Origin of Low-Frequency (Intraseasonal) Oscillations in the Tropical Atmosphere.
Part I: Basic Theory

K.-M. LAU
Laboratory for Atmospheres, NASA/Goddard Space Flight Center, Greenbelt, MD 20771

L. PENG
Universities Space Research Association, Columbia, MD 21044

(Manuscript received 12 March 1986, in final form 6 October 1986)

ABSTRACT

A theory of the origin of intraseasonal oscillations of the tropical atmosphere is presented and tested by simple model experiments. This study focuses on the validation of the basic theory against key features of the observed 40–50 day oscillation. It is shown that the observed eastward propagation of intraseasonal oscillation in the tropical atmosphere arises as an intrinsic mode of oscillation resulting from an interaction of convection and dynamics via the so-called "mobile" wave–CISK mechanism. Through this mechanism, the heat source feeds on the east–west asymmetry of forced equatorial waves. As a result, Kelvin waves are selectively amplified, which in turn causes the heat source to propagate eastward. This mechanism also prevents small-scale waves from immediate destabilization, contrary to the results of traditional wave–CISK theory. The "mobile" wave–CISK establishes a new dynamical equilibrium state between convection and the wind field to form a wave packet or collective motion with relatively fixed horizontal and vertical structure. Relative to the steady state solutions with stationary heat source, the new equilibrium state has suppressed Rossby-wave response to the west and enhanced Kelvin-wave response to the east of the propagating heat source.

Results also suggest that the periodicity of the oscillation is determined by the time taken for the Kelvin wave to complete one circuit around the globe in the equatorial region. The propagation speed (~19 m s⁻¹) of the model disturbance, which is about twice as fast as the observed, is found to coincide with the real part of the complex phase speed of the model's unstable normal mode modified by internal heating. The speed and the growth rate are dependent on the vertical structure of the heating profile and the static stability of the basic state. In addition to the eastward propagation, many observed features, such as pressure and wind distribution, amplitude modulation by SST, and dominance of low wavenumber response, are well simulated in the idealized experiments. The theory also predicts that the low-frequency disturbance should have a westward tilt with height. This is partially confirmed in real observation and in GCM simulations. While the basic theory appears to explain some fundamental features of the 40–50 day oscillation, large discrepancies still exist. The possibility of examining further detailed features of the oscillation in the present theoretical framework is also discussed.

1. Introduction

In recent years, the family of phenomena associated with the so-called 40–50 day oscillation has become the focal point for studies of the intraseasonal variability of the tropical atmosphere. The 40–50 day oscillation is the strongest low-frequency (intraseasonal) signal so far found in the tropical atmosphere. Its potential impact on long-range weather or short-term climate predictions appears to be far reaching. In their pioneering work Madden and Julian (1971, 1972) first described the oscillations as global-scale eastward-propagating zonal circulation cells along the equator. Several years later, Zangvil (1975), Yasunari (1979, 1980) and Maruyama (1982) reported the existence of quasi-40 day cloudiness oscillations over the summer monsoon region of East Asia and India and suggested that they may be related to those found by Madden and Julian. All of the above work was based on observations that are often limited in space and time. Comprehensive descriptions of the 40–50 day oscillation did not emerge until the early 1980s; these descriptions were based on the results from FGGE. Some examples are the work of Lorenz (1984), Murakami and Nakazawa (1984, 1985), Krishnamurti and Subrahmanyan (1983), Krishnamurti (1985) and others. These authors found that the 40–50 day oscillation is linked to regional characteristics such as monsoon onsets, breaks and fluctuation of low-level jets over India/Southeast Asia.

The global nature of the oscillation is revealed by studies of long-term record of wind and radiation data. Weickmann et al. (1985) and Knutson et al. (1986) reported possible relationships of tropical convection with global scale circulation anomalies in the 30–60 day time scale. Lau and Chan (1983a,b) first noted large fluctuations in east–west dipole anomalies in monthly means of outgoing longwave radiation over the Indian Ocean and the western Pacific. This was
later found to be connected with the large persistence of the 40–50 day oscillation. In a series of papers, they subsequently documented the relationship between the global 40–50 day waves and the regional characteristics of the winter and summer monsoon, and the winter-time extratropical circulation anomalies (Lau and Chan, 1985, 1986a; Lau and Phillips, 1986). It was also pointed out that because of the similar spatial and relative temporal evolution of atmospheric anomalies associated with the 40–50 day oscillation and those with the El Niño–Southern Oscillation, the two phenomena may be closely related (Lau, 1985a,b; Lau and Chan, 1986b).

Up to the present, observational knowledge has far outpaced our theoretical understanding of the 40–50 day oscillation. In spite of a number of hypotheses that have been proposed (see discussion in section 2), still lacking is a tangible theory that may explain the myriad facets of the phenomenon. Undoubtedly, such a theory is necessary to provide a basis upon which observational and GCM results of the phenomenon can be interpreted. In this paper, a preliminary theory of the origin of intraseasonal oscillation in the tropical atmosphere is presented. Because the phenomenon is extremely complex, at present we only focus on the key features of the oscillation that are understandable within the basic framework of this theory. In section 2 we first synthesize the main observational features and then discuss the theoretical background concerning the intraseasonal oscillations in the tropics. In section 4, the basic formulation of the model and its essential physics are described. Sections 5 through 9 are devoted to the model results and theoretical interpretations. The limitation of this theory and its potential in providing further theoretical understanding of other aspects of the phenomenon are discussed in section 10. We conclude in section 11 by pointing out some implications of this work on future modeling of the low-frequency variability of the tropical ocean–atmosphere system.

2. Preliminaries

a. Observational background

From the results of the studies cited above, the key features of the intraseasonal variability of the tropical atmosphere are synthesized as follows:

(i) There is a predominance of low-frequency oscillations in the broad period range from 30 to 60 days (referred to interchangeably in the paper as the 40–50 day oscillations or low-frequency oscillations).
(ii) The oscillations, which have predominant zonal scales of wavenumbers 1 and 2, propagate eastward along the equator year-round.
(iii) Strong convection is confined to the equatorial regions of the Indian Ocean and western Pacific sector, while the wind pattern appears to propagate around the globe.

(iv) There is a marked northward propagation of the disturbance over India and East Asia during the northern summer monsoon season and, to lesser extent, southward penetration over northern Australia during northern winter.

(v) Coherent fluctuation between extratropical circulation anomalies and the tropical 40–50 day oscillation may exist, indicating possible tropical–midlatitude interactions in the above time scale.

(vi) The 40–50 day oscillation appears to be phase-locked to oscillations of 10–20 day periods over India and the western Pacific. Both are closely connected with monsoon onset and break conditions over the above regions.

Of the above, the perennial eastward propagation along the equator and the slow time scale of 30–60 days appear to be the most fundamental features of the oscillation. In this paper, we concentrate on a theoretical investigation of these features including the modulation in convection, i.e., items (i) through (iii).

b. Theoretical background

The origin of the 40–50 day oscillation has remained somewhat of a mystery since its discovery. Chang (1977) suggested that the eastward propagation is the manifestation of convectively driven equatorially trapped Kelvin waves. He showed that the observed large vertical scale and slow motions are consistent with the theory of equatorial waves in the presence of cumulus friction and Newtonian cooling (cf. Stevens and White, 1979). Webster (1983) suggested that quasi-biweekly oscillations, which may be related to the 40–50 day oscillations, can result from feedback between hydrological cycle and dynamics over the monsoon regions. Goswami and Shukla (1984) showed that the interaction between convection and dynamics in a zonally symmetric atmosphere can result in low-frequency oscillations. Yamagata and Hayashi (1984) suggested that some aspects of the oscillation are consistent with the response of the tropical atmosphere to an oscillating but stationary heat source of 40-day period. Anderson (1984) found that slow tropical modes can arise by including a zonally symmetric Hadley-type circulation in the basic flow. However, the origin of the apparent eastward propagation of the heat source and its accompanying circulation and the strong spatial modulation of the heat source are not explained in any of the above studies. At this writing, there is a large number of theoretical studies focusing on the mechanisms of the eastward propagation. A forthcoming paper by Chang and Lim (1987) is particularly relevant to the work reported here.

3. Two scenarios

Based on the above considerations, we propose two scenarios for the origin of the eastward propagation
associated with the 40–50 day oscillation. They will serve as the working hypotheses for this investigation.

a. Scenario I: Local oscillating heat source

Convection associated with the 40–50 day oscillation is mostly confined within the Indian Ocean and the western Pacific because the heat source is induced by local processes. The slow time scale of the oscillation arises from monsoon-related fluctuations of wind, convection, soil moisture, etc., possibly involving some kind of feedback mechanisms among radiation, convection and dynamics (e.g., Webster, 1983; Goswami and Shukla, 1984). Since the heat source associated with the oscillation is confined within the deep tropics, transient wave disturbances generated by the heating will propagate away from the source region in the form of trapped equatorial waves. By some yet unknown process, possibly resonance with viscous Kelvin wave modes (Chang, 1977), these wave modes become selectively excited and hence the eastward propagation.

b. Scenario II: Internally forced heat source

An initial disturbance generated near the equator will disperse away from the source region, eastward by Kelvin waves and westward by Rossby waves. In the absence of internal heating, these waves will be dissipated. However, the presence of an internal heating mechanism, such as Conditional Instability of the Second Kind (CISK), may maintain or even amplify these disturbances away from the source region. In the transient case, it is expected that the Kelvin wave will be affected more strongly than the Rossby wave because the former is a divergent wave and the latter is largely dominated by the rotational wind component. For the sake of argument, let us assume that the Kelvin waves become preferentially enhanced, and the self-maintained disturbance will propagate eastward. In an idealized sense, the interval of recurrence of the above disturbance at a particular location is determined by the time the Kelvin waves take to go around the equatorial latitude circle once.

The question remains: Where does the Kelvin wave get the energy to maintain its seemingly “perpetual” motion? One possibility is the following. When the eastward disturbance passes over warm water, it is expected to amplify by CISK due to enhanced supply of moisture from the ocean surface. Likewise, over cold water, its amplitude will decrease due to reduced moisture supply. This may be the reason why observed strong convection is confined only to the warmer part of the ocean, i.e., the equatorial Indian Ocean and the western Pacific. If the eastward propagating wave(s) can maintain itself over dissipation for one circuit around the equator, it is possible to continue the process indefinitely, provided it is rejuvenated by CISK over the warm water with enough energy to go around the next cycle. In this scenario, the origin of slow oscillation is the result of the interaction between dynamics and convection and does not require complex monsoon physics. To test this, we shall exclude all monsoonal effects such as land–sea thermal contrast, soil moisture feedback, etc. and concentrate on ocean-only boundary conditions.

In the following sections, we examine the validity of the above scenarios using a simple dynamical model. It should be noted that the two scenarios are not necessarily mutually exclusive. A discussion of this will be presented later.

4. Model description

a. Basic formulation

The model is formulated in terms of a system of primitive equations in σ-coordinates. The prognostic variables are the vertical component of vorticity $\zeta$, horizontal divergence $D$, surface pressure and temperature. The barotropic version of the model is described in detail in Lau and Lim (1984). For this study, five levels in the vertical are used. The levels are evenly discretized in the vertical with $\Delta \sigma = 0.2$. The $\sigma$-velocity is staggered in the vertical with the temperature, vorticity, divergence and geopotential field. The $\sigma$-velocity is set equal to zero at the ground and at the top of the atmosphere. The vertically discretized equations are as follows:

$$\zeta = -2\Omega (\sin \varphi \sigma D + \nu \cos \theta/a) + N_f$$

$$D = 2\Omega (\sin \theta \xi - u \cos \theta/a) - \nabla^2 (\phi + RT_0 \ln P_s) + N_D$$

$$\ln P_s = -\Delta^T D + N_c$$

$$T + SD = Q + N_T$$

$$\phi = GT$$

where $\xi, D, T, \phi, u$ and $v$ are column vectors of the vertically discretized relative vorticity, horizontal divergence, temperature, geopotential height, zonal and meridional velocity, respectively. The $N$s represent nonlinear terms as well as dissipative terms, $a$ the radius of the earth, $P_s$ the surface pressure, $Q$ the thermal forcing, $\Delta^T$ a constant row vector with the superscript $T$ denoting the transpose, and $G$ and $S$ matrices related to the thermal stratification and the vertical discretization scheme.

The horizontal representation of each dependent variable consists of a rhomboidally truncated series of spherical harmonics up to wavenumber 15. For time integration, the model adopts the semi-implicit scheme and the time filter of Roberts (1966) and Asselin (1972) commonly used by spectral general circulation models. In all experiments to be discussed in this paper, all nonlinear terms in the system are omitted, temperature perturbations are deviations from a global mean, and wind and geopotential perturbations are deviations from a basic state at rest. Surface topography is not included. Dissipative processes are represented in the form of Rayleigh friction and Newtonian cooling with
a time scale of five days. The only physics of the model is in the form of convective heating in the tropics.

b. Convective heating parameterization

Two types of convective heating are used in the model: 1) external heating with no feedback from the circulation and 2) internal heating that interacts with the circulation. For simplicity, both types of heating profile assume the form of the discrete vertical heating profile shown in Fig. 1, which is an approximation to the continuous heating profile derived from GATE observations and is commonly used in many simple model studies of the tropical circulation (e.g., Geisler, 1981). For the external heating, the horizontal pattern is also specified.

For the internal heating, we used a simple wave–CISK type parameterization. It is assumed that heating occurs only in regions of low-level convergence and that the heat released by condensation is proportional to the upward flux of moisture from the lower troposphere due to low-level moisture convergence. The heating rate, \( Q(\sigma) \), is given by

\[
Q(\sigma) = \begin{cases} 
-\eta(\sigma) r \sigma q_{\text{sat}}(T_3) D_3 \Delta \sigma / C_p, & D_3 < 0 \\
0, & D_3 \geq 0
\end{cases}
\]

where \( \eta \) is the normalized vertical profile (i.e., its vertical integral is unity); \( r \) relative humidity (=0.75); \( q_{\text{sat}}(T_3) \) saturation specific humidity at temperature \( T_3 \) at the lowest model level; \( D_3 \) divergence at the lowest level; \( m \) the moisture availability factor, which is the ratio of the total amount of moisture available for condensation to the total amount of moisture convergence at the 900 mb level within an atmospheric column; \( L \) latent heat of condensation; and \( C_p \) specific heat at constant pressure. As a result of the heating scheme in (6), only positive heating with low-level convergence is allowed. Since the actual heating is a spectral representation of (2), convective heating induced by one zonal circulation mode is not restricted to the same zonal wavenumber but may have a significant heating amplitude in other wavenumbers, i.e., energy can be exchanged between different wave scales. Thus the heating parameterization is essentially nonlinear. This is different from the wavelike heating parameterization, which assumes that dynamic forcing arises from both heating and cooling, traditionally used in linear analyses of wave–CISK (e.g., Hayashi, 1970; Lindzen, 1974). We shall return to this point later in the discussion.

5. Experiment for Scenario I

In this scenario, it is first assumed that a local oscillatory heat source of 40-day period can arise as a result of monsoon interaction in the manner described by previous authors, e.g., Goswami and Shukla (1983) and Webster (1983). An experiment is designed to study the transient response of the tropical atmosphere to a given oscillatory heat source.

Lau and Chan (1985) showed that the heat source associated with the 40–50 day oscillation appears in the form of an east–west dipole which has maximum intensity over the Indonesia–western Pacific region. For simplicity, we choose an idealized heating pattern symmetric about the equator. The horizontal shape of the heat source used in the following experiment is shown in Fig. 2. The period of the oscillation is specified at 44 days. The atmosphere is spun up from rest and the heat source turned on at Day 0. Figure 3 shows the time–longitude section of the zonal wind at 300 mb near the equator (2.2\(^\circ\)N). It can be seen that the strongest response is confined to the source region and the entire tropical atmosphere adjusts to the heating in a matter of a few days. Both eastward and westward expansion away from the source region can be discerned, but the amplitudes are rather weak. Beyond Day 20, the entire tropical atmosphere pulsates with a period of 44 days and no transient is present to any appreciable extent. This result is similar to that of Yamagata and Hayashi (1984), although they only used a monopole heat source.

Figure 4 shows the cross section of the 300 mb zonal wind at 15.5\(^\circ\)N. The amplitude of the response is significantly reduced, indicating that the disturbance is largely trapped near the equator. The eastward extension is not apparent, and the wind perturbations show a predominant westward propagation due to Rossby

---

**Fig. 1.** Vertical structure of heating profile (adopted from Geisler, 1981). Actual values used in the model layer are shown as discrete solid lines. Units in °C day\(^{-1}\).
waves. Again, the zonal wind adjusts to the oscillating heat source within about 5 days. Thereafter, the entire tropical atmosphere is oscillating in coherent phase with the dipole heat source. Figures 5a to 5c show respectively the horizontal distribution of surface pressure, 700 and 300 mb wind at Day 77, when the phase of the dipole corresponds to heating at 180° and cooling at 120°. The east–west asymmetry with Rossby (Kelvin) response to the west (east) of the source is obvious and compares well with the steady response of Gill (1980) and Lau andLim (1982). The wavy structure along 15°N and 15°S and along the equator is due to combined heating and cooling effect associated with the dipole source. The wind response is dynamically consistent with the surface pressure field with low-level convergence (divergence) over the heating (cooling) region. A wind reversal is observed at 300 mb (Fig. 5c).

The Walker-type circulation associated with the heating pattern can be depicted in the zonal mass flux streamfunction shown in Fig. 6. Here the streamfunction is defined by

$$M_\phi(p) = a \Delta \phi / g \int_{p_0}^{p} u^* dp$$  \hspace{1cm} (7)

where $u^*$ is the deviation of the zonal wind from its zonal average, $a$ the radius of the earth, $g$ the gravitational acceleration, $p_0$ the surface pressure, and $\Delta \phi$ an increment of latitude ($=10^\circ$). The above streamfunction is only an approximation to the true streamfunction, provided the divergence associated with the meridional component of the wind is small (Hartmann et al., 1984). This is the case for a heat source symmetric about the equator, where the zonal wind response is much stronger than the meridional wind. The Walker-type circulation with concentrated rising motion over the heating region and sinking motion over the cooling region is evident. Weaker secondary cells are found west and east of the main circulation cell with the former much stronger than the latter.

Overall, the above results suggest that, in the absence of a basic flow, the entire tropical atmosphere oscillates coherently with a large-scale oscillatory heat source. Free waves propagating away from the source region are not present to any significant extent. Equilibration is established in a matter of a few days, with the amplitude of the response decaying monotonically away from the source region. The horizontal scale of the main circulation is determined by the scale of the heating, and the extent of the east–west expansion is dependent on the dissipation. The major disagreement between the above result and reality is that both eastward and westward expansion of the wind perturbation are seen in the model, whereas only eastward propagation is observed in reality. Further, the eastward expansion observed in the model is far too weak to account for the eastward propagation found in observations. Another major weakness of this scenario is the assumption of stationary forcing, which excludes the dynamical effects associated with a mobile heat source.
6. Experiments for Scenario II

Three experiments are designed for this scenario. Experiment 1 (E1) is a control run in which the atmosphere is allowed to relax, with no internal heating from a specified initial condition. In Experiment 2 (E2), relaxation from the same initial condition is allowed but with CISK-like internal heating as prescribed in (6). In E2 all environmental parameters (i.e., relative humidity, mean temperature at 900 mb) are zonally symmetric. In Experiment 3 (E3), the same procedure is repeated, including internal heating as in E2 and an effect of an east–west distribution in SST. The SST effect is incorporated by including in the saturation moisture $q_{sat}(T_3)$ a geographically fixed wavenumber-1 distribution in the mean temperature field of the lowest model level, $T_3$.

The initial zonal wind distribution for E1 is the same as that obtained for Scenario 1 at Day 55 when the phase of the dipole corresponds to positive heating near 120° and negative heating at 180°. The relaxation process is initiated by removing the external heat source abruptly at a time denoted by Day 0. Figure 7 shows the time–longitude section of the 300 mb zonal wind along the equator. The dispersion and decay of the initial wave packet away from the source region is clearly seen. The speeds of the eastward and westward dispersion are consistent with those expected from equatorial Kelvin and Rossby waves at approximately 34 and 11 m s$^{-1}$, respectively. As expected, in the absence of any internal heating, the disturbances are
damped and reduced to insignificant amplitude by about 10 days.

Figure 8 shows the time–longitude section of the 300 mb zonal wind for E2. It can be seen that when internal heating is included, a mobile disturbance is produced with a constant eastward phase speed estimated to be approximately 19 m s\(^{-1}\) throughout the 36 days shown. At this speed, the disturbance makes one complete circuit around the globe in about 24 days. Results from further experiments (not shown) indicate that this eastward propagation disturbance will be amplified indefinitely when the moisture factor, \(m > 0.8\). These results indicate that the value of \(m = 0.8\) used in the experiment shown in Fig. 8 is close to the marginally unstable condition for the internal heating parameterization in (6). It is important to note that the westward-propagating (Rossby) branch is not significantly affected by the internal heating. There appear to be some westward-propagating transients, but they are much less organized. This result shows vividly the east–west asymmetry in the effect of CISK on transient wave propagation along the equator. The same eastward propagation is apparent in the surface pressure, the temperature and geopotential height field at all levels near the equator (not shown), suggesting that the disturbance has a coherent three-dimensional structure.

Figure 9 shows the space–time variation of the heating rate associated with the propagating disturbance for E2. The heating is estimated in terms of a rainfall rate, assuming that the total latent heat release by CISK over the atmosphere column results in rain. The eastward propagation is again evident. The spatial scale of the heating (\(\sim 10^3\) km) is small compared with that of the circulation (\(\sim 10^4\) km) but is large compared with that of cumulus cloud or cluster (\(\sim 10^2\) km). Thus, one may imagine the eastward movement of the mobile disturbance as that associated with a super-cloud cluster. The initial adjustment in the heating field to the onset of CISK is essentially complete within the first 5 days, after which the rainfall rate is maintained at a background level of approximately 4 mm day\(^{-1}\) over a small area that propagates eastward along with a surface low pressure center (not shown). The maximum precipitation rate reached is over 7 mm day\(^{-1}\).
The horizontal and vertical scale of the circulation driven by the above interactive heat source can be seen in Fig. 10, where the instantaneous mass-flux stream-function at Day 24 is shown. The convective updraught near the center of the heating is substantial up to about 300 mb. The associated subsidence is weaker but extends over a longitudinal span of about 180°. The eastern branch of the circulation appears to be somewhat stronger than the westward branch. A westward tilt of the axis of the circulation can be discerned. The zonal mass flux appears to be stronger above (below) the 600 mb level for the eastern (western) branch of the circulation. These east–west asymmetries in the vertical circulation are maintained throughout its journey around the globe.

In E3, a wavenumber-1 distribution in the mean temperature at 900 mb is superimposed on the global mean temperature to mimic the effect of east–west variation in SST on convective heating. The peak-to-peak amplitude of the temperature wave is 8°C, with the maximum located at 270° and the minimum at 90° longitude. Hereafter, we shall refer to the longitude span from 180° to 360° as the warm sector and 0° to 180° as the cold sector. Figures 11 and 12 show, respectively, the time–longitude section of the 300 mb zonal wind and the convective heating for the first 36 days. The amplitude of the eastward–propagating disturbance is seen to be strongly modulated by SST effect. Over the warm sector, CISK–induced response is more pronounced, as is evident in the simultaneous increase in rainfall and zonal wind over that region. There is a lag of about 3–4 days between the passage of the rising branch of the disturbance over the warmest water at 270° and the maximum wind response aloft, which occurs downstream near 330°. Over the cold sector, a corresponding reduction in the intensity of the disturbance is observed with a temporal lag similar to that over the warm sector. The disturbance appears to be rejuvenated as it reenters the warm ocean from Day 24 to Day 36.

Figures 13 and 14 show the instantaneous maps of the zonal mass flux along the equator, the surface pressure and 300 mb wind perturbations for Day 12 and Day 24 respectively for E3. At Day 12, the vertical ascent over the region of strong convection near the eastern edge of the warm sector is very concentrated and intense (Fig. 13a). The vertical phase tilt at the center of the disturbance is again westward. This westward tilt appears to be a consistent feature for all the model eastward–propagating disturbances in all the experiments we have tried. Also noticed is a downstream subsidence associated with a secondary circulation that appears to be breaking away from the main Walker cell. This secondary disturbance is transient in nature and has opposite vertical tilt to that of the primary disturbance, suggesting that away from the source region the upper level disturbance is moving faster than the lower level.

It is interesting to note that the surface pressure (Fig. 13b) has a large zonally symmetric component, even though the heating itself has a much smaller spatial scale. This is apparently due to our heating parameterization, which allows only positive heating and therefore a nonzero zonally symmetric component. The maximum low pressure center is located immediately to the east of the ascending motion. The pressure tendency (not shown) indicates falling pressure ahead of the surface low consistent with the eastward
propagation of the disturbance. In the upper level wind field (Fig. 13c), it is noted that the zonal wind response is much stronger east than west of the heat source. This is different from the steady state solution of Gill (1980), where the response to the west generally has larger amplitude. The reduced Rossby wave response is also apparent in the much weaker amplitude of the two anticyclones to the northwest and southwest of the forcing than those obtained for the steady state.

At Day 24, the rising branch of the wave has moved to the edge of the cold sector (Fig. 14a). Here, as a result of the reduction in available moisture over the cold ocean, the intensity of the disturbance is substantially reduced. However, the spatial structure of the zonal circulation is similar to that at Day 12 except for a slight increase in the westward vertical tilt. In this case, the transient features downstream of the main Walker cells disappear. The surface pressure and upper level wind (Fig. 14b and c) show that, except for a reduction in intensity, the spatial structure of the primary disturbance is essentially unchanged from Day 12.

In summary, the eastward propagation appears to be well simulated by the effect of wave-CISK on wave dispersion near the equator. The modulation of the intensity of the disturbance by the idealized SST effects appears to have captured some features of the observed amplitude modulation. The main problem is in the phase speed of the propagation. While the observed phase speed of the most dominant low-frequency oscillation is in the range from 5 to 10 m s\(^{-1}\), the model disturbance propagates with a speed about a factor of two faster. To better understand the above in terms of the model framework, the eastward propagation and the phase speed will be examined in detail in the following.

7. Mechanism for eastward propagation

It was noted in the discussion of the instantaneous maps of 300 mb wind distribution (e.g., Figs. 13c and 14c) that the Rossby wave response is suppressed compared to the steady state response. This provides a clue
and for Rossby waves

$$q_{n+1}(x, t) = (nQ_{n+1} + Q_{n-1}) \exp(-\epsilon x/c_n) \times [L_n(x) - L_n(x + c_n t)], \quad \text{for } n \geq 1,$$

where $\epsilon$ is the dissipation constant, $Q_n$ are the components of the heating function, $q_{n}$ is a linear function of pressure and zonal wind as defined in the Appendix, $c_n = (gh)^{1/2}$ and $c_p = c_0/(2n + 1)$ are the phase speeds of the Kelvin and Rossby waves, respectively; $L_n$ are shape functions dependent on the horizontal distribution of the heat source. The index $n$ represents the order of the parabolic cylinder function. All other symbols are defined in the Appendix. Equations (8) and (9) represent Kelvin waves ($n = 0$) moving eastward with speed $c_0$ and Rossby waves ($n \geq 1$) moving westward with speed $c_n$. The divergence field for each wave mode can be computed from (8) and (9) (see Appendix).

The analytic form of the divergence associated with the transient waves to a heating function that is Gaussian-shaped in the horizontal at different times is shown in Fig. 15. To compare with the relaxation experiments discussed previously, the transient response for the switch-off of the heating from the steady state condition is shown. The divergence field shown corresponds to the lower level. At Day 2, the low-level convergence at the source associated with the eastward (Kelvin) branch and that associated with the westward (Rossby) branch begins to separate, with the former having a local maximum at the equator and the latter two local maxima located symmetrically away from the equator. Near the equator, the ratio of the magnitude of the Kelvin- to the Rossby-wave divergence is about 3 to 1, as expected from the calculation shown in the Appendix. Most of the convergence is associated with the portion of the circulation that propagates eastward. By Day 4, the Kelvin and Rossby waves are clearly separated. At the equator, the convergence associated with the Rossby wave is still relatively small. Calculation in the Appendix shows that the maximum is located at about 1.7 times the Rossby radius of deformation (approximately 12°) away from the equator. The convergence associated with the Kelvin wave remains strong up to Day 6. This east-west asymmetry appears to be maintained throughout the relaxation process. Therefore, in the presence of CISK, which is initiated by low-level convergence, the Kelvin wave is expected to be selectively amplified. This is seen in the following results.

b. Divergence field for E1 and E2

The instantaneous divergence field at $\sigma = 0.9$ for E1 and E2 is shown respectively in Figs. 16 and 17. The result for E1 is similar to that for the analytic solution. The predominance of the equatorial divergence field associated with the eastward-propagating disturbance is apparent. The difference in the shape of the pattern compared with Fig. 15 is due to the use of dipole heat-

a. Analytic solution to switch-on heating

The transient response for shallow-water waves in the equatorial $\beta$-plane to a localized heating can be represented by the following general expressions (cf. Lau and Lim, 1982, and Appendix). For Kelvin waves,

$$q_0(x, t) = Q_0 \exp(-\epsilon x/c_0) \times [L_0(x) - L_0(x - c_0 t)], \quad (8)$$

to the cause of the eastward propagation in the model. Because a Kelvin wave is composed solely of divergent wind, it is possible that the associated low-level convergence is larger than a Rossby wave, which is largely rotational. Therefore, the former will be enhanced by wave–CISK more than the latter. As will be shown later, the divergence due to a symmetric Rossby wave has weak maxima away from the equator and very small amplitude along the equator. Therefore, if the initial disturbance is located near the equator, the divergence due to the Kelvin wave will predominate. In the presence of wave–CISK, the above effects combine to give a selective enhancement of the Kelvin wave. Since the heat source moves eastward along with the Kelvin-wave convergence, it continuously excites new Kelvin-wave fronts to its east. The Rossby waves, while also being continuously excited to the west, because of their slower response time compared with the Kelvin waves, never have sufficient time to attain full response before a new heat source establishes itself farther eastward. In a Lagrangian sense, a new equilibrium state is achieved as a result of the mobile heat source interacting with the wind field. The above arguments can be demonstrated quantitatively by the following analyses.
ing in the initial condition for the numerical experiment. For E2, the effect of CKS is noticed by Day 2 and becomes very pronounced by Day 4 (Fig. 17). During this period, the atmosphere is still undergoing readjustments to the initial condition and CKS. It is noticed that during the first two days the eastward propagation of the divergence pattern is similar to that for E1. The speed of the eastward propagation is estimated to be close to that of the free waves in E1, i.e., approximately 35 m s⁻¹. During the equilibration between Day 2 to Day 6, the eastward propagation speed is reduced. By Day 6, the adjustment is essentially complete and a new low-level convergence pattern emerges to the east while the divergence to the west is much reduced. As we have shown, this new divergence propagates eastward with a speed of about 19 m s⁻¹. It is noticed that at Day 4, a Rossby wave couplet is generated poleward and westward of the mobile heat source. But this couplet appears to trail behind the moving heat source and eventually dissipate. As the disturbance propagates eastward, weak Rossby-wave couplets appear to be generated continuously at the trailing edge but rapidly “shedded” by the mobile heat source. This is because Rossby waves, once generated, will propagate westward away from the source region at the same time the source moves eastward. This gives rise to the characteristic appearance of the divergence pattern for the matured disturbance at Day 6. As we have seen before, the circulation features associated with this convergence pattern remain basically unchanged as the disturbance propagates eastward.

8. Mechanism for slow time scales

The major problem with the results in E2 and E3 is that the model disturbance propagates about twice as
fast as the observed. While it is believed that, because of the lack of a realistic moist process, the observed phase speed is unlikely to be simulated using the present form of the model (see discussion in section 8), it is instructive to see why and how the model interactive disturbance selects its preferred structure and propagation speed. It is hoped that this will shed light on the real cause of the slow motion. In this section, the dependence of the phase speed on the basic model structure and physics will be examined.

a. Model normal modes

For linear nondissipative motions, the Ns in (1) to (4) vanish and the vorticity and divergence equations are not vertically coupled. Equations (3) to (5) may be combined into a single vector equation of \( h \)

\[
h_t + \mathbf{C} \mathbf{D} = \mathbf{G} \mathbf{Q}/g
\]

where

\[
gh = \phi + RT_0 \ln P_s
\]

\[
\mathbf{C} = (\mathbf{G} \mathbf{S} + RT_0 \Delta^T)/g.
\]

The vertical normal modes of the model are given by the eigenvectors \( \mathbf{e}_j \) of the matrix \( \mathbf{C} \) for \( j = 1, \ldots, 5 \). The corresponding eigenvalues, \( H_j \), are the equivalent depths of the vertical modes whose phase speed is given by \( c_j = (gh_j)^{1/2} \). Any vertically discretized variable may be expressed in terms of the vertical modes as

\[
\mathbf{u} = \sum_{j=1}^{5} \mathbf{u}_j \mathbf{e}_j
\]

where \( \mathbf{u}_j \) is the projection of vector \( \mathbf{u} \) on the \( m \)th normal mode.
Figure 18 shows the vertical structure of the five normal modes of the model numbered according to the magnitude of its eigenvalue \( H_j \). In the figure, they are identified by the letters A to E. The basic phase speed, \( c_j \), for each mode, which is the same as that of the equatorial Kelvin waves, is given in Table 1, row 1. The first mode is the external mode with a barotropic structure and a large phase speed close to that of sound waves. The remaining are baroclinic modes with gradually decreasing vertical scale as the mode number increases. The second and third modes, respectively, have single amplitude maximum near the top of the atmosphere and at 300 mb. The fourth mode has a phase reversal above and below 400 mb. The fifth mode has two reversals in the vertical. Table 1, row 1 (\( m = 0 \)) shows that in the absence of internal heating the above-normal modes contain fast and slow motions with a range of possible phase speeds from approximately 300 to 6 m s\(^{-1}\). The phase speed (about 34 m s\(^{-1}\)) of the eastward dispersion in E1 is close to that of the third mode, suggesting large wind response in this mode. This is reasonable because the external heating profile used to generate the initial disturbance has a maximum between 400 and 600 mb close to where the third mode changes sign.

The structure of the eastward-propagating disturbance is now examined in terms of the normal modes in the model. Figure 19 shows the projection, \( u_j \), of the zonal wind into each of the five normal modes at all longitudes and for Day 12 and Day 24 in E3, as given in (13). It is recalled that at Day 12 the disturbance is over the eastern end of the warm sector with enhanced convection, while at Day 24 it is at the eastern edge of the cold sector with suppressed convection. This is evident in the overall smaller amplitude of the normal-mode projection at Day 24 compared with that at Day 12. It is noted that near the center of the disturbance the response is largely in the third and the fourth normal modes. The larger response in the third (fourth) mode at the leading (trailing) edge of the disturbance is consistent with the east–west asymmetry in vertical structure of the zonal mass circulation noted earlier. Away from the disturbance region, the responses for all modes have either small amplitudes or large cancellation. The relative contribution by the various modes near the disturbance center remains unchanged, indicating that, except for modulation of amplitude by the underlying SST distribution, the vertical normal modes are phase-locked, resulting in a constant vertical structure as the disturbance propagates eastward. From
Fig. 16. Instantaneous low-level convergence at Day 0 through Day 6 at 2-day intervals for E1. Contour interval is $2.3 \times 10^{-2}$ s$^{-1}$.

Fig. 17. As in Fig. 15 except for E2.
Table 1, row 1, it can be seen the two largest normal modes that make up the disturbance have phase speeds of 34.0 and 15.3 m s⁻¹. At the two speeds, the time required for the disturbance to complete one circuit around the globe is 13.6 and 30 days. However, the model results for E2 and E3 suggest a period of approximately 24 days. A plausible explanation of the difference is that the inclusion of internal heating may modify the normal modes of the system and hence alter the phase speeds of its eigenmodes. This modification is a characteristic of the model and the specific heating parameterization used. In the following, this problem will be addressed.

**b. Normal modes modified by internal heating**

Since the internal heating in (6) is proportional to the horizontal divergence at level 5, it may be rewritten as

\[
Q = -LD
\]  

(14)

where the matrix \( L \) is given by

\[
L = mLq_{vol}(T_e) \Delta \sigma C_p^{-1} \times \eta J^T, \quad \text{for} \quad j = 1, \ldots, 5.
\]

(15)

where \( J^T \) is the row vector \((0, 0, 0, 0, 1)\) and \( \eta \) is a vector representing the vertical heating profile. Substituting for \( Q \) into (10), the vertical structure equation can be written as

\[
h_t + C^*D = 0
\]

(16)

\[
gC^* = G(S + L) + RT_0 \Delta^T.
\]

(17)

Comparing (17) with (12), it can be seen that the effect of internal heating is to modify the effective static sta-
bility of the system and hence the equivalent depth and vertical modes of the system. As a result of the heating effect, the original eigenmodes are mixed to produce new eigenmodes. Before the results for the modified normal modes can be compared to the numerical integration, a clarification is essential. In the numerical integration, positive-only heating is used. Mathematically, it is equivalent to introducing a nonzero zonal averaged heating and retaining amplitude in the same wavenumber equal to half of that in the original wavelike heating plus a wave packet containing higher wavenumbers (cf. Lindzen, 1974). For example, a heating of the form

\[ Q = \begin{cases} Q_0 \cos kx, & Q > 0 \\ 0, & Q \leq 0, \end{cases} \]  

(18)

can be expressed as

\[ Q = Q_0 / \pi + \frac{1}{2} Q_0 \cos kx + \ldots. \]  

(19)

From (19), in order to compare the linear analysis to the previous results, the total heating amount used should be twice that used in the numerical integration. Furthermore, in making direct comparison between the linear analysis and the numerical integration, we must also keep in mind the possible effect of dissipation which is not included in the normal mode analysis. The interpretation of the simulated model disturbance discussed in the following is similar to that suggested by Chang and Lim (1987).

In general, results shows that the overall changes in structure of all but the third and fourth eigenmodes are small. Figure 20 shows the changes in these two normal modes as the heating parameter \( m \) changes. It can be seen that the two modes draw close to each other and coalesce into a single mode when at the value of \( m = 0.8 \). At this value of \( m \), the phase speed associated with this normal mode is complex and contains an eastward, growing component. The normal mode structure shown in Fig. 20 for \( m = 0.8 \) is the amplitude of the complex vertical structure.

Table 1 shows the phase speeds of the modified normal modes as a function of the magnitude of the moisture availability factor \( m \). As a result of increased heating, a reduction (increase) in phase speed is found in the third (fourth) modified normal mode. The tendency for the dominant phase speeds to draw closer as a result of heating has been noted in other studies of CISK (Crum and Stevens, 1983). At \( m = 0.8 \), the two modes coalesce and a growing mode with speed of 21.8 m s\(^{-1}\) is excited. The phase speed of the growing mode decreases with increasing \( m \). At \( m = 1.5 \) (approximately twice that used in the numerical integration), it reduces to the same as in the model. For a given wavenumber, the vertical phase shift resulting from the nonzero imaginary part of the phase speed has been calculated and is found to be consistent with the westward phase tilt with height observed in the model disturbance. Thus, for a growing Kelvin-CISK mode, the westward tilt with height appears to be an essential feature. This interpretation is consistent with earlier description in terms of the unmodified normal modes. As noted before, two dominant unmodified modes are required for a complete description of the east–west asymmetry of the vertical circulation.

The above situation is considerably more complicated than single-mode systems, e.g., shallow water systems, where it has been shown that the inclusion of heating proportional to convergence reduces the static stability and therefore always lowers the phase speed (Gill, 1982). For a multilevel system, the heating effect takes the form of mixing of vertical modes. In general, from (17) the modified normal mode phase speeds are functions of both the static stability and the vertical scale of the heating. A more detailed discussion of the above in terms of the present model results is reported in Chang and Lim (1987).

The sensitivity of the phase speeds of the modified normal modes to the vertical structure of the heating has been studied by repeating the normal mode calculations for various heating profiles. The results for \( m = 0.8 \) are summarized in Table 2. For all heating profiles whose level of maximum heating is at or above 500 mb, the third or the fourth original normal mode coalesce to form a single unstable mode with a phase speed range of 24 to 16 m s\(^{-1}\). In general, as the heating profile is lowered, the phase speed decreases. A very slow unstable mode of phase speed 7.6 m s\(^{-1}\) is excited when the level of maximum heating is lowered to 700 mb. Thus, it is possible to generate very slow phase

![Fig. 20. Vertical structure of the third (C) and fourth (D) normal modes of the model atmosphere modified by internal heating for different values of m. The two modes coalesce at m = 0.8.](image-url)
Table 2. Phase speed (m s⁻¹) of the model normal modes as a function of shape of the vertical heating profile.

| Vertical heating profile shown as normalized weights | Mode |
| --- | --- | --- | --- | --- |
| 900, 700, 500, 300, 100 mb | 1 | 2 | 3 | 4 | 5 |
| (0.00, 0.01, 0.45, 0.51, 0.03) | 309.3 | 76.4 | 24.4* | 24.4* | 5.2 |
| (0.01, 0.18, 0.40, 0.38, 0.03)# | 309.2 | 77.9 | 21.8* | 21.8* | 5.6 |
| (0.01, 0.23, 0.39, 0.31, 0.06) | 309.2 | 78.3 | 20.8* | 20.8* | 6.3 |
| (0.02, 0.28, 0.40, 0.28, 0.02) | 309.2 | 78.8 | 19.6* | 19.6* | 6.6 |
| (0.06, 0.31, 0.39, 0.23, 0.01) | 309.2 | 79.4 | 18.3* | 18.3* | 6.7 |
| (0.03, 0.38, 0.40, 0.18, 0.01) | 309.1 | 79.8 | 15.9* | 15.9* | 10.2 |
| (0.03, 0.51, 0.45, 0.01, 0.00) | 309.1 | 81.1 | 20.9 | 7.6* | 7.6* |

# Heating profile used in this model.
* Indicates real part of complex phase speed for the unstable mode.

Speeds provided the heating has a large component at low levels. In the real atmosphere, such heating may come from surface evaporation or boundary layer effects. At present, work is being undertaken to include these effects in the present model.

9. Comparison with traditional wave-CISK theory

The foregoing results suggest that, except for the phase speed, the most basic feature (i.e., eastward propagation) and, to some extent, the amplitude modulation of the low-frequency oscillation in the tropics are fairly well described by Scenario II. These features can be explained in terms of a mechanism of interaction between dynamics and convection not considered in previous studies of wave-CISK (e.g., Hayashi, 1970; Lindzen, 1974; Chang, 1976; Stevens and Lindzen, 1978; Mak, 1981; and many others). In most traditional linear instability studies of wave-CISK, the amount of heating is effectively made proportional to the wave divergence at one or more levels irrespective of its sign. This means that the heating function itself is wave-like. As a result, physically unreasonable negative heating (or negative rainfall amount) is also included. Although negative heating can sometimes be thought of as a perturbation in the actual heating due to the convergence in the basic state, unless the basic-state atmosphere is raining heavily everywhere in the region of interest, the negative heating can still run into problems. On the other hand, the heating parameterization we used here assumes that the atmosphere has no convergence in the basic state (or everywhere dry except in the heating region) so that only positive heating is present. While the real tropical atmosphere is somewhere in between the two situations, our heating parameterization brings out the above asymmetry in its most idealized form.

As a result of the inherent nonlinearity present in the positive-only heating parameterization (6), disturbance of a single wavenumber can lead to heating and response in other wavenumbers. In the purely linear case, i.e., the heating is also wave-like, the heat forcing for a given zonal wavenumber can only affect the response in the same wavenumber. The different atmospheric response due to positive-only heating and the wave-like heating is clearly shown in Figs. 21a, b. Here a blowup of the precipitation time–longitude section at the equator for the first 12 days after relaxation is shown for E2 and a similar experiment performed with wavelike heating but with $m = 0.4$. For E2, the organization of the eastward-propagating super-cloud cluster becomes fully established in about 5–6 days. Westward-propagating gravity-wave-like rainbands are also excited, but with rapid growth and decay rates. These high-frequency westward-moving rainbands do not organize into super-cloud clusters (see also Fig. 9). In contrast, the wavelike heating fails to establish a highly organized eastward-propagating super-cloud cluster. With wavelike heating, results (see following discussion) indicate that increasing the precipitation efficiency ($m \geq 0.6$) leads to immediate destabilization of the initial disturbance into small-scale waves. Also, these waves do not show organized eastward propagation.

Figure 22 shows the wavenumber spectrum of the 300 mb zonal wind as a function of the wavenumber for E2. It can be seen that the initial disturbance is dominated by planetary scale waves (wavenumbers 1 to 4), with largest amplitudes in wavenumbers 2 and 3. As a result of the positive-only CISK heating, there is a noticeable redistribution of energy in the wavenumber domain with strong reduction in wavenumbers 2 to 3 up to Day 12 and a buildup of amplitudes in other wavenumbers that are not present in the initial disturbance. In particular, the buildup of the zonally symmetric component (wavenumber 0) should be noted. This is because of the nonzero zonal-averaged heating component when negative heating is eliminated as shown in (19). When the disturbance reaches some form of dynamic equilibrium at Day 36, the planetary waves regain some energy, with the largest amplitude now appearing in wavenumber 1 and significant amplitude in wavenumbers 2 and 3.

The energy distribution for an identical experiment with $m = 0.8$, but with wavelike heating, is dramatically
two results shows that positive-only heating can channel energy from high to low wavenumbers. As a result, the most unstable mode is a wave-packet disturbance with a red-noise type distribution having its largest contribution from planetary scale waves. The wavenumber 1 amplitude is largest because it is the largest wave component that can possibly fit on a finite equator.

The net effect of the positive-only heating on the scale of the disturbance is estimated as follows. The wavenumber spectrum \( A \) for the vertical velocity at the \( \sigma = 0.9 \) level is computed as a function of time. The corresponding spectrum \( B \) for the upward-only \( (i.e., \text{if } \sigma > 0, \sigma \text{ is set equal to zero}) \) vertical velocity is computed. The latter represents the spectrum of the positive-only heating function for each wavenumber. The net effect between the positive-only heating function and the wavelike heating on each wavenumber is then obtained by the quantity \( \delta_n = B_n - A_n \) for each wavenumber \( n \). Figure 24a shows \( \delta_n \) for wavenumbers 0 and 1 as a function of time. It can be seen that during the initial period of adjustment (Day 0 to Day 6) fast gravity-wavelike oscillations take place with a period on the order of 2 days. Throughout the integration period, the positive-only heating contribution to these two wavenumbers stays positive, except for brief intervals during the first 6 days for wavenumber 1. Beyond Day 6, there appears to be a steady rise in the net heating effect for these two wavenumbers. The effect of the net heating for other wavenumbers is best illustrated by the “cumulative” net heating, which is given by

\[
\Delta_n(I\Delta t) = \sum_{i=0}^{I} \delta_n(i\Delta t),
\]

where \( \Delta t = \frac{1}{4} \) day. Figure 24b shows \( \Delta_n \) for wavenumbers 0 to 7. A clear separation between wavenumbers 0 and 1 and the rest can be seen. The slopes of the cumulative curves indicate that the net effect on waves 0 and 1 are always positive but are negative for all other wavenumbers. Thus, relative to the wavelike heating, the positive-only heating inhibits the growth of the small-scale waves by effectively reducing the wave forcing for these waves as well as allowing an upscale transfer of energy. In particular, the positive-only heating may also account for the large zonally symmetric response in the model results noted in the preceding discussions. Zonally symmetric components in upper level temperature anomalies associated with localized tropical heating have been observed (Horel and Wallace, 1981) and may be related to the above mechanism.

10. Further discussion

To distinguish from the traditional wave-CISK, we refer to the mechanism discussed in the preceding sections as “mobile” wave-CISK. It is interesting to note that as a result of mobile wave-CISK, the equilibrium
Fig. 22. Amplitude of zonal wind response as a function of wave-number for E2 at different simulated days after relaxation. Units in m s$^{-1}$.

Fig. 23. As in Fig. 22, for an identical experiment, but with wavelike heating.
the equilibrium state may be viewed as a stationary response in the presence of mean easterlies in a frame moving along with the disturbance, i.e., a Lagrangian steady state. The westward tilt with height of the mobile disturbance is the result of an eastward-propagating unstable mode generating by wave-CISK. Westward vertical tilt of the 40–50 day wave has been observed by Murakami and Nakazawa (1985).

Because the structure remains essentially unchanged during propagation, the disturbance is apparently nondispersive and the propagation can also be described as a group velocity phenomenon. This may be a further reason why Kelvin waves are favored over Rossby waves. The nondispersive nature of the former would seem to make it easier for cooperative effect between convection and circulation to operate.

In the real atmosphere, the convection associated with the 40–50 day oscillation is likely to be made up of a large number of super-cloud clusters, which form and decay respectively over the warm and cold parts of the ocean. Mobile wave-CISK favors the development of planetary waves, in particular wavenumbers 0 and 1. The predominance of wavenumber-1 response in the observed wind field for the 40–50 day waves may be due to the further enhancement in this wave scale by an ensemble of these super-cloud clusters whose life cycles are strongly modulated by the underlying large-scale SST distribution as they propagate eastward around the globe. Results from recent GCM “aqua-planet” model integrations confirm the above result (Hayashi and Sumi, personal communication, 1986).

The apparent success in simulating the eastward propagation of the low-frequency oscillation does not necessarily preclude the relevance of Scenario I in a regional description of the oscillation. Yamagata and Hayashi (1984), using a shallow-water model, showed that by using different coefficients in the Newtonian cooling and wind dissipation, the atmospheric response shows a distinct poleward propagation from an oscillating tropical heat source in the tropics. Observations by Lau and Chan (1985) showed that the dipole heat source associated with the 40–50 day oscillation appears to be amplified and the eastward propagation somewhat stalled over the maritime continent/equatorial central Pacific region. The poleward propagation is most pronounced over the summer monsoon region of East Asia and India and at times even more so than the eastward propagation along the equator (Lau and Chan, 1985a). Over these regions, the 40–50 day oscillation appears also to be phase-locked to a quasi-20-day oscillation (Krishnamurti et al., 1985). Thus, the description of a stationary oscillating heat source may not be entirely inappropriate over these regions. It is likely that monsoon dynamics, including the interaction between soil moisture and cloud radiation, may play an important role in shaping the regional characteristics of the 40–50 day oscillation.

---

FIG. 24. (a) Differential amplitude of the 900 mb vertical velocity for positive-only CISK, and for zonal wavenumbers 0 and 1 as a function of time. See text for detailed explanations. Units of $10^{-3}$ mb s$^{-1}$. (b) Cumulative curves for differential amplitude of 900 mb vertical velocity for positive-only CISK and for zonal wavenumbers 1 to 7. Units of $10^{-3}$ mb s$^{-1}$.

state obtained in the model is different from that of the steady state solution with no mean wind (Gill, 1980; Matsuno, 1966) but compares well with the transient solution for localized heat source in the tropics in the presence of mean easterly wind (Lau and Lim, 1982). In Lau and Lim, it was observed that in the presence of mean easterlies (westerlies) the Rossby wave response to the west of the forcing is substantially reduced (enhanced). The effect of mean wind on the Kelvin wave is opposite in sense to that of the Rossby wave. Thus,
Recently, Lau and Lau (1986) successfully identified low-frequency oscillation in the GFDL GCM similar to that described in this paper. While we were writing this paper, it came to our attention that such oscillations have been found in many more GCMs, such as those at NCAR, the United Kingdom Meteorological Office, ECMWF, the Meteorological Research Institute in Japan and others. Results from idealized experiments with the so-called aqua-planet model integrations, such as those performed by Hayashi and Sumi, compared very favorably with those predicted by the present theory. Others who have carried out similar integrations include Dr. T. Palmer at ECMWF and Drs. N. C. Lau and I. Held at GFDL. In general, these models produce eastward propagation faster than the observed 40–50 day oscillation. Thus, the fast phase speed appears to be a problem common to these models. Current work at GFDL suggests that this discrepancy can be alleviated by increasing the horizontal resolution of the model. Within our present simple model framework, further understanding of the mechanism of the slow time scale can be gained by including a realistic moisture budget with surface heating and a realistic basic flow. Recycling time of moisture in convection is likely to be important in limiting the growth of instability and altering the phase speed of propagation. Since the real atmosphere has mean upper level zonal easterlies of about 5 m s\(^{-1}\), including a realistic zonal flow should significantly reduce the mean ground speed and hence the recurrence interval of the disturbance.

11. Conclusion

A theory for the origin of low-frequency oscillations in the tropical atmosphere is proposed and tested by an idealized model. It is found that the eastward propagation of low-frequency oscillations arises as a result of the interaction between convection and dynamics by the so-called mobile wave-CISK mechanism. Via this mechanism, the heat source feeds on the intrinsic east–west asymmetry of equatorial waves. As a result, Kelvin waves are selectively amplified, which in turn causes the heat source to propagate eastward. However, the phase speed of the propagation is much faster than observed and is dependent on the shape of the vertical heating profile. The model mobile disturbance is likened to a propagating super-cloud cluster having the following characteristics:

1) Under perfectly zonally symmetric boundary conditions, it propagates eastward with constant phase speed of about 19 m s\(^{-1}\), while essentially maintaining the same thermally direct zonal circulation structure; i.e., it is nondispersive.

2) The disturbance has a westward tilt with height. Its phase speed and vertical structure is consistent with that predicted from the most unstable heat-modified normal mode of the system. This result is consistent with that suggested by Chang and Lim (1987).

3) The mobile wave-CISK mechanism gives rise to a new equilibrium (or a Lagrangian steady state) between the heat source and the circulation. In this Lagrangian steady state, the Rossby wave response to the west of the heat source is suppressed and the Kelvin wave response to the east is enhanced. The new state can be compared to the steady state response in the presence of mean easterlies.

4) The horizontal scale of the zonal circulation is mainly determined by a combination of dissipation and the positive-only heating parameterization. The latter allows an upscale transfer of energy, resulting in enhanced response in low wavenumbers, especially wavenumbers 0 and 1. It also inhibits unstable growth for large wavenumbers. As a result, the most unstable mode is best described as a wave packet and its propagation as a group velocity phenomenon.

5) The amplitude of the CISK-induced mobile heat source/circulation system is strongly modified by the east–west SST distribution. When the disturbance passes over warm water, its intensity is enhanced substantially due to the increased moisture supply. The reverse occurs when the disturbance is over cold water.

It was also found that a quasi-stationary oscillatory heat source (Scenario 1) alone cannot give rise to the observed eastward propagation. Nonetheless, this kind of heat source may be important in accounting for the regional characteristics of the low-frequency oscillation over the monsoon region.

Finally, it was noted from observation that, except for the time scale, the eastward propagation of the intraseasonal oscillation discussed here appears to belong to the same family of phenomena observed during ENSO (Lau and Chan, 1985, 1986a,b). The results in this study and in GCM simulations (e.g., Lau and Lau, 1986) suggest that low-frequency (intraseasonal) oscillations are intrinsic atmospheric phenomena that exist without a dynamically interactive ocean. On the other hand, ENSO is known to be a strongly coupled ocean–atmosphere phenomenon. For these two phenomena to be linked, one possibility is that the ocean interacts with the atmosphere, resulting in an amplification and further slowing down of the 40–50 day disturbance. Lau (1985a,b) has hypothesized that the 40–50 day waves may act as stochastic forcings that trigger instability in the ocean–atmosphere system leading to the onset of ENSO. This hypothesis can only be tested in coupled ocean–atmosphere models. However, except for coupled GCMs, most present simple coupled models of ENSO (e.g., Cane and Zebiak, 1985; Anderson and McCreary, 1985) do not include active atmospheric components. It is expected that the stability property of the coupled system will be very different when low-frequency atmospheric components are included. This may be the reason why present coupled models can only produce "deterministic" ENSOs, which occur at much more regular intervals than nature. It is likely that the inclusion of intrinsic atmospheric variability
in these models is necessary to understand the onset mechanism and the observed intermittency in the occurrence of ENSO.

Acknowledgments. This work was supported by the NASA Global Scale Program. All of the computation was done at the NASA Vector Processing Facility, Goddard Space Flight Center. The National Science Foundation also provided partial support through Grant ATM-8414834 to the Department of Meteorology, University of Maryland. The authors wish to thank Drs. C. H. Sui, M. Suarez, Professors A. Sumi and T. Matsuno for stimulating discussions. In particular, we thank Professor C. P. Chang for reading the draft manuscript and making suggestions that lead to a better interpretation of the results.

APPENDIX

Divergence For the Kelvin Wave

The nondimensional time-dependent equations for heat-induced motions in the equatorial $\beta$-plane are given by the following (Gill, 1980):

\[ u_t + eu - \frac{1}{2} y v = -p_x \]  
(A1)

\[ \frac{1}{2} y u = -p_y \]  
(A2)

\[ p_t + e(p_x + u_x + v_y) = -Q \]  
(A3)

where the length and time scales have been normalized by the basic scales $(c/2\beta)^{1/2}$ and $(2\beta c)^{-1/2}$, respectively. The velocity $c = (gh)^{1/2}$ is the basic speed for a shallow-water system with equivalent depth $h$. Other symbols have their usual meanings. The time-dependent solution of the above to a switch-on heating has been obtained by Lau and Lim (1982). The following is a brief summary of the solution. By expanding the above equations in terms of parabolic cylinder functions, they can be reduced to

\[ (q_0)_t + (q_0)_x + e q_0 = Q_0 \]  
(A4)

\[ (q_{n+1})_t - (2n + 1)^{-1}(q_{n+1})_x + e q_{n+1} \]

\[ = (m Q_{n+1} + Q_{n-1})/(2n + 1) \]  
(A5)

\[ v_x = 2(n + 1)/(2n + 1)(q_{n+1})_x \]

\[ r_{n-1} = (n + 1)q_{n+1}, \text{ for } n \geq 1 \]  
(A7)

\[ q_{1,0} \text{ and } v_0 = -Q_1 \]  
(A8)

where $q = p + u$ and $r = p - u$, the subscripts $t, x$ refer to differentiation, and the subscript $n$ denotes the index of the expansion coefficients of the above variables in terms of the parabolic cylinder functions (Gill, 1980).

For any switch-on heating function, $Q(x)H(t)$, where $H(t)$ is the Heaviside step-function, the transient solution can be separated into Kelvin waves,

\[ q_n(x, t) = Q_0 \exp(-e x/c_0) \times [L_0(x) - L_0(x - c_0 t)], \]  
(A9)

and Rossby waves,

\[ q_{n+1}(x, t) = (n Q_{n+1} + Q_{n-1}) \exp(-e x/c_0) \times [L_n(x) - L_n(x + c_n t)], \]  
(A10)

where $c_n = c_0/(2n + 1)$, $c_0 = 1$ and $L_n$ are shape functions dependent on the exact distribution of the heat source (cf. Lau and Lim, 1982). These solutions represent Kelvin waves moving eastward with speed $c_0$ and Rossby waves moving westward with speed $c_n$. The divergence associated with the transient response is given by

\[ D = -p_x - e y - Q. \]  
(A11)

In (A11), the term $p_x$ may be considered as the transient part of the divergence. For a Gaussian-shaped switch-on heating function,

\[ Q(x, y, t) = \exp[-(x^2 + y^2)/4]H(t), \]  
(A12)

the divergence associated with the transient Kelvin wave is given by

\[ D_{\text{Kelvin}} = \frac{1}{2} [q_0(x, t)]_y D_0(y) = \frac{1}{2} [q_0(x, t)]_y \exp(-y^2/4). \]  
(A13)

The divergence associated with the Rossby wave is given by

\[ D_{\text{Rossby}} = \frac{1}{2} \{ q_2(x, t, y) D_2(y) + [r_0(x, t)]_y D_0(y) \}

\[ = \frac{1}{2} [q_2(x, t)]_y (y^2 + 1) \exp(-y^2/4). \]  
(A14)

From the $y$-dependence in (A13) and (A14), it can be seen that the divergence associated with the transient Kelvin wave is a maximum at the equator, whereas that associated with the Rossby wave is a maximum at $y = \sqrt{3}$ times the Rossby radius away from the equator. For the heating function given in (A12), from (A4) and (A5), the only nonzero forcing functions for $q_0$ and $q_2$ are $Q_0$ and $Q_0/3$, respectively. Hence, the magnitude of the divergence for the Kelvin and Rossby waves at the equator is also approximately in the ratio of 3 to 1.

REFERENCES


Chang, C. P., 1976: Vertical structure of tropical wave maintained
by internally induced cumulus heating. J. Atmos. Sci., 33, 729–739.


