma_c &= 2.0
\end{align*}
$$

(47)

again for $m = 1$ and $A_m^s$ and $A_m^t$ representing surface amplitudes at $60^\circ$. Amplitude and phase structures of the stationary and traveling waves are shown in Fig. 7. These are not unrepresentative of observed planetary waves in the stratosphere and, as will be seen in section 5, structures which emerge from Primitive Equation calculations in realistic winds. The stationary wave peaks at 700 gpm near 6 scale heights (45 km) and the traveling wave at 750 gpm somewhat lower. Phase behavior at these levels is similar to that of the previous case.

Fig. 6. Time–height section of instantaneous EP flux magnitude and direction.
As was true in Fig. 4, the transient component (not shown) dominates the instantaneous wave amplitude in the lower to middle stratosphere, with the stationary wave eventually prevailing at upper levels. Only now, both components decay above 8 scale heights (56 km).

Modulation in the instantaneous wave field (Fig. 8) is rather dramatic. It may be compared with the time-mean picture shown in Fig. 7a. A wavepacket migrating up from below completely alters the time-mean behavior. What begins as a simple wave structure growing steadily up to 5 scale heights and decaying above, surges upward and eventually splits, ejecting a second capsule upwards into the upper stratosphere and lower mesosphere. Owing to the decay of both components at these levels, this part of the wave field eventually collapses. Phase behavior accompanying this modulation is also striking, with a rapid change, if not a phase reversal, following the minimum or node between the two capsules.

Local EP flux orbits in this case (not shown) are similar to those in Fig. 5. However, because the stationary and traveling wave structures now resemble one another more closely, the orbits of $F'$ are nearer to linear polarization. Likewise, upward migration is again evident in the height–time section of $F$ (not shown), as was seen previously in Fig. 6. Changes in the direction of $F$ are substantial in the lower stratosphere, where the transient wave is large. However, in this case, the influence of the transient component of $F$ drops sharply above 6 scale heights due to the decay of both waves in the upper stratosphere.

A sequence of latitude–height sections of $F$ and the

Fig. 8. Latitude–height cross sections of instantaneous wave field during the vacillation. Time increments in cycles. Amplitude (solid); phase (dashed).
instantaneous eddy forcing of the mean flow are presented in Fig. 9. The localization of EP flux into a packet propagating upwards emerges clearly. Rising values of $F$ at the leading edge of this wavepacket lead to deceleration of the mean flow. At the trailing edge, just the opposite behavior is observed. As the wavepacket moves into the upper stratosphere, the associated flux of wave activity collapses, followed shortly thereafter by the dipole in mean flow forcing and eventually the emergence of a new capsule from below.

Over the lower and middle stratosphere, modulation of the time-mean EP flux and its divergence is substantial. As for $F'$, transient forcing of the mean flow can also be thought of in terms of amplitude and phase. These are shown in Fig. 10 as functions of latitude and height. As can be seen from the phase tilt, the vacillation proceeds upward and equatorward, but is confined to an envelope at high latitudes. The high latitude character of this transient contribution is derived from the $\cos^{-1}(\phi)$ dependence of the eddy forcing of the mean flow (24), ultimately from the poleward focusing of wave activity when $F$ is diverted poleward and equatorward.

5. Stationary wave and normal mode traveling wave in realistic mean flow

As the final case to be considered, we examine the interference signature with a stationary wave and an
atmospheric normal mode, both derived under realistic conditions. These are obtained numerically with a global, linear Primitive Equations calculation. The numerical scheme has been described elsewhere (Salby, 1981a; Garcia and Salby, 1987). Both waves are generated by surface forcing. A Gaussian mountain, centered at $\phi = 45^\circ$ and having a sigma of 15° latitude is used to force the stationary wave. The procedure described in Salby (1981a) is used to locate in frequency the second symmetric normal mode of wavenumber 1 (i.e., the 16-day wave).

The mean flow field is taken from a simple analytic seasonal march (Garcia and Salby, 1986) corresponding to 5 January and is shown in Fig. 11. Easterlies occupy the summer stratosphere and mesosphere with remaining features typical of solstice conditions.

Resulting structures of the stationary and traveling waves, assigned surface amplitudes of 9 and 300 gpm at 60°, respectively, are shown in Fig. 12. These values, although large in a statistical mean sense, produce disturbance fields quite similar to the stationary and traveling waves observed during January 1979 (Madden and Labitzke, 1981). The stationary wave maximizes at 1200 gpm near 6 scale heights (45 km) and 75°. Its structure is confined to the winter hemisphere, both in the troposphere and in the stratosphere. Strong phase tilt is evident near the tropopause above the region of generation, signaling vertical radiation into the stratosphere. In the middle stratosphere, this phase behavior is transformed into an equatorward but still upward tilt, indicating the refraction of wave activity into the tropical easterlies where it is presumably absorbed.

The normal mode also exhibits a significant equa-
Fig. 13. As in Fig. 4, but for wave fields derived from the linearized Primitive Equations under January-like conditions shown in Fig. 12.
torward phase tilt in the middle and upper stratosphere where, like the stationary wave, it is largely confined to the winter westerlies. However, in the troposphere (not shown) and lower stratosphere the mode’s structure is global, hemispherically symmetric, and virtually barotropic—in contrast to the stationary wave. All of these features are consistent with the characteristics of atmospheric normal modes in realistic conditions (Salby, 1981b). The traveling wave peaks at 1000 gpm near 5 scale heights (37 km) and 70°. As for the previous analytical cases, the traveling wave dominates at lower levels, here below about 4 scale heights, with the stationary wave prevailing above due to the different vertical growth rates of the two components.

The evolution of wave amplitude at 60° for selected levels is shown in Fig. 13. At 2 and 4 scale heights (15 and 30 km) the complex amplitude of the combined wave orbits completely about the origin, signaling systematic retrogression throughout the interference cycle. At 4 scale heights, however, there is near cancellation of the instantaneous wave during destructive interference, so the wave vacillates between zero and twice its time-mean strength. Again, this gives the appearance of a burst in wave activity, followed by a nearly complete collapse of the wave field. At 6 scale heights the stationary wave prevails, and the complex amplitude vector no longer orbits completely about the origin. Instead, the phase of the instantaneous wave is retrograde over half of the cycle and prograde over the other half (cf. Fig. 1). Both stationary and traveling waves decay at greater altitudes, and the instantaneous amplitude behavior is either that of pure interference or slight predominance by the transient component. Of course, this will vary with latitude.

Evolution of the instantaneous wave field is shown in Fig. 14. As in the analytical cases, there is again

![Fig. 14. As in Fig. 8, but for wave fields shown in Fig. 12.](image-url)
INSTANTANEOUS EP FLUX

\[ \phi = 60^\circ \]

(a) \(Z = 2\)

(b) \(Z = 4\)

(c) \(Z = 6\)

(d) \(Z = 8\)

(e) \(Z = 10\)

(f) \(Z = 12\)

Fig. 15. As in Fig. 5, but for wave fields shown in Fig. 12.
rather striking modification of both the amplitude and phase of the time-mean wave field (cf. Fig. 12). However, because the stationary and traveling waves exhibit similar phase behavior in the middle and upper stratosphere, the interference at these levels proceeds with near simultaneity. Below these altitudes a bulging up from below can be seen. The peak amplitude, occurring at about 40 km, intensifies over the course of the cycle from 650 gpm to in excess of 2100 gpm. The region influenced by the wave expands both upward and equatorward. During this evolution, the phase changes from a regular vertical phase tilt, indicating simple upward radiation, to one virtually barotropic at peak amplitude, suggesting vertical trapping. Neither of these characterizations is completely correct, as the refractive properties of the zeroth order flow are unchanged during the cycle. What actually appears to occur is that the barotropic region of the traveling wave below bulges up into the stratosphere, driving the region of phase tilt equatorward and to higher altitudes. Following peak amplification the entire pattern collapses, with the phase tilt being restored to the middle and upper stratosphere.

The instantaneous Eliassen–Palm flux at the same locations as in Fig. 13 is shown in Fig. 15. At 2 scale heights both magnitude and direction of $\mathbf{F}$ are profoundly altered. Peak fluxes in wave activity occur simultaneously with upward and poleward propagation, underscoring the importance of both the inverse density dependence and poleward focusing of eddy driving.
(24). At 4 scale heights $F$ is modulated between zero and twice its time-mean value, giving again the appearance of a sequence of pulsations or bursts in wave activity. Vacillations at higher altitudes approach linear polarization, following from the similarity of structures of the stationary and traveling waves at these levels. Nevertheless, even at these altitudes the modulation is near 100%.

Evolution of the instantaneous EP flux field and eddy driving of the mean flow are given in Fig. 16. Over much of the stratosphere the time-mean component of eddy driving is dwarfed by the transient component, which peaks at a value nearly four times that of the steady contribution. During the initial phase of the cycle, wave activity radiates vertically out of the troposphere above the source region. It is refracted both poleward and equatorward, but rapidly attenuates with altitude as a result of dissipation. As the traveling wave constructively interferes with the stationary wave, the flux of wave activity swells upward out of the troposphere, penetrating to substantially greater heights. Following peak surge, the EP flux field veers equatorward and subsequently collapses.

The eddy driving of the mean flow has maximum values at upper levels, due to the inverse density effect, and at high latitudes, due to the polar focusing mechanism. Initially, there is a region of strong mean flow deceleration (about $-70 \text{ m s}^{-1} \text{ day}^{-1}$) in the middle polar stratosphere. This gradually breaks down and is replaced by an intensifying lobe of mean flow acceleration which migrates up from below. The sequence at middle and upper stratospheric levels proceeds with near simultaneity due to the similarity of phase structures of the stationary and traveling waves at these altitudes.

As before, these characteristics describe a vacillation which propagates through and is confined by an envelope. Amplitude and phase structures of the transient EP flux driving are shown in Fig. 17. The high latitude character of the amplitude envelope, seen earlier, emerges again. It is intrinsic to the veering of $F$ and polar focusing/spreading of wave activity at high latitudes. However, the phase of the vacillation now varies appreciably only in the lower stratosphere where the traveling wave is barotropic. The strongest eddy driving actually occurs synchronously across much of the polar stratosphere. There is also a weaker region of transient eddy driving at tropical latitudes which is nearly out of phase with that at polar latitudes.

Fig. 18. Amplitude and phase of transient zonal-mean wind reaction (m s$^{-1}$) to the eddy forcing shown in Fig. 17. Although upward migration is evident in the lowest levels, the major strengthening and weakening of the flow in the middle stratosphere occurs with near simultaneity.

Fig. 19. Amplitude and phase of transient zonal-mean temperature response (K). Maximum values are achieved directly over the pole, with a 180$^\circ$ phase reversal separating the two maxima in the stratosphere and mesosphere.
6. Reaction of the mean flow

With the wave fields shown in Fig. 12, the instantaneous eddy driving of the mean flow is dominated by the transient contribution at high latitudes. As we shall explore in more detail shortly, these amplitudes are large enough, given the mean flow in Fig. 11, to render the validity of linear and quasi-linear descriptions dubious. Nevertheless, it is important to examine how much of the behavior, observed during January 1979, can indeed be captured by such lower order descriptions, and how much cannot. It so doing, we may provide insight into which facets of the evolution are truly indicative of strong nonlinearity.

We have seen that, with wave amplitudes representative of observed values, interference leads to a transient contribution to $F$ which prevails over the time-mean component in the polar stratosphere. It is natural to expect this transient component to result in an equally prominent response in the basic flow. The reaction of the mean flow to this fluctuating driving is governed by (26) and may be derived by applying suitable boundary conditions. We have done just this by constructing a global boundary value problem, requiring boundedness at the poles and upward and downward radiation of the zonal mean at 2 and 12 scale heights, respectively. The Hough spectral scheme described by Plumb (1982) was modified to apply to frequency components, $\Psi$, and include thermal and mechanical damping (Garcia, 1987). Newtonian cooling and Rayleigh friction of the same magnitudes as in the wave calculations were prescribed. The problem was
due to the positive EP flux driving occurring shortly before. [From (18a) it can be inferred that, if the balance of the zonal-mean momentum equation is dictated approximately by the zonal wind tendency and the eddy forcing, the zonal flow response should lag the EP flux driving by roughly 90°.] By one-quarter cycle, a strong easterly flow has developed almost simultaneously between 6 and 10 scale heights (45 and 70 km). This subsequently spreads upwards into the mesosphere and is replaced by renewed acceleration in the polar stratosphere which comes up from below. By comparison with Fig. 14 it can be seen that the mean flow reaction is nearly out of phase with the instantaneous wave field at these altitudes, in agreement with observed variability during winter 1978/79 (Smith, 1985). Tropical behavior, albeit much less pronounced, is approximately out of phase with the sequence of events at polar latitudes. This is easily confirmed by monitoring the migration of the zero wind line. During the polar acceleration phase, easterlies advance nearly 20° into the winter hemisphere, whereas at maximum easterly polar flow the zero wind line retreats towards the equator.

7. Synoptic behavior of geopotential height

We now examine the interference signature in synoptic fields of geopotential height. Questions have been raised regarding the ability of wave-mean flow descriptions to characterize the behavior of the polar night due to the positive EP flux driving occurring shortly before. [From (18a) it can be inferred that, if the balance of the zonal-mean momentum equation is dictated approximately by the zonal wind tendency and the eddy forcing, the zonal flow response should lag the EP flux driving by roughly 90°.] By one-quarter cycle, a strong easterly flow has developed almost simultaneously between 6 and 10 scale heights (45 and 70 km). This subsequently spreads upwards into the mesosphere and is replaced by renewed acceleration in the polar stratosphere which comes up from below. By comparison with Fig. 14 it can be seen that the mean flow reaction is nearly out of phase with the instantaneous wave field at these altitudes, in agreement with observed variability during winter 1978/79 (Smith, 1985). Tropical behavior, albeit much less pronounced, is approximately out of phase with the sequence of events at polar latitudes. This is easily confirmed by monitoring the migration of the zero wind line. During the polar acceleration phase, easterlies advance nearly 20° into the winter hemisphere, whereas at maximum easterly polar flow the zero wind line retreats towards the equator.

7. Synoptic behavior of geopotential height

We now examine the interference signature in synoptic fields of geopotential height. Questions have been raised regarding the ability of wave-mean flow descriptions to characterize the behavior of the polar night due to the positive EP flux driving occurring shortly before. [From (18a) it can be inferred that, if the balance of the zonal-mean momentum equation is dictated approximately by the zonal wind tendency and the eddy forcing, the zonal flow response should lag the EP flux driving by roughly 90°.] By one-quarter cycle, a strong easterly flow has developed almost simultaneously between 6 and 10 scale heights (45 and 70 km). This subsequently spreads upwards into the mesosphere and is replaced by renewed acceleration in the polar stratosphere which comes up from below. By comparison with Fig. 14 it can be seen that the mean flow reaction is nearly out of phase with the instantaneous wave field at these altitudes, in agreement with observed variability during winter 1978/79 (Smith, 1985). Tropical behavior, albeit much less pronounced, is approximately out of phase with the sequence of events at polar latitudes. This is easily confirmed by monitoring the migration of the zero wind line. During the polar acceleration phase, easterlies advance nearly 20° into the winter hemisphere, whereas at maximum easterly polar flow the zero wind line retreats towards the equator.

7. Synoptic behavior of geopotential height

We now examine the interference signature in synoptic fields of geopotential height. Questions have been raised regarding the ability of wave-mean flow descriptions to characterize the behavior of the polar night due to the positive EP flux driving occurring shortly before. [From (18a) it can be inferred that, if the balance of the zonal-mean momentum equation is dictated approximately by the zonal wind tendency and the eddy forcing, the zonal flow response should lag the EP flux driving by roughly 90°.] By one-quarter cycle, a strong easterly flow has developed almost simultaneously between 6 and 10 scale heights (45 and 70 km). This subsequently spreads upwards into the mesosphere and is replaced by renewed acceleration in the polar stratosphere which comes up from below. By comparison with Fig. 14 it can be seen that the mean flow reaction is nearly out of phase with the instantaneous wave field at these altitudes, in agreement with observed variability during winter 1978/79 (Smith, 1985). Tropical behavior, albeit much less pronounced, is approximately out of phase with the sequence of events at polar latitudes. This is easily confirmed by monitoring the migration of the zero wind line. During the polar acceleration phase, easterlies advance nearly 20° into the winter hemisphere, whereas at maximum easterly polar flow the zero wind line retreats towards the equator.

7. Synoptic behavior of geopotential height

We now examine the interference signature in synoptic fields of geopotential height. Questions have been raised regarding the ability of wave-mean flow descriptions to characterize the behavior of the polar night
vortex during disturbed conditions. We establish here the connection between latitude–height sections of mean flow and wave fields described previously and the implied behavior in synoptic maps. As before, we will concentrate on the solutions derived under realistic conditions with the linearized Primitive Equations.

Figure 21 shows the zonal-mean time-mean geopotential height field at 4 scale heights (30 km), referenced to the equatorial height. Negative values have been arbitrarily shaded, although they demarcate the edge of the vortex with reasonable fidelity. A maximum depression of 1500 gpm is achieved.

The time-mean height field is given in Fig. 22. As a result of the stationary wave, the vortex has become
displaced slightly off the pole and deformed by the introduction of a weak ridge at 0° longitude. The height minimum, just off the pole, is 1700 gpm. Of particular interest is the comma-like shape assumed by the shaded region. This pattern is associated with the lateral phase tilt of the stationary wave. Reference to Fig. 12 indicates a latitudinal tilt at and equatorward of 50°. This means that trough and ridge axes will shift westward with decreasing latitude, giving the appearance of the wave spiraling outward from the pole.

When the transient contribution (traveling wave plus reaction of the mean flow) is added, the instantaneous synoptic pattern (Fig. 23) exhibits a rather familiar signature (cf. Fig. 2). The phase of the vacillation during the cycle depends upon the level chosen. We shall take the initial phase to be at 0.75 cycles and discuss the evolution forward from there. The vortex, centered initially over the pole, is quite intense, having a maximum depression of 2600 gpm. One-quarter cycle later a ridge of 300 gpm builds in, and the vortex is simultaneously displaced off the pole and diminished in amplitude to 1400 gpm. A quarter-cycle later the ridge has propagated westward and amplified to 1000 gpm. At the same time, the vortex has rotated about the pole opposite the ridge and has become further displaced, its amplitude falling to 1200 gpm. As a result of the marked intensification of the ridge and simultaneous displacement and weakening of the vortex, a strong cross-polar flow is established.

Accompanying the retrograde motion is a consider-

FIG. 24. Instantaneous geopotential height field at 6 scale heights and \( t = 0.50 \) cycles.

TABLE

ZONAL-MEAN TIME-MEAN ERTEL POTENTIAL VORTICITY
(\( \theta = 850 \) K)

![Diagram](image)

FIG. 25. Zonal-mean time-mean potential vorticity field on the 850 K isentrope.

erable deformation of regions of high and low geopotential, each taking on comma shapes and spiraling equatorward about one another (cf. Fig. 2). As the cycle moves into the final phase, this wobbling about the pole and distortion continues. The ridge now collapses and the vortex becomes restored to the pole and reenforces. During the process the trough axis becomes elongated, spiraling equatorward about the ridge and ultimately ending up in the tropics. This too is a consequence of the lateral phase tilt of the instantaneous wave (Fig. 14). As the wave intensifies, the phase variation of the disturbance manifests itself in the combined synoptic field. The behavior of the trough thus reflects not only the vortex but the wave depression as well.

The intensification sequence signals the passage of a capsule or packet of wave activity (Fig. 3b) through the level shown. Because of its equatorward tilt, the eddy geopotential (surfacing in the combined field) spirals westward with decreasing latitude. At this altitude, the traveling component of the instantaneous wave predominates (Fig. 13), resulting in systematic retrogression throughout the cycle. Consequently, the ridge and vortex move continually westward. Together with the equatorward spiraling of ridge and trough axes, the retrogression gives the wavepacket the appearance of a corkscrew propagating through the level shown.

At 6 scale heights (45 km) the traveling wave no longer predominates (Fig. 13). As a result, the vortex-anomaly pattern first moves eastward and then west-
The time-mean $q$ map is given in Fig. 26. When the stationary wave is introduced, a reasonably well-defined boundary to the region of high potential vorticity emerges. There is a slight displacement of the $q$ maximum off the pole, consistent with the behavior of the time-mean geopotential field (Fig. 22). The comma-like distortion of the vortex is exaggerated from that evident in geopotential, spiraling clockwise away from the main body of the vortex. All this is consistent with the relationship between potential vorticity and geopotential. Under quasi-geostrophic conditions, disturbance potential vorticity is simply the Laplacian of the streamfunction. It is well known that spatial differentiation has the effect of high-pass filtering spatial variability. Therefore, the comma-like distortion of the vortex, induced in geopotential by the lateral phase tilt of the wave, is more pronounced in the higher order field of potential vorticity.

The evolution of potential vorticity resulting from the combined field is shown in Fig. 27. Beginning at $t = 0.0$, the vortex is fairly well centered over the pole, attaining a maximum in $q$ of $1.1 \times 10^{-2}$ (SI units). The region of poleward $q$ gradient is readily distinguished from the flat behavior equatorward of $30^\circ$. At $t = 0.25$ a depression in $q$ appears near $30^\circ$ longitude at high latitudes, reflecting the buildup of the anticyclonic ridge in geopotential (Fig. 23). This $q$ minimum actually can be traced back along an axis around the periphery of the vortex to low latitudes. Accompanying the high latitude depression is the maximum in $q$ being displaced off the pole and falling in magnitude to $0.75 \times 10^{-3}$. A tongue of high $q$ begins to emerge at the boundary of the vortex, spiraling clockwise away from its main body. By $t = 0.50$ this tongue is well developed, giving a clear impression of material being drawn out from the main body of the vortex and spiraling anticyclonically about the depression in $q$. Exchanging with this high vorticity air is a tongue of low $q$ which has intruded into the vortex from low latitudes. The central body of the vortex remains displaced off the pole, retrogressing opposite the $q$ minimum, and has dropped in magnitude to $0.62 \times 10^{-3}$. At $t = 0.75$ the vortex has moved back towards the pole, although continuing to retrogress, and the peak value has increased to $1.3 \times 10^{-2}$. The outer shell of the high $q$ tongue has now spiraled nearly halfway around the globe, while the inner core has returned to the main body of the vortex. Ultimately the outer shell separates from the main body, giving the impression of having sheared off and left a region of high potential vorticity at low latitudes evident at $t = 0.0$.

Insofar as the material is concerned, detachment in a strict sense cannot occur. Instead of truly separating from the main body of the vortex, the two regions would remain connected by a narrow but nonvanishing
filament. No doubt this is the case for the material field. The apparent separation of regions of high potential vorticity is introduced by the narrowing and ultimate collapse of the meridional scale of the tongue below that which can be resolved in this simple calculation. Given greater resolution, e.g., more wave-numbers and frequencies, the $q$ distribution would be capable of remaining intact throughout.

Perhaps equally important is that potential vorticity is not truly conserved. Erosion of $q$ by radiative dissipation, e.g., through induced mass influx across isentropic surfaces and concomitant dilution (Haynes and McIntyre, 1987), may lead to a genuine break in the potential vorticity distribution, even though no such discontinuity occurs in the material field. Similarly, the return of the inner shell of the tongue to the main body of the vortex may also not accurately reflect material displacements. Radiative dissipation can be ex-
Fig. 28. Instantaneous eddy potential vorticity on the 850 K isentrope during the vacillation cycle, derived under realistic conditions.

expected to play an important role here, because relaxation times at this level are of the order of 5–10 days, comparable to the time scale for material advection from polar to tropical regions. In particular, as horizontal gradients of potential vorticity are exaggerated in the advection process by thermal wind balance so too may be vertical temperature gradients. These would be expected to lead to increased radiative dissipation. Such diminution of $\theta$ is presumably responsible for the eventual collapse of the trailing segment of the tongue, or debris, seen to have spiraled further at one quarter cycle later ($t = 0.25$).

We have previously established a connection between the spiraling of the vortex about the anticyclonic anomaly and lateral phase tilt of the instantaneous wave field. This connection emerges dramatically in the behavior of eddy potential vorticity. Figure 28 shows the evolution of disturbance potential vorticity, derived in the same manner as before. The spiraling of high and low potential vorticity about one another
is striking. It may be thought of in terms of the enstrophy of the wavepacket. Specifically, in three dimensions, the potential vorticity distribution of the wavepacket describes a helix (localized in the vertical), which winds through the isentropic level shown. Beginning at $t = 0.25$, a tongue of high $q'$ radiates equatorward from the polar maximum near 60°, completing nearly one revolution about the globe. One-quarter cycle later it has joined with subtropical high $q$ and extends even further. The progression continues until by $t = 0.0$ the region of high $q$ has elongated and spiraled nearly two complete revolutions about the globe, ending in a somewhat irregular manner near the equator.

9. Conclusions

We have explored the interference produced when a traveling planetary wave propagates across a stationary forced wave, examining the signature in a variety of diagnostics. It is most easily recognized in complex polars of instantaneous wave amplitude. There, a transient wave vector simply orbits about the time-mean component, causing the instantaneous amplitude to vacillate in both magnitude and phase. If the transient component dominates, the phase of the instantaneous wave will propagate continually throughout the cycle, indicating, for example, systematic retrogression of the combined wave. If, on the other hand, the stationary wave is dominant, the combined wave will reverse direction during the vacillation. When traveling and stationary wave amplitudes are comparable, the instantaneous wave pulsates between zero and twice its time-mean amplitude.

The transient disturbance modulates all of the transport properties of the stationary wave—even if the traveling wave alone fluxes nothing. In particular, the Eliassen–Palm flux, or flux of wave activity, is modulated in both magnitude and direction. For transient and stationary waves of comparable amplitude the fluctuating component of $F$, which orbits about the time-mean component, can readily drive the instantaneous EP flux vector through zero—completely altering its strength and direction. This temporal variability follows from the EP flux field being modulated in space and the resulting pattern migrating with time.

Spatial modulation of the wave field converts what may be, for the stationary wave alone, a simple uniform stream of wave activity into one which is corpuscular, eddy activity organized into wavepackets. These capsules of wave activity propagate upward and equatorward in a manner similar to the phase of the time-mean wave. They are attended by a change in both magnitude and direction of $F$. As a result, the signature realized at a fixed location emerges as a series of bursts in wave activity. Time lags between different locations depend upon the difference in phase structures of the transient and stationary waves.

For a barotropic traveling wave and a vertically tilting stationary wave, a clear upward migration of these disturbances emerges. Such is the case for the 16-day wave, calculated under solstice conditions, in the troposphere and lower stratosphere. The behavior is in agreement with observed phase lags of eddy heat and momentum fluxes (Madden, 1983). In regions where the traveling and stationary waves both tilt westward, the phase lag between points vanishes, and the vacillation proceeds with near simultaneity. This is indicative of calculations with the 16-day wave in the middle and upper stratosphere and in agreement with observed wave amplitude vacillations at these levels (Smith, 1985).

The spatial modulation of the EP flux field and propagation of the ensuing pattern produces similar behavior in the eddy driving of the mean flow. The leading edge of one of the EP flux packets is marked by rising values of $F$ and thereby mean flow deceleration, whereas the trailing edge is characterized by just the opposite behavior. For a transient wave of comparable amplitude to the stationary wave, the time-mean eddy forcing of the mean flow can be profoundly altered. This is true even if the traveling wave is purely barotropic.

Greatest influence is exerted at high latitudes in the middle and upper stratosphere, stemming from the inverse dependence of eddy driving on density and cosine latitude. The mechanism actually responsible for this high latitude variability is the directional modulation of $F$ by the transient wave. Specifically, veering of $F$ to and away from the pole, where meridians converge, causes repeated focusing and spreading of wave activity. This leads to exaggerated values of mean flow forcing. A weaker vacillation at tropical latitudes is nearly out of phase with the polar fluctuation.

When this wave forcing is introduced into the zonal-mean equations, the result is a vacillation in the basic flow. Wave amplitudes representative of January 1979 lead to a response in the mean which is considerable. For these anomalously strong amplitudes, a zonal wind reversal occurs in the polar stratosphere, preceded by an acceleration of the mean flow. Zonal-mean temperature increases markedly, maximizing over the pole, and is attended by cooling in the mesosphere. Alternating acceleration and deceleration of the zonal flow near the pole was indeed observed during January 1979. Palmer and Hsu (1983) refer to the acceleration phase as stratospheric cooling. The cycle appears in their Fig. 4 as a back and forth movement of the vortex, first poleward then equatorward as easterlies build into the polar stratosphere. The reversed flow in fact appears first at lower levels.

The synoptic signature of the vacillation consists of two basic elements:

(i) displacement and wobbling of the vortex about the pole, associated with a ridge which builds in from below;

(ii) distortion of the vortex into a comma-like shape, its axis spiraling anticyclonically and equatorward about the ridge.
The first of these is a consequence of the antisymmetry of wavenumber 1 about the pole and simultaneous deceleration of the zonal flow accompanying wave amplification. The second characteristic results from the increased latitudinal phase tilt of the wave during the interference cycle. Though evident in geopotential, the spiraling is exaggerated in synoptic fields of potential vorticity. This can be simply understood by considering the equatorward tilt of the streamfunction as a spectrum over latitudinal wavenumber: \( \psi' = \int_{-\infty}^{\infty} \psi \, e^{i\theta} \, dl \). Applying the Laplacian results in

\[
q' \sim \int_{-\infty}^{\infty} -l^2 \times \psi \, e^{i\theta} \, dl,
\]

which has the effect of suppressing contributions from small \( l \) and exaggerating those from large \( l \), i.e., high-pass filtering. Thus if \( \psi' \) oscillates equatorward at a certain rate, corresponding to a given rate of spiraling about the pole, similar behavior in \( q' \) will be more pronounced.

In combination, the equatorward spiraling and rotation of the pattern about the pole suggest material being drawn out of the vortex. To the extent that potential vorticity is conserved, these motions do indeed reflect material displacements. The behavior may also be interpreted in terms of the enstrophy carried aloft by the wavepacket. In particular, eddy potential vorticity on an isentropic surface exhibits a conspicuous and pronounced spiral. If vertical phase tilt is taken into account, the three-dimensional distribution of eddy enstrophy associated with the wavepacket is in fact helical and localized in the vertical. The spiraling pattern is achieved when this helix of eddy potential vorticity, propagated upwards with the group velocity, intersects a given \( \theta \) surface. It is manifest in the combined field initially as distortion of the vortex into a comma-like shape and ultimately as potential vorticity being drawn anticlockwise around the ridge to low latitudes. As potential vorticity spirals equatorward, it becomes elongated and eventually appears to shear off at low latitudes. We have associated the apparent separation of regions of potential vorticity with the limited resolution inherent to this simple calculation and with radiative dissipation of \( q \). Episodes where potential vorticity appears to reattach to the vortex, as has been noted in observations (Clough et al., 1985), occur as well and may be attributed to the same mechanisms.

This alternate interpretation of potential vorticity behavior, namely, viewed in terms of eddy enstrophy being carried upwards with the wavepacket, is perhaps complementary to the interpretation predicated on material motions. Because the eddy potential vorticity field is of zero mean, it contributes no net change to the total hemispheric potential vorticity. Instead, it acts to disturb the distribution of \( q \) on a \( \theta \) surface, in effect rearranging the potential vorticity. It may, therefore, be viewed as a perturbing influence which redistributes, but neither creates nor destroys, potential vorticity. To the degree that \( q \) is conserved, the same argument can be applied to the material field, in which case the wavepacket may be thought of as inducing advection along the \( \theta \) surface.

There is an important distinction between the wave breaking and interference descriptions. For the interference process, in the absence of damping, small scales in the distribution of a tracer cannot be generated permanently. Material elements are restored to their original position following such an episode. However, in wave breaking strong nonlinearity presumably leads to a cascade of variance to higher wavenumbers associated with material elements not returning to their original positions. Small scales in potential vorticity can be generated by interference, through diabatic or other nonconservative effects, as we have discussed. The same would hold for any species whose lifetime is comparable to a characteristic time scale for advection. Likewise, genuine transport can be accomplished in the presence of dissipation, without the intervention of strong nonlinearity. The debris in \( q \) remaining after passage of the wavepacket may genuinely reflect a net exchange of material between high and low latitudes. This would follow from diabatic effects which prevent parcel trajectories from forming closed orbits so that material elements do not return to their original positions. A detailed analysis of material motions and comparison with potential vorticity, needed to faithfully resolve these issues, is planned.

Undoubtedly, details of the various signatures we have examined depend critically upon the zero-order mean flow, particularly for higher order quantities such as EP flux divergence and hence the magnitude of the zonal response. Damping, which has been incorporated here in the form of Newtonian cooling and in reality might be introduced through nonlinear effects, is not well understood. Such influences could mitigate the strength of the response. However, the precise values actually recovered (e.g., of the polar wind response) are inherently sensitive to the nature of the wave fields and the mean flow. This follows from the singularity of the mean flow equations at the pole and the high latitude character of the transient forcing. Consequently, small changes in eddy driving, achieved through changes in either wave forcing or the time-mean flow, can lead to significant changes in the zonal-mean response.

Nevertheless, the high latitude character of eddy

\footnote{An important property of this and all truncated (Fourier) representation is that, while the fully nonlinear equations conserve potential vorticity in an individual or material sense, approximate representations such as this one conserve potential vorticity exactly only in a global sense. However, it should be noted that in no calculation is potential vorticity truly conserved, nor as we have discussed is it conserved in the atmosphere. The meaningful question then is just how well is it conserved in this approximate description, and is this approximate conservation adequate in the presence of realistic damping? Preliminary calculation (not presented here) indicates reasonably good correspondence between material motions and potential vorticity.}
driving and mean flow reaction is intrinsic to the interference process and concomitant switching of $F$ to and away from the pole. These are robust ingredients of the process. Therefore, whenever significant modification of $F$ at high latitudes results through interference, it appears that the behavior described here will follow, at least qualitatively. Because of polar magnification, the mean flow response at high latitudes may be dramatic even for modest amplitudes. This conclusion is underscored for winter conditions, because wave amplitudes peak at very high latitudes, higher than during other seasons. The polar night stratosphere would thus appear to be particularly vulnerable to traveling waves of appreciable amplitude at middle and high latitudes, e.g., the 16-day wave.

Zonal-mean features such as the wind reversal and polar warming, accompanied by out of phase behavior in the tropics and mesosphere, are, of course, the hallmark of stratospheric warmings. In some respects these findings parallel the hypothetical scenario envisaged originally by Matsuno (1971). Matsuno speculated on the amplification of a tropospheric anomaly, whose influence was communicated into the stratosphere. The upward migrating wavefront, associated with this disturbance, induced a divergence of wave flux and consequently a deceleration of the mean flow.

The situation here involves the excitation of transient planetary waves, which in the troposphere are predominantly barotropic. These presumably disperse horizontally out of some localized source region. Through modulation of the stationary wave pattern they lead to features analogous to wave fronts which propagate upward into the stratosphere. A fundamental consideration is just what percentage of planetary wave activity generated near the surface actually radiates out of the troposphere versus that which goes into the horizontally propagating barotropic response. Although this issue cannot be addressed here, it is well recognized that only a small fraction of energy can actually be introduced into the upper atmosphere without having the dire consequences first pointed out by Charney and Drazin (1961).

We have concentrated on a simple monochromatic traveling wave, inspired by the discrete character of the 16-day wave in observational statistics. Consequently, the ensuing signature is steady oscillatory. However, an actual episode would be of limited duration and hence be composed of many frequencies. Likewise, several wavenumbers would be involved. Whereas wavenumber 1 displaces the vortex off the pole, wavenumber 2 would act to split the vortex. It is unlikely that a transient episode would lead to the excitation of any single mode (Salby, 1984b; Hirooka and Hirota, 1985); rather several components would be expected in the response. Consequently, the amplitude and EP flux orbits would be more complex than those presented here.

Whether or not the 16-day wave can be thought of everywhere as a discrete mode is a matter of some subtlety. In the troposphere and lower stratosphere where its energy is trapped, the structure is barotropic, and a discrete frequency response dominates. However, in the upper stratosphere a westward phase tilt appears, indicating the presence of an internal component. Depending upon the relative contribution of the two, the frequency response at these levels may or may not be sharp. While a discrete signature may prevail near the surface, variance at upper levels may be spread over a wide range of frequencies. There is an indication of such a progression in the amplitude polars for January 1979 (Fig. 1). Regardless, the interference mechanism is divorced from the spectral makeup of the traveling wave, i.e., whether the wave transience is concentrated in a single mode or involves many components.

Not only is transience of the eddy field admitted in this manner, but the mean flow behavior would no longer be constrained to be cyclic. When displaced off the pole, the vortex would presumably become subject to dissipative agents, dynamical (such as mixing) or diabatic. In either event, these would prevent the initial vorticity from being restored as is prescribed in the simple monochromatic description. This scenario is presumably equivalent to what Palmer and Hsu (1983) term "preconditioning", whereby the vortex is left more vulnerable to subsequent disturbances (see also Quirzo, 1979; Kanzawa, 1980; McIntyre and Palmer, 1984). A notable candidate for introducing such hysteresis is the asymmetric radiative drive resulting when the vortex is displaced out of circular symmetry of the polar night. Differential cooling across the vortex would then act to destroy the circular temperature gradient and thereby the vorticity. It is worth noting that this mechanism is equivalent to the thermal absorption of planetary wave activity described by Dickinson (1969).

We have presented calculations based on rather large amplitudes, because they are indicative of a highly documented event (January 1979). However, with smaller more typical values, not presented in the interests of brevity, the tendencies are basically the same. Only net effects on the mean flow are weaker. Nevertheless, the morphology which emerges in this simple calculation, with a single wavenumber and only two frequencies, captures the essential character of quasi-periodic disturbances in the stratosphere as reported by Madden and others. It thus supports Madden's speculation that interference might play an important role in disturbed conditions of the stratosphere. Indeed, the fluctuating response of the mean flow, observed for several cycles during winter 1978/79 to be out of phase with the wave field (Smith, 1985), is reasonably well predicted.

What is remarkable is that, even for the large amplitude event of January 1979, one of the strongest on record, the low order formalism is successful in capturing many of the salient aspects of the overall unsteady behavior—whether viewed from the perspective of mean and eddy components or from synoptic fields. This is in accord with the enstrophy budget for the
period, which indicates the principal agent for the warming was in fact wave transience (Smith, 1985).

At such amplitudes, particularly when the mean flow is strongly decelerated, the mathematical underpinnings of linear and quasi-linear descriptions break down, at least locally. That is to say, they cannot be justifiably applied to the stationary and traveling waves would be expected to modify the refractive behavior so that wave activity is focused even more strongly into the pole, at least initially. Subsequent absorption of wave activity would then lead to an advance of easterlies, similar to the classical Matsuno (1971) model, which would operate on a time scale dictated by the group velocity. This unsteady behavior would accompany that imposed by the wave transience, which operates on a time scale largely divorced from the former. As long as the transience time scale is short compared with that following from wave absorption, the latter would be expected to have minimal effect, as the mean flow will switch back and forth before the region of wave absorption has advanced appreciably.

If anything, the mean flow deceleration in this particular example is too strong. Some of the discrepancy may be attributed to the breakdown of geostrophy (i.e., of the zonal-mean) near the pole. The important point to recognize, though, is that the numerical details of the eddy forcing and mean flow response are fundamentally sensitive to the zero order flow and wave forcing, due to the singular nature of the problem. That is to say, modest changes in the wave structures can result in substantial changes in the EP flux and its divergence near the pole. What does emerge robustly in these calculations is an inherent vulnerability of the polar jet to interference, viz. through F being diverted to and away from the pole.

The quasi-linear description may be regarded as an asymptotic series solution to the complete problem, neglecting contributions third order and higher in eddy amplitude. Under the aforementioned conditions, where eddy and mean velocities are comparable, the rate at which this series converges, i.e., how much information is lost through omission of higher order terms, is cast into doubt. A fundamental issue is just how much and which aspects of the actual behavior can be captured by the lowest orders of the complete description. Our results indicate that the gross aspects of the evolution are indeed represented in the low-order formalism. In particular, the displacement and wobbling of the vortex and its distortion and equatorward spiraling are all predicted.

Our results do not preclude the existence of strong nonlinearity in such episodes. On the contrary, there are suggestions that such behavior might well arise out of amplification and from the synoptic configurations realized during the interference process. For instance, when the vortex buckles under distortion, regions of reversed potential vorticity gradient are created. These may be unstable under local considerations analogous to Rayleigh's criterion, or in three dimensions Charney and Stern's (1962). Ensuing behavior might lead to the mixing of potential vorticity and concomitant depletion of the vortex as proposed by McIntyre and Palmer (1983). However, by the same token, such features would be expected to be of smaller dimension than the large-scale pattern from which they evolve. McIntyre and Palmer's interpretation is complicated by the fact that there exist close parallels in the quasi-linear behavior. Elements of potential vorticity, through limited resolution and thermal dissipation, can easily give the appearance of shearing off at low latitudes and at times reattaching to the main body of the vortex. What is rather strongly indicated by our results is that the gross morphology of the January 1979 warming, specifically the distortion and spiraling of potential vorticity to low latitudes and arguably all that is genuinely resolved in satellite measurements, is not in itself a reflection of strong nonlinearity and eddy mixing.

Acknowledgments. The authors are grateful to J. Holton, R. Madden, and M. McIntyre for providing valuable comments on a preliminary version of the manuscript and to W. Grose for providing the maps used to construct Fig. 2. Calculations were performed on the NCAR Cray-1, and figures were drafted by the NCAR Graphics Department. Ms. L. McGinley generated the final manuscript. This work was sponsored by NASA Grant NAGW-772.

REFERENCES

Dunkerton, T., C. P. Hsu and M. McIntyre, 1981: Some Eulerian


