The Evolution of Three-Peak Raindrop Size Distributions in One-Dimensional Shaft Models. Part II: Multiple Pulse Rain

GREG M. McFARQUHAR AND ROLAND LIST

Department of Physics, University of Toronto, Toronto, Ontario, Canada

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ABSTRACT

The release of multiple pulses of rain with durations of 120 s < T < 600 s and pulse repetition periods of 240 s < r < 1200 s at the top of one-dimensional shafts, followed by drop coalescence and breakup during the fall, leads to the arrival of drops in overlapping packages at the ground, with the largest drops first in the individual packages. When the drop spectra are averaged over time, three-peak distributions (3PDs) are found. More frequent and shorter pulses produce 3PDs that resemble three-peak equilibrium distributions (3PED) for steady warm rain closer than less frequent longer pulses because overlapping pulses lead to more interactions between large and small drops. When a 3PED is released at shaft top, it disappears because different size drops fall at different speeds, and then must be recreated by drop interactions.

These multiple pulses replicate trends measured during nonsteady tropical rain better than single pulses. A "pulse interference diameter," based on the parameterization of single pulse rain, describes how much interaction occurs between pulses.

1. Introduction

To simulate observed temporal and spatial inhomogeneities in raindrop-size distributions, processes affecting raindrops must be modeled in shafts having a nonsteady input of drops at the top. Warm rain modeling has shown that an equilibrium distribution evolves at the ground, independent of input spectra, after sufficient time in both box models (Donaldson 1984; Valdez and Young 1985; List et al. 1987; Brown 1988; Feingold et al. 1988; and List and McFarquhar 1990a) and in shaft models with steady input (Donaldson 1984; List et al. 1987; Tzivion et al. 1989) when the Low and List (1982) parameterization of laboratory raindrop collisions is used. This equilibrium distribution has three maxima in number density per logarithmic diameter when plotted against the logarithm of drop diameter and is termed the three-peak equilibrium distribution (3PED).

List et al. (1987) modeled nonsteady rain by sinusoidally varying the initial rainfall rate at the shaft top and determined its influence on the raindrop-size distributions. Levin et al. (1988) studied the effect of imposing sinusoidal variations in the small-to-large drop component ratio in the initial exponential spectra. List and McFarquhar (1990b, henceforth LMI) used an Eulerian model to examine the evolving drop-size distributions at all levels throughout the shaft when a single pulse of drops was released at the top for a certain length of time. Even though the drop populations arriving at the ground at any one time were dominated by drops of a particular size, the cumulative time-integrated distributions had three peaks in number concentration at the same diameters as for the 3PED. However, the relative peak heights were quite different, and hence the spectra were called three-peak distributions (3PDs).

Several field studies have investigated the nature of raindrop spectra arriving at the ground. Willis (1984), Steiner and Waldvogel (1987), Asselin de Beavville et al. (1988), and Zawadzki and de Agostino Antonio (1988) have all observed distributions with peaks, some of whose locations are quite similar to those of the 3PED. However, there are also measurements of raindrop spectra without the presence of multiple peaks, and some observations may have misleading biases due to the instrumentation difficulties discussed by List (1988) and Sheppard (1990). Temporal and spatial inhomogeneities in nonsteady rain have also been recorded. Waldvogel (1974) described the variations in the raindrop spectra as jumps in the intercept of the exponential distributions commonly used to characterize drop-size distributions (Marshall and Palmer 1948). Other authors (Warner 1969; Joss and Gori 1978) have observed single modes in the spectra varying with time and altitude.

Figure 1 shows the number of differently sized drops arriving at the ground, as measured by a disdrometer, varying with time during a typical nonsteady rain event during Phase 1 of the Warm Rain Experiment in Pen
ang, Malaysia. Several other nonsteady rain cases observed displayed trends similar to the ones seen here. No correction was made for the number of smaller diameter drops not counted during heavier rainfalls because none were detected below a threshold that varied with rainrate. The drops arrived in packets, with the largest drops first, followed by the smaller drops. Asselin de Beauville et al. (1988) observed similar examples in Guadeloupe, but found smaller drops arriving at the ground before the larger ones. The behavior of single drop packets has been qualitatively replicated in Part I (LMI). This analysis is now expanded because interference between packets was frequently observed, as large drops of one pulse arrived at the ground simultaneously with smaller drops of a preceding pulse. This is seen, for example, at times of 17, 26, and 39 minutes in Fig. 1. Evidence for the equilibrium peaks could be demonstrated by summing the number of drops of each size category over time.

This pulsating behavior of warm rain with pulse repetition periods between 5 and 15 min can be qualitatively replicated by releasing a series of square-wave rain pulses at the shaft top. The results are compared with Part I to determine if single pulse results can be extended to multiple pulses. Time-averaged distributions at various altitudes, the interference rate between different packets of drops and the time required for a repeating pattern to develop are also studied. This type of model assumes that the source cloud produces rain, followed by a collapse and regeneration of the precipitation zone in the cloud. Mixing of rain packages, originating from different clouds or cells, by shear may be another cause for pulse overlap. In all cases, larger drops reach the ground before most smaller drops due to fall velocity. The first possibility is examined in this paper because it may be simulated using a one-dimensional model; a two- or three-dimensional model with shear or treatment of phenomena inside the cloud would require substantially more computational time. Phase II of the Tropical Rain Experiment in Malaysia, executed in October–November 1990, may also help differentiate between these two possibilities by observing, with a Doppler radar, the origin and fall of rain packages.

2. The model

Two processes are included in the shaft model: the collisional breakup and coalescence of raindrops, and the fall of the drops. The collisional breakup of drops is based on the parameterization of the fragment size distributions produced by laboratory raindrop collisions (Low and List 1982); the drop fall algorithm uses the simple, positive, definite advection scheme of Smolarkiewicz (1983) that limits numerical diffusion. Details of the model are found in LMI. The model used in this study is identical to that used in Part I, with the exception of the upper boundary. As in Part I, the microphysics–dynamics feedback found by Clark and List (1971) and thermodynamic effects are not considered here. Thus, pressure perturbations and induced air flow around the drop ensembles, which alter the fall of particle zones and the arrival times of differently sized drops at the ground, are neglected in this conceptual investigation of nonsteady rain.

List and McFarquhar (1990b) released raindrops at the shaft top in single pulses for varying lengths of time, called the pulse length (T). Here, the numerical experiments with a continuous release of pulses of drops with various pulse repetition periods (τ) and lengths (T) are summarized in Table 1 together with the input spectra. For each T and τ, a Marshall–Palmer distribution with rainrate 54 mm h⁻¹ (MP54) and a three-peak equilibrium distribution (3PED54) with the same rainrate were input in different simulations. A case with the input of a Marshall–Palmer distribution with rainrate 10 mm h⁻¹ (MP10) was also performed. The heavy rainrate of 54 mm h⁻¹ is used because there are more raindrop interactions, and hence the equilibrium peaks should be easier to see. This rate corresponds to heavy rain in Malaysia, but no attempt was made to replicate particular rain showers.

<table>
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MP54: Marshall–Palmer 54 mm h⁻¹ rainrate distribution.
3PED54: three-peak equilibrium distribution with same rainrate.
MP10: Marshall–Palmer 10 mm h⁻¹ rainrate distribution.
3. Simulated pattern of drop arrival at the ground

Figure 2 shows how the number density flux per logarithmic diameter measured at the ground varies with time for the case $T = 120$ s, $\tau = 600$ s, and MP54 input. This case shows trends typical of the other cases, and qualitatively replicates most Malaysian features (Fig. 1). The drops arrive in packets, with the large drops first in each (in Fig. 1, this most clearly seen for drops smaller than 1.7 mm, with prominent pack-
ages beginning at 27 min and 51 min); and the drop-size distributions are dominated by a small range of drop sizes at each particular time. Smaller drops accompany the arrival of larger drop sizes at all times in both the observed and modeling cases. Evidence for a 3PD is also seen when the time-averaged drop-size distributions are plotted, as will be shown later. The interference between two packets can be increased or reduced to match particular observations by adjusting $T - \tau$. The drop diameters at which the interference between pulses begins is the same for cases with constant $T - \tau$ since it takes the same time for the larger drops to overtake the smaller drops.

The average rainrate for the numerical case (Fig. 2) is 11.4 mm h$^{-1}$, while it was only 0.9 mm h$^{-1}$ for the case of Fig. 1. Other cases, included in the data catalogue, showed similar features with much higher average rainrates. The difference is not significant because trends in the data are being compared rather than numerical values. It is only suggested by the similarity of the model and field study that the multipulse release is a good explanation of this arrival pattern of rainfall. The pattern of rainfall arrival, such as the interference rates between the packages or the time of arrival of drops at the ground, is not affected by the lower rainrates (a simulation with average rainrate 2 mm h$^{-1}$ was performed; figure not shown). There would be fewer collisions, and, therefore, the assumption of a quasi-stochastic coalescence/breakup scheme (LMI) is not as good, and the three peaks are not as prominent because it takes more time for their evolution.

The typically observed period between packets of 5 to 15 min is comparable to the Brunt–Väisälä period, suggesting that buoyancy oscillations could cause these packages, but only more complete cloud models could verify this. Another possibility is that a critical amount of liquid water may be needed before cloud droplets are effectively converted to precipitation-size particles and fall to the ground, like in Kessler’s (1969) autoconversion mechanism. Time would then be required between pulses to replenish the water supply and regenerate the precipitating cloud. More research is needed to develop the relationship between successive pulses and phenomena inside the cloud.

4. Time-averaged drop-size distributions

Drop-size distributions arriving at the ground are added over one complete cycle (i.e., $\tau$), and then normalized (as in LMI) so that differing amounts of precipitation do not dominate the comparison. The last complete cycle calculated in a row of pulses (e.g., 3001–3600 s for $\tau = 600$ s and a total simulated time of 3600 s) is chosen for the spectra addition so that the steady-state effects rather than the initial effects of the drop interactions are shown.

Figures 3a–d show the time-averaged drop-size distributions at ground level for cases with $R = 54$ mm h$^{-1}$. There is evidence for three-peak distributions (3PDs) averaged over time for 3PED54 input, but, besides two peaks, there is only a slight shoulder at the location of the large-drop peak for MP54 input. The peaks for MP54 input are in the range of 0.22 to 0.3 mm and 0.75 to 0.95 mm, with a shoulder at 1.4 to 2.0 mm; for 3PED54 input the peaks are at 0.22 to 0.3 mm, 0.75 to 0.95 mm, and 1.45 to 1.55 mm. These peak locations are almost identical to those observed by LMI for single pulses of rain and contain the peak positions of the initial normalized equilibrium distributions (3PED54) at 0.27 mm, 0.92 mm, and 1.72 mm (List and McFarquhar 1990a). A smaller peak around diameter 0.45 mm is only observed at the ground for the cases $T = 120$ s, $\tau = 1200$ s (MP54 and 3PED54 input) and $T = 240$ s, $\tau = 1200$ s (MP54 input especially). No evidence for this subpeak is seen at midlevel in the shaft, and no physical or numerical explanation for its existence has been found yet.

The height of the normalized small-drop peak is significantly smaller for multipulse input as compared to single pulse input (LMI), with an average maximum number density per logarithmic diameter, $a(l)$, of 0.75 m$^{-3}$ compared to 1.0 m$^{-3}$ for single pulse input. Conversely, the medium-drop peak (0.35 m$^{-3}$ compared to 0.2 m$^{-3}$) and the large-drop peak or shoulder (0.2 m$^{-3}$ compared to 0.1 m$^{-3}$) are higher for the multipulse input. At the end of the pulse for single-pulse input, there are many small drops and no large drops, meaning the small drops have a large weighting in the cumulative distribution because all drops reaching the ground are included in it (see LMI). For multipulse input, the small drops interact with the large drops of the next pulse as well as those within the same pulse; these increased interactions promote the growth of the
3PED. Therefore, the heights of the peaks of the time-averaged drop-size distributions for multipulse input are closer to the heights of the peaks for the normalized steady-state 3PED. Hence, peak heights (and positions) must be close to those of the 3PED for the drop-size distribution to be near equilibrium.

Asselin de Beauville et al. (1988) measured similarly shaped spectra with the same peaks at different altitudes below cloud base. In this modeling study, the 3PDs are also seen at several different altitudes; in fact, for the input of a 3PED54, a 3PED is initially found at the shaft top because it is released there. It disappears below the top as the different drop sizes separate due to their differing fall speeds, and must be recreated from drop interactions. For MP54 input, the two smallest diameter peaks become visible 150 m below the shaft top, with the large-drop peak not being seen so clearly. However, for smaller rain rates (MP10 input, $T = 120$ s, $\tau = 600$ s), these two peaks are not seen clearly until 2 km below shaft top; the largest diameter peak never becomes visible. This occurs because there are not enough interactions.

Before attempting to explain the trends in Fig. 3, a measure of distance between two drop-size distributions is defined using Eq. (14) of LMI:

$$p(\bar{x}, \bar{y}) = \left[ \sum_{i=1}^{40} (x_i - y_i)^2 \right]^{1/2},$$

where $\bar{x} = (x_1, \cdots, x_{40})$, $\bar{y} = (y_1, \cdots, y_{40})$; $x_i$, $y_i$ denote the number densities per logarithmic diameter in the $i$th bin, where there are 40 bins in total. Figure 4 shows how $p$, the distance between the spectrum vectors representing the normalized time-averaged drop-size distribution and the normalized 3PED, varies with height for different $T$, $\tau$, and input spectra. For all MP54 inputs, $p$ rapidly decreases below shaft top, then increases, and finally approaches a constant value asymptotically.

For 3PED54 input, there is an initial increase in $p$ below shaft top because the initially present equilibrium distribution disappears before being approached near the shaft bottom, where $p$ asymptotically approaches different small values ($p < 0.25$ m$^{-3}$). For comparison, in 40-dimensional phase space, the distance from the origin to the tip of the normalized MP54 vector is 1.60 m$^{-3}$ and is 1.95 m$^{-3}$ to the tip of the normalized 3PED54 vector; the length of the normalized 3PED vector does not change with rainrate because of the linear relation between the drop-size distribution and rainrate discussed by List (1988).

Information is also obtained by looking at the variation with height in the number of small, medium, and large drops under each of the peaks in the cumulative distributions, with the dividing diameters at 0.575 mm and 1.448 mm, as in List and McFarquhar (1990a). Figure 5 shows the variation with height of the three size groups after normalization for the
raindrop interactions. In the steady case, rain would not reach the ground in pulses, meaning instantaneous and time-averaged distributions are the same. The 3PDs would be seen at most times and most height levels (figure not shown) with the exception of the lower rainrate cases as previously discussed. The 3PEDs are always present throughout the shaft for 3PED input.

Trends for the variation of $\tau$ while keeping $T$ constant ($120$ s) may also be explained. The more frequent release of drops, that is smaller $\tau$, at the shaft top yields a drop-size distribution closer to the 3PED (Fig. 4) because there are more interactions between drop packages. A visual comparison of the time-averaged drop-size distributions at the ground (Fig. 3) also shows that the $\tau = 240$ s case is closest to the 3PED, followed by the $\tau = 600$ s and $\tau = 1200$ s cases.

An attempt was made to determine a scaling relation when the ratio of pulse length to pulse repetition period was constant. This is a different scaling relation than that used by Srivastava (1988). When $\tau = 2T$, the normalized time-averaged drop-size distributions in Fig. 3 are similar, but not exactly the same. There are more small drops when $T = 600$ s ($\tau = 1200$ s) followed by $T = 120$ s ($\tau = 240$ s) and $T = 240$ s ($\tau = 480$ s). The ordering for the number of medium and large drops under the peaks in the normalized distributions is opposite. In general, drop-size distributions from the shorter, more frequent pulse input cases ($\tau = 240$ s and 480 s) seem closer to equilibrium, indicating that interactions between pulses are more effective than interactions within a pulse at forcing the drop-size distribution to approach the 3PED over this range of $T$ and $\tau$. This is reasonable since the differential fall speeds of various size drops allow large drops to overtake small and medium ones in other pulses causing more raindrop collisions. Breakup events caused by two colliding drops from the same pulse colliding are less frequent because the drops become separated due to their different fall speeds. Thus, further attention will be given to how raindrops from different pulses or packages interfere with each other.

$T = 120$ s, $\tau = 600$ s cases with MP54 and 3PED54 input. The similarity of these numbers to the numbers of small, medium, and large drops in the 3PED serves as further justification for comparing the cumulative distributions to the 3PED, as in Fig. 4. The evolution trends are similar for both input spectra, with the exception being that the equilibrium number of large drops (labeled $N_L$ on the axis) is approached more closely for 3PED54 input. This suggests that there are not sufficient interactions to establish an equilibrium population of large drops for MP54 input. The number of small drops initially increases below shaft top and then asymptotically approaches a constant value, whereas the number of large drops decreases slightly below shaft top. There is also a decrease in the number of medium-size drops until around 2500 m. The decreases occur due to the normalization procedure; the normalized number of medium drops decreases because the total number of drops increases due to the small drop increase. Total number is not conserved because pulsed rain does not represent an equilibrium situation. The actual number of medium-size drops increases slightly before asymptotically approaching a constant value (figure not shown); the number of large drops remains approximately constant.

As $T$ approaches $\tau$, the case of steady rain is approached (see List et al. 1987). For MP54 input the 3PED would not be present throughout the entire shaft (as for 3PED54 input) because of lack of sufficient

![Figure 4](image-url)

**Fig. 4.** (a) Distance, $\rho$, between normalized time-averaged (over last cycle) drop-size distribution and 3PED vectors, defined using Eq. (1) versus height above ground, for MP54 input at the shaft top for various $T$ and $\tau$. (b) Represents the same quantities when there is 3PED54 input.

![Figure 5](image-url)

**Fig. 5.** Number of small, medium, and large drops under respective peaks versus height for normalized time-averaged drop-size distributions for $T = 120$ s, $\tau = 600$ s, and MP54 and 3PED54 input. $N_S$, $N_M$, $N_L$ represent the normalized number of large, medium, and small drops, respectively, for the 3PED.
5. Interference rates between pulses

List and McFarquhar (1990b) investigated the time at which different size drops from a single pulse started and ceased to arrive at the ground. Provided $T$ was greater than or equal to 240 s, and excluding the smallest drops, they found that the initial arrival time of differently sized drops at the ground was about the same. The time at which drops ceased arriving at the ground was dominated by the sedimentation of the final drops of that size in the pulse. To estimate the degree of drop package interference, parameterizations developed by LMI for single pulses can be used to compare with the results from the multipulse models.

The pulse interference diameter, $D_i$, is defined as the largest drop size in a pulse that has a local maximum for number density per logarithmic diameter, $a(l)$, [i.e., the adjacent bins have lower $a(l)$] when the largest drops from the next pulse first reach the same volume element. The $D_i$ gives a quantitative measure of how much interference occurs between pulses. A large $D_i$ indicates that there are many interactions between pulses. Obviously $D_i$ depends on height, so ground level is used for comparing different cases.

Figure 6 shows how $D_i$ varies with $T$ and $\tau$ when an MP54 distribution is released at the shaft top. The values for $D_i$ are obtained directly from the simulations. The curves are almost identical for the 3PED54 case. The logarithm of $D_i$ is a linear function of $T$ when $\tau$ is constant, and vice versa, for the range of values used in the simulations. The $D_i$ is a function of both $T$ and $\tau$ when their ratio is constant. These relations are parameterized by

\[ T = 120 \text{ s: } \log D_i \]
\[ = -5.119 \times 10^{-4} \tau + 0.258, \quad r = -0.990 \quad (2) \]
\[ \tau = 1200 \text{ s: } \log D_i \]
\[ = 4.181 \times 10^{-4} T - 0.391, \quad r = 1.000 \quad (3) \]
\[ \tau = 2T: \log D_i \]
\[ = -5.788 \times 10^{-4} T + 0.196, \quad r = -0.961 \quad (4) \]

where $r$ is the correlation coefficient of the fit, $T$ and $\tau$ are both in seconds, and $D_i$ is in millimeters. When $T$ increases and $\tau$ decreases, $D_i$ increases. The dependence on $\tau$ is stronger than that of $T$ because $D_i$ decreases when $T = 2\tau$. Equations (2), (3) and (4) are considered valid only within the tested range (i.e., 120 s $< T < 600$ s, and 240 s $< \tau < 1200$ s). As $T$ approaches $\tau$, the situation of steady input and a three-peak equilibrium distribution is approached, which would produce a $D_i$ near the maximum drop size; $D_i$ is undefined for steady input, when $T = \tau$.

Very similar relations for the pulse interference diameter can be obtained by using the parameterizations for single pulses for the initial arrival time, $t_i$, of drops at the ground and for the time at which drops of a particular size are arriving at the ground in their maximum concentrations, $t_m$. The $t_i$ was defined by LMI as the time before the number density per logarithmic diameter became greater than a critical value of 1 m$^{-3}$. They determined $t_i$ as a function of diameter, using

\[ t_i = m \log D + b, \quad (5) \]

with $t_i$ measured in seconds, where $m = -86.88$, $b = 419.5$, and 0.15 $< D < 4.5$ mm for the case of MP54 input. The $t_i$ did not vary greatly with diameter over this range, thus an average initial arrival time of 6.08 min = $t_i$ was set by substituting $D = 4.09$ mm in (5); this was representative of $t_i$ for the largest drops in the pulse that attained the critical number density per logarithmic diameter of 1 m$^{-3}$.

Here $t_m$ can be parameterized as a function of $T$ and $D$ according to

\[ t_m = A + \frac{B}{1 - \exp[-(D/1.77)^{1.47}]} \quad (6) \]

This is identical to Eq. (13) of LMI, which determined the final arrival time of different drop sizes at the ground [i.e., last time for which $a(l)$ is greater than 1 m$^{-3}$]. The denominator of (6) is proportional to the terminal velocity of a raindrop of size $D$, suggesting that $t_m$ was dominated by the final sedimentation of drops in the pulse, with lesser effects due to coalescence. For cases of large $T$, $t_m$ was chosen as the last time at which $a(l)$ was a local maximum. Coefficients $A$ and $B$ were determined as functions of $T$ as shown in Table 2. The fits seem good as $\chi^2$ indicates. Coefficient $B$ is weakly dependent on $T$, and $A$ is strongly dependent on $T$ since sedimentation dominates $t_m$.
The pulse interference diameter is obtained using $t_i^*$ and the parameterization of $t_m$ by equating

$$t_i^* + \tau = t_m(D, T).$$  

(7)

Solving for $D$ should give a value close to $D_i$.

Table 3 shows $D_i$ obtained from numerical simulations of multipulse rain compared to those calculated using the parameterizations from superpositions of single pulses. There is at most a 10 percent difference between the values indicating that the parameterization of $t_m$ and $t_i^*$ obtained from the results of the single pulse input can be used to approximate the pulse interference diameter at the ground. This conclusion can be extended to any height level in the rain shaft. Thus, as generally expected in the first place, the numerical simulations for multipulse input are only needed to determine the shape of drop-size distributions and not the pulse interference diameter.

Table 3 also shows that $D_i$ is virtually constant when $\tau - T$ is constant. This is expected because there is a constant time interval between pulses in which the large drops can overtake the small ones. However, the cumulative 3PDs at the ground are not the same for different $T$ and $\tau$ (figure not shown) because there are more drop interactions within a pulse and a longer interaction time between pulses (even though initial interactions are the same) for larger $T$. Hence, the 3PDs for larger $T$ are usually closer to the 3PD, with the exception of a few height levels where the reversals in evolving drop-size distributions discussed by LMI and Brown (1990) are visible.

Figure 2 shows that a cyclic pattern of repeating number flux density per logarithmic diameter, $a_i(\lambda)$, is quickly established at ground level. By comparing $a_i(\lambda)$ at time $t$ with $a_i(\lambda)$ at time $(t - \tau)$ for the same drop size, the time at which a repeating pattern for each drop size is established can be determined. But, individually sized drops arrive in clusters according to size bins, and their concentration quickly drops to zero. Hence, it is easier to look at the variation of total number. Figure 7 shows the temporal evolution of total number density at ground level for the case $T = 120$ s, $\tau = 600$ s, with input spectra MP54. Similar results are obtained for a 3PED54. This verifies that a repeating pattern of drop-size distributions is quickly established at the ground when the same distribution is periodically input at the top of the shaft.

6. Summary and conclusions

Studies of pulsating rain in still air have been performed using the model of List and McFarquhar (1990b) that simulates the temporal evolution of warm rain spectra in one spatial dimension by including effects due to the sedimentation of raindrops and their collision-induced breakup. Drop spectra were released at the shaft top in multiple pulses, characterized by the pulse length ($T$) and the pulse repetition period ($\tau$). Both Marshall–Palmer (MP54) and three-peak equilibrium distributions (3PED54) were used as input spectra. The principal conclusions are:

1) With 3-km high rainshafts, drops reached the ground in 2 or 3 overlapping packets originally from different pulses. The large drops always preceded the arrival of small drops within an individual packet.

2) For reasonable pulse lengths ($T > 120$ s), the time-averaged drop-size distributions usually appear

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Table 2. Parameters for fit of the time of particular size drops to arrive at ground in their maximum concentration $t_m$ against diameter performed using Eq. (6).

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Table 3. Pulse interference diameter, $D_i$, obtained from numerical simulations of multipulse rain compared to $D_i$ calculated using single-pulse parameterizations. All cases correspond to release of MP54 spectra at the shaft top (3PED54 input cases similar).

<table>
<thead>
<tr>
<th>Pulse length (s)</th>
<th>Pulse period (s)</th>
<th>$D_i$ from simulations (mm)</th>
<th>$D_i$ from parameterization (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>240</td>
<td>1.45</td>
<td>1.45</td>
</tr>
<tr>
<td>120</td>
<td>600</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>120</td>
<td>1200</td>
<td>0.46</td>
<td>0.49</td>
</tr>
<tr>
<td>240</td>
<td>480</td>
<td>1.02</td>
<td>1.09</td>
</tr>
<tr>
<td>240</td>
<td>720</td>
<td>0.81</td>
<td>0.77</td>
</tr>
<tr>
<td>240</td>
<td>1200</td>
<td>0.51</td>
<td>0.50</td>
</tr>
<tr>
<td>600</td>
<td>1080</td>
<td>0.81</td>
<td>0.77</td>
</tr>
<tr>
<td>600</td>
<td>1200</td>
<td>0.72</td>
<td>0.65</td>
</tr>
</tbody>
</table>

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Fig. 7. Number density integrated over all sizes near the ground (142.5 m) against time for $T = 120$ s, $\tau = 600$ s, and MP54 input. The dashed line corresponds to the percentage difference between the number density at time $t$ to that at $t - \tau$. 
to have three peaks in number concentration (3PDs) at the same diameters as the three-peak equilibrium distribution (3PED) for steady rain. These 3PDs are much closer in shape to the 3PED than the cumulative 3PDs produced by the input of a single drop pulse at the shaft top.

3) Longer pulse lengths and shorter pulse repetition periods produce more drop interactions causing the time-averaged drop-size distribution to more closely replicate the 3PED.

4) A quantitative indication of the amount of interference between pulses is obtained from the pulse interference diameter, the diameter at which the drops of a pulse have a maximum number density per logarithmic diameter when drops of the next pulse start to reach the same volume element. As confirmed by one-dimensional modeling of multiple pulse rain, the pulse interference diameter may be estimated by using parameterizations of when raindrops start and cease to arrive at the ground when a single pulse of drops is input at the shaft top.

5) Pulsed rain quickly reaches a steady repeating pattern of total number density at ground level after two or three pulses of rain with the same spectra are periodically released at the top of the shaft.

6) The simulated pattern of drop arrival at the ground qualitatively replicates the major features seen during a field study on tropical rain in Malaysia. More details will have to be added to the model before a more quantitative replication can be expected.

This work has provided a more comprehensive model of observed nonsteady state warm rain in the tropics by releasing series of rain pulses at the shaft top. Because observed patterns of rainfall at the ground are replicated, it may be speculated that this multipulse model is a good explanation of nonsteady rain. How such pulses are initiated at the shaft top must be established in field experiments. Future work could concentrate on a more quantitative comparison of the modeling results to observations made in the field. The model could also be improved by adding thermodynamic effects and/or a microphysics–dynamics feedback, and deal with the whole rain formation process. In this way, a quantitative replication of field observations could be accomplished.

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REFERENCES


