SHORTER CONTRIBUTIONS

THE MAGNITUDE OF RADIATIVE HEATING IN THE LOWER STRATOSPHERE

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1. Introduction.—It has often been a convenient meteorological practice to assume that nonadiabatic temperature changes due to radiative processes are negligible in the troposphere. The order of magnitude of radiative temperature changes is important in the lower stratosphere since in that region other nonadiabatic effects such as turbulent exchange and condensation should reach a minimum because of the stability and general dryness of these layers. If it can be shown that changes with time of the potential temperature in the lower stratosphere due to the radiative mechanism are of a smaller order of magnitude than those due to advective changes then, as a first approximation, the time rate of change of potential temperature of a given moving air parcel can be taken as zero. Using this assumption certain hitherto unknown information concerning meteorological processes in the lower stratosphere can be obtained.

2. Method.—A. Model. If Dobson and others [1] are correct in assuming that water vapor, carbon dioxide, and ozone contribute equally to the radiative heat balance of the lower stratosphere then at least the order of magnitude of radiative temperature changes can be determined in spite of the fact that only incomplete information exists as to the vertical distribution of these absorbers. The following model of vertical distribution of the parameters involved was constructed as a basis for calculation:

1. The temperature-height curve is that given by Pennendorf [6]. The means of these data are tabulated in the second column of table 1.

2. The ozone concentration-height curve is parallel to that given by Meetham and Dobson [5] for a total vertical path length of 0.300 cm STP of ozone at Trondheim, Norway. The means of these data are tabulated in the fourth column of table 1.

3. The carbon dioxide concentration-height curve was computed on the basis that the concentration of this gas would remain constant at 0.0003 by volume through the lower stratosphere. The assumption seems to be a valid approximation in light of the results of Glickauf [3]. The means of these data are tabulated in the fifth column of table 1.

4. Adequate and comprehensive data on the vertical distribution of water vapor above the tropopause are not available. The sum of the net heat flux as a result of the presence of ozone and carbon dioxide was considered representative of the magnitude of the net heat flux in accordance with Dobson’s assumption. Data offered by Dobson [1] indicate very low values of water-vapor content in the lower stratosphere.

5. The pressure for each level was computed from pressure-height data given by Pennendorf [7]. These data are tabulated in the third column of table 1.

In order to facilitate the computation the atmosphere was divided into layers which could be considered as isothermal. From ten to thirty-five kilometers the layers were chosen with a thickness of five kilometers each, from thirty-five to fifty kilometers the layers were chosen with a thickness of three kilometers each. Through each layer a mean value for the path length of the two absorbing gases as well as for the temperature and pressure was computed (table 1).

B. Equations. The following symbols are used:

\( B_\nu \) = black-body intensity at frequency \( \nu \)
\( c_p \) = specific heat at constant pressure
\( F \) = heat flux
\( I_\nu \) = intensity at frequency \( \nu \)
\( dh \) = heat imparted per unit mass
\( k_\nu \) = coefficient of absorption at frequency \( \nu \)
\( p \) = pressure
\( T \) = absolute temperature
\( z \) = vertical coordinate
\( \alpha \) = specific volume
\( \theta \) = potential temperature in degrees absolute
\( \kappa \) = Poisson’s constant
\( \nu \) = frequency
\( \varphi \) = longitude angle of polar coordinates
\( \psi \) = minimum angle between beam of radiation and normal to the surface
\( \rho \) = density of absorber.

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Table 1. Distribution of elements of synthesized radiation model.

<table>
<thead>
<tr>
<th>Layer boundaries in km</th>
<th>Mean temperature of layer in deg abs</th>
<th>Mean pressure of layer in mb</th>
<th>Total O(_3) path length in cm STP</th>
<th>Total CO(_2) path length in cm STP</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-15</td>
<td>224</td>
<td>193.6</td>
<td>0.0355</td>
<td>34.93</td>
</tr>
<tr>
<td>15-20</td>
<td>227</td>
<td>88.9</td>
<td>0.0750</td>
<td>15.83</td>
</tr>
<tr>
<td>20-25</td>
<td>280</td>
<td>40.4</td>
<td>0.0868</td>
<td>7.08</td>
</tr>
<tr>
<td>25-30</td>
<td>233</td>
<td>18.9</td>
<td>0.0465</td>
<td>3.28</td>
</tr>
<tr>
<td>30-35</td>
<td>256</td>
<td>8.6</td>
<td>0.0235</td>
<td>1.48</td>
</tr>
<tr>
<td>35-40</td>
<td>246</td>
<td>4.6</td>
<td>0.0103</td>
<td>0.45</td>
</tr>
<tr>
<td>40-45</td>
<td>265</td>
<td>3.1</td>
<td>0.0000</td>
<td>0.28</td>
</tr>
<tr>
<td>45-50</td>
<td>281</td>
<td>2.1</td>
<td>0.0007</td>
<td>0.18</td>
</tr>
<tr>
<td>50-55</td>
<td>298</td>
<td>1.5</td>
<td>0.0042</td>
<td>0.12</td>
</tr>
<tr>
<td>55-60</td>
<td>315</td>
<td>1.0</td>
<td>0.0025</td>
<td>0.08</td>
</tr>
</tbody>
</table>

* Tropopause taken at 10 km.
† Based on assuming total of 0.300 cm STP ozone between 10 and 30 km.
‡ Based on 0.0003 by volume of CO\(_2\).
The general differential equation for radiative transfer

\[ \frac{dI_s}{dz} = k_s \sec \psi (B_s - I_s) \]  

(1)
is transformed by change of variable \( dv = k_p dz \), to

\[ \frac{dI_s}{(B_s - I_s)} = \sec \psi \, dv. \]

If the upward intensity at \( r_0 \) is \( I_s^+ \) and the downward intensity at \( r_1 \) is \( I_s^- \), and for a given layer we assume \( B_s \) independent of \( r \), we find that the differential equation has the solutions:

\[ I_s^+ \text{ (upward intensity at level } \tau \text{)} = +B_s + (I_s^+ - B_s)e^{-(\tau - \tau_0)} \sec \phi \]

and thus:

\[ I_s = I_s^+ + I_s^- = \beta e^{-(\tau - \tau_0)} \sec \phi - \delta e^{-(\tau_1 - \tau)} \sec \phi, \]

(2)

where \( \beta = I_s^+ - B_s \) and \( \delta = I_s^- - B_s \).

The flux of radiation at a level \( \tau \) is defined by

\[ F_s = \int I_s \cos \psi d\psi, \]

where \( d\psi \) is infinitesimal angle in polar coordinates, and this yields upon integration of \( \psi \) from 0 to \( 2\pi \),

\[ F_s = 4\pi \left[ \beta \int_0^{\pi/2} e^{-(\tau - \tau_0)} \sec \phi \sin \psi \cos \psi \, d\psi - \delta \int_0^{\pi/2} e^{-(\tau_1 - \tau)} \sec \phi \sin \psi \cos \psi \, d\phi \right], \]

and letting \( \eta = \sec \psi, \)

\[ F_s = 4\pi \beta E_{\eta \eta} (\tau - \tau_0) - \delta E_{\eta \eta} (\tau_1 - \tau), \]

(3)

where \( E_{\eta \eta} (\tau) = \int_0^{\pi/2} \eta e^{-\eta \eta} d\eta \) is the \( n \)th exponential integral. Tables of \( E_{\eta \eta} (\tau) \) for certain values of \( \eta \) are given by several writers in radiation theory [2].

Using equation (3) the net radiative flux for each layer was calculated, and assuming that the flux divergence \( \partial F / \partial \eta = \Delta F / \Delta \eta \), the radiative contribution to the change of potential temperature was calculated by means of the relationship

\[ \left( \begin{array}{c} \frac{\partial \theta}{\partial t} \\ \frac{\partial \varphi}{\partial t} \end{array} \right)_{\text{rad}} = \frac{\partial}{\partial \eta} \left( \begin{array}{c} \theta \\ \varphi \end{array} \right). \]

(4)

C. Absorption coefficients. It was assumed that mean values of the absorption coefficients could be used over the absorption bands concerned. The values used are those given by Sutherland and Callendar [8], as shown in table 2.

It is necessary to mention that the computations considered only the effect of the infrared bands of ozone and carbon dioxide. Data offered by Goetz [4] indicate that for the layers between 10 and 25 km the contributions to the net heat flux due to ultraviolet bands should be of a lower order of magnitude than that calculated for the infrared bands. Although Goetz's data are for ozone alone it seems not too unlikely that the same effect would hold for carbon dioxide and for water vapor.

D. Black-body intensities. Values of the black-body intensities \( B_s \) for the various layers and absorption bands were computed by integrating the Planckian intensity over the wave numbers concerned:

\[ B_s(\nu) = \int_1^\infty d\nu R \int_1^\infty \nu^2 (e^{\nu^2} - 1)^{-1} d\nu, \]

where \( \nu = q \nu / T \) (\( \nu \) is wave number), \( q = 1.436 \) cm deg, and \( 4\pi R = 2.264 \times 10^{-9} \) cal (3 hr)\(^{-1} \) cm\(^2 \) deg\(^{-4} \). A table of the integral \( \int_1^\infty \nu^2 (e^{\nu^2} - 1)^{-1} d\nu \) is given by Eissasser [2].

3. Results.—Results of the calculations are presented in table 3. The flux-divergence values together with the mean values of \( (\partial \theta / \partial t)_{\text{rad}} \) for each layer are shown. It must be borne in mind that above approximately 25 km (depending on the vertical distribution of the absorbing gases) the contributions of ultraviolet radiations cannot be neglected.

4. Remarks.—The results obtained represent a calculation based on superposition of several sets of representative data, with certain simplifying assumptions. It is the opinion of the writer that only the orders of magnitude of the results are correct and possess significance. It should be borne in mind that these calculations represent the instantaneous radiative potential temperature changes and do not account for kinetic changes.

Table 3 indicates support for at least two of Dobson's [1] contentions since:

1. The flux divergences of ozone and of carbon dioxide are of approximately the same order of magnitude.
2. In the lower stratosphere ozone-concentration variations would more profoundly affect the rate of change of temperature than would those of carbon dioxide since the latter is in black-body concentration up to the 25-km level at least.

5. Summary.—The problem presented is the determination of the order of magnitude of nonadiabatic temperature changes in the lower stratosphere due to radiative processes. Calculations of heat-flux divergence are made on a synthesized model by dividing the atmosphere between 10 and 50 km into layers with assumed uniform temperature and absorber distribution. Results of the calculations, which are made for three bands of \( \text{O}_3 \) in the infrared and for two bands of \( \text{CO}_2 \) in the infrared, indicate that the time rate of change of potential temperature due to radiative

<table>
<thead>
<tr>
<th>Gas</th>
<th>Band center in ( \mu )</th>
<th>Band width in ( \mu )</th>
<th>( \bar{k}_\nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{O}_3 )</td>
<td>4.75</td>
<td>4.5-5.0</td>
<td>3.0</td>
</tr>
<tr>
<td>( \text{O}_3 )</td>
<td>9.65</td>
<td>9.4-9.8</td>
<td>3.0</td>
</tr>
<tr>
<td>( \text{O}_3 )</td>
<td>14.1</td>
<td>12.3-15.5</td>
<td>0.2</td>
</tr>
<tr>
<td>( \text{CO}_2 )</td>
<td>4.3</td>
<td>4.0-4.5</td>
<td>2.0</td>
</tr>
<tr>
<td>( \text{CO}_2 )</td>
<td>15.0</td>
<td>14.0-16.0</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Table 3 Calculated infrared flux, flux divergence, and heating effect due to O₃ and CO₂.

<table>
<thead>
<tr>
<th>Layers in kms</th>
<th>Net infrared flux due to O₃ in cal (3 hr)⁻¹ cm⁻³</th>
<th>Net infrared flux due to CO₂ in cal (3 hr)⁻¹ cm⁻³</th>
<th>Total net infrared flux due to O₃ + CO₂ in cal (3 hr)⁻¹ cm⁻³</th>
<th>ΔF/Δθ in cal (3 hr)⁻¹ cm⁻³</th>
<th>(dθ/dF)ad in deg cal sec⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>10–15</td>
<td>+0.480</td>
<td>+0.100</td>
<td>+0.580</td>
<td>+0.422 × 10⁻⁴</td>
<td>+1.41 × 10⁻⁴</td>
</tr>
<tr>
<td>15–20</td>
<td>+0.737</td>
<td>+0.054</td>
<td>+0.791</td>
<td>-0.622 × 10⁻⁴</td>
<td>-6.62 × 10⁻⁴</td>
</tr>
<tr>
<td>20–25</td>
<td>+0.538</td>
<td>-0.058</td>
<td>+0.480</td>
<td>-0.202 × 10⁻⁴</td>
<td>-5.65 × 10⁻⁴</td>
</tr>
<tr>
<td>25–30</td>
<td>+0.180</td>
<td>+0.199</td>
<td>+0.379</td>
<td>+0.454 × 10⁻⁴</td>
<td>+3.59 × 10⁻⁴</td>
</tr>
<tr>
<td>30–35</td>
<td>+0.053</td>
<td>+0.553</td>
<td>+0.606</td>
<td>-0.244 × 10⁻⁴</td>
<td>-5.13 × 10⁻⁴</td>
</tr>
<tr>
<td>35–38</td>
<td>-0.022</td>
<td>-0.348</td>
<td>-0.370</td>
<td>-0.369 × 10⁻⁴</td>
<td>-1.39 × 10⁻⁴</td>
</tr>
<tr>
<td>38–41</td>
<td>-0.064</td>
<td>-1.412</td>
<td>-1.476</td>
<td>-0.295 × 10⁻⁴</td>
<td>-1.99 × 10⁻⁴</td>
</tr>
<tr>
<td>41–44</td>
<td>-0.106</td>
<td>-2.254</td>
<td>-2.360</td>
<td>-0.993 × 10⁻⁴</td>
<td>-1.14 × 10⁻⁴</td>
</tr>
<tr>
<td>44–47</td>
<td>-3.104</td>
<td>-2.554</td>
<td>-2.658</td>
<td>+0.020 × 10⁻⁴</td>
<td>+3.84 × 10⁻⁴</td>
</tr>
<tr>
<td>47–50</td>
<td>-0.057‡</td>
<td>-2.395‡</td>
<td>-2.652</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The troposphere was, for purposes of the calculation, considered devoid of ozone and a temperature of 290A taken for the earth's surface.
† The tropopause was, for purposes of the calculation, considered as a black body radiating with the tropopause temperature (223A).

The processes decreases from 10⁻⁶ (degrees absolute per second) in the lower 15 km to 10⁻² in the layers from 35 to 47 km and decreases to 10⁻⁴ in the uppermost 3 km.

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REFERENCES