Linear Diagnosis of Stationary Waves in a General Circulation Model

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ABSTRACT

A linear stationary wave model is used to diagnose the causes of stationary waves in integrations of a general circulation model (GCM) and to indicate the sources of differences between the stationary waves of separate integrations.

The GCM generates solutions to the equations of motion. The linear model, constructed to be as similar as possible in structure to the GCM, is employed in various attempts to approximate and understand the time averages of the GCM solutions. When GCM values for internal dissipation time constants are used in the linear model, significant differences between the linear model and GCM solutions are found. These differences can be interpreted as errors due to the linear approximation. The linear simulation is improved somewhat by enhancing the scale selective horizontal diffusion.

The linear model with enhanced dissipation is used to simulate the differences between the stationary waves of two consecutive months of a GCM integration. Transient forcing turns out to be the major cause for these differences, according to the linear model.

The phase structure of the errors of the linear solutions indicates that the error source is located primarily in the tropics and subtropics. Possible explanations for the errors are inaccurate representation of the topographic forcing and reflections from critical latitudes. The latter possibility is subjected to a crude test and cannot be rejected.

1. Introduction

Charney and Eliassen (1949), using a simple linear stationary wave model, showed convincingly that the maintenance of stationary waves by orographic forcing is necessary for accurate short term numerical weather prediction. Since that pioneering investigation, increasingly more complex and computationally demanding linear stationary wave models have been solved in the effort to better define the processes that maintain atmospheric stationary waves. It is necessary to make certain assumptions to apply these models to the atmosphere. In particular, the numerical linear models have a bounded computational domain in the vertical and are not amenable to the application of what is thought to be the correct upper boundary condition—a radiation condition for vertical propagation. Choices must be made for internal and boundary layer dissipative parameters, and these choices are guided in part by tuning of the linear results. Additionally, the thermal forcing for stationary waves cannot be directly measured, and indirect evaluation suffers from lack of data of sufficient quality. If the linear model is sensitive to small changes in the basic state, accurate data for this field are needed also.

If, on the other hand, the stationary wave model is designed for diagnosis of the time mean fields of a general circulation model, some of the above problems can be avoided. The formulation of the upper boundary condition, the dissipative parameterizations, the thermal forcing, and the time means of the predicted fields are known. A suitably designed stationary wave model applied to general circulation model produced data is then subject to none of the uncertainties found in application of similar models to the atmosphere, and the results should have quantitative significance.

The diagnosis of the maintenance of the general circulation model stationary waves has only indirect bearing on the understanding of the atmosphere. Presumably, the better the general circulation model, the more relevant understanding its results will be in relation to the atmosphere. If the stationary wave model produces a good enough simulation, its diagnosis could help in understanding systematic errors in the general circulation model and the causes of success or lack thereof in long range weather predictions. However, if the stationary wave model approach fails in its application to the general circulation model, then it cannot succeed when applied to the atmosphere.

The stationary wave model is derived by time averaging the equations of motion and considering the time averaged variables to be the dependent variables. Forcing due to heating and time dependent motions is taken to be externally specified for diagnostic pur-
poses. The zonal means of the dependent variables (the basic state) are also taken to be given. Valdes and Hoskins (1989) discuss this derivation schematically in their introductory section. The stationary wave equations derived in this manner involve no approximations, only the categorization of terms into knowns and unknowns. The system, however, is nonlinear, and in order to find solutions the stationary wave equations are linearized by considering the solutions to be small perturbations about the basic state. The linear stationary wave model is the lowest order problem that arises from the perturbation expansion. Higher order corrections are generally not examined.

In some circumstances the perturbation expansion can be shown to converge. However, problems arise at critical latitudes, where the basic state zonal winds change sign, because the linear system then becomes singular in many cases. In the linear barotropic case in the limit of zero dissipation, a critical latitude located in a region of meridionally increasing zonal mean absolute vorticity absorbs or partially reflects incident stationary waves (Dickinson 1970; Tung 1979). The nonlinear inviscid critical layer in the barotropic case acts as perfect reflector (Benny and Bergeron 1969; Haynes and McIntyre 1987). In the nonlinear case, the zonal mean winds are not taken to be fixed, and are modified by wave-mean flow interactions in the critical layer. Whether the critical latitude is absorbing or reflecting could have significant consequences for the stationary waves.

When a representation of the tropical Hadley circulation is introduced in the basic state, the critical latitude singularity is removed and an additional possibility is introduced, that Rossby waves propagate across critical latitudes in the direction of the mean meridional circulation, while Rossby waves propagating in the opposite direction are absorbed (Schneider and Watterson 1984). However, addition of a mean meridional circulation to the basic state introduces a new singularity at the latitudes of zero mean meridional velocity.

Another type of problem with the linear approximation to the stationary wave equations has been studied by Chen and Trenberth (1988). They examine the application of the linearized lower boundary condition when orography is finite amplitude, which they claim leads to inaccurate topographically induced divergence. They argue that the configuration in which the winds blow around the mountains with a much smaller divergence is not represented in the zonally symmetric basic state linear model, leading to errors in the rotational part of the solution.

The calculations presented in this paper examine the usefulness of the linear stationary wave model with zonally symmetric basic state for simulating and diagnosing the processes maintaining the stationary waves of a general circulation model. A winter season integration of a general circulation model with initial conditions for 15 December 1985 and observed sea surface temperature boundary conditions was made. The general circulation model was the National Meteorological Center medium range forecast model, which was operational in 1985, and which is similar to the 1986 operational model used in the DERT experiments (Tracton et al. 1989). The seasonal means of the integration are described in Kinter et al. (1988). A steady zonally symmetric basic state linear model with the same spatial differencing schemes and the same form of the turbulent dissipation parameterizations as those in the prediction model is applied to the diagnosis of the results of the prediction experiments. The linear model is used to examine the maintenance of the January and February stationary waves in the prediction model integration.

The prediction model simulates stationary waves with what appears to be a degree of realism. First, the linear model forcing is prescribed to be that which would maintain the prediction model time mean flow in a nonlinear stationary wave model identical to the linear model except for the inclusion of the wave-wave interactions. The linear model solution for this forcing is found with diffusion coefficients taken to be of approximately the same magnitudes as those occurring in the prediction model. By construction, the error in the linear simulation is then due only to the linear approximation. The same calculation with the basic state meridional circulation included is also described. The errors in either case are substantial, but the simulations are not entirely unsuccessful.

Next, an attempt is made to improve the linear simulations by increasing the damping due to the scale selective horizontal diffusion. The simulations are improved by this modification, but the remaining errors are still large. The increased horizontal diffusion linear model is used to examine the relative roles of orographic, thermal, and transient forcing in the maintenance of the prediction model’s stationary waves. Also, the causes of the differences between the prediction model’s January and February stationary waves are diagnosed with this version of the linear model. The diagnostic calculation shows some skill, and the difference in this case is diagnosed to be due to transient forcing.

The structure of the linear model errors due to the neglect of stationary nonlinearity is suggestive of an origin in the tropics and subtropics, which is the location of the basic state critical latitudes and also of the Himalayas. The contribution of stationary nonlinearity in the lower latitudes to the total error is found by forcing the linear model with the diagnosed lower latitude stationary nonlinearity only. This contribution is shown to be significant, and suggests that the effect of a reflecting critical layer be examined. A linear reflecting critical latitude is produced by altering the basic state. The reflected response when forcing is confined to the Northern Hemisphere extratropics is evaluated.
While there is some resemblance between the reflected response and the linear model error, there is no clear evidence in favor of the reflecting critical latitude.

Quantitative measures, including differences, correlations, and normalized root mean square errors are introduced into the discussion where appropriate.

Our use of a linear stationary wave model for diagnosis of general circulation model generated data follows the approach developed by Nigam et al. (1986, 1988) and Held et al. (1989). The linear model simulation of the general circulation model appeared to be very successful in those studies, which suggests that quantitative applications are possible. In Nigam et al. (1986, 1988) and Held et al. (1989), the vertical structure of the linear model was identical to that of the general circulation model, but the horizontal discretization was performed by finite differencing, while the general circulation model used a spectral formulation. Here, the general circulation and linear models are fully spectral in the horizontal, and both have the same vertical structure.

A recent study by Valdes and Hoskins (1989) applied a linear model very similar in structure to the one used here to the diagnosis of the maintenance of winter (December through February) mean stationary waves in the atmosphere as defined by European Centre for Medium Range Forecasting initialized analyses. They have also attempted to estimate the importance of stationary nonlinearity using methods similar to those used here. We have chosen to present 200 mb streamfunction in some of the Figures, using the same contour interval as Valdes and Hoskins (1989) in order to facilitate comparison.

2. The model
   a. General description

The linear model is designed to diagnose the stationary waves produced by integrations of a general circulation model (GCM). The GCM uses sigma coordinates and finite differences in the vertical with 18 equally spaced levels. Horizontal discretization of the GCM is performed using the spectral formulation. The spectral prognostic variables, vorticity, divergence, virtual temperature, log of surface pressure, and water vapor mixing ratio are represented in the GCM using a rhomboidal 40 (R40) truncation.

The design of the linear model began with the goal of making the linear model as similar as possible to the GCM, so that data produced by the GCM could be used directly by the linear model without interpolation. The basic state of the linear model is zonally symmetric. The linear model equations are derived from the GCM discretized sigma coordinate primitive equations by writing the prognostic variables as the sum of a time mean component and the deviation from the time mean (the transient component). The resulting set of equations is averaged over time, and terms that remain involving the transients are considered to be externally specified. The time mean vorticity, divergence, virtual temperature, and log of surface pressure are taken to be the dependent variables of the system. At this point the equations are nonlinear in the dependent variables, which are written as the sum of a zonally symmetric component and the deviation from the zonal mean (the eddies), and substituted into the nonlinear stationary wave equations. The zonal means are taken to be externally specified, and the eddies are taken to be the unknowns. Terms involving products of the eddy variables are discarded in obtaining the linear stationary wave model system.

The linear model sigma levels and finite differencing are the same as those used in the GCM. The linear model dependent variables are vorticity, divergence, virtual temperature, log of surface pressure, geopotential, and vertical sigma velocity. The latter two are auxiliary variables introduced to increase computational efficiency. The horizontal discretization of the linear model is spectral. The equations are projected onto the spectral components by computing the interaction coefficients. The linear model equations and solution algorithm are described in Schneider (1989).

b. Parameterizations

The GCM employs del to the fourth damping in the horizontal for vorticity, divergence, and virtual temperature with coefficients independent of space and time. This scale selective damping is a parameterization of the effects of the unresolved scales. The parameterization is heuristically based on spectral energy-entropy transfer arguments, and is believed to improve simulations and predictions made with the GCM. In the GCM the del to the fourth damping was applied to the upper half of the rhomboid, while in the linear model it is applied to all of the spectral components. The role that this damping plays in the linear model is to prevent the governing matrix from becoming singular or nearly singular due to the presence of critical latitudes.

The other parameterizations of the GCM are nonlinear, and cannot be directly incorporated into the linear model. Initial experiments with the linear model were performed with only the del to the fourth damping. Forcing due to the other, nonlinear, processes was obtained from the GCM output and specified for the linear model. The upper tropospheric solutions were reasonably well behaved, but excessive amplitudes, with velocities on the order of 100 m s⁻¹, occurred in the near-surface response. Evidently a parameterization of the boundary layer friction is necessary in this linear model to produce a reasonable response. A representation of the surface boundary layer was not needed in the linear stationary wave model of Jacqmin and Lindzen (1985), possibly because their heating was
small near the surface. The GCM employs vertical diffusion of momentum, potential temperature, and moisture. The transfer coefficients depend on the local properties of the model atmosphere, and the surface fluxes are calculated from a similarity model. The transfer coefficients and the drag coefficients that result are strongly variable in space and time. There is no possibility of faithfully representing the GCM vertical diffusion scheme in a zonally symmetric basic state steady linear model.

Vertical diffusion of momentum and heat are included in the linear model, with coefficients varying in height only. The coefficients were calibrated from a one month integration of a later version of the GCM than that described in Kinter et al. (1988), also with 18 levels but a different spacing in the vertical. The form of the tendency due to vertical diffusion used in the linear model is

$$\frac{\partial X}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho K \frac{\partial X}{\partial z} \right)$$

where $X$ is relative vorticity, divergence, or virtual potential temperature. At the lower boundary, the condition

$$\rho K \frac{\partial X}{\partial z} = -\rho c |v| (X_b - X)$$

is applied, where $X_b = 0$ for vorticity and divergence, and $X_b$ is the ground temperature in the case of sensible heat flux. However, since the ground temperature does not enter into the linear model as a variable, the term $-\rho c |v| X_b$ is taken to be specified. The values used for $K$ and $c |v|$ in the linear model are given in Table 1.

No attempt is made to enhance the damping at the top of the model to produce a sponge layer, as was done, for example, by Valdes and Hoskins (1989). The GCM has no special provisions to prevent reflections from the top; therefore, neither does the linear model. Indeed, one of the potential applications of the linear model (not pursued here) is to evaluate the influence of this deficiency on the quality of the GCM's simulations and predictions. First, however, it must be shown that the linear model can produce a realistic enough simulation of the GCM for this application to be made.

Parameterization of radiative cooling in the linear model by Newtonian damping, as used in many linear stationary wave models, is not included. The GCM output was examined to determine an effective radiative time constant for the zonally asymmetric part of the temperature field. It was found that the effective damping time in the GCM was on the order of 20 days for the extratropical troposphere and the stratosphere. However, in the tropical troposphere, the zonally asymmetric part of the time mean radiative cooling was negatively correlated with the temperature. Zonal variations in the radiative cooling in the tropical troposphere in this GCM (which assumed that the cloud field that interacted with radiation was specified and zonally symmetric) were dominated by zonal variations in the moisture field, which turned out to be negatively correlated with the temperature. It does not seem appropriate to use a Newtonian cooling based on temperature in this case.

### Table 1. The values of sigma ($p/p_x$, where $p$ is pressure and $p_x$ is surface pressure) at layer interfaces, the relevant vertical diffusion parameters, and the values of those parameters in the linear model vertical diffusion formulation. The surface exchange coefficient, $c |v|$, is given in m s$^{-1}$, and the vertical eddy viscosity–diffusivity $K$ is in m$^2$ s$^{-1}$.

<table>
<thead>
<tr>
<th>Sigma</th>
<th>Parameter</th>
<th>Value</th>
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<td>$K$</td>
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<tr>
<td>0.06</td>
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</table>

3. Accuracy of the linear approximation

The equations governing the maintenance of the stationary waves contain products of the eddy variables. In the linear model, these products are set to zero. The accuracy of this procedure is examined here for a particular case. The linear model is used to simulate the February mean of the GCM integration. The forcing that would produce the GCM solution in a nonlinear stationary wave model with the dissipative parameterizations of the linear model, which will be called the "total forcing," is found as a residual by substituting the time mean GCM solution into the appropriate nonlinear stationary wave equations. The GCM solution is then also the solution to the nonlinear stationary wave model forced by the total forcing. The linear model differs from the nonlinear stationary wave model only in the neglect of stationary nonlinearity due to wave–wave interactions.

In the results described in this section, the horizontal diffusion coefficients are given the value $10^{-16}$ m$^4$ s$^{-1}$, which is close to the value used in the GCM. The for-
ing due to stationary nonlinearity alone is also evaluated from the GCM solution. If the linear model response to forcing by the stationary nonlinearity is small, or equivalently if the linear solution is close to the GCM solution, then the linearized equations are a useful approximation to the nonlinear stationary wave model. Agreement between the linear and GCM solutions in this case would imply that linear dynamics are sufficient to describe the dynamics of critical latitudes in this GCM. For example, the critical latitudes in the GCM solution could be absorbing due to the strength of the internal dissipation that is used in the GCM. Then the linear model would obviously be a useful tool for understanding the behavior of the GCM.

With the above horizontal diffusion coefficients, it was found that 31 Legendre functions were sufficient to give a good approximation to the linear solution. The GCM horizontal diffusion is large enough with this meridional resolution to resolve the linear critical layer, which is primarily absorbing in the absence of the mean meridional circulation for the basic states obtained from the GCM solutions. Linear solutions found with a meridional resolution of 31 functions were close to those found with 41 functions. A meridional truncation of 21 functions produced significant differences from the 31 or 41 function solution. A 15 wave meridional truncation was totally inadequate. The calculations described below were performed with 31 function meridional resolution and zonal wavenumbers 1–7.

The February means of the GCM integration for zonal velocity, \( u \), temperature, \( T \), and meridional velocity, \( v \), zonally averaged at constant sigma are shown in Fig. 1. These fields form the basic state for the linear model together with the zonal mean of the log of surface pressure, \( q \), and the vertical sigma velocity, \( \sigma \). In cases in which the basic state meridional circulation is not included, \( \bar{v} \) and \( \bar{\sigma} \) (overbars for zonal mean) are set to zero. The spectral expansion of \( \bar{\sigma} \) is found from the continuity (surface pressure tendency) equation using time mean variables only. The contribution of transients to \( \bar{\sigma} \) evaluated with daily mean or daily snapshot data is small. No attempt is made to modify the basic state for geostrophic balance. Imbalances in the basic states obtained from the GCM are due to advections by the mean meridional circulation, forcing in the zonal mean time averaged meridional momentum equation by transients, and friction in the planetary boundary layer. Within the context of a fixed basic state for the stationary waves, there is no inconsistency in this procedure.

In the following, two quantitative measures that will be applied to the comparisons are the correlation coefficient (with the usual definition), which will be referred to as the “cc,” and the normalized root mean square (rms) error, which will be referred to as the “nrmse.” The rms error is normalized by the rms value of the GCM field over the domain of interest to obtain the nrmse. A zero solution would have an nrmse of one. These coefficients have been calculated over four regions: global, Northern Hemisphere (20°N–90°N), tropics (20°S–20°N), and Southern Hemisphere (90°S–20°S).

### a. Linear response with nondivergent basic state

The linear response to the total forcing with the nondivergent basic state was found. The difference between this response and the GCM solution is due only to the neglect of stationary nonlinearity, and may be forced by the stationary nonlinearity evaluated from the GCM solution. The linear response is a good approximation to the near surface wind fields, but contains significant differences from the GCM solution above the surface layer. For the zonal and meridional winds near the surface, global and regional cc’s are on the order of 0.9 or higher, with the nrmse’s on the order of 0.4. The agreement of the winds is particularly high in the tropics. The near surface temperature field has a cc of 0.99 and a nrmse of order 0.1. The linear model is relatively successful in the near surface layer because this is the region in which vertical diffusion is important and wind magnitudes are small, and hence, where the dynamics of the GCM are linear.

The sea level pressure of the GCM, the linear model, and the linear model error are shown in Fig. 2. This measure of the atmospheric mass distribution shows more significant differences than the near surface wind fields, but still qualifies as a good simulation. The Southern Hemisphere is well simulated by the linear model (cc = 0.8, nrmse = 0.63), and the Northern Hemisphere simulation is slightly less successful (cc = 0.77, nrmse = 0.70). The tropical simulation of the linear model is again the best (cc = 0.87, nrmse = 0.54). The high over south-central Asia is too strong in the linear model, the low pressure region over the north Pacific is too weak, particularly near the southern coast of Alaska. The major sea level pressure errors appear to originate near the Himalayas and extend in a wavelike pattern up the east coast of Asia and across Canada.

In the upper troposphere, the nonlinearity due to stationary wave–wave interactions produces significant differences between the linear and GCM solutions. Figure 3 shows the 200 mb streamfunction for GCM, linear model, and the linear model error. Agreement is much less satisfactory than in the lower troposphere. The zonal and meridional components of the rotational horizontal winds, which dominate the divergent components, may be deduced from this figure. The cc for the Northern Hemisphere is 0.73, with the tropics and Southern Hemisphere 2 points higher. The nrmse is large, 0.86 for the Northern Hemisphere and tropical regions, but decreases to 0.76 in the Southern Hemi-
Fig. 1. February zonal means of GCM integration, which provide a basic state for the linear model. The horizontal coordinate is latitude in degrees and the vertical coordinate is pressure in mb. (a) Temperature, contour interval 7 K. (b) Zonal wind, contour interval 3 m s⁻¹. (c) Meridional wind, contour interval 0.6 m s⁻¹.
The Northern Hemisphere extratropical zonal winds bear a strong resemblance to those found in the GCM, but the subtropical westerly jets do not extend far enough to the east, and the easterlies are too strong in central Asia. The most obvious error in the linear simulation of the zonal winds at 200 mb is excessive magnitude in the tropics and subtropics, although the positions of the major features in those regions seem to correspond to those in the GCM. Some imagination is required to draw a correspondence between the linear simulation and GCM 200 mb zonal winds in the Southern Hemisphere extratropics. The linear model produces 200 mb meridional winds magnitudes that are reasonable except for excessive amplitudes near Antarctica. Geopotential height scores for the linear model simulation are: cc = 0.8 and nrmse = 0.75 in the Northern Hemisphere at 200 mb, cc = 0.61 and nrmse = 0.88 in the Southern Hemisphere at 200 mb, and slightly worse scores at 500 mb.

The northwest to southeast tilt of streamfunction error constant phase lines gives the impression of a general poleward propagation of error out of the tropics and into the Northern Hemisphere via Rossby waves. The structure of the GCM streamfunction suggests that the reverse, propagation from midlatitudes into the tropics, is the most important process transmitting the GCM stationary waves. This result indicates that the stationary wave–wave interactions are most important in the tropics, and their neglect leads to large errors there. These errors then apparently influence the extratropics by a process that resembles linear Rossby wave propagation from the error source region. We will return to this point in section 6.

In the stratosphere, tropical solutions compare well, but the linear simulation in the Southern Hemisphere extratropics is poor. The largest amplitude by far in the stratospheric stationary wave geopotential is found in the North Polar Region in both the GCM and the linear model. In this region the GCM solution is dominated by zonal wavenumber 1, while in the linear model, the amplitude of zonal wavenumber 2 is dominant, with a significant contribution by zonal wavenumber 1 as well. The linear simulation produces much too small an amplitude in zonal wavenumber 1 in the 50 mb geopotential. The Northern Hemisphere 50 mb geopotential error can be traced downwards to corresponding features at 200 mb, 500 mb, and sea level pressure, with a barotropic vertical structure. The scores at 50 mb for the streamfunction are: 0.45, 0.66, and 0.35 for the cc and 0.95, 0.76, and 1.14 for the nrmse in the Northern Hemisphere, the tropics, and the Southern Hemisphere, respectively.
Fig. 2. February mean sea level pressure with zonal mean removed. (a) From the GCM integration. (b) The linear simulation of response to February total forcing with low horizontal diffusion and nondiagnostic basic state. (c) Difference linear simulation minus GCM result. Contour interval is 4 mb.
b. Linear response with divergent basic state

The response of the linear model to the total forcing with the mean meridional circulation included in the basic state was found. Inclusion of the basic state mean meridional circulation has a significant impact on the quality of the simulation, but in a negative sense. In the frictional boundary layer, little difference can be seen between the linear solutions with and without the basic state meridional circulation. However, in the upper troposphere and stratosphere, the linear solution with nondivergent basic state were closer to the GCM solution. This result is illustrated in Fig. 4, which shows the streamfunction at 200 mb for the linear solution with divergent basic state. The main effect due to the basic state meridional circulation is intensification of the amplitude of the linear model response in the subtropics of the Northern Hemisphere. The concentration of high amplitudes in the subtropics of the linear simulation with basic state meridional circulation could be associated with the zero wind line for the basic state meridional velocity at that latitude in the upper troposphere (Fig. 1c). While inclusion of the basic state meridional circulation removes the singularity of the linearized equations at the zeros of the basic state zonal wind, new singularities are introduced in a shallow water model where the basic state meridional wind is zero (Watterson and Schneider 1987).

The linear model scores with divergent basic state for the 200 mb streamfunction are 0.61, 0.74, and 0.75 for the cc in the Northern Hemisphere, the tropics, and the Southern Hemisphere; and the nrmse is 1.11, 1.15, and 0.83, respectively. These scores show a significant deterioration in the linear model simulation when the mean meridional circulation is included in the basic state.

The GCM produces a better simulation of the atmosphere than does the linear model of the GCM in the sea level pressure, upper tropospheric geopotential, and streamfunction (see Kinter et al. 1988, their Figs. 4.7 and 4.9). Verification criteria for prediction models are more stringent than those being applied here (e.g., anomaly correlation coefficients rather than correlation coefficients).

These linear model experiments demonstrate that stationary nonlinearity is important in the maintenance of the GCM time mean flow above the boundary layer. This occurs despite the fact that the parameterized diffusion in the GCM is of sufficient strength to produce primarily absorbing critical layers in the linear approximation. The importance of stationary nonlinearity is even more apparent from experiments with an R15, 18 level version of the GCM. We routinely run long integrations with this lower horizontal resolution model, using the same value of the del to the fourth horizontal diffusion coefficient as in the linear model.
Fig. 3. As in Fig. 2 for the 200 mb streamfunction, contour interval $2.2 \times 10^6$ m$^2$ s$^{-1}$. 
Fig. 4. Low horizontal diffusion divergent basic state linear simulation of response to February total forcing for 200 mb streamfunction. Contour interval as in Fig. 3.
of this section, and obtain realistic looking stationary waves. The 15 meridional wave linear model does not resolve the critical layers with this value for the horizontal diffusion coefficient and produces completely unrealistic solutions. We may safely conclude that it is necessary to understand the role of stationary nonlinearity in order to understand the maintenance of the stationary waves in the R15 GCM.

c. Response to nonlinear forcing

The response of the linear model to forcing by the GCM’s stationary nonlinearity applied separately to each of the equations: vorticity, divergence, thermodynamic, and continuity, was also calculated.

We found that stationary nonlinearity is not important in the divergence equation. Linearization of the divergence equation is a posteriori a valid approximation to the steady nonlinear divergence equation.

Forcing by stationary nonlinearity in the continuity equation, on the other hand, produces an extremely large response. The zonal wind response at 200 mb reaches magnitudes of \(-110 \text{ m s}^{-1}\) and \(+85 \text{ m s}^{-1}\). These magnitudes are much larger than in the linear response to the total forcing or the GCM solution.

However, the response to forcing by stationary nonlinearity in the thermodynamic equation is also very large and, for the most part, cancels the response obtained by forcing the continuity equation with the stationary nonlinearity. The result is that the response to forcing both the continuity and thermodynamic equations by the stationary nonlinearity produces a much smaller response than forcing only one of the two equations.

Solutions found to forcing the sigma-coordinate primitive equation linear model vorticity equation by stationary nonlinearity and to forcing both the thermodynamic and continuity equations by stationary nonlinearity also show significant cancellation. The magnitudes of the separate responses are not as large as those found with the forcing of the continuity equation, and the cancellation is not as complete. The structure of the error due to linearization resembles the structure of the response to forcing the vorticity equation alone with the stationary nonlinearity.

It has been pointed out by Dr. I. Held (private communication) that the cancellation of the responses to forcing by stationary nonlinearity is probably closely analogous to a result that may be derived from the steady adiabatic inviscid quasi-geostrophic equations. The lower boundary condition on this system for topographically forced stationary waves is that potential temperature is constant along streamlines. The vertical velocity at the lower boundary is related to the horizontal velocities and surface topography by the kinematic condition that there is no flow normal to the lower boundary. It can be demonstrated that the streamfunction and potential temperature of a solution to the linearized lower boundary condition also form a solution to the nonlinear lower boundary condition, provided that the basic state zonal wind does not vary in the meridional direction. However, the linearized vertical velocity at the lower boundary is not equal to the vertical velocity of the nonlinear solution, which may be obtained by using the linear-nonlinear solution for the streamfunction in the nonlinear kinematic boundary condition.

If the quasi-geostrophic lower boundary condition is represented by the two coupled equations, the thermodynamic equation and the kinematic boundary condition, the result then corresponds to the behavior found in our sigma-coordinate linear model. The linear system response to forcing the kinematic boundary condition by stationary nonlinearity, which is evaluated from the linear (and in this case exact) solution, is exactly canceled by the response to lower boundary thermodynamic equation forcing by stationary horizontal temperature advection responses. Using the linearized kinematic boundary condition for vertical velocity produces a more accurate solution than using the correct vertical velocity in the linear thermodynamic equation.

The situation in the interior of the quasi-geostrophic system is similar. When the basic state zonal winds are independent of latitude, a linear solution for the streamfunction of the adiabatic, inviscid, stationary, quasi-geostrophic potential vorticity equation, which results from the topographic forcing problem, is also a nonlinear solution. Again, the linear solution for the vertical velocity must be adjusted a posteriori by substitution of the linear streamfunction solution into the nonlinear vorticity or thermodynamic equation to complete the exact solution. The quasi-geostrophic potential vorticity equation is equivalent to a system consisting of the quasi-geostrophic vorticity and thermodynamic equations. As the linear solution is not a nonlinear solution of either the vorticity or thermodynamic equation separately (due to the above-mentioned property of the linear solution for the divergence), forcing either equation with stationary nonlinearity will produce responses that are nonzero and that exactly cancel.

The sigma-coordinate primitive equation model appears to behave similarly to linear solutions of the quasi-geostrophic system vis a vis the cancellation of the responses to forcing the vorticity, thermodynamic, and continuity equations separately by the stationary nonlinearity. However, the forcing due to stationary nonlinearity was calculated from the GCM solution rather than from the linear solution, therefore, the correspondence in the behaviors of the two systems is not direct.

4. Enhanced dissipation

a. Choice of dissipation

Experiments were performed to examine whether the linear response to the total forcing could be made
FIG. 5. (a) High horizontal diffusion nondivergent basic state linear simulation of response to February total forcing for 200 mb streamfunction. (b) Error of this linear simulation. Contour interval as in Fig. 3.
Fig. 6. Heating field from February mean of GCM used to force linear model. (a) Structure of vertical mean with zonal mean removed. Contour interval 0.5 K day$^{-1}$. (b) Longitude–sigma cross section averaged between 20°S and 20°N with zonal mean included. Contour interval 0.7 K day$^{-1}$. (c) Longitude–sigma cross section averaged between 35°N and 45°N with zonal mean included. Contour interval 1 K day$^{-1}$. 
to more closely resemble the GCM solution by enhancing the dissipation in the linear model. Nigam et al. (1986, 1988) have had some success with strong damping in the tropics and polar latitudes. In another linear sigma-coordinate stationary wave model patterned after a GCM, Simmons (1982) used strong damping near critical surfaces. We found that increasing the del to the fourth horizontal diffusion coefficients improved the linear model simulation of the GCM solution. The February total forcing was used, and the best fit to the GCM solution 200 mb zonal and meridional wind response was sought. The horizontal diffusion coefficients that yielded the greatest improvement in the linear solution for this case had values 7 \times 10^{-17} \text{ m}^4 \text{ s}^{-1} for vorticity and divergence (an increase of a factor of 70 from the value used in the experiments described in section 3) and 1 \times 10^{-16} \text{ m}^4 \text{ s}^{-1} for temperature (no change). With the enhanced horizontal diffusion, 16 meridional waves are sufficient to resolve the solution, but the calculations shown below were all done with 31 meridional waves as in the previous cases. The damping time scale for total wavenumber 31 in the vorticity and divergence equations then turns out to be nearly the same in our enhanced horizontal diffusion model as in the model used by Valdes and Hoskins (1989), although they choose a del to the sixth representation for horizontal diffusion.

Inclusion of the basic state mean meridional circulation in the high horizontal diffusion linear model has negligible impact.

The increase in the horizontal diffusion for the horizontal wind fields is particularly successful in damping out the short horizontal length scale for high amplitude zonal wind field variations that occur in vicinity of the tropical critical latitudes in the low diffusion calculations. The occurrence of this type of behavior near linear critical latitudes is well understood (see Schneider and Watters 1984, for a brief explanation). The manner in which stationary nonlinearity prevents such behavior in the GCM and the real atmosphere is not so well understood, although two major possibilities have been suggested: 1) stationary nonlinearity acts like frictional damping near critical latitudes (the implicit assumption behind all linear stationary wave models that are tuned by some type of friction), and 2) that stationary nonlinearity acts to produce reflecting critical latitudes. The latter hypothesis will be examined briefly in section 6.

Increasing the horizontal diffusion of temperature was found to degrade the linear simulation, particularly in regions of significant topographic variation such as Antarctica. As horizontal diffusion was performed on sigma surfaces in the version of the GCM that generated the data; the same form was chosen for this linear model.

The 200 mb streamfunction of the high dissipation linear model and the error of the linear simulation of this field are shown in Fig. 5. The low and high dissipation linear simulations are broadly similar. Comparison of Fig. 5b with Fig. 3c shows that the reduction

**Fig. 6. (Continued)**
Fig. 7. Eddy sea level pressure response of high horizontal diffusion linear model to (a) topographic forcing, (b) thermal forcing, (c) transient forcing, and (d) total forcing. Contour interval as in Fig. 2.
Fig. 7. (Continued)
Fig. 8. Eddy 200 mb streamfunction response of high horizontal diffusion linear model in February case to (a) topographic forcing, (b) thermal forcing, and (c) transient forcing. Contour interval as in Fig. 3.
in the error is noticeable but not spectacular. The cc’s for the 200 mb streamfunction (0.82, 0.85, and 0.84 for the Northern Hemisphere, the tropics, and the Southern Hemisphere, respectively, for the high diffusion case) are 9 to 10 points higher than for the low diffusion simulation. The nrmse’s of the 200 mb streamfunction for the high diffusion case are 0.64, 0.64, and 0.60 for the same regions; a significant improvement, but still large. For sea level pressure in the high diffusion case, the cc’s are 0.79, 0.92, and 0.88, and the nrmse’s are 0.63, 0.43, and 0.49, again a noticeable improvement from the increase in dissipation. The enhanced dissipation improves the simulation primarily by reducing the magnitude of the errors. The error patterns are not significantly affected.

As far as can be ascertained, the errors found in the high diffusion simulation seem to be typical of those found in other stationary wave simulations (Valdes and Hoskins 1989, Nigam et al. 1988), both in structure and amplitude.

b. Composition of the linear response

Next we examine the linear model diagnosis of the relative contributions due to topographic forcing, heating, and transient forcing to the stationary waves of the February mean of the GCM. Together, these three components make up the total forcing for the nonlinear stationary waves. The linear model tuned with high dissipation was used, as it produces a more satisfactory simulation of the total response. The heating field used to find the thermal response was retrieved from the diagnostic output routinely saved in the GCM integrations and consists of the time mean of the sum of the temperature tendencies due to parameterizations of deep and shallow convection, other condensation of moisture, radiation, and vertical diffusion. The contribution from each type of heating was accumulated at each time step in the integrations to avoid sampling error.

The vertical mean of the February heating with the zonal mean removed is shown in Fig. 6a. The sea surface temperature used in the GCM integration was the observed for the period of integration, which was an El Niño situation. The strong heating in the eastern Pacific is a result of this choice of sea surface temperature. While the broad scale structure of the heating seems reasonable, it is obvious that this version of the NMC model experienced significant problems in the distribution of heating over land. For example, the strong heating over Ethiopia and Northeast Brazil is due to release of the latent heat of condensation and is associated with heavy rainfall. In the real world both of these areas are very dry. The central Amazon was very deficient in rainfall in the GCM integration. Additionally, convective heating is zero above 300 mb in this version of the model. A tropical longitude-height
section of the heating, averaged between 20°S and 20°N, is shown in Fig. 6b, which demonstrates the effect of the cutoff of convective heating in the upper troposphere. A midlatitude section of the heating, averaged between 35°N and 45°N is shown in Fig. 6c. The structure of the heating in midlatitudes is similar to that found for the atmosphere by Valdes and Hoskins (1989, Fig. 2c).

Transient forcing is defined for the purposes of this study to be the total forcing (the tendency required to produce the GCM solution in the low diffusion nonlinear stationary wave model) for the vorticity, divergence, and continuity equations, and the difference between the total forcing and the heating for the thermodynamic equation. This residual technique assures that the sum of the topographic, thermal, and transient forcing equals the total forcing. The sum of the linear solutions for topographic, thermal, and transient forcing is equal to the linear solution for the total forcing. However, since the linear model vertical diffusion would not give the vertical diffusion tendencies of the GCM when applied to the GCM solution, the transient forcing as defined here contains effects that could be ascribed to stationary nonlinearity; this also corrects for the fact that the vertical diffusion heating is included twice: once in the specified heating, and once in the linear model's vertical diffusion parameterization. The differences between the transient forcing calculated from the large scale fields and as defined here could be significant near the ground.

It would perhaps be more consistent to refer to the residual forcing rather than the transient forcing. One consistent method of calculating the transient forcing for the GCM integration, including that due to the physical parameterizations, would be to find the tendencies of the GCM for one time step using the time mean fields for the dynamical quantities, and then to find the transient tendencies as residuals.

The contributions from the topographic, thermal, and transient forcing to the linear solution for the February case are shown in Fig. 7 for the sea level pressure and in Fig. 8 for the 200 mb streamfunction. The topographic streamfunction response is very similar in structure and amplitude to that produced by Nigam and Held (1983) with a barotropic model. The 200 mb streamfunction response to topographic forcing of Valdes and Hoskins (1989) also shows a close correspondence in structure to that shown in Fig. 8a but significantly weaker amplitude over eastern Asia and the north Pacific. The response to heating (Fig. 8b) is also very similar to that shown by Valdes and Hoskins (1989, their Fig. 7), with the major difference a somewhat stronger tropical–subtropical streamfunction response in their study, particularly in the Atlantic.

Both the topographic (Fig. 7a) and transient (Fig. 7c) forcing produce large contributions to the eddy sea level pressure, while the sea level pressure response to heating (Fig. 7b) is small and basically in phase with the transient component.

The diagnosis produced by the linear model at 200 mb indicates that topographic forcing (Fig. 8a) and transients (Fig. 8c) are dominant and of similar magnitude in the extratropics, while heating (Fig. 8b) is responsible for producing the stationary waves in the tropics and subtropics. Over the western Pacific in the Northern Hemisphere tropics and subtropics, transients are also important, especially in extending the subtropical jet stream from the east coast of Asia across the dateline. The topographic response at 200 mb is positively correlated with the GCM solution, but the correlation is not high (see below).

At 50 mb (figures not shown), the responses in the high latitudes of the Northern Hemisphere to topography and transients are in phase with the GCM response, while the heating response is out of phase with the GCM response. The heating response in this region reaches values of −150 m and +80 m.

Quantitative measures of the relative contributions of the topographic, thermal, and transient forcing, and error of the linear simulation to the GCM solution are ambiguous. The variances of the components of the linear simulation of a field do not add up to the GCM variance, since the components have nonzero covariance. One linear measure of the relative contribution of a component of the linear response to the GCM field is the covariance of that component with the GCM field, normalized by the variance of the GCM field. The contributions of the components will add up to one by this measure. The linear measures of the contributions of the components of the linear solution to the GCM solution are (−0.03, 0.04, 0.71, and 0.28) for sea level pressure and (0.24, 0.25, 0.40, and 0.11) for 200 mb streamfunction, in the order (topography, heating, transients, and error) for the region from 25°N to 90°N. The rms amplitudes of the components of the sea level pressure response over this region, normalized by the amplitude of the GCM solution are (0.66, 0.20, 1.22, and 0.64) in the same order. The 200 mb streamfunction has normalized amplitudes (0.55, 0.58, 0.71, and 0.63). It is interesting that the error contribution to the 200 mb streamfunction has large amplitude but low projection on the GCM solution. The fact that the normalized amplitudes do not add up to one is due to the nonzero covariances between the separate components. Transient forcing appears to be the most important with regard to sea level pressure, while no component is dominant with regard to the forcing of the 200 mb streamfunction.

5. Diagnosis of the causes of differences in the stationary waves between two periods

The linear model can be used diagnostically to understand the causes of the differences between the sta-
Fig. 9. Simulation of January minus February difference in the GCM stationary waves by the high horizontal diffusion linear model, for eddy sea level pressure. (a) Difference in the GCM. (b) Difference in the fully forced linear model. Contour interval as in Fig. 2.
tionary waves of GCM integrations. The high horizontal diffusion model is used as it produces the better simulation. The procedure is the same as followed above. The GCM time mean fields for the cases are substituted into the nonlinear stationary wave model equations with low horizontal diffusion to determine the total forcing. The linear model solutions with the appropriate basic state are found for the total forcing, topographic forcing, and thermal forcing. The linear response to transient forcing is found to be the difference between the responses to the total forcing and the topographic plus thermal forcing. The linear solutions are interpolated to pressure coordinates, and differences are taken between the cases to obtain the diagnosis.

This procedure is illustrated using the monthly means of the output from the GCM integration for January and February. The difference in the heating between the two months was a strong in-place intensification of the tropical heating with time. The sea level pressure difference for January minus February for the GCM is shown in Fig. 9a, and the difference produced by the linear model in response to the total forcing is shown in Fig. 9b. The linear model produces a reasonable simulation of the GCM results. The measures of skill for the sea level pressure of the linear simulation of the differences are cc’s of 0.67, 0.71, and 0.77 and nrmse’s of 0.77, 0.80, and 0.67 for the Northern Hemisphere, the tropics, and the Southern Hemisphere, respectively. These scores are somewhat worse than those that result from the comparison of the individual months with the GCM, but the simulation of differences is a more stringent test of the linear model. The sea level pressure difference in the linear model is forced predominantly by transients.

The results for the 200 mb streamfunction are similar. The linear simulation of the intermonthly differences due to the total forcing has some skill, but less than the monthly simulations, with cc’s of 0.68, 0.35, and 0.56 and nrmse’s of 0.83, 1.16, and 0.89 for the Northern Hemisphere, the tropics, and the Southern Hemisphere, respectively. Transient forcing explains most of the correlation between linear model and GCM, and dominates the amplitude of the linear response difference, even in the tropics.

The linear diagnosis of the stationary wave differences was unsuccessful in the stratosphere.

This case demonstrates that a linear model with a zonally symmetric basic state can provide guidance as to the causes of differences between stationary waves in GCM integrations. However, the errors in the linear simulation of the differences are too large for the linear diagnosis to be considered unambiguous. It would be of interest to use the linear model for cases in which the differences in the topographic and thermal forcing would be expected to be larger, such as the diagnosis of the causes of differences between winter and summer stationary waves.

6. Tropical influences

As noted in section 3a, the 200 mb Northern Hemisphere streamfunction error (Fig. 3c) in the low horizontal diffusion case is suggestive of a tropical or subtropical source. In order to investigate this behavior in more detail, a further experiment was performed for the February case with the forcing due to stationary nonlinearity of the GCM solution in the low horizontal diffusion linear model. The nonlinear forcing was set to zero north of the Northern Hemisphere sub tropics and the linear response was calculated (Fig. 10). Comparing these results with the total error (Fig. 3c) it is seen that cutting off the forcing at 20°N produces a response that is of the correct structure to cancel the error, and explains about a third of the error in high northern latitudes. Cutting off the nonlinear forcing north of 30°N yields a Northern Hemisphere extratropical response that is somewhat stronger than the error. Setting the forcing to be zero south of 20°S does not change the Northern Hemisphere response in either case, as would be expected with linear absorbing critical latitudes. The response of the 50 mb geopotential in the 30°N cutoff experiment produces a similar result; about 300 m of the 600 m error in the North Polar regions is due to stationary nonlinearity in the tropics and sub tropics.

These experiments provide some support to the idea that the stationary waves obey approximately linear dynamics, except in the tropics and sub tropics (and the Antarctic region). The errors in the Northern Hemisphere extratropics of the low diffusion linear model are in large part the result of the remote influence of effects due to stationary nonlinearity in the tropics and sub tropics. This influence is transmitted to the extratropics by linear stationary waves. Two possible explanations for this result are:

1) Transmission of the tropical and subtropical forcing to the extratropics is somehow enhanced by local stationary wave–wave interactions in the forcing region. This possibility includes topographic effects.

2) The stationary waves forced in the extratropics that propagate into the tropical belt are reflected or over-reflected from that region.

One type of manifestation of the first possibility was addressed by Schneider and Watterson (1984) and Watterson and Schneider (1987), who investigated the effect of the basic state mean meridional circulation on the propagation of stationary waves into and out of the tropics. In the model described here, the inclusion of the basic state mean meridional circulation either led to a poorer simulation (low horizontal diffusion) or no effect (high horizontal diffusion). Sardeshmukh and Hoskins (1988) have suggested that stationary wave–wave nonlinearity due to vorticity advections by the divergent flow can transmit the effect of tropical
Fig. 10. Response of low horizontal diffusion linear model 200 mb eddy streamfunction to forcing by stationary nonlinearity evaluated from February GCM solution. (a) Forcing set to zero for latitudes north of 20°N. (b) Forcing set to zero for latitudes north of 30°N. Contour interval as in Fig. 3.
forcing to regions where its influence may propagate to midlatitudes. This effect and others represented by the first possibility cannot be investigated in the framework of a zonally symmetric basic state linear stationary wave model. However, experiments that estimate the possible magnitude and structure of the response due to the second possibility can be performed.

Experiments on the effects of critical latitudes in linear barotropic models with a zonally symmetric basic state (e.g., Geisler and Dickinson 1974) show that the linear stationary waves incident from the propagating region (westerlies) are either absorbed, reflected, or over-reflected at a critical latitude. The type of behavior obtained depends on the sign of the meridional gradient of the absolute vorticity at the critical latitude and the magnitude of the dissipation. For low dissipation, the incident linear stationary waves are either absorbed, reflected, or over-reflected if the meridional gradient of the absolute vorticity at the critical latitude is positive, zero, or negative, respectively. Case studies of nonlinear critical latitudes in barotropic flow (Benny and Bergeron 1969; Killworth and McIntyre 1985) indicate that stationary waves are reflected from the vicinity of critical latitudes in the limit of vanishing dissipation. The nonlinear barotropic theory says effectively that the incident stationary waves mix absolute vorticity in a region surrounding the critical latitude (the critical layer) and reduce the meridional gradient of the absolute vorticity to near zero in the critical layer. The nonlinear critical layer then reflects the incident waves. Haynes and McIntyre (1987) show that the linear stationary wave model gives a good approximation to the behavior of the fully nonlinear steady solution when the basic state contains the correct critical layer structure.

Figure 11 displays the total zonal wind and absolute vorticity of the GCM at 300 mb for the February time mean. It is of interest that the zero lines of the zonal wind lie in regions of very small or zero meridional absolute vorticity gradient. This coincidence is plausibly explained by a local application of the nonlinear critical layer results. A similar relationship between the zonal wind and absolute vorticity is found at all levels, except near the ground and in the stratosphere. In the latter regions, the tropical easterlies form a continuous belt around the globe, which seems to be associated with the highly distorted absolute vorticity surfaces not occurring.

One way to simulate the effect of reflection of locally incident wave packets by the nonlinear critical latitude in a zonally symmetric basic state linear model is to modify the basic state so that the zero wind lines are embedded in regions of zero meridional gradient of absolute vorticity. A crude experiment along these lines is described here. The absolute vorticity of the basic state was set to zero between 10°S and 15°N (zero is approximately the mean value of the basic state absolute vorticity in that region) for all levels above 700 mb. The basic state zonal winds are calculated from the adjusted zonal mean relative vorticity. Additionally, the total forcing and the topography were set to zero everywhere south of 20°N. The response to this Northern Hemisphere extratropical forcing was found both with the original and the adjusted basic state in the low horizontal diffusion linear model.

The adjusted basic state zonal wind is shown in Fig. 12. Ruosteenoja (1989) has produced reflecting critical layer solutions by modifying the basic state zonal wind so that the meridional gradient of the basic state absolute vorticity is zero at the critical latitude. Here, the experiment does not require a careful treatment of the basic state to the south of the critical latitude, as only the response to forcing from the north is considered. The basic state zonal winds will remain continuous if the mean of the absolute (or relative) vorticity of the region being adjusted is preserved. The vorticity adjustment used here leads to discontinuities of absolute vorticity at 10°S and 15°N, which of course are not resolved by the linear model. The effect of the basic state vorticity discontinuities in the solutions discussed below is not known.

The result of this experiment for the 200 mb streamfunction is shown in Fig. 13. The expected poleward propagating signal is clearly seen in the difference field. The reflecting critical latitude signal tends to cancel the error in the linear simulation north of 50°N (cf. Fig. 3c). It is interesting that reflected signal in Fig. 13 appears to emanate from the region between India and Malaysia where Fig. 11 shows that in the GCM simulation the zero zonal wind line is crossing a region of uniform absolute vorticity. However, the strong low to the east of the Himalayas intensifies the negative streamfunction error in that region. The reflected signal is obviously dominated by the topographic forcing from the Himalayas. The signal reaches the critical latitude in the Bay of Bengal and is reflected up the east coast of Asia and possibly as far east as Greenland. It is not clear which forcing is responsible for the low over northern Europe. The constant phase surfaces of the reflected signal show the northwest to southeast tilt expected for a tropical source, but the magnitude of their slope (change in longitude with increasing latitude) is significantly smaller than found in the linear simulation error of Fig. 3c. This experiment shows that critical layer reflections could have significant extratropical impact, but does not fully support the hypothesis that the errors of the linear simulation can be explained as being due to critical layer reflection.

The structure of the results shown in Fig. 13 is very similar to that found by Nigam and Held (1983, their Fig. 12b) for critical layer reflections in a barotropic model with topographic forcing, aside from a multiplying factor of −1. The amplitude of the response found by Nigam and Held (1983) is somewhat smaller than found here, particularly in the east Asian and western Pacific region. The 180° phase shift may be
Fig. 11. Structure of February mean GCM solution at 300 mb with zonal mean included. (a) Zonal wind, contour interval 5 m s$^{-1}$. (b) Absolute vorticity, contour interval 20 $\times$ 10$^{-6}$ s$^{-1}$. Note coincidence of regions of uniform absolute vorticity and zero zonal wind lines.
Fig. 12. Basic state zonal wind for linear model with reflecting critical layers. Contour interval 4 m s⁻¹. The February basic state of Fig. 1b has been modified by setting the absolute vorticity to be zero for pressures less than 700 mb and between latitudes 15°N and 10°S.

an indication of the sensitivity of the reflected response to the structure and position of the critical layer.

The reflecting critical latitude experiment produces ambiguous but suggestive results. The results also suggest an indirect mechanism by which tropical influences could be felt in midlatitudes. If local zero zonal wind lines in the tropics provide reflecting surfaces for stationary wave packets, then the magnitude and position of the reflected signal will depend on the position (and perhaps orientation) of the reflecting surface relative to the forcing for the signal. The tropical zonal winds are primarily forced by heating due to deep moist convection, and the position of the heating is tied to the properties of the lower boundary. A shift in the position of the convective heating would change the position of the easterlies and hence the reflecting surface. The stationary waves forced in midlatitudes could be refracted towards the easterlies and the interaction of the extratropically forced waves with the tropical thermally forced regime could cause the flow at the critical surface to adjust until the reflection condition is met. In this manner zonal asymmetries in the tropical heating could indirectly affect midlatitude stationary waves, even if the direct response to the tropical heating were equatorially trapped.

In a zonally asymmetric basic state linear model, it may be difficult to distinguish whether anomalous extratropical stationary waves produced in response to tropical heating anomalies are due directly to the heating or indirectly to the effect of the heating on the basic state enhancing reflection of the extratropically forced waves.

7. Summary

The properties of a linear stationary wave model with a zonally symmetric basic state and the discretization employed by and data generated by a GCM have been investigated. The purpose of the study was to determine whether the linear model with its inherent approximations is a useful quantitative tool for diagnosis and understanding of the maintenance of the GCM stationary waves. Quantitative measures were applied to determine the accuracy of the linear simulations.

The main approximation in the linear model is the neglect of stationary wave–wave interactions. Due to this approximation, the linear model could be inaccurate in the case of finite amplitude forcing and in the presence of critical layers. In comparing the linear model solutions to the GCM solution, it was found that the linear model solutions were qualitatively similar to the GCM solutions but the errors were not small according to the quantitative measures. Three types of
Errors were found above the surface boundary layer: too large a response in the tropics with reasonably accurate phase; errors that appear to originate in regions of large topographic variations; particularly Antarctica and the Himalayas; and errors that appear to originate in the tropics and propagate into the higher latitudes. The Antarctic errors are trapped in that region, but the errors originating in the Himalayas seem to propagate northward and eastward producing significant remote impact.

The errors were reduced significantly by increasing the scale selective horizontal damping of the velocity. The greatest improvement was found in the tropics and subtropics, where the amplitude of the linear response was reduced. It is difficult to separate topographic and critical latitude contributions to the total error in the linear simulation, due to the proximity of the Himalayas to the critical surfaces. However, an experiment with a reflecting Northern Hemisphere critical layer showed that the reflected component of the waves forced in midlatitudes has structure and amplitude such as to significantly reduce the error in the higher northern latitudes. Critical layer reflection cannot be rejected as an explanation of the result that the higher latitude error in the linear simulation appears to originate in the lower latitudes. Large errors in the geopotential in the North Polar stratosphere were not much improved by either the enhanced damping or critical latitude reflection.

Despite the shortcomings of the linear model, a linear simulation of the difference between two contiguous monthly means of a GCM integration showed positive skill and was not much worse than the linear simulations of the two months separately. The linear model diagnosed the difference in the case considered to be due almost entirely to transient forcing. The success of the diagnosis in this case may have been due to the predominance of the transient forcing in the inter-monthly variations of the GCM stationary waves. It would be useful to study cases in which topographic effects (changes in the basic state zonal winds) or heating are more important in producing the differences, such as the comparison of stationary waves in different seasons.

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