Diabatic Sources of Potential Vorticity in the General Circulation

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ABSTRACT

A form of the potential vorticity (PV) budget is proposed that facilitates analysis on the role of global heat sources and sinks in the general circulation. A local diabatic source of PV occurs due to vertical variations of heating. Additionally, since the irrotational mass circulation in isentropic coordinates is uniquely linked to diabatic heating, the associated horizontal advection of PV may be viewed as a diabatic source. The sum of these processes constitutes an effective "baroclinic wave source" due to diabatic processes, and is analogous to the effective barotropic Rossby wave source due to divergence as discussed by Sardeshmukh and Hoskins. Diagnostic results are presented for the upper-tropospheric PV balance at 350 K during northern winter. When the PV budget is diagnosed in its conventional form, the midlatitude flow appears insulated from the influence of tropical heating in the sense that diabatic sources and sinks are mainly due to vertical variations of extratropical heat sources and sinks. In the NH, these sources/sinks are balanced by the mean horizontal advection of PV by the total flow, which acts to transport PV from reservoirs of large values over eastern Asia and Canada to small values over the central North Pacific and western North Atlantic oceans.

Analysis of the effective baroclinic wave source reveals that the midlatitude PV balance depends strongly on the distribution of tropical heating, a result that agrees more favorably with empirical and numerical studies on tropical-extratropical interactions. Sinks due to horizontal PV advection by the irrotational flow occur throughout the eastern hemisphere along 30° latitude, and exceed the local sources associated with in situ diabatic cooling. The implied poleward transport of low PV air from the tropics occurs in the outflow branch of the regional Hadley circulation, revealing the large influence of the Australasian monsoon.

1. Introduction

The condition that diabatic processes are the only source of the potential vorticity (PV) of an inviscid fluid makes it a particularly attractive property for studying the role of heat sources and sinks in the general circulation. In the presence of nonzero PV, a local source occurs due to vertical variations in heating. Horizontal advection effectively acts to redistribute the potential vorticity between its source and sink regions. However, since the irrotational flow is in part related to heating, a portion of the advective effect is itself diabatically forced. This relationship is particularly strong in isentropic coordinates where the time-averaged irrotational mass circulation is identically equal to the vertical variation of the time-averaged heating (e.g., Johnson et al. 1985). Thus, it is reasonable to treat the horizontal advection of PV by the irrotational isentropic mass circulation as an "effective" diabatic source.

Such an interpretation of the potential vorticity budget may be viewed as a generalization of the Sardeshmukh and Hoskins (1988, hereafter SH88) analysis of the kinematic vorticity budget. In analogy to their formulation of a barotropic Rossby wave source due to a horizontal distribution of divergence, a relation is developed herein for a baroclinic wave source due to a three-dimensional distribution of heating. Diagnostic results are compared for potential and kinematic vorticity balances, and in particular with regard to understanding the interaction between tropical monsoons and extratropical circulation.

The dynamical influence of heating on the atmosphere has been previously demonstrated by the results from simple numerical models. Stationary wave motions are sensitive to the three-dimensional distribution of heating in extratropical (e.g., Smagorinsky 1953; Hendon and Hartmann 1982) and tropical latitudes (e.g., Hoskins and Karoly 1981; Simmons 1982). The numerical results are consistent with empirical studies on the influence of heating on the observed flow (e.g., Weickmann 1983; Liebmann and Hartmann 1984), although budget studies of the observed circulation have been more difficult to interpret with regard to the role of heating.

Diagnostic budget studies on the dynamical influence of heating have generally employed a vorticity approach based on application of Bjerknes' (1937) circulation theorems. Petterssen (1950) examined the low-level budget of kinematic vorticity (the curl of the velocity) in order to understand the influence of extratropical heat sources. However, since the low-level
balance is largely between friction and the associated vortex stretching, the direct diabatic effect is unclear. Lau’s (1979) analysis of the upper-level vorticity balance also sheds little insight into the role of heating. The extratropical balance occurs between mean horizontal advection of absolute vorticity and vortex stretching, processes that can be attributed to kinematic properties of the flow due to finite-length jets and gradient balanced motions (e.g., Blackmon et al. 1977). Sardeshmukh and Hoskins (1985) examined the tropical time-mean vorticity balance during the 1982–83 El Niño–Southern Oscillation in order to diagnose the influence of anomalous heating on atmospheric circulation. An interesting result from their study was that despite anomalously intense tropical convection, local divergent sources in the tropics were small.

An alternative diagnostic approach has involved application of Bjerknes’ (1937) dynamic circulation theorem. Hoerling and Johnson (1991) analyzed the wintertime budget of dynamic vorticity (the curl of the momentum), and showed that maintenance of the lower-tropospheric circulations in the Northern Hemisphere is a direct response to differential heating of continents and oceans. However, the dynamic vorticity balance is considerably more complicated in the upper troposphere, and the role of diabatic processes becomes less certain (Hoerling 1987).

The budget results would appear to be at odds with the numerical investigations previously cited, which suggested a strong sensitivity of the atmosphere’s rotational flow to diabatic processes. Sardeshmukh and Hoskins addressed this apparent paradox by demonstrating that the advection of vorticity by the divergent wind, a process that tends to be small compared to advection by the rotational wind, exerts a large influence on the steady-state response to tropical divergence. Their results and others demonstrate the danger of inferring the importance of a physical process solely from the magnitude of its contribution to a time-mean local budget.

Implied also is the limitation of diagnosing the dynamical effect of thermal forcing from kinematic and dynamic vorticity budgets alone. A major difficulty is that both properties possess large sources that are not directly related to diabatic processes. Such is not the case for potential vorticity whose primary source in the free atmosphere is due to heating.

Potential vorticity budgets of the general circulation have not been extensively studied due in part to our incomplete knowledge of the three-dimensional distribution of heating. However, recent analyses of global data produced by sophisticated assimilation systems have yielded physically realistic patterns of the climatological heating (e.g., Hoskins et al. 1989; Schaeck et al. 1990). Indeed, these climatological distributions have been used by Chen and Trenberth (1988) and Valdes and Hoskins (1989) for modeling the stationary wave response to heating.

The purpose of this study is to utilize the three-dimensional distribution of heating diagnosed from the National Meteorological Center’s (NMC) daily global analyses in order to diagnose the diabatic sources of potential vorticity, and thereby quantify the role of heating in maintaining the general circulation. In view of the fact that our understanding of atmospheric heating remains imperfect, and that estimates of heating based on analysis of datasets produced at different assimilation centers differ, the results presented herein should be viewed as preliminary.

Section 2 presents the dataset and methods of analysis. Potential vorticity is defined in section 3, and the budget equation used in the diagnostic analysis is derived. The wintertime potential vorticity balance is presented in section 4, the results of which are contrasted with the kinematic vorticity balance. A discussion is given in section 5.

2. Dataset and methods of analysis

The diagnostics are based on analyses of once-daily NMC global initialized data. Winter periods (defined as 1 December–28 February) for 1986/87, 1987/88, and 1988/89 are studied. This relatively short sample has been selected owing to the significant changes in NMC’s Global Data Assimilation System introduced in May 1986. The NMC analyses since May 1986 are believed to offer a consistent and superior estimate of the global climate, although uncertainties remain in the tropics and the Southern Hemisphere (Trenberth and Olson 1988).

NMC data were available on a $2.5^\circ \times 2.5^\circ$ latitude/longitude grid at the 12 mandatory pressure levels from 1000 mb to 50 mb. Several conversions of the isobaric gridpoint data are performed. First, in order to take advantage of diagnostic software available in NCAR’s Community Climate Model (CCM) modular processor (Wolski 1987), the NMC data are first converted into CCM history tape format. The data are vertically interpolated to 13 sigma levels with the assumption that properties vary linearly with the natural log of sigma. The data are also interpolated to a T31 resolution Gaussian grid that corresponds approximately to a $3.7^\circ \times 3.7^\circ$ latitude/longitude grid. In order to study the PV budget, a sigma to isentropic interpolation is performed (again assuming a linear variation with the natural log of sigma). The isentropic dataset extends from 220 to 400 K with 5-K vertical resolution.

The potential and kinematic vorticity budgets are diagnosed in isentropic coordinates once daily, and then time averaged for the three winter seasons. The diabatic heating is derived through a vertical integration of the isentropic mass continuity equation (see Wei et al. 1983, section 2) with the boundary constraint that the diabatic mass flux $(\rho J_\theta)$ vanishes at 400 K.

The results presented in section 4 have been spatially filtered in order to emphasize large-scale features. First,
a one-dimensional zonal Fourier filter is applied that truncates harmonics having wavelengths less than 6000 km. Four passes of a low-pass filter having weights (2, 3, 2) and one pass of an inverse filter having weights (−1, 5, −1) are applied meridionally. The response to the meridional smoothing is shown in Fig. 1 of Schaack et al. (1990).

3. Potential vorticity and its time rate of change
   a. Lagrangian sources of potential vorticity

Potential vorticity in isentropic coordinates is given by

\[ P = (\zeta_0 + f) / \rho J_0, \]  

where \( \zeta_0 \) is the kinematic vorticity on isentropic surfaces, \( f = 2\Omega \sin \theta \), \( \rho \) is density, \( J_0 = |\partial z / \partial \theta| \) is the Jacobian transformation, and the hydrostatic isentropic mass density is

\[ \rho J_0 = -\frac{1}{g} \frac{\partial \rho}{\partial \theta}. \]  

(2)

The potential vorticity (PV) equation in isentropic coordinates is derived from a combination of the mass continuity and vorticity equations. The isentropic mass continuity equation is given by

\[ \frac{\partial (\rho J_0)}{\partial t_0} + \nabla_\theta \cdot (\rho J_0 V) + \frac{\partial (\rho J_0 \hat{\theta})}{\partial \theta} = 0, \]  

(3)

where \( \hat{\theta} = d\theta / dt \) is the diabatic heating rate. With the definition of the Lagrangian derivative, the conservation of mass may also be expressed as

\[ \frac{d}{dt} (\rho J_0) = -\rho J_0 \nabla_\theta \cdot V - \rho J_0 \frac{\partial \hat{\theta}}{\partial \theta}. \]  

(4)

The isentropic vorticity equation is given by

\[ \frac{d(\zeta_0 + f)}{dt} = -(\zeta_0 + f) \nabla_\theta \cdot V \]

\[ + \left( \frac{\partial \hat{\theta} \partial u}{\partial y \partial \theta} - \frac{\partial \hat{\theta} \partial v}{\partial x \partial \theta} \right) + k \cdot \nabla_\theta \times F, \]  

(5)

where \( F \) is the friction force. Elimination of the velocity divergence terms between (4) and (5) with the definition of PV yields

\[ \frac{dP}{dt} = P \frac{\partial \hat{\theta}}{\partial \theta} + \frac{1}{\rho J_0} \left[ \left( \frac{\partial \hat{\theta} \partial u}{\partial y \partial \theta} - \frac{\partial \hat{\theta} \partial v}{\partial x \partial \theta} \right) + k \cdot \nabla_\theta \times F \right]. \]  

(6)

In its Lagrangian form (6), the potential vorticity budget reveals that sources of PV are associated with the thermal forcing by a three-dimensional heat source and the mechanical forcing by a horizontal curl of the friction force. With the scaling approximation that tilting effects are small, (6) becomes equivalent to Eq. (73) of Hoskins et al. (1985). With this simplification, the diabatic source of potential vorticity is a function only of the vertical distribution of heating and cooling.

b. The effective “baroclinic wave source” due to heating

A form of the kinematic vorticity equation for studying the effect of tropical divergence on midlatitude motions appears in Kang and Held (1986) and Saraneshmukh and Hoskins (1988). Equation (2) of SH88 modified for an \((x, y, \theta)\) system is

\[ \left( \frac{\partial}{\partial t_0} + V_x \cdot \nabla_\theta \right) (\zeta_0 + f) = S_\tau + k \cdot \nabla_\theta \times F, \]  

(7)

where vertical advection and tilting have been neglected, and the velocity has been expressed in terms of rotational \((\nabla_\theta V)\) and irrotational \((V_x)\) components. Here \( S_\tau \) expresses the source of Rossby waves associated with divergence

\[ S_\tau = -V_x \cdot \nabla_\theta (\zeta_0 + f) - (\zeta_0 + f) \nabla_\theta \cdot V. \]  

(8)

The Rossby wave source incorporates the advection of absolute vorticity by the divergent component of the velocity, and the local stretching of vortex tubes by horizontal divergence.

As suggested in the Introduction, a diagnostic PV model for studying the effect of heating must represent the portion of horizontal PV advection that is explicitly coupled to diabatic processes. Consider a mass weighting of (6) and an expansion of the Lagrangian derivative

\[ \rho J_0 \frac{\partial P}{\partial t_0} + \rho J_0 V \cdot \nabla_\theta P + \rho J_0 \frac{\partial P}{\partial \theta} \]

\[ = (\zeta_0 + f) \frac{\partial \hat{\theta}}{\partial \theta} + k \cdot \nabla_\theta \times F, \]  

(9)

where tilting has been neglected.\(^1\) In this mass-weighted form, the individual terms represent the total process (per unit volume) for an isentropic layer, and the individual terms acquire the units of kinematic vorticity tendency.

The mass transport vector \( \rho J_0 V \) is expressed in terms of rotational and irrotational components according to

\[ \rho J_0 V = (\rho J_0 V)_\theta + (\rho J_0 V)_\chi, \]

(10)

where the Poisson equation for \( \chi \) is given by

\[ \nabla^2 \chi = \nabla_\theta \cdot \rho J_0 V. \]  

(11)

The irrotational component of the mass transport is related to diabatic heating through the isentropic mass

\(^1\) The vertical advection of PV has been retained in (9) because it exerts a large effect in the local budget near the tropopause. In this region, PV becomes nearly discontinuous in the vertical.
balance (3). For short time periods, the isentropic mass tendency may be large, thereby short circuiting the link between irrotational mass transport and heating. The isentropic mass balance simplifies, however, for time scales beyond a few days for which the local mass tendency becomes small. Thus, within the steady, time-averaged structure the link between \( \rho \mathbf{J}_e \mathbf{V}_e \) and diabatic heating is exact (Wei et al. 1983; Johnson et al. 1985).

A combination of (9) with (10) and rearrangement yields

\[
\left( \rho \mathbf{J}_e \frac{\partial}{\partial \theta} + (\rho \mathbf{J}_e \mathbf{V}_e) \cdot \nabla \theta \right) P = S_p + k \cdot \nabla \theta \times \mathbf{F},
\]

(12)

where \( S_p \) expresses the baroclinic wave source due to heating,

\[
S_p = -(\rho \mathbf{J}_e \mathbf{V}_e) \cdot \nabla \theta P - \rho \mathbf{J}_e \frac{\partial P}{\partial \theta} + (\zeta_e + f) \frac{\partial \theta}{\partial t}.
\]

(13)

Here \( S_p \) is explicitly linked to the three-dimensional distribution of heating through the effects of 1) horizontal advection of PV by the irrotational isentropic mass transport, 2) vertical advection of PV by the diabatic mass flux, and 3) a vertically varying heat source.

The horizontal advective component represents a nonlocal effect in the sense that computation of the irrotational mass transport involves solving (11). While on the one hand this may complicate interpretation of the local balance, it has the attribute of offering a perspective on the "remote" influence of diabatic processes and the implied tropical–extratropical interactions. Figure 1a presents the 350–355-K winter-mean distribution of mass transport potential derived from the isentropic mass divergence. Note that the meridional scale of \( \rho \mathbf{J}_e \mathbf{V}_e \) depends upon the distribution of tropical heat sources and extratropical heat sinks (see Fig. 1c). A nonlocal influence of tropical convection is implied by the advection of PV by the irrotational mass transport over the North Pacific (Fig. 1b).

Since (3) predicts that the time-mean distribution of \( \rho \mathbf{J}_e \mathbf{V}_e \) is uniquely related to heating, the associated horizontal advection of PV can be interpreted as a diabatic source. Thus, one overcomes a conceptual difficulty encountered in SH88, namely, that vorticity advection by \( \mathbf{V}_x \) is not strictly a source related to heating since \( \mathbf{V}_x \) itself is not uniquely related to heating.

Some ambiguity remains, however, because the instantaneous distribution of \( \rho \mathbf{J}_e \mathbf{V}_e \) does not balance the heating exactly. In this regard it becomes important to distinguish between the interpretation of \( S_p \) as a source in the context of a time-mean budget as proposed herein, and its interpretation as a source in the context of the time-dependent response to heating. In the former case, \( S_p \) may be viewed as a source related to heating if the time-mean process \( \rho \mathbf{J}_e \mathbf{V}_e \cdot \nabla P \) occurs mainly through advection by the time-mean irrotational flow, \( \rho \mathbf{J}_e \mathbf{V}_e \cdot \nabla P \). The importance of PV advection by the climatological-mean distribution of \( \rho \mathbf{J}_e \mathbf{V}_e \) is implied in Fig. 1, and is confirmed by the budget results shown in the following section. For the time-dependent problem, \( S_p \) represents a source only if the irrotational mass transport responds to heating on a faster time scale than the potential vorticity responds to the source, \( S_p \). The time scales of interest are those of gravity wave and Rossby wave modes. A

\[ \text{FIG. 1. DJF-mean distributions of (a) 350–355-K mass transport potential (10}^4 \text{ kg K}^{-1} \text{s}^{-1}), (b) 350–355-K potential vorticity (10}^{-6} \text{ kg}^{-1} \text{ m}^2 \text{s}^{-1} \text{ K}), \text{ and (c) 350-K heating (} \theta \text{) (K day}^{-1} \text{). Irrotational mass transport vectors shown in (a) and (b). Heating has been filtered to emphasize wavelengths greater than 6000 km.} \]
satisfactory resolution of the latter issue would require time integration of a potential vorticity-conserving model subjected to a heat source, a task that is beyond the scope of this study.

4. Results

Results are presented for the 350–355-K layer, which is near the location of maximum mass outflow from tropical deep convection. Analyses performed for other layers, not to be presented herein, reveal that the PV balance for 350–355-K is representative of the layer-averaged balance between 330 and 355 K. Section 4a presents the PV balance based on the diagnosis of (12), while section 4b presents the kinematic vorticity balance based on the diagnosis of (7).

a. PV balance in the 350–355-K layer

Figure 1 illustrates several key climatological features of the 350–355-K layer as concerns the large-scale PV budget. These include a wavy distribution of potential vorticity in midlatitudes with maximum values in troughs over eastern Asia and Canada and minimum values in ridges over the northeastern Pacific and Atlantic (Fig. 1b); a strong meridional gradient of PV near 30° latitude (Fig. 1b); and a mean poleward irrotational mass transport that couples the tropical heat sources and midlatitude heat sinks (Figs. 1a and 1c).

With a dynamical definition of the tropopause corresponding to 2–4 PV units (1 PV unit = 1 × 10^{-6} K m^2 kg^{-1} s^{-1}), it is seen from Fig. 1b that the 350–355-K layer resides in the upper troposphere equatorward of 20° latitude and in the lower stratosphere poleward of 40° latitude. The strong meridional gradient of PV in midlatitudes thus marks the mean location of the tropopause within this upper isentropic layer. The mass distribution at 350 K (Fig. 2a) yields a similar interpretation, with small values indicative of stable stratospheric air at high latitudes and large values indicative of less stable tropospheric air at low latitudes. Coincident with the tropopause are the westerly jets with maxima located downstream of Asia and North America (Fig. 2b). Easterlies cover much of the tropical eastern hemisphere, and nearly overlap the region of mean tropical heating extending from Africa to the date line (see Fig. 1c). It is important to note that the irrotational mass transport associated with the convection extends far into the prevailing westerlies.

The principal terms of the time-averaged potential vorticity budget are shown in Fig. 3. A residual analysis, computed as the sum of the first four terms in (9), is presented in panel (d) in order to assess the quality of the climatological balance. Recall that the potential vorticity is mainly positive in the NH and negative in the SH due to the cyclonic absolute vorticity of each hemisphere. In the following figures, a contribution to local "cycloic" potential vorticity tendency (i.e., positive tendency in the NH and negative in the SH) is depicted by solid contours in the NH and dashed contours in the SH. For ease of description, these will be subsequently referred to as local contributions to increasing potential vorticity.

The total horizontal advection of PV (−ρj0 V · ∇_P) (Fig. 3a) is largest between 30° and 40° latitude in the vicinity of the tropopause. Contributions to increasing potential vorticity are located over the central North Pacific and western North Atlantic, while contributions to decreasing potential vorticity are found over the Middle East, the western Pacific, western North America, and over much of the midlatitude SH. The two prominent dipoles of horizontal advection in the NH stem in part from the condition that the upper-level circulation flows from the reservoirs of high PV over eastern Asia and North America to low PV over the adjacent oceans. The implied PV transport lies along the jet axes, which in the time-mean-experience decreasing PV in their entrance regions and increasing PV in their exit regions due to advection by the mass transport.

Mean horizontal PV advection is balanced largely by the local source due to the vertical variation of heat-
ing \( (\phi + f \partial / \partial \phi) \) (Fig. 3b). Since this process is weighted by the absolute vorticity, the role of extratropical heat sources is greatly emphasized relative to the role of tropical heating. Sink regions of PV occur over the central North Pacific and western North Atlantic where mean diabatic heating occurs near the tropopause (see Fig. 1c). Vertical profiles over the wintertime storm tracks (Schaack et al. 1990) reveal maximum heating rates in the low to middle troposphere, which decrease to weak heating or cooling in the upper troposphere. The local destruction of PV is thus associated with a tendency to destabilize the 300–350-K layer due to convergence of the diabatic mass flux above the level of maximum heating. Local PV source regions cover North Africa, much of Asia, western North America, and a nearly continuous zonal band of the SH between 20° and 40°S. These areas are the major wintertime heat sink regions, and are characterized by maximum cooling rates in the middle troposphere (see Schaack et al. 1990). The decrease of radiational cooling with height stabilizes the upper isentropic layers, thereby acting as a source of potential vorticity.

Mean vertical (or diabatic) advection of potential vorticity (Fig. 3c) exerts a significant influence in the tropics at 350 K. Over the monsoonal heat source within the South Pacific convergence zone the diabatic advection contributes to decreasing PV, and is the primary term in the local budget. Elsewhere in the tropics the relative importance of individual processes is less clear owing to the large value of the residual (Fig. 3d) in comparison to other budget terms. This is to be contrasted with the extratropical budget where the diagnosed large-scale balance is sufficiently good to permit meaningful physical interpretation.

The results in Fig. 3 would imply that the upper-level midlatitude circulation is somewhat insulated from the tropics in the sense that the major PV sources and sinks are linked to extratropical diabatic forcing. However, since the monsoons are characterized by large-scale mass outflow, the tropical heat sources exert a dynamical influence through horizontal advection of PV by the irrotational mass transport (Fig. 4a). A prominent feature of this process is the contribution to decreasing potential vorticity (i.e., a sink) at 30° latitude that spans the longitudinal extent of the Australasian monsoon. Somewhat weaker PV sinks occur
symmetric about the equator at the longitudes of the African and South American winter monsoons.

Further analysis (not shown) reveals that the stationary component of advection \((\rho J_0 V) \cdot \nabla \bar{P}\) at 350 K is nearly an order of magnitude larger than the transient \((\rho J_0 V)_t \cdot \nabla \bar{P}\) effects. Since the time-averaged irrotational flow is uniquely related to heating as discussed in section 3, the previously stated results confirm that the time-mean advection of PV by \((\rho J_0 V)_t\) shown in Fig. 4a is intimately related to diabatic forcing.

An important result is that the implied remote influence of tropical heating is comparable to the direct influence of the extratropical heat sources or sinks (see Figs. 4a and 3b). For example, the contribution to decreasing PV due to advection by \((\rho J_0 V)_x\) over eastern Asia exceeds the generation of PV associated with the local heat sink. In the central North Pacific, both processes contribute equally to a decreasing tendency of PV, while in the western North Atlantic their contributions are nearly equal and opposite.

The relative role of tropical and extratropical heat sources in maintaining the climatological PV is more clearly illustrated in Fig. 4b, which presents the baroclinic wave source, \(S_p\), due to diabatic effects [see Eq. (13)]. In the NH, two major sink regions are found during winter. One sink over the central North Pacific is associated with both the local influence of latent heating in baroclinic disturbances and the remote tropical influence due to a poleward advection of low PV air in the regional Hadley circulation. A second sink over Southeast Asia is primarily associated with the remote influence of the winter monsoon. The source regions over western North America, the Middle East, and western Asia are mainly due to the local influence of diabatic cooling over the continents.

It is of interest to compare the distribution of \(S_p\) with the distribution of stationary waves that are shown in Fig. 4c in terms of the 350-K eddy potential vorticity. In the NH, the baroclinic wave source lags the eddy PV by approximately a quarter-wavelength. For example, the maximum PV sink located south of the Aleutians occurs near the zero eddy PV contour separating high PV within the East Asian trough from low PV within the northeast Pacific ridge. Similarly, the
PV source over western North America is situated between the upstream low PV of the northeast Pacific ridge and the downstream high PV within the Hudson Bay trough. The phase relationship over subtropical latitudes is less clear, although the PV sink centered over east-central China nearly coincides with the zero eddy PV contour separating low PV within the west Pacific subtropical anticyclone from high PV over the Middle East. Thus, there is a suggestion that the time-mean diabatic processes contribute to a retrogressive tendency of the quasi-stationary troughs and ridges. This influence is compensated by the horizontal PV advection by the rotational flow, which acts to induce a downstream shift of the eddies (Fig. 4d).

b. Kinematic vorticity balance in the 350–355-K layer

The results of the wintertime kinematic vorticity budget at 350–355 K (Fig. 5) are broadly in agreement with Lau’s (1979) diagnosis of the isobaric vorticity budget at 300 mb. A large compensation between mean horizontal advection \(-\nabla \cdot \nabla (\zeta \theta + f)\) (Fig. 5a) and mean stretching \((- \zeta \theta + f) \nabla \theta \cdot \nabla \theta \) (Fig. 5b) illustrates the well-known condition that the upper-tropospheric vorticity balance is an expression of near-zero horizontal flux divergence of absolute vorticity.

A comparison of kinematic and potential vorticity balances is useful for providing insight on the role of static stability (as measured by \(\rho J_\theta\)) and its variations in the maintenance of circulation. The degree to which \(\rho J_\theta\) is spatially and temporally uniform determines the extent to which the rotational flow can be described in terms of the kinematic vorticity dynamics alone. Note that the horizontal potential vorticity advection can be expressed as a sum of advections

\[-\rho J_\theta \nabla \cdot \nabla \theta P = -\nabla \cdot \nabla (\zeta \theta + f) + PV \cdot \nabla \theta \rho J_\theta,\]

while the local diabatic source of PV can be expressed as

\[(\zeta \theta + f) \frac{\partial \theta}{\partial t} = -(\zeta \theta + f) \nabla \theta \cdot V
- \rho_\theta \left( \frac{\partial \rho J_\theta}{\partial t} + V \cdot \nabla \theta \rho J_\theta + \frac{\partial \rho J_\theta}{\partial \theta} \right).\]

![Figure 5](unavailable)

**Fig. 5.** DJF-mean contributions to the 350–355-K kinematic vorticity budget by (a) horizontal advection of absolute vorticity, (b) local stretching, (c) horizontal advection of absolute vorticity by the divergent component of velocity, and (d) the Rossby wave source, \(S_\theta\). Contoured every \(4 \times 10^{-11} \text{ s}^{-2}\). Filtered as in Fig. 3.
In the absence of horizontal mass variations, (14) reveals that the mass-weighted horizontal PV advection simplifies to the horizontal absolute vorticity advection. If in addition the mass is distributed uniformly in the vertical and is constant in time, (15) reduces to an equality between local sources of potential and kinematic vorticities.

That the horizontal mass distribution is not uniform is clearly evident in Fig. 2a, which reveals the strong meridional mass variations across the sub-tropics. As anticipated from (14) and (15), differences are evident between distributions of the mean horizontal advection of potential and absolute vorticities (cf. Figs. 3a and 5a), and between local diabatic and divergent sources (cf. Figs. 3b and 5b). Over the NH mid-latitudes, maximum contributions in the PV budget are nearly in quadrature with those in the kinematic vorticity budget. This displacement is illustrative of the baroclinic structure of the NH planetary waves, within which a mean advection of mass occurs due to the phase shift between temperature and height fields. In the SH, the fields of individual terms in the potential and kinematic vorticity budgets nearly overlap, a result that is consistent with the nearly equivalent barotropic structure of the SH stationary waves (e.g., Trenberth 1980).

Much of the pattern and amplitude of the climatological absolute vorticity advection by the total flow is attributable to the advection by the rotational flow, $\mathbf{V}_\phi$ (not shown). An important local exception is found over eastern Asia and Australia where advection by $\mathbf{V}_\phi$ is large (Fig. 5c). This advective contribution to decreasing vorticity is linked in part with the velocity divergence occurring over the wintertime monsoon. The implied remote tropical influence is analogous to that inferred from the PV advection by $(\rho J_\theta \mathbf{V})_x$ (see Fig. 4a). It should be recalled, however, that since the velocity divergence $\nabla \cdot \mathbf{V}$ is not uniquely linked with diabatic effects, advection by $\mathbf{V}_\phi$ cannot be strictly interpreted as a Rossby wave source due to diabatic forcing. In contrast, since the isentropic mass divergence $\nabla \cdot \rho J_\theta \mathbf{V}$ is explicitly linked to the time-mean heat sources and sinks, advection by $(\rho J_\theta \mathbf{V})_x$ is uniquely coupled to diabatic effects.

An additional distinction between the two advective contributions associated with rotational motions involves the intensity of potential and absolute vorticity gradients. The magnitude of the horizontal PV gradient is significantly influenced by the abrupt change of static stability associated with the tropopause, while the magnitude of the absolute vorticity gradient is determined principally by the meridional variation of the zonal wind. With the definition of PV given by (1), horizontal gradients of absolute and potential vorticity are related according to

$$\nabla_a (\xi + f) = \rho J_\theta \nabla_a P - P \nabla \rho J_\theta. \quad (16)$$

For the midlatitude circulation at 350–355 K, the mass-weighted PV gradient (Fig. 6a) exceeds in magnitude the absolute vorticity gradient (Fig. 6b) by up to 100%, demonstrating the important effect of horizontal variations in mass.

The comparatively large magnitude of the midlatitude PV gradient clarifies why advection by the rotational flow exerts a proportionately greater influence in the PV budget than in the kinematic vorticity budget. Note that the amplitude and sign distribution of the effective Rossby wave source (Fig. 5d) is determined almost entirely by the mean stretching in the NH. This result is to be contrasted with the effective baroclinic wave source (see Fig. 4b) whose distribution from the Himalayas to the central Pacific is largely attributable to advection by $(\rho J_\theta \mathbf{V})_x$. A striking difference occurs between the two source terms over eastern Asia where the net effect of divergence is to generate kinematic vorticity, while the net effect of diabatic processes is to destroy potential vorticity. In this region the Australasian monsoon appears to exert a major influence on the midlatitude circulation through the mean poleward transport of tropical low PV air. This advective influence would appear somewhat secondary from analysis of the kinematic vorticity budget.
5. Discussion and conclusions

The present study has attempted to set forth a diagnostic framework for studying the dynamical role of diabatic processes in the general circulation. A modification to the isentropic potential vorticity budget is proposed, which facilitates an investigation on the local and remote influence of the three-dimensional structure of atmospheric heating. An "effective" baroclinic wave source due to diabatic processes is derived that incorporates contributions by the local vertical variation in heating and by the horizontal advection of PV by the irreversible mass circulation. Due to the simple balance requirement for mass in isentropic coordinates (Johnson et al. 1985), the latter is explicitly linked to the three-dimensional variation of heating, and is thus interpreted as an effective PV source. This approach is formally equivalent to that of Sardeshmukh and Hoskins (1988), who argued that the correct form of the Rossby wave source associated with a region of divergence must include the absolute vorticity advection by the divergent velocity.

Diagnostic results were presented for the boreal winter potential vorticity balance at 350–355 K. An important feature of the midlatitude balance is that the distribution of the effective baroclinic wave source reveals a large remote influence of tropical heating comparable to the local influence of in situ heat sources. Mean PV advection by \((\rho J/V)\) attains maximum values near 30\(^\circ\) latitude where the poleward mass circulation encounters the intense PV gradient at the tropopause. This advective contribution to the PV budget is oriented nearly symmetric about the equator, spans the extent of the Australasian monsoon from the Indian Ocean to the central Pacific, and acts as a strong sink of PV in midlatitudes.

Suggested here is that the extratropical circulation is particularly sensitive to the behavior of convection over the Indian and western Pacific oceans. Such an interpretation of the wintertime PV budget would appear to support the barotropic modeling results of Simmons et al. (1983), which examined the dynamics of teleconnection patterns of the 300-mb January mean flow. They found that one of the most effective regions for forcing the midlatitude zonally varying flow is located over the tropical northwest Pacific and Southeast Asia. The PV budget results are also consistent with the numerical study of Valdes and Hoskins (1989). Using a linear baroclinic steady-state model, they examined the role of observed forcings due to orography, diabatic processes, and transient fluxes in maintaining the wintertime climatological stationary waves. In midlatitudes at 200 mb, all forcings were found to be important. Of relevance to the current study, Valdes and Hoskins examined the forcing by extratropical and tropical heating separately and found that the midlatitude response to heating over the storm tracks was comparable to that due to tropical heating alone (see also Jacqmin and Lindzen 1985). This agreement with the behavior of the wintertime PV budget is not surprising since the response in their baroclinic model could in principle be completely described by the potential vorticity dynamics.

Perspectives on tropical–extratropical interaction arising from analysis of potential and kinematic vorticity balances were contrasted. Qualitatively similar results emerge from both budgets with regard to the inferred remote influence of the tropical monsoons. The subtropical sinks of potential and kinematic vorticity that occur through advection by the irrotational flow are locally important effects in their respective budgets. In this regard, the diagnostics support the modeling results of SH88, who demonstrated that the extratropical response to tropical divergence is significantly altered by incorporating the divergent component of absolute vorticity advection in the Rossby wave source. Quantitatively, however, horizontal PV advection by \((\rho J/V)\) exerts a proportionately larger effect in the PV budget than does horizontal absolute vorticity advection by \(V_x\) in the kinematic vorticity budget. The greater influence in the PV budget is due in part to the near discontinuous meridional variation of potential vorticity near 30\(^\circ\) latitude, which acts to amplify the dynamical effect of divergent outflow from the winter monsoon.

In conclusion, the current study has attempted to combine the attributes of an isentropic perspective of the general circulation especially regarding the thermodynamics of monsoonal circulations (see Johnson 1989), with the dynamical insight offered by application of potential vorticity principles (see Hoskins et al. 1985). Such an approach has recently been employed by Hoskins (1991), who briefly considered various benefits of a "PV-\(\theta\)" view of the general circulation. In addition to lending insight on the role of climatological heating in maintaining the time-mean circulation, the diagnostic model described herein may also prove useful in understanding interannual variability of the circulation in relation to anomalous tropical heating. A comparison of the potential vorticity dynamics associated with the observed extreme phases of the Southern Oscillation is currently being performed, the results of which will be presented elsewhere.

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