The Evolution of Three-Peak Raindrop Size Distributions in One-Dimensional Shaft Models. Part I: Single-Pulse Rain

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ABSTRACT

Pulsed input of raindrop packages at the top of one-dimensional shafts, followed by coalescence and breakup during the fall produces, when integrated over time of the whole rainfall, three-peak drop-size distributions (3PDs) at all levels throughout the shaft. The 3PDs are different from the three-peak equilibrium distributions (3PEPs) that develop with steady sources of rain. However, for long pulse lengths, the 3PD becomes closer to the 3PED; for short pulse lengths, the large-drop peak is not very prominent for the input of Marshall–Palmer distributions. The diameters corresponding to the maximum concentrations are approximately the same as those of the 3PED. For 3PED input, the biggest differences between the number concentrations for the 3PED and 3PDs occur for pulse lengths between 240 and 600 s while the radar reflectivity concentrations of 3PDs steadily approach that of 3PEPs with increasing pulse length. All trends are the same whether a 3PEP or a Marshall–Palmer distribution is used as input.

In nature, the raindrops often arrive at the ground in packets, with the largest drops followed by progressively smaller ones. This, and the presence of 3PDs at the ground, when the drop-size distributions were integrated over time, were observed in Malaysia by List et al.

1. Introduction

The Low and List (1982) parameterization is a detailed scheme giving the average fragment-size distributions resulting from laboratory raindrop collisions that simulated natural events. Using this, several warm rain modeling studies have investigated the time evolution of drop-size distributions. Most have been done in box (zero spatial dimensions) models (Donaldson 1984; Valdez and Young 1985; List et al. 1987; Brown 1988; and Feingold et al. 1988) rather than in shaft (one spatial dimension) models (Donaldson; List et al. 1987; Tzivion et al. 1989). An equilibrium distribution, independent of input spectra, was found to develop for steady state conditions. Studies using earlier parameterizations of collisional breakup also yielded an equilibrium distribution (see Gillespie and List 1978; Srivastava 1982). The equilibrium distribution obtained from the Low and List parameterization has three peaks in number concentration, and is called the three-peak equilibrium distribution (3PED). Brown (1989 and 1990) investigated reversals in evolving raindrop size distributions due to the effects of coalescence and breakup as the 3PED was approached and List and McFarquhar (1990) examined the interactions and breakup types producing and maintaining the 3PED. This complemented previous work of Valdez and Young (1985) and Brown (1988).

Drop sedimentation must be added to the models to explain observed patterns of nonsteady rain. Both the precipitation intensity and the shape of the raindrop size distribution may vary with time for natural rain events. Waldvogel (1974) observed sudden variations in the raindrop spectra that were recognized as jumps in N0, the intercept of the Marshall–Palmer (henceforth MP, Marshall and Palmer 1948) distribution commonly used to describe drop size distributions. Joss and Gori (1978) observed single modes in the spectra that varied with time and altitude, and Donnadieu (1988) found growth and dissipation stages in rain showers corresponding to the arrival of large and small drops respectively. Further, List et al. (1988) found that drops tended to reach the ground in packets, each having larger drops first, followed by smaller drops. They also found evidence for the three peaks when the cumulative drop-size distributions were examined.

Brazier-Smith et al. (1973), Gillespie and List (1978), and Le Cam and Isaka (1989) numerically modeled the time evolution of raindrop spectra with sedimentation, but none included a breakup scheme as detailed as that of Low and List. Only List et al. (1987) and Tzivion et al. (1989) have incorporated shaft models with the Low and List parameterization; List et al. (1987) using a semi-Lagrangian scheme to describe the drops’ vertical motions, and Tzivion et al. (1989) using the method of moments. List et al. (1987)

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and Levin et al. (1988) have also used their models to examine the case of nonsteady rain. In the model presented here, the case of nonsteady rain is studied using an Eulerian framework so that the trajectories of individual drops do not have to be computed and so that the model may be more easily extended to two dimensions in the future. To reduce computational costs, to limit the amount of numerical diffusion and to maintain the positiveness of all quantities, the simple positive definite advection scheme of Smolarkiewicz (1983) is used.

The evolution of drop spectra during rainfall through still air is modeled here. The additional complication of microphysics—dynamics feedback is not included. This may be important because airflow induced by particle weight through pressure perturbations (Clark and List 1971) and thermodynamic responses (Girard and List 1975) can significantly increase the fall speeds of particle zones. However, the model still allows an investigation of some of the conceptual questions of nonsteady rain.

The time evolution of drop size distributions throughout the one-dimensional shaft and the arrival time of different drops at the ground are examined after different input spectra are released at shaft top for a limited pulse length. The use of these input pulses is justified because trends in the data collected by List et al. (1988) are qualitatively replicated by these simulations. A comparison of these results with future modeling and field studies will further assess the model’s utility for simulating warm rain. Part II of this paper will study the nonsteady rain that occurs when a series of these pulses is added at the top of the shaft.

2. The model

The model used to describe the temporal evolution of drop size distributions in warm one-dimensional rain shafts is very similar to the box model by List and McFarquhar (1990), except that the sedimentation of drops is included with the collisional breakup and coalescence of raindrops. Other aspects, such as electrical effects and aerodynamic breakup, are not included for the reasons given by McTaggart-Cowan and List (1975). Evaporation is also excluded at this time because List et al. (1987) found it did not significantly alter the shape of the 3PED.

The overall equation for the evolution of the drop spectra is expressed by the quasi-stochastic coalescence/breakup equation

$$\frac{dn(m, t)}{dt} = \int_{m/2}^{\infty} \int_{m-x}^{\infty} K(m; x, y)n(x)n(y)dydx,$$

where $n(m, t)$ represents the number density in mass coordinate and $K(m; x, y)$ the “kernel” that describes the mean number of fragments of sizes $m$ to $m + dm$ produced or lost by a single collision between drops of masses $x$ and $y$. Equation (1) is called the quasi-stochastic coalescence/breakup equation because the Low and List parameterization provides a quasi-stochastic solution. In reality not all collisions between identically sized drops produce exactly the same fragments. However, in the present model all identical drop interactions are assumed to give the same result, as described by the average of all possible fragment size distributions. Thus, the word “quasi” is used and fractional numbers of drops may be produced by collisions in this model.

The Lagrangian derivative in Eq. (1), $d/dt$, follows the particle motion. Since the size distribution in the rainshaft is studied as a function of particle mass, height, and time, this equation is solved using partial derivatives that will be computed numerically,

$$\frac{\partial n}{\partial t} + V(m) \frac{\partial n}{\partial z} = I.$$  (2)

Here, $I$ is the double integral on the right-hand side of Eq. (1).

Equation (2) may be simplified for two special cases. When there is vertical invariance caused by constant initial conditions through some vertical column of air, the advective term can be dropped, yielding

$$\frac{\partial n(m, t)}{\partial t} = I.$$  (3)

This is the equation used for zero-dimensional box models. Steady state distributions, not changing with time, exist when the upper boundary condition does not vary with time. If the independent variables are changed from $m$, $z$, and $t$ to $m$, $z$, and $(t - z/V)$, Eq. (2) becomes

$$V(m) \frac{\partial n}{\partial z} = I.$$  (4)

A shaft model with no time dependence, computationally equivalent to a box model, may be used to obtain solutions to (4) (Donaldson 1984).

Neither of these simplifications are used in the present study because the effect of a time varying source is being investigated. Pulses of drops, with a constant input spectrum, are added at the shaft top for varying amounts of time. The distributions at each height are functions of time because of the initial condition and sedimentation, which cause the concentrations to first rise and then eventually decay since the pulse does not have infinite duration.

The fall of the drops and the action of coalescence and breakup are treated independently. The mean free path of raindrops is sufficiently large (List et al. 1970) that most drops do not interact with others during the 1-second time step used in this model. Thus, the drop interactions are computed after the fall of the drops for each time step. However, the mean free path of the drops is small enough so that most drops experience
interactions during their fall through the shaft for the heavy rain events modeled here, showing that drop interactions must be included. Equation (2) is solved to determine the product of the interactions. All details concerning the use of the nonlinear Low and List parameterization with this equation and its reduction to an algebraic form are found in List and McFarquhar (1990). Forty bins, logarithmically spaced with the mass evenly distributed in each, are used to describe the drop-size distribution, with the smallest bin centered at 0.1 μg, corresponding to a diameter of 0.06 mm, and the largest bin at 72 μg, corresponding to 5.2 mm.

The numbers of particles and the continued creation and destruction of drops make it difficult to follow the trajectory of each particle. Therefore, an Eulerian model, where drops fall between volume elements according to an advection scheme, is used. To compute the drops’ free fall, the simple positive advection scheme of Smolarkiewicz (1983) is used since it reduces computational time compared to other similar schemes, minimizes numerical diffusion, prohibits negative concentrations that would destabilize the solution of the nonlinear quasi-stochastic coalescence/breakup equation, and can be easily extended to two and three spatial dimensions (see Smolarkiewicz and Clark 1986).

Treating the fall process separately from the breakup process, the change in number concentration is written:

\[ \frac{\partial n}{\partial t} + V_T(m) \frac{\partial n}{\partial z} = 0. \]  

(5)

Drops fall at their terminal velocity, \( V_T(m) \), determined by the balance of gravitational and drag forces. The velocities are based on the data of Gunn and Kinzer (1949) and fitted to an expression by Best (1950). The terminal velocity is the same for all drops in one bin, determined by substituting the mean mass for that bin into the Best formula. The height dependence of the drops’ terminal velocities is ignored and the density of air is assumed constant, thereby neglecting any height dependence of the collisional coefficients. For the three kilometer shaft used, the difference between velocities at the top and bottom of the shaft would be only 12 percent. Updrafts and downdrafts are not considered.

Using Smolarkiewicz’s approach, the solution to (5) is written as the standard “upstream” advection equation solution plus a corrective or antidiffusive term, estimated by expanding the standard solution in a second-order Taylor series. This is summarized by

\[ kQ_i^* = kQ_{i+1}^* + \left[ F(kQ_i^*, kQ_{i+1}^*, V_T) \right] \]

(6a)

\[ kQ_i^{N+1} = kQ_i^* - \left[ F(kQ_i^*, kQ_{i+1}^*, \bar{u}_{i+1/2}) \right] \]

(6b)

where \( kQ_i^* \) represents the total mass in bin size \( k \) for volume element \( i \) and time step \( N \), \( kQ_i^* \) is the solution to the “upstream” advection scheme, and the antidiffusive velocity, \( \bar{u}_i \), is given by

\[ \bar{u}_{i+1/2} = \frac{(|V_T| \Delta z - \Delta t V_T^2)(Q_{i+1}^* - kQ_i^*)}{(Q_i^* + kQ_{i+1}^* + \varepsilon) \Delta z}. \]  

(7)

In (7), \( \varepsilon \) is a small value used to ensure there are no undefined values. The mass fluxes of drops, \( F \), are determined for points staggered between the volume elements:

\[ F(kQ_i^N, kQ_{i+1}^N, v) = \frac{[(v + |v|)kQ_i^N + (v - |v|)kQ_{i+1}^N]}{2\Delta z}, \]  

(8)

where \( \Delta t \) and \( \Delta z \) are the time and space increments. The top volume element is labeled number one, and \( z = 0 \) corresponds to ground level. This solution is positive definite (Smolarkiewicz 1983) provided

\[ \frac{v \Delta t}{\Delta z} < 1 \]  

for all \( v \).  

(9)

The collisional algorithm consists of significantly more computations than the free fall algorithm, but finer resolution is needed for the free fall scheme because there is a larger change in the mass distribution in a volume of air due to the fall of drops than due to collisional breakup. Thus, each volume element is divided into ten subvolume elements to increase the resolution of the free fall interactions only. The time step is the same for both processes. The total mass, \( kQ_i \), in bin size \( k \), in subvolume element \( i \) of principal volume element \( j \), changes because of drop free fall and because of coalescence/breakup interactions. First, the free fall of the drops is calculated using the Smolarkiewicz advection scheme. Then, the mass change in bin size \( k \), in each subvolume element \( i \), of principal volume element \( j \), due to collisional interactions, denoted \( \Delta kQ_{i,j} \), is calculated as

\[ \Delta kQ_{i,j} = \frac{\sum_{k=1}^{k=40} kQ_i}{\sum_{k=1}^{k=40} kQ_j}, \]  

(10)

where

\[ \sum_{i=10}^{i=1} kQ_{i,j} = kQ_j. \]  

(11)

Here, \( kQ_j \) represents the total mass of drops in bin size \( k \) in all nested volume elements constituting the larger volume element \( j \). This method prevents negative concentrations. The change in mass in each bin size is distributed into the subvolume elements in this way because the mass change would be expected to be dis-
tributed in a similar manner as the mass itself. Other schemes for distributing the mass changes into the subvolume elements, such as dividing the total mass change by the number of subvolume elements, were tested and provided similar results.

Experiments were performed to determine the largest possible resolution: 20 volume elements, each having 10 subvolume elements, were the fewest possible that provided sufficient detail for the 3 km rainshaft. A time step greater than one second could not be used because of Smolarkiewicz’s (1983) stability requirement. Hence, for one hour of simulated time, more than 360 million computations must be performed.

3. Drop-size distributions occurring beneath the shaft top

Single pulses of drops were added at the top of an initially empty 3-km tall shaft for varying lengths of time. The pulse length ($T$), defined as the length of time drops were added, was set at 10, 120, 240, 600, 1800, 3600 and 7020 seconds. Two different simulations were performed for each $T$, with the input spectra being either the Marshall–Palmer (MP) distribution or the 3PED.

The number concentration for the MP distribution is given by

$$N(D) = N_0 e^{-AD},$$

(12)

where $N(D) dD$ is the number of drops per unit volume with diameters between $D$ and $D + dD$,

$$\Lambda = 41 R^{-0.21} \text{[cm}^{-1}],$$

(13)

with $R$ being the nominal rainfall rate (in mm h$^{-1}$); here, $N_0$ is set equal to 0.08 cm$^{-4}$. With $R = 50$ mm h$^{-1}$, the integrated rainrate is 54.0 mm h$^{-1}$, and hence this distribution is denoted by MP54. The 3PED that evolves from such a distribution in a box model, where mass is conserved, has a rainrate of 50.5 mm h$^{-1}$. However, MP54 is compared with 3P54 in order to have the same mass flux into the shaft.

Figure 1 shows the temporal evolution at two height levels of the total rainfall rate ($R$), total radar reflectivity ($Z$), total number density ($N$) and total mass density ($M$) for the case when an MP54 distribution is added at the shaft top for 240 seconds. These trends are typical for all cases with pulse lengths of 120, 240 or 600 seconds. The quantities dominated by the concentrations of the larger drops, namely $R$, $Z$, and $M$ reach maximum values at similar times and quickly drop to zero. These maxima correspond to the approximate arrival times of larger drops falling due to sedimentation alone. This suggests that most large drops fall through the shaft without losing their identity, probably because most large drops are reappearing after breakup as large breakup fragments or as part of a coalesced drop after a collision (see Valdez and Young 1985; List and McFarquhar 1990). The absence of large drops after this initial wave is due to insufficient interactions between the smaller, more slowly falling drops, which are unable to produce drops substantially larger than themselves.

The quantity $N$ is relatively larger than $M$, $Z$ or $R$ at later times because the slower falling small drops make significant contributions to $N$. Most small drops reaching the ground have been created by the collisions of other drops; after being produced by collisional breakup, they lag behind the larger drops and fall to the ground unperturbed. Because small drops are created at all different height levels and because there are different size small drops, the arrival of the small drops at the ground is spread out over a wide range of time. The first peak in $N$ happens at times similar to the peaks of other quantities; this helps show that smaller drops accompany the arrival of other size drops at the ground.

The presence of small peaks or fluctuations in the $N$ curves in Fig. 1 is caused by insufficient resolution of the mass scale. Since all drops of the same bin size are assumed to travel at the same speed, the drops tend to cluster together causing some oscillations in concentration with height and time. The problem is most noticeable for small $T$ and for $N$ curves. The paucity

![Figure 1](image-url)
of drop interactions for short pulse lengths magnifies the isolation of the different drop sizes and the smaller drops fall more slowly and thus there is greater time between the arrival times of drops in different bin sizes.

To verify this situation, the resolution was reduced by two to see if the problem intensified (increasing the resolution would have increased the computational time significantly). The magnitude of the irregularities in the number density curve increased considerably, but did not affect the general trends described in this paper (Fig. not shown). This showed that the original resolution is adequate.

Figure 2 shows how the number density flux per logarithmic diameter interval at the ground varies with time. The contours of number density flux are logarithmically spaced, so that the arrival of the leading and trailing edge of the packets is more clearly seen. The number density flux is dominated by drops of one particular size at each time, with the larger drops arriving first, followed by the smaller ones. This qualitatively replicates the pattern of drop arrival observed at the ground in Malaysia by List et al. (1988), who observed many such streaky packages. The interference between these packages will be discussed in Part II of this series to provide a more comprehensive description of the warm rain process.

The drop-size distributions are dominated by the arrival of one particular size at each time. The time for the maximum number density flux for each drop size closely matches the time at which those drops would reach the ground if only sedimentation of the initial drops in the pulse was considered (dashed line).

![Contour Plot](image)

**Fig. 2.** Contours of constant number flux density per logarithmic diameter interval arriving at the ground for drop size plotted against time after input of MP54 distribution of drops at top of 3 km shaft \( T = 240 \) s. The dashed line represents the expected time of arrival of the first drops in the pulse if they fell due to sedimentation alone; the dotted line, the arrival of the final drops in the pulse if they fell due to sedimentation alone. The contour intervals are logarithmically spaced.
The effect of small drop creation by breakup, representing the area to the left of this line, is highly prominent because different size drops start to reach the ground at approximately the same time. The coalescence of intermediate size drops into larger drops and of smaller drops into intermediate size drops leads to the arrival of some drops of a particular size after the main pulse, since they fall at slower speeds until they coalesce. This is represented by the area to the right of the final sedimentation line (arrival of last drops in pulse due to sedimentation alone).

In comparing number densities to number flux densities the weighting of the size classifications needs to be considered. Smaller drops travel more slowly, so number densities overemphasize their contributions compared to number flux spectra measured in the field. Number densities are examined in this paper hereafter because the 3PED has been based on number density in past works. For comparison with observed raindrop spectra, the number density is easily converted to number flux density because it is proportional to the terminal velocity of the appropriate drops.

To determine how drop size distributions vary with $T$ and input spectra, the number densities are summed over time because variations in the cumulative dropsize distributions (i.e., time integrated) can be more easily seen. The summed drop size distribution is normalized (area under the curve set equal to 1) so that comparison of the cases is easier. Figure 3 shows the cumulative normalized number densities at the ground for 120, 240, 600 and 1800 s pulse lengths for MP54 and 3P54 input, along with the input spectra itself. The 10 s pulse is not shown because there had not been enough interactions to allow for any trends to be seen in the data. Some of the cases are difficult to distinguish from each other because the drop size distributions are quite similar. In general, three peaks are noticeable for any $T$ and both input spectra, but the large peak is just a shoulder for the MP input since there are not sufficient interactions to generate a sharp peak.

The peaks for MP54 input are in the range of 0.22 to 0.27 mm and 0.7 to 0.93 mm, with a shoulder at 1.3 to 2.0 mm; for 3P54 input the peaks are at 0.22 to 0.27 mm, 0.8 to 0.93 mm, and 1.5 to 1.85 mm. These compare with the peaks of the initial 3PED at 0.268 mm, 0.920 mm, and 1.724 mm. The 3PED is a little different from that of List et al. (1987) because slightly different numerical methods were employed to calculate the interaction matrix (see List and McFarquhar 1990). List et al. (1988) also observed drop size distributions that had three noticeable peaks in number density in the field when they were cumulated with respect to time.

There are more small drops in the cumulative distributions at different heights than in the 3PED because, even for the 1800 s case representing an almost steady state event, there are many small drops and no large drops in the tail end of the pulse to collect them. Hence, they have a larger weighting in the cumulative distribution because they are included in the sum for a longer period of time. An increase of $T$ beyond 1800 s would bring the normalized drop size distribution closer to the 3PED by diminishing the relative importance of the front and tail ends of the pulse (case of 3600 s pulse not shown).

A measure of distance between two drop size distributions is defined by using Eq. (10) of List and McFarquhar (1990),

$$\rho(\vec{x}, \vec{y}) = \left( \sum_{i=1}^{40} (x_i - y_i)^2 \right)^{1/2}, \quad (14)$$

where $\vec{x} = (x_1, \ldots, x_{40}), \vec{y} = (y_1, \ldots, y_{40}); x_i, y_i$ denote the number density per logarithmic diameter interval in the $i$th bin size. This is based on the representation of a given drop spectrum by a vector, whereby every bin size represents a space coordinate and the number density per logarithmic diameter interval of that size class gives the magnitude of that coordinate. The quantity $\rho$ then becomes the distance of the endpoints of two vectors from each other. Figure
4 shows how $\rho$, the distance between a normalized cumulative drop size distribution and the normalized 3PED, varies with height for different $T$ for both MP54 and 3P53.5 input at the shaft top. For comparison, the distance from the origin to the unnormalized 3P54 is $8094 \text{ m}^{-3}$, to the normalized 3P54 is $1.95 \text{ m}^{-3}$, and to the normalized MP54 is $1.60 \text{ m}^{-3}$. The drop size distributions at each height level are added for all times up to 7020 s; any precipitation reaching the ground after this time does not make a substantial contribution to $\rho$ (e.g., for $T = 3600 \text{ s}$ and MP54 input, the forty-dimensional distance between the spectra averaged over $7200 \text{ s}$ compared to that averaged over $10\,800 \text{ s}$ is $0.07 \text{ m}^{-3}$). For the $T = 7020 \text{ s}$ case, the drop size distributions are also cumulated for the first 7020 s indicating that this is just the steady state case. Here, $\rho$ increases as the distance below shaft top increases for most cases, except for an initial decrease at the top of the shaft for MP54 input. This final increase is at least partly due to increased weighting of smaller drops. However, when $T$ is increased to 3600 seconds, $\rho$ asymptotically approaches a constant value near the bottom of the shaft showing that steady state behavior is being approached. It has been suggested that the reversal in $\rho$ that occurs is caused by the reversals in evolving raindrop size distributions discussed by Brown (1989, 1990). This possibility will be discussed in more detail later.

The value of $\rho$ at the bottom of the shaft for the 240 and 600 second pulses is just as large as the initial distance between the normalized MP54 and 3P54 distributions. However, Fig. 3 shows that the cumulative distributions at the shaft bottom for these cases do have three peaks in number concentration. The location of the peaks is approximately the same as for the 3PED, but the relative heights are different. This indicates that a careful distinction must be made between three-peak distributions (3PDs) and the three-peak equilibrium distribution (3PED). This difference partly occurs because there are more small drops in the tail of the pulses when the spectra are integrated over time. Hence, the number of medium and large drops is reduced due to the normalization procedure. This might also explain why three-peak drop-size distributions cumulated over time at the ground in nature for nonsteady rain (e.g., List et al. 1988) do not agree in detail with the trimodal equilibrium distribution predicted by the box model.

In Fig. 3, for both MP54 and 3P54 input, the small peak is largest for $T = 600 \text{ s}$, followed closely by $T = 240 \text{ s}$, and then by $T = 1800 \text{ s}$ and $T = 120 \text{ s}$. The middle peak is largest for the 1800 s input pulse, followed by the 120 s, 240 s, and 600 s length pulses. The ordering for the highest diameter peak is virtually identical, except that the peak corresponding to $T = 600 \text{ s}$ is higher than for $T = 240 \text{ s}$, with the height being similar for the 120 s, 240 s, and 600 s pulse inputs.

These trends might be explained by recalling that the smallest two peaks form first (List et al. 1987) when an MP distribution evolves towards a 3PED. For the shortest $T$, there have probably not been enough breakup interactions to produce large surpluses of small drops seen for the $T = 240 \text{ s}$ and $T = 600 \text{ s}$ cases. For $T = 1800 \text{ s}$, the relative number of small drops is reduced because the fractional importance of the small drops in the tail end of the pulses is less (i.e., the time during which drops of all sizes in a package are arriving at the ground accounts for a larger fraction of time so large and medium drops have larger fractional importance). This trend continues for the 3600 s pulse case. This explanation is valid for 3P54 input as well as MP54 input because the different drop sizes initially become segregated from each other due to their different fall speeds. Hence, the 3PED is not present at any level and time even for the case of 3P54 input; the time-averaged distribution approaches the 3PED when the pulse length increases to very long times (3600 s).

A critical pulse length at which the concentration of small drops is a maximum, and that of the large drops is a minimum for the normalized distributions, exists. The reduction in the numbers of medium and large drops for these cases may occur because the total number of drops increases as a result of the small drop increase; hence, it is only the fractional importance of the medium and large drops that is reduced. This can be seen from the cumulative drop size distributions at the ground plotted in Fig. 3, and from the behavior of

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Fig. 4. (a) Distance between cumulative drop size distributions at each height level and 3PED defined using Eq. (10) of List and McFarquhar (1990), $\rho$, versus height above ground level when an MP54 distribution is added at the top of the shaft for varying $T$. (b) Represents the same quantities when there is 3P53.5 input.
in Fig. 4. The maximum value of \( \rho \) occurs for \( T \) between 240 and 600 seconds at most heights, but there is not sufficient data to predict the exact value of the critical pulse length; extensive computational time would be required.

These trends are somewhat similar to the reversals in evolving raindrop size distributions due to the opposing effects of coalescence and breakup that Brown (1989, 1990) has observed. He found that distributions evolving from MP spectra rapidly overshoot their equilibrium position, reverse direction, and then settle slowly towards equilibrium. List and McFarquhar (1990) also showed that a distribution evolving from an MP54 distribution moved further away from the 3PED between 120 s and 350 s before approaching it steadily. They found many small drops were created in the first few minutes of evolution, reaching a maximum concentration 4 min. after the start of interactions, as they overshoot their equilibrium value. Thus, it is reasonable that the \( T = 240 \) s and \( T = 600 \) s cases have the most small drops; for \( T = 240 \) s, there are exactly 240 s of time for interactions between all drop sizes after which the various drop sizes become separated and the interactions are not as plentiful. A critical pulse length, between 240 s and 600 s, representing the furthest distance from equilibrium, is also consistent with Brown’s (1990) estimate that the small-drop concentration reaches a maximum 300 s after an MP distribution is input in a box model. In Fig. 4, for MP54 input, \( \rho \) rapidly decreases before it increases again. This could also be due to these reversals. Thus, the cases for \( T \) around the critical pulse length may represent those cases that have overshot equilibrium values at given peaks and have yet to start their slow approach towards overall equilibrium.

Measures of distance between drop-size distributions can be defined by using quantities other than the number density, such as the number flux density, the mass density or the radar reflectivity density. Equation (14) is again used to define \( p \) with \( x_i \) and \( y_i \) representing the quantities listed above. The results and trends seen using \( p \) based on number flux density are similar to those previously discussed. However, there are differences when \( p \) is based on the other quantities, which are dominated more by contributions from the larger drops. For the normalized radar reflectivity curves, for example, \( p \) continually decreases from a value of 3.3 \( \mu m^3 \) at the shaft top to 2.5 \( \mu m^3 \) at the bottom for an MP input with \( T = 120 \) s. There is no evidence for a critical pulse length. For higher \( T \), \( p \) decreases to lower values at the ground (2.0 \( \mu m^3 \) for \( T = 240 \) s; 1.1 \( \mu m^3 \) for \( T = 600 \) s; and 0.4 \( \mu m^3 \) for \( T = 1800 \) s) from the same value at the shaft top. For 3PED input, \( p \) steadily rises to 1.0 \( \mu m^3 \) at the ground from zero at the top for \( T = 120 \) s, and to steadily decreasing maxima at the bottom as \( T \) increases. These results show that the drop size distribution within the shaft does indeed evolve away from the MP and towards the 3PED as \( T \) increases and as the drops fall through the shaft, especially in terms of large drop concentration. Because the critical pulse length no longer occurs, the small drop concentration must be primarily responsible for this pattern. The trends seen for \( p \) based on mass density are similar to for \( p \) for radar reflectivity.

There is not a large difference between the cumulative normalized drop size distributions that are obtained for different \( T \). Thus, some of the small differences between drop size distributions could also be caused by mass resolution difficulties; for example, if the peak position was to shift to the left or right by half a bin size, this could appear as a reduction in the peak height. However, the principal conclusions should not be altered even if some of these shifts exist. Thus, given enough time and fall distance to evolve, the behavior of the drop size distributions does not depend on the input spectra (Markov property of stochastic processes, see Valdez and Young 1985). In addition, a 3PED is produced by drop interactions when there is a sufficiently large pulse length, and 3PDs are always present at any level in the shaft for \( T \) greater than or equal to 120 s when the drop size distributions are integrated over time.

4. Arrival time of different size drops

The time at which different sized drops begin to arrive and cease to arrive at ground level may give some interesting information. Comparison of such times with field data may help assess how well this “simple” model replicates observed phenomena. Figure 5 shows the time at which different size drops begin to arrive at ground level for four different pulse lengths and two different input spectra. The 3600 s pulse case is not shown because increasing \( T \) beyond 1800 s does not affect the arrival time of drops. For MP54 input, the \( T = 600 \) s and \( T = 1800 \) s cases are not clearly seen because they are identical. The initial arrival time of drops of a certain size at the ground, \( t_i(D) \), is defined as the time step before the number density per logarithmic diameter becomes greater than a critical value of 1 \( m^{-3} \). This value ensures that the arrival of the drops is physically realistic; the use of a critical value of 0 \( m^{-3} \) would give the same \( t_i \) for all drop sizes because of small amounts of numerical diffusion. The use of large critical values, such as 10 \( m^{-3} \), yields almost identical results as that for 1 \( m^{-3} \). Here, \( t_i \) is not defined for those drop sizes where the critical number is not reached, such as very large or very small drops.

The choice of input spectra has little effect on the initial arrival time of drops. \( T \) has little effect on the \( t_i \) of medium and large size drops because \( t_i \) depends on the first drops in a pulse, and the first larger drops fall through the shaft without experiencing many interactions. Here, \( T \) affects \( t_i \) for very small drops (\( D < 0.15 \) mm); these first small drops at the ground are breakup
$b = 6.992$, and $0.15 < D < 4.5$ mm ($x^2 = 0.79$) for the case with MP54 input, and $m = -1.856$, $b = 7.257$ and $0.15 < D < 3.7$ mm ($x^2 = 0.48$) for 3P54 input. This indicates that most drop sizes begin reaching the ground at approximately the same time.

After the release of drops at the shaft top has stopped, drops will eventually cease to arrive at the ground. The time at which a drop size stops reaching the ground is called the final arrival time, $t_f$. It is defined like $t_i$, except it is the time at which the number density per logarithmic diameter interval becomes less than $1$ m$^{-3}$.

Figure 6 shows $t_f$ versus the diameter for different $T$ after an MP54 distribution is added at the shaft top. The case with a 3P54 input is virtually identical. The curves have a similar shape, like that of the curve describing drop arrival of drops when sedimentation alone is considered.

Using this information $t_f$ was parameterized as a function of diameter

$$t_f = A + \frac{B}{1 - \exp[-(D/1.77)^{1.47}]}$$

with $t_f$ measured in minutes. The denominator of (13) is proportional to the terminal velocity of a raindrop of size $D$. The quantity $A$ should be a function of $T$, and $B$ should be independent of $T$, provided sedimentation dominates $t_f$ (i.e., no drops of diameter $D$ produced by coalescence of other drops follow after these original drops). Table 1 shows the values of $A$ and $B$ and $x^2$ determined for this fit. The fit seems to be very good for small $T$, with $x^2$ increasing for larger $T$, and then decreasing again for the nearly steady state 3600 s case. As expected, $B$ is only weakly dependent on $T$.

The value of the constant $B$ can be calculated theoretically. Using a three-kilometer high shaft, a value of $B = 5.15$ min is obtained. This is a little lower than the $B$ values obtained from the fit. This suggests that coalescence of intermediate size drops is playing at least

![Fig. 5. (a) Initial arrival time of drops at ground, $t_i$, versus drop size for inputs of different pulse length MP54 distributions added at the top of the shaft. (b) Represents the same thing for cases with a 3P54 input. Some lines are not seen because other lines are superimposed in the same position ($T = 600$ s and $T = 1800$ s); $t_i$ is that time when $a(t)$, the number density per logarithmic diameter, for that size drop is greater than $1$ m$^{-3}$.](image)

![Fig. 6. Time at which drops cease to arrive at the ground, $t_f$, versus diameter for inputs of different pulse length MP54 distributions added at the top of the shaft. The value of $t_f$ for a shaft, where sedimentation alone is calculated, is also shown (negligible pulse length); $t_f$ is the time at which $a(t)$ ceases to be larger than $1$ m$^{-3}$.](image)
TABLE 1. Parameters for fit of $t_f$ against diameter performed using Eq. (13).

<table>
<thead>
<tr>
<th>Input spectra</th>
<th>Pulse length (s)</th>
<th>$A$ (min)</th>
<th>$B$ (min)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP54</td>
<td>10</td>
<td>-0.330</td>
<td>5.641</td>
<td>0.10</td>
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<tr>
<td>MP54</td>
<td>120</td>
<td>3.195</td>
<td>5.470</td>
<td>0.53</td>
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<td>MP54</td>
<td>240</td>
<td>3.736</td>
<td>5.574</td>
<td>3.29</td>
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<td>MP54</td>
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<td>12.508</td>
<td>5.604</td>
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</tr>
<tr>
<td>MP54</td>
<td>1800</td>
<td>30.755</td>
<td>5.836</td>
<td>55.49</td>
</tr>
<tr>
<td>MP54</td>
<td>3600</td>
<td>63.047</td>
<td>5.550</td>
<td>1.90</td>
</tr>
<tr>
<td>3P54</td>
<td>10</td>
<td>-0.079</td>
<td>5.915</td>
<td>0.23</td>
</tr>
<tr>
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<td>120</td>
<td>3.443</td>
<td>5.545</td>
<td>2.33</td>
</tr>
<tr>
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<td>6.407</td>
<td>5.506</td>
<td>4.37</td>
</tr>
<tr>
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<td>600</td>
<td>12.107</td>
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<td>3P54</td>
<td>3600</td>
<td>62.637</td>
<td>5.558</td>
<td>1.83</td>
</tr>
</tbody>
</table>

MP54 = Marshall-Palmer 54 mm h$^{-1}$ rainrate distribution.
3P54 = three-peak distribution with same rainrate.

some role in providing the final drops of most size classifications because $t_f$ increases as $B$ increases.

The investigation of the arrival times may have to be reexamined at some later time due to the current restrictions of this microphysical model. Pressure perturbations and induced air flow caused by falling drop ensembles may shorten arrival times considerably (Clark and List 1971). However, the ability of smaller drops (in a parcel of drops with a spectrum of particles) to follow larger drops faster because they gain more speed by downdrafts induced by larger drops (List and Clark 1973) may be less important because the coalescence/breakup process maintains a local mix of drops of different sizes more effectively.

These fits of $t_i$ and $t_f$ may be useful in the future when experiments with an input of multiple pulses are conducted (part II of this series). If the parameterizations of both $t_i$ and $t_f$ are known, then both the time and height at which different pulses will meet and the size of drops involved in the interference can be predicted. Only the effect of the interference on the drop-size distributions should have to be investigated for the case of a multipulse input, as will be done in Part II.

5. Conclusions

A numerical simulation of the interactions of raindrops in one-dimensional rain shafts with still air is used to calculate the changes in number concentration of different size drops during coalescence and collision-induced breakup and the fall of drops. The coalescence/breakup was based on the parameterization of laboratory experiments (Low and List 1982), and the fall of drops was calculated using Smolarekiewicz’s (1983) advection scheme. A single pulse of drops was released at the top of an initially empty shaft, and the evolution of raindrop spectra during fall was examined. The main conclusions for these initial conditions are

1) The three-peak equilibrium distribution (3PED) is not present at any time or any level throughout the shaft, even for the case of 3PED input, because at first drops of different sizes rapidly separate from each other due to their different fall speeds.

2) The time-integrated drop-size distributions at any level in the shaft have three peaks in number concentration (3PD) for pulse lengths larger than 120 s (Fig. 3). The 3PD peaks occur in the diameter ranges 0.22 to 0.27 mm, 0.7 to 0.93 mm, and 1.3 to 2.0 mm. These peaks are at positions similar to those of the 3PED, but there is a difference in the relative heights of the peaks.

3) The forty dimensional distance between the number concentrations of 3PDs and the 3PED can be almost as large as that between an MP distribution and the 3PED itself, showing that a careful distinction between a 3PD and the 3PED must be made. The time-averaged 3PD does approach the 3PED for large pulse lengths of 3600 s and more. Cumulative drop size distributions examined by List et al. (1988) for the case of nonsteady rain in Malaysia also displayed higher numbers of drops at the equilibrium peaks' positions.

4) There is a critical pulse length, between 240 and 600 s, at which the cumulative concentration of the small drops in the normalized distribution and the value of the metric $\rho_3$ describing the forty dimensional distance between a drop size distribution and the 3PED, are maxima. However, the radar reflectivity concentration of 3PDs steadily approaches that of 3PEDs with increasing pulse length.

5) The trends for how the drop size distributions vary with pulse length are the same for 3PED and MP distribution input.

6) Provided that the pulse length $T$ is larger than about 240 s and excepting small drops, $T$ does not significantly affect the initial arrival time of different sized drops at the ground.

7) The times at which drops cease arriving at the ground are slightly larger than those due to sedimentation alone because such drops may be created later by smaller ones.

This work is a good starting point for studying the behavior of drop size distributions in warm rain shafts. Future work will extend these results to the case of a multipulse input (Part II). Further insight about the warm rain process may be gained by going beyond the spectra evolution during rainfall in still air through full modeling of the whole rain formation process with cloud dynamics, starting from nucleation and ending with the falling rain. Such sophisticated modeling with complete microphysics–dynamics feedback would have to include pressure perturbations induced by the drops and the resulting alteration of the velocity field of the air that forms the environment for the freely falling drops (List and Lozowski 1970; Clark and List 1971). Comparing model results to data collected from field
studies may indicate if the full rigor for the treatment of pulsed rain is necessary. However, the simplistic study reported here may clarify some conceptual questions of nonsteady rain.

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REFERENCES


