Semiprognostic Tests of the Arakawa–Schubert Cumulus Parameterization Using Simulated Data

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ABSTRACT

The Arakawa–Schubert (A–S) cumulus parameterization is evaluated by performing semiprognostic tests against data simulated by a cumulus ensemble model (CEM). The CEM is a two-dimensional cloud model for simulating the formation of an ensemble of cumulus clouds under prescribed large-scale conditions. Three simulations, two with vertical wind shear and one without, are performed with identical (time-varying) large-scale advective effects.

The semiprognostic tests follow a procedure similar to that used by Lord except that simulated data averaged over the entire domain or selected subdomains of the CEM provide the “observed” large-scale conditions. Detailed comparisons were made between the results of simulation and parameterization. The results include comparisons of surface precipitation rate, apparent heat source, apparent moisture sink, updraft mass flux, and downdraft mass flux. Two different sets of tests were performed. One is the standard A–S parameterization with the cloud work function (CWF) quasi equilibrium, and the other allows CWF nonequilibrium by taking into account the simulated time change of the CWF. The tests show that the A–S parameterization is basically valid in spite of the existence of mesoscale organization in cumulus convection. In particular, the assumption of CWF quasi equilibrium is more accurate for inputs averaged over smaller subdomain sizes that resolve some mesoscale processes. On the other hand, errors due to the nondiagnostic aspect of cumulus convection appear to be more significant for inputs averaged over larger subdomain sizes. Errors due to the inherent nondeterministic aspect of cumulus convection appear to be more significant for inputs averaged over smaller subdomain sizes.

A modified A–S parameterization with a convective-scale downdraft formulation was also tested against the simulated data. The inclusion of downdrafts slightly improves the results of semiprognostic tests. The impact of downdrafts on the subcloud layer may depend significantly on the subdomain size.

1. Introduction

Cumulus convection is the product of complicated interactions of moist-convective turbulence with large-scale circulations, radiation, and cloud microphysical processes. Because of its significant effects on large-scale motion, it is necessary to account for the collective effect of cumulus convection in a quantitative way in large-scale numerical models or general circulation models (GCMs). Practically no individual clouds are resolved on the scale of the computational grid, and therefore, their collective effect must be formulated in terms of resolvable-scale variables. This problem is known as cumulus parameterization and has been intensively studied for almost three decades. Various aspects of this problem have recently been reviewed by Arakawa and Chen (1987), Tiedtke (1988), and Cotton and Anthes (1989), among others.

Many attempts have been made to understand the interaction of cumulus convection with large-scale circulations under the general assumption that there exists a scale separation between convective-scale and large-scale processes. Under such situations, it may be possible to describe the collective effect of the clouds, rather than the effect of each individual cloud, to predict the time change of the large-scale disturbance. One of the theories developed for this purpose was presented by Arakawa and Schubert (1974, hereafter A–S). Their theory was based on the cloud work function (CWF) “quasi-equilibrium” hypothesis, in which the thermodynamic structure of the atmosphere is gravitationally neutral; that is, the generation of CWF by large-scale processes is approximately balanced by its destruction due to cumulus convection itself.

Although the A–S theory has been proven to be very successful (Lord 1978, 1982; Suarez et al. 1983; Kao and Ogura 1987; Randall et al. 1989; Cheng and Arakawa 1990; Grell et al. 1991), one must keep in mind that the A–S cumulus parameterization, like most existing parameterizations, is “deterministic” and “diagnostic” (i.e., it determines the collective effect of cumulus convection uniquely and it does not introduce additional prognostic equations for cumulus clouds.

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themselves). As a consequence of the diagnostic assumption, the parameterized cumulus clouds have no history of their own. This can be expected when cumulus clouds are in a statistical equilibrium with large-scale processes without any free fluctuations.

The nondeterministic and nondiagnostic effects are, however, likely to be more important when mesoscale organization of clouds occurs (Arakawa and Chen 1987; Xu 1991; Xu et al. 1992).

To examine these effects it seems necessary to evaluate existing cumulus parameterizations against data that accurately resolve mesoscale organization. Unfortunately, no such dataset is available because the collective effects of cumulus convection are not directly or extensively observed. A suitable dataset can be generated with a numerical cumulus ensemble model (CEM), as described by Xu (1991) and Xu et al. (1992). CEMs cover a large horizontal area but resolve individual clouds and their mesoscale organization.

The evaluation is performed with semiprognostic tests of the A–S cumulus parameterization. A semiprognostic test is a one-step prediction of the cumulus activity based on given large-scale conditions (Lord 1978, 1982). It isolates errors caused by the parameterization from those caused by other components of the large-scale numerical model. The computed cumulus effects, however, do not influence the large-scale variables as they would in a fully prognostic test.

This study is similar to Lord (1978, 1982) and some recent studies (e.g., Kao and Ogura 1987; Cheng and Arakawa 1990; Grell et al. 1991), except that the simulated data are used to provide the “observed” large-scale conditions. CEMs can provide such data as the updraft and downdraft mass fluxes, which are not directly available from observations. In addition, the simulated data have a much higher resolution than observed data. Such a dataset can provide a very detailed test of a cumulus parameterization.

The objectives of the present study are twofold: (i) to evaluate the validity of the A–S cumulus parameterization and its modified versions, which include convective-scale downdrafts, in the presence of mesoscale organization, and (ii) to assess the significance of nondiagnostic and nondeterministic effects in cumulus parameterization.

The organization of this paper is as follows. Section 2 describes the results of simulations. Section 3 shows the results of semiprognostic tests of the original A–S parameterization. Section 4 presents the semiprognostic tests of the modified A–S parameterization. Conclusions and discussion are presented in section 5.

2. Numerical simulation of cumulus convection

The numerical simulations using the UCLA CEM (Krueger 1985, 1988; Xu and Krueger 1991) have been described in Xu et al. (1992). Three from these simulations, Q02, Q03, and Q04, are used for the semiprognostic tests in this study. Briefly, two simulations (Q02 and Q04) with a sheared $x$ component of geostrophic wind and one simulation (Q03) without shear were performed with identical (time-varying) large-scale advective effects. The period of time variation was chosen to be 27 h. The domain size used in Q02 and Q03 is 512 km, while Q04 is identical to Q02 except with a larger domain size (1024 km). Because of the periodic nature of the imposed large-scale advective effects, an ensemble averaging with respect to the phase of large-scale processes is used to simplify the presentation of the results. The results of simulations that will be used for comparison with results of parameterization are presented as follows.

The simulated hourly rates of surface precipitation averaged over the entire CEM domain of Q02, Q03, and Q04 were presented by Xu et al. (1992, Figs. 4 and 5). The results show that cumulus activity is rather strongly modulated by large-scale processes in spite of the existence of some nonmodulated high-frequency fluctuations. Differences between the sheared (Q02 and Q04) and nonsheared (Q03) simulations appear mainly in the amount of systematic phase delays in the modulation.

Figure 1a shows the ensemble mean of the apparent heat source ($Q_1$) and apparent moisture sink ($Q_2$) for

![Figure 1](https://example.com/figure1.png)

FIG. 1. The ensemble mean of the apparent heat source ($Q_1$) and apparent moisture sink ($Q_2$) for (a) Q03 and (b) Q02 simulated by the CEM. The abscissa is the phase of the imposed large-scale advective processes. Contours over 10 K d$^{-1}$ are hatched.
Q03. [See Yanai et al. (1973) for the definition and physical interpretation of \( Q_1 \) and \( Q_2 \).] The level of the maximum \( Q_1 \) is about 3 km higher than that of the maximum \( Q_2 \). The time variations of the mean \( Q_1 \) and \( Q_2 \) have several maxima while the prescribed large-scale advective effects only have one maximum at 13.5 h. In addition, the standard deviations of \( Q_1 \) and \( Q_2 \) (not shown) are not negligible.

Figure 1b shows similar diagrams as in Fig. 1a but for Q02. The ensemble means of \( Q_1 \) and \( Q_2 \) in Q02 are not quite the same as in Q03, especially in their time evolution, due to the existence of mesoscale organization in Q02 (Xu et al. 1992). For Q02, the maxima of \( Q_1 \) and \( Q_2 \) appear at 13 h and the secondary maxima appear at 19 h. Furthermore, both \( Q_1 \) and \( Q_2 \) are very small before 6 h. The standard deviations (not shown) are larger than those in Q03. These results show that the existence of mesoscale organization significantly influences the response of cumulus convection to large-scale processes.

Before the simulated updraft mass flux \( (M_u) \) and downdraft mass flux \( (M_d) \) are shown, the definition of updraft and downdraft areas is given. An updraft is assumed to occupy the entire CEM grid box (2 km wide) if the sum of the cloud-water and cloud-ice mixing ratios \( (q_c + q_i) \) exceeds 1% of the saturation water-vapor mixing ratio over water \( (q^*) \). Otherwise a fractional updraft area of the grid box is defined as the ratio of the sum to 0.01 \( q^* \) (Xu and Krueger 1991). In this definition, \( M_u \) does not include the upward vertical motion in cloud-free regions and the cloudy areas have positive vertical velocity. The downdraft area is defined similarly except using the sum of rainwater \( (q_r) \) and graupel \( (q_g) \) mixing ratios and 0.1 g kg\(^{-1}\) as the criterion to take into account the precipitating downdrafts only. The remaining areas are treated as the environment.

The ensemble means of \( M_u \) and \( M_d \) for Q03 and Q02 are shown in Fig. 2. The time evolution of \( M_u \) and \( M_d \) is similar to that of \( Q_1 \) and \( Q_2 \) except for the vertical location of maxima. The maxima of \( M_u \) and \( M_d \) are located at about 2.5 km and 1.5 km, respectively. The downdraft mass flux is very weak above 5 km. The difference of \( M_u \) between Q03 and Q02 is due to the fact that more deep clouds exist in Q02. The difference of \( M_d \) between Q02 and Q03 is, however, very small. When only active drafts \((|w| > 1 \text{ m s}^{-1})\) are considered (Fig. 3), \( M_d \) in Q03 is less than half of that in Q02, indicating that downdrafts in Q03 are less active than in Q02. On the other hand, the updraft mass fluxes are dominated by strong, active drafts in both Q02 and Q03.

3. Semiprognostic tests of the A–S parameterization

Arakawa and Schubert (1974) presented a theory for cumulus parameterization in which cumulus clouds
are assumed to modify the large-scale environment by hypothetical subsidence between the clouds that compensate the upward mass fluxes and by detrainment of cloud air containing sustained liquid water droplets. An ensemble of cumulus clouds is represented by a spectral of subensembles with different fractional entrainment rates. Each subensemble is modeled by a statistically steady, entraining cloud model. The solution for the subensemble thermodynamical properties from the vertical distribution of large-scale thermodynamical variables is achieved by (i) normalization of the subensemble mass flux at cloud base, (ii) assumption of nonbuoyancy of cloud air at the cloud top, and (iii) specification of the cloud-base moist static energy and water vapor mixing ratio.

Thus, the parameterization is reduced to the determination of the remaining unknown, cloud-base mass flux $M_B$, and is closed by assuming the quasi equilibrium of CWF. The CWF is defined as the rate of generation of cloud-scale kinetic energy due to work done by the buoyancy force per unit cloud-base mass flux. Let $A$ denotes the CWF. Then the time derivative of the CWF may be schematically written as

$$\frac{dA}{dt} = -kM_B + F, \quad (1)$$

where $F$ is the large-scale forcing and $-kM_B$ is the adjustment of CWF by a cumulus subensemble. When the left-hand side (lhs) in (1) is neglected, $M_B$ is determined for a given $F$. This is the standard A-S parameterization with the CWF quasi equilibrium. We call the semiprognostic test using this scheme the “control” test. In another test, the lhs in (1) is calculated from a known time sequence of the CWF and $dA/dt$ is treated as a portion of $F$. Then $M_B$ is determined for given $F$ and $dA/dt$. This test is referred to as the "nonequilibrium" test.

The nonequilibrium test, however, does not have any prognostic capability. Because the time change of CWF ($dA/dt$) is not explicitly formulated, the parameterization is not closed. Such a test is nevertheless instructive to show the impact of the CWF quasi-equilibrium assumption in the parameterization.

Specifically, the control test includes (i) all forcing mechanisms, such as large-scale destabilizing and moistening, turbulence, and radiative heating effects, (ii) the quasi-equilibrium assumption of the CWF, and (iii) a planetary boundary layer (PBL) condition based on the mass-weighted averaging of the variables over the lowest five CEM layers up to 566 m. In the previous studies (e.g., Lord 1982; Cheng and Arakawa 1990), a "fixed" PBL test was performed, in which all PBL processes such as turbulence and large-scale vertical motion were ignored by assuming that the total effect of PBL processes is balanced by the effect of cumulus clouds and, therefore, no net time change of PBL properties is produced. This study, however, retains the forcing associated with all PBL processes with one exception; that is, a fixed depth of PBL is used as in the previous studies to simplify the calculation, although the PBL depth diagnosed from the CEM generally varies with time. The diagnosed PBL depth averaged over 512 km varies slightly (<50 m). For shorter averaging distances, however, the PBL depth varies greatly from a convectively active period to a depressed period. Thus, the forcing mechanism associated with the temporal variability of the PBL depth is likely to be more important for short averaging distances.

The inputs to the parameterization consist of the vertical distributions of the advective cooling and moistening, turbulent fluxes, temperature, humidity, and radiative heating averaged either over 512 km, 256 km, 128 km, or 64 km, as well as over one hour in time.

a. Semiprognostic tests applied to the 512-km domain

The semiprognostic tests were first performed with the inputs averaged over 512 km, which corresponds to the entire domain size in Q02 and Q03 and half of the domain size in Q04. These tests correspond to situations when the parameterization is applied to a rather coarse horizontal grid, in which mesoscale processes are not resolved at all. Figure 4a shows the ensemble mean and standard deviation of parameterized precipitation rates from the control test for Q02 and Q03.

![Fig. 4. The ensemble mean of parameterized surface precipitation rate (thick solid line) and associated standard deviation (error bars) of Q03 and Q02 from (a) the control test and (b) the nonequilibrium test of the A-S parameterization. The abscissa is the phase of the imposed large-scale advective processes. The thin solid line is the ensemble mean of the simulated surface precipitation rate.](image_url)
The ensemble means agree very well with those obtained from the CEM (thin solid line), especially in Q03. The standard deviation around the ensemble mean is very small, compared to that of simulation (Xu et al. 1992). The reasons are that (i) the parameterization is diagnostic, as mentioned in the Introduction, and (ii) the inputs to the parameterization are almost entirely determined by the imposed time-varying, large-scale advective effects. The combined effect of both gives the parameterized variables with almost identical time sequences for all cycles. The large standard deviation between 18 h and 24 h in Q02 is, however, due to the failure of the parameterization to predict the existence of clouds in later stages of some cycles, since a criterion used in the parameterization for the existence of clouds in terms of the PBL relative humidity and the upper-level thermodynamic structure is not satisfied.

The results for the half-domains of Q04 (Fig. 5) also indicate that the A–S parameterization performs very well. Note that this test differs from that in Fig. 4a, in which the inputs to the parameterization that primarily consist of the large-scale advective effects are model generated rather than imposed, and therefore, it fluctuates from one cycle to another. Consequently, the standard deviation of parameterized precipitation rates agrees very well with that of simulation in Q04 (compare Fig. 5a with Fig. 5b).

Figure 4b shows the parameterized precipitation rates from the nonequilibrium test for Q02 and Q03. The ensemble means closely resemble those of simulations (thin solid line). The standard deviations are comparable to those of simulations (Xu et al. 1992). The overestimate of the ensemble mean between 3 h and 11 h in Q02 of the control test is significantly reduced in the CWF nonequilibrium test. The underestimate after 18 h remains about the same. Thus, for the entire domain, the major source of the differences between the parameterized and simulated cumulus convection is in the use of the CWF equilibrium assumption in the parameterization. This conclusion is still valid even when convective-scale downdrafts are included in the A–S parameterization (see section 4).

The precipitation rate is one of the final products of a cumulus parameterization. More importantly, a parameterization must correctly predict the vertical structure of the atmosphere that is modified by cumulus convection through $Q_1$ and $Q_2$. For simplicity, the results are hereafter shown only for Q02 unless mentioned otherwise.

The ensemble means of parameterized $Q_1$ and $Q_2$ from the control test are shown in Fig. 6a for Q02. The parameterization duplicates well the overall distribution of the mean $Q_1$ and $Q_2$ (compare to Fig. 1b), except for excessive drying and heating in the lower troposphere (mainly below 1.5 km). This shows that the A–S parameterization performs reasonably well even without downdrafts (Lord 1982). On the other hand, the time evolution of $Q_1$ and $Q_2$ is similar to that of the surface precipitation rate.

Next, Fig. 6b shows the ensemble means of parameterized $Q_1$ and $Q_2$ from the nonequilibrium test for Q02. The ensemble means of parameterized $Q_1$ and
$Q_2$ are very similar to those of the simulation (Fig. 1b), as seen from the magnitude and timing of maxima. In particular, the magnitude of the primary maxima at 13 h agrees very well with that of simulation. The overestimate or underestimate of $Q_1$ and $Q_2$ is smaller than that with $dA/dt = 0$ (Fig. 6a) and is basically within 5 K d$^{-1}$. Although the underestimate after 18 h is still significant, the overestimate between 1 h and 11 h is not. In addition, the standard deviation is comparable to that of simulation (not shown). These results, which are free from the CWF quasi-equilibrium hypothesis, indicate that the statistically steady cloud model used in the A−S parameterization is basically valid.

The ensemble mean of parameterized $M_u$ from the control test is shown in Fig. 7a for Q02. In existing semiprognostic tests with the observed data (Laud 1982; Kao and Ogura 1987; Cheng and Arakawa 1990; Grell et al. 1991), the updraft mass flux has never been examined because it is not extensively observable. As seen from Fig. 7a, the overall distribution of $M_u$ is well parameterized except that the magnitude is underestimated, especially in the lower troposphere. This underestimate is not entirely caused by the possible overestimate of the simulated $M_u$ due to the two-dimensionality. Instead, $M_u$ in A−S parameterization is really the net upward cloud mass flux (i.e., $M_c = M_u + M_d$). By comparing $M_c$ of the simulation (Fig. 2b) with the parameterization (Fig. 7a), it is found that the magnitude of $M_c$ is well parameterized. The time variation of $M_u$ is better parameterized in the nonequilibrium test, especially for the period between 1 and 14 h (Fig. 7b).

The time evolution of the large-scale forcing (Arakawa and Schubert 1974; Lord 1982) for the deepest cloud type, which corresponds to the smallest entrainment rate, is shown in Fig. 8. Figure 8 shows that the time variation of the ensemble mean of the large-scale forcing in Q02 and Q03 is almost entirely determined by that of the imposed large-scale advective (destabilizing and moistening) effects and that the standard deviation around the ensemble mean is very small. Although the imposed large-scale advective effects are identical, the peak values of the large-scale forcing in Q02 and Q03 is somewhat different ($\sim 6000$ J kg$^{-1}$ d$^{-1}$ in Q03 versus $\sim 7000$ J kg$^{-1}$ d$^{-1}$ in Q02). On the other hand, the standard deviation is relatively large between 18 h and 24 h in Q02. This may again be a consequence of the failure of the parameterization to predict the existence of clouds in later stages of some cycles.

In Fig. 8, the component of large-scale forcing due to moistening is also shown. The difference between these two is the component due to steepening of temperature lapse rate (destabilizing) by large-scale processes. The fact that this component of large-scale forcing can dominate the total large-scale forcing is overlooked in many parameterization schemes (e.g., Kuo 1965, 1974) and in physical interpretation of the interaction of cumulus clouds with their large-scale environment.

b. Semiprognostic tests applied to small subdomains

To evaluate the A−S parameterization for small subdomain sizes and to show the sensitivity of the A−S parameterization to the horizontal resolution, the semiprognostic tests were repeated for various subdomains of the CEM with the widths of 256, 128, and 64 km. In these tests, the input to the parameterization is the advective cooling and moistening rates and thermodynamical variables averaged over the subdomains being considered. The input is dominated by those effects due to internally determined mesoscale circulations rather than the imposed large-scale effects uniform over the entire domain. Consequently, these tests correspond to situations in which the parameterization is applied to rather fine horizontal resolutions, which partly resolve mesoscale processes.

1) RESULTS

Figure 9 shows the lag correlation coefficients between surface precipitation rates of simulation and pa-
rameterization for the control test (solid line) and nonequilibrium test (dotted line) of Q02 and Q03. The lag time from −5 h to +5 h is chosen. A positive lag time indicates that the parameterized cumulus convection precedes the simulated cumulus convection. The correlation coefficient is calculated for four horizontal averaging distances.

The control test of Fig. 9 shows the following common features: (i) the maximum correlation coefficient decreases as the horizontal averaging distance decreases; (ii) the amount of lag of maximum correlation is about 2–3 h for the 512-km averaging and less than 1 h for the rest of averaging; (iii) the correlation coefficient at a longer lag decreases as the horizontal averaging distance decreases; and (iv) the difference between Q02 and Q03 is not significant except for the 512-km averaging distances.

The nonequilibrium test of Fig. 9 (dotted line) shows the following features: (i) the correlation coefficient at zero lag is always larger than that of the control test in all averaging distances; (ii) the amount of increase is larger for long averaging distances; (iii) there is a small
amount of lag remaining, however, even with small subdomain sizes; and (iv) there is the near equality of the correlation coefficients at zero lag and 1-h lag, suggesting that the maximum correlation coefficient appears at about 0.5 h.

Figure 10 shows the lag correlation coefficients for \( Q_1 \) and \( Q_2 \) and \( M_a \) of Q02. Most of the features shown in Fig. 9b appear again in Fig. 10 with a few exceptions: (i) the maximum correlation coefficients in \( Q_1 \) and \( Q_2 \) for the control test appear at about zero lag, except for
the 512-km averaging distance; and (ii) the maximum correlation coefficients in $Q_1$ and $Q_2$ for the nonequilibrium test are closer to the zero lag for the 512-km averaging distance but are virtually unchanged for the rest of the averaging distances.

It should be noticed that the correlation coefficients shown in Fig. 10 are not independent of the height. For instance, there is a negative correlation above 11 km in $Q_1$ (Fig. 10b), due to excessive detrainment of liquid water of parameterized cumulus convection re-
sulting in moistening. It is insignificant due to the small magnitude of moistening (Fig. 6). It is also noticeable that the increase of correlation coefficients at zero lag is relatively significant in the middle and upper troposphere due to the inclusion of $dA/dt$.

2) DISCUSSION

Figures 9 and 10 show that the maximum correlation coefficients in the control and nonequilibrium tests tend to slightly decrease as the averaging distances de-
crease. This decrease is clearly not related to the CWF quasi-equilibrium assumption. It is possible that the inherent nondeterministic nature of cumulus convection begins to become relevant for short averaging distances because smaller-scale phenomena become resolvable. This does not mean that for a specific phenomenon of a fixed scale the combined nondiagnostic and nondeterministic error can be minimized. We expect that nondeterministic errors are the largest for the smallest resolvable scale and those for a fixed larger scale are not sensitive to the grid size used.

Figures 9 and 10 also show that the increase of correlation coefficients at zero lag due to the inclusion of $dA/dt$ decreases as the averaging distance decreases. This indicates that the inclusion of $dA/dt$ is more important for long averaging distances, especially in the presence of mesoscale organization (Q02). In other words, the CWF quasi-equilibrium assumption is more accurate when mesoscale processes are at least partially resolved. There is a small amount of lag remaining ($\sim 0.5$ h), however, even with small subdomain sizes. This small amount of lag is apparently not related to the inclusion of $dA/dt$. It is possible that the storage of liquid water/ice inside clouds or some cloud microphysical processes cause this lag.

The relatively large increase of correlation coefficients at zero lag in the middle and upper troposphere due to the inclusion of $dA/dt$ (Fig. 10) implies that the deviation from CWF quasi equilibrium is more significant for deep and middle clouds than shallow clouds.

It might be puzzling that the importance of the CWF nonequilibrium effect decreases as the averaging distance decreases since one would expect that the time change of the CWF is larger for shorter averaging distances. To interpret this, the scatterplots of $dA/dt$ versus the large-scale forcing for deep clouds (their tops extend from 7.9 to 14.1 km) are shown for both Q02 and Q03 (Fig. 11). Note that each data point represents a 3-hour time average. As can be seen from Fig. 11, the magnitude of the large-scale forcing increases as the averaging distance decreases but that of $dA/dt$ remains about the same or is slightly larger for shorter averaging distances. The ratio of $dA/dt$ to the large-scale forcing, however, decreases as the averaging distance decreases. It is very likely that the large-scale forcing, which is mainly due to advective processes, becomes more dominant for smaller scales. On the other hand, the CWF, which is completely determined by a sounding, is not significantly scale dependent. Grell et al. (1991) obtained a similar result using observed data in midlatitude.

By comparing Q02 with Q03, it is found that the magnitude of $dA/dt$ is larger in the sheared experiment for a given averaging distance, and especially for 256 km and 512 km. Thus, the inclusion of $dA/dt$ is more important when mesoscale organization is not even partially resolved. When mesoscale processes are resolved, as for the 128- and 64-km averaging distances, quasi equilibrium of the CWF is a much better approximation.

Furthermore, the importance of the inclusion of $dA/dt$ is a function of cloud type. For middle cloud types, with tops extending from 3.5 to 7.9 km (Fig. 12), and shallow cloud types, with tops extending from 0.5 to 3.5 km (not shown), the CWF quasi-equilibrium assumption is even better for all horizontal averaging distances.

4. Semiprognostic tests of the A-S parameterization with downdrafts

Cheng and Arakawa (1990) incorporated a combined updraft–downdraft model into the A–S parameterization. Convective-scale downdrafts associated with precipitation are the only type of downdrafts being considered (Cheng 1989). The rainwater originates from the tilting updraft and the downdraft is initiated by the weight of hydrometeors. The properties of downdrafts can be determined from the rainwater and vertical momentum budgets of tilting updrafts.

Downdrafts can significantly modify the thermodynamic structure of the subcloud layer (SCL) by detraining below cloud base (e.g., Betts 1976; Echternacht and Garstang 1976; Krueger 1988). The extent to which the downdraft air is mixed with the preexisting SCL air is not known. The spreading cold outflow induced by the detrainment of downdrafts may locally enhance the surface turbulent fluxes. Thus, it is assumed that the enhanced surface turbulent fluxes locally compensate the cooling and drying effects of downdrafts on the SCL environment. This is called the ASC-b scheme. In the ASC scheme, the downdraft air is assumed to completely mix with the preexisting SCL air over the entire grid. When the grid size is large, it is anticipated that the complete mixing is unrealistic. In nature, the effects of downdrafts on the SCL environment are confined in relatively narrow regions near convective cells. Thus, as Cheng and Arakawa (1990) argued, the ASC-b scheme might be superior to the ASC scheme using inputs averaged over long distances. [See Xu (1991) for a detailed comparison of the results using ASC and ASC-b schemes.] In the following results for the 512-km domain from the control test of the ASC-b scheme will be presented. A brief comparison of the results between ASC and ASC-b will be made.

Figure 13 shows the ensemble mean and standard deviation of parameterized surface precipitation rates from the ASC-b scheme for Q02 and Q03. The inclusion of downdrafts in the A–S parameterization only slightly modifies the parameterized precipitation rate. The time variation of the ensemble mean is very close to that parameterized by the A–S scheme (Fig. 4a).

The parameterized $Q_1$ and $Q_2$ from the ASC-b scheme for Q02 (Fig. 14) are very close to those from the A–S scheme (Fig. 6) except for less drying and heating in the lower troposphere (mainly below 1.5
The level of maximum $Q_1$ and $Q_2$, however, remains the same. Thus, the vertical structures of the parameterized $Q_1$ and $Q_2$ from the ASC-b scheme resemble those of simulation more closely than those from the A-S scheme. As expected, these results generally agree with those of semiprognostic tests using observed data in spite of the significantly different inputs to the parameterization (Cheng and Arakawa 1990) and different downdraft formulations (Grell et al. 1991).

Figure 15a shows the parameterized $M_u$ from the ASC-b scheme for Q02. Comparing with that parameterized by the A-S scheme (Fig. 7a), it is found that (i) the magnitude of maximum $M_u$ is almost doubled and (ii) the vertical structure with the maximum at 3 km is very similar to that of the simulation (Figs. 2b and 3b). The improvement in the parameterized $M_u$ is significant, especially in view of the contribution from shallow clouds. It can be concluded that $M_u$ is well parameterized by the ASC-b scheme.

For $M_d$ the ASC-b scheme cannot duplicate the overall distribution very well (compare Figs. 2b with 15b), notably the vertical position of maximum $M_d$. The maximum of parameterized $M_d$ is always located at the lowest model level but it appears around 1.5 km in the simulation (Figs. 2b and 3b). This discrepancy
is caused by adopting the one-dimensional cloud model with no explicit horizontal component of the motion. On the other hand, the ASC-b scheme can qualitatively capture the different magnitude of $M_d$ associated with active drafts ($|w| > 1$ m s$^{-1}$) between Q02 and Q03 (Fig. 3). The parameterized $M_d$ in Q02 is approximately twice of that in Q03 (not shown) which is consistent with the simulation (Fig. 3). Therefore the inclusion of convective-scale downdrafts in the A–S parameterization is reasonably successful.

To summarize the comparison among the schemes for the 512-km averaging distance, the time sequence of the ensemble mean of cloud-base mass fluxes ($M_b$) for Q02 is shown as a function of the cloud-top heights, that is, cloud types (Fig. 16a). The most impressive result is that the A–S parameterization and its modified versions behave very well when applied to such a high-resolution (30 layers) dataset. As seen from Fig. 16a, A–S produces many deep clouds but lacks shallow clouds. In ASC and ASC-b shallow clouds are abundant. Shallow clouds cause more moistening and possibly less heating in the lower troposphere. Therefore, the excessive heating and drying disappear when convective-scale downdrafts are included. A comparison of $M_b$ between ASC and ASC-b suggests that there is too much stabilization in ASC as far as the inputs from the 512-km averaging are concerned. This stabilizing effect, which can significantly reduces the strength of the deep clouds, may well be a function of the horizontal averaging distance.
Figure 16b is identical to Fig. 16a except for the "fixed" PBL test. This type of test was performed with the observed data (e.g., Lord 1978, 1982; Cheng and Arakawa 1991). Comparison of this type of test with the control test in this study can reveal the role of PBL forcing in cumulus parameterization. Figure 16b shows that (i) the magnitude of $M_d$ for deep clouds is significantly reduced and (ii) the cloud-top heights for clouds with maximum $M_d$ are lower. Consequently, the parameterized precipitation rate is significantly reduced ($\sim 20\%$). The effect of reduced $M_d$ on the deep clouds is that the parameterized $M_d$, $Q_1$, and $Q_2$ have relatively large discrepancies from the simulation in the middle and upper troposphere (not shown). Thus, the PBL forcing is important for properly parameterizing the intensity of deep cumulus convection in the A-S parameterization.

From the preceding comparison it is concluded that the inclusion of convective-scale downdrafts improves the performance of the A-S parameterization.

5. Conclusions and discussion

The results presented in this study represent a first attempt to quantitatively and systematically evaluate the validity of a cumulus parameterization by performing semiprognostic tests against simulated data averaged over a range of subdomain sizes. The cumulus ensemble model (CEM) provides consistent datasets that are not readily available from observations. For example, the CEM provides high-resolution inputs that include the effects of mesoscale organization of cumulus convection.

The main finding of this study is that the Arakawa-Schubert (1974) cumulus parameterization is basically valid in spite of the existence of mesoscale organization in cumulus convection. In particular, the assumption of cloud work function (CWF) quasi equilibrium is more accurate for inputs averaged over shorter distances than can partially resolve mesoscale processes. This finding is not surprising because the large-scale forcing increases more than does the time change of the CWF as the horizontal averaging distance decreases. Grell et al. (1991) obtained a similar conclusion from observed data.

As in most existing cumulus parameterizations, the Arakawa-Schubert cumulus parameterization is diagnostic in the sense that no additional prognostic equations for subgrid-scale variables is introduced and deterministic as far as the collective effects of the clouds are concerned. These nondiagnostic and nondeterministic aspects can be examined by testing the sensitivity of the parameterization to the horizontal grid resolution. This is the approach adopted in this study. It is found that (i) errors due to the nondiagnostic aspect are more significant for coarser resolutions that do not resolve mesoscale processes even partially and (ii) errors due to the nondeterministic aspect increase as the grid resolution becomes finer. The latter conclusion means that the inherent nondeterministic nature of cumulus convection becomes more relevant for smaller grid sizes due to smaller differences in scales between individual clouds and grid size.

This study also brings up the question of what roles mesoscale organization of clouds play in cumulus parameterization. Mesoscale organization is usually associated with strong vertical wind shear. The existence of mesoscale organization causes some phase delays in the modulation of cumulus activities by large-scale processes (Xu et al. 1992). As illustrated by a semiprognostic test, in which the CWF quasi equilibrium is relaxed using a known time sequence of CWF
Fig. 16. The ensemble means of cloud-base mass fluxes estimated by the A-S, ASC, and ASC-b schemes of (a) the control test and (b) the fixed PBL test for the 512-km averaging distance of Q02. The abscissa is the phase of the imposed large-scale advective processes. Contours over 2 mb h⁻¹ are hatched.

(dA/dt), the phase delays are somewhat captured by the A-S parameterization. Therefore, the effect of the mesoscale parameterization can be included in a prognostic parameterization assuming that dA/dt can be formulated.

The other finding of this study is that the inclusion of convective-scale downdrafts improves the results of semiprognostic tests as revealed by studies using observed data (Cheng and Arakawa 1990; Grell et al. 1991). It is found that the impact of downdrafts on the subcloud layer (SCL) of the large scale is not negligible. The mixing between the downdraft air and the preexisting SCL air results in significant stabilization of the SCL, which in turn influences the intensity of parameterized cumulus convection. This stabilization effect might be reduced as the horizontal averaging distance decreases presumably due to the enhanced surface turbulent fluxes. This result also points out the importance of understanding the interaction between downdrafts and the SCL, as well as the SCL processes. The SCL processes themselves are important for properly parameterizing the intensity of deep cumulus convection.

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REFERENCES

Krueger, S. K., 1985: Numerical simulation of tropical cumulus clouds and their interaction with the subcloud layer. Ph.D. dis-
ertation, University of California, Los Angeles, CA 90024, 205 pp.


