Representation of the Equatorial Stratospheric Quasi-Biennial Oscillation in EOF Phase Space

JOHN M. WALLACE
Department of Atmospheric Sciences, University of Washington, Seattle, Washington

R. LEE PANETTA
Department of Atmospheric Sciences, Texas A&M, College Station, Texas

JERRY ESTBERG
Department of Physics, University of San Diego, San Diego, California

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ABSTRACT

A 35-year record of monthly mean zonal wind data for the equatorial stratosphere is represented in terms of a vector (radius and phase angle) in a two-dimensional phase space defined by the normalized expansion coefficients of the two leading empirical orthogonal functions (EOFs) of the vertical structure. The tip of the vector completes one nearly circular loop during each cycle of the quasi-biennial oscillation (QBO). Hence, its position and rate of progress along the orbit of the point provide a measure of the instantaneous amplitude and rate of phase progression of the QBO. Although the phase of the QBO bears little if any relation to calendar month, the rate of phase progression is strongly modulated by the first and second harmonics of the annual cycle, with a primary maximum in April/May, in agreement with previous studies based on the descent rates of easterly and westerly regimes.

A simple linear prediction model is developed for the rate of phase progression, based on the phase of the QBO and the phase of the annual cycle. The model is capable of hindcasting the phase of the QBO to within a specified degree of accuracy approximately 50% longer than a default scheme based on the mean observed rate of phase progression of the QBO (1 cycle per 28.1 months). If the seasonal dependence is ignored, the prediction equation corresponds to the "circle map," for which an extensive literature exists in dynamical systems theory.

1. Introduction

Over 30 years have elapsed since the discovery of the quasi-biennial oscillation (QBO) in zonally averaged zonal wind in the equatorial stratosphere (Reed 1960; Ebdon 1960; Reed et al. 1961). Among its notable characteristics are

- a sharp spectral peak with a period slightly longer than 2 years, centered near the 30-hPa (24-km) level over the equator where peak-to-peak amplitudes are on the order of 45 m s\(^{-1}\). The QBO is detectable all the way from the tropopause (~17 km; 100 hPa) up to approximately 50 km (Reed 1965);
- downward phase propagation of successive easterly and westerly wind regimes at a rate of about 1 km month\(^{-1}\), such that zonal wind fluctuations in the lower stratosphere (~70 hPa) tend to occur out of phase with those in the middle stratosphere (~10 hPa); and
- more rapid downward propagation of westerly wind regimes than easterly regimes.

These properties are clearly evident in Naujokat's (1986) extended time–height section of zonal wind over the equator and in the composite cycle that she constructed by averaging the data from 14 individual cycles.

Individual cycles of the QBO have been observed to vary in length from 2 to 3 years and exhibit a variety of phases relative to the climatological mean annual cycle. Dunkerton (1990) has noted that much of the variability in the length of individual cycles of the QBO is due to the fact that the descent of easterly wind regimes slows down or sometimes even stalls briefly during the calendar months July through February. This behavior is readily apparent in time–height sections of the QBO. Dunkerton examined the modulation of the descent rate by the annual cycle in more detail by calculating (separately) the average easterly and westerly accelerations at the 10-, 30-, and 50-hPa levels at three different tropical stations. He found evidence of an annual modulation, particularly for the easterly acceler-
ations at the lower levels, with weakest accelerations toward the end of the calendar year. Maruyama (1991) has shown conclusive evidence of an annual cycle in the amplitude of Kelvin waves and mixed Rossby–gravity waves at one equatorial station, which he views as being related to the rapid descent of easterly wind regimes in March–June and the slow descent in July–February in the 30–50-hPa layer. It has been suggested by Maruyama and Tsuneoka (1988) that the El Niño–Southern Oscillation phenomenon might also contribute to the variability in the length of individual cycles of the QBO.

The approach taken in this paper is to specify the instantaneous state of the QBO in terms of variables that concisely define the vertical structure of the zonal wind, as opposed to using the wind itself at one (or more) selected level(s). This formalism brings the irregularities in the QBO into sharper focus and establishes an interesting connection between the QBO and the “circle map” studied extensively in connection with the theory of nonlinear dynamical systems. We represent the QBO by a single point in a two-dimensional phase space whose coordinates are the projections on the two leading empirical orthogonal functions (EOFs) derived from slightly smoothed, deseasonalized zonal wind data at 70, 50, 40, 30, 20, 15, and 10 hPa. During each cycle of the QBO, the point executes a nearly circular orbit in phase space. The amplitude and phase angle of the vector in a polar coordinate representation serve as measures of the instantaneous amplitude and phase of the QBO. By taking the time derivative of the phase angle, we obtain a measure of the instantaneous rate of phase progression. This formulation reveals clearly the dependence of the rate of phase progression of the QBO upon the phase of the QBO itself (i.e., the difference between the descent rates of westerly and easterly regimes), and upon the phase of the annual cycle. These two factors, acting in concert, account for almost half the temporal variance in the rate of phase progression of the QBO in 5-month running mean data. We will explore the feasibility of statistical prediction of the phase of the QBO based upon these relationships.

The study is based on the dataset of Naujokat (1986) updated to include Singapore observations through June 1990, which she has graciously provided. The data were deseasonalized by fitting and removing the first harmonic of the annual cycle at each of the seven levels. This procedure was performed independently on the three station records (Canton Island, 3°S, 171°W, January 1956–September 1967; Gan, 1°S, 73°E, September 1967–December 1975; and Singapore, 1°N, 104°E, January 1974–June 1990) that were spliced together to form a single continuous record. The deseasonalized time series were smoothed with a centered 5-month running mean filter. The removal of the annual cycle and the smoothing have only a modest effect upon the time series. For example, the rms amplitude of the time series of zonal wind at the 30-hPa level is reduced from 17.6 to 16.1 m s⁻¹; the correlation coefficient between the deseasonalized, smoothed time series and the raw time series at that level is 0.972. Figures 1a and 1b show time series of the raw and deseasonalized and smoothed 30-hPa wind data. Because the 10-hPa data are available only from January 1956 onward, the deseasonalized and smoothed data used in this paper begin with March 1956.

2. Scatter diagrams for individual levels

In order to motivate the mode of representation of the QBO used in this study, we show in Fig. 2 a sample of a scatter diagram constructed by plotting the zonal wind at one level versus zonal wind at another level. If the time series for the higher level is used as the

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**Fig. 1.** Upper panel: monthly mean zonal wind at 30 hPa based on a composite of equatorial stations compiled by Naujokat (1986) and updated with data from Singapore (1°N), in units of meters per second. Lower panel: 5-month running mean deseasonalized data based on the same record.
abscissa and the one for the lower level as the ordinate, data points for successive months fall on an orbit that circulates about the origin in a counterclockwise sense. In the case of a simple cosine wave propagating in the vertical monotonically (but not necessarily uniformly) with constant amplitude, it is readily verified that the orbit would be an ellipse, with the following characteristics:

- orientation of the major axis along either $+45^\circ$ or $-45^\circ$, depending upon whether the separation between the levels is closer to an even or odd number of half-wavelengths,
- eccentricity ranging from zero for separations of odd numbers of quarter-wavelengths to unity for separations of even numbers of quarter-wavelengths, and
- counterclockwise (clockwise) circulation for separations ranging from $n$ to $n + \frac{1}{2} (n - \frac{1}{2}$ to $n)$ wavelengths, where $n$ is any integer.

For the seven levels included in this study, 21 such plots can be constructed, which exhibit a variety of shapes. Consistent with the signs of the correlation coefficients between the various time series of zonal wind, the clouds of data points on scatterplots for adjacent levels (10 vs 15 hPa, etc.) exhibit positive slopes, whereas those for distant levels (10 or 15 vs 70 hPa) exhibit negative slopes (not shown). The more interesting plots are the ones for levels at which the zonal wind fluctuations in the QBO tend to be more nearly in quadrature (10 with 40 hPa, 15 with 50 hPa). The orbits on these plots tend to be triangular in shape, as seen in Fig. 2. The clustering of points near the vertices of the triangle is a reflection of the narrowness of the shear zones: much of the time the accelerations are small at both levels and the QBO, as represented in this two-dimensional phase space, is not progressing around its orbit. The onset of westerly winds at 10, 15, and 20 hPa is accompanied by acceleration in the opposite sense at lower levels (indicated by the slanted path in phase space from $A$ to $B$), whereas the onset of easterly winds (the horizontal path from $C$ to $A$) is not. This asymmetry reflects the differing vertical structure of easterly and westerly wind regimes of the QBO. Because of these marked departures from pure oscillatory behavior, it is not obvious how to characterize the properties of individual cycles of the QBO. To describe them in detail is cumbersome: to summarize them in terms of a few statistics requires subjective choices that may not be easy to justify.

3. EOF analysis of the QBO

If we are willing to sacrifice some of the details concerning the vertical structure of the QBO, we can distill much of the information inherent in the time series of zonal wind through the use of an eigenvector analysis based on the correlation matrix between zonal wind at each of the seven levels (70, 50, 40, 30, 20, 15, and 10 hPa). The first two modes, whose vertical structure can be seen in Fig. 3 and Table 2, account for 95.5% of the normalized variance of the seven deseasonalized smoothed series combined. The first mode (hereafter referred to as $Z_1$) reflects the negative correlations between zonal wind fluctuations at 10 and 70 hPa, and the second ($Z_2$) picks up the variability at middle levels. The time series of the expansion coefficient of $Z_1$ [denoted as $e_1(t)$] is correlated with the time series of 10-hPa zonal wind minus 70-hPa zonal wind at a level of 0.98, and $e_2(t)$ is correlated with 30-hPa zonal wind at a level of 0.96.

In order to illustrate how well these EOFs represent the variability in time–height sections of the QBO, we
contrast, in Fig. 4, an actual time–height section based upon the deseasonalized, smoothed wind data, and a reconstruction based on linear combinations of \( e_1(t) \) and \( e_2(t) \). (To obtain smoother contours, the time series at the seven pressure levels were interpolated to levels equally spaced in log pressure.) It is evident that some of the details are lost or obscured in the reconstruction, particularly at the upper levels, and the duration of the westerly wind regimes at the upper levels is substantially shortened. Despite these deficiencies, however, the reconstructed section captures quite well the character of the individual cycles of the QBO.

Figure 5 shows the time variation of the normalized expansion coefficients associated with these modes in two different forms: (a) a conventional scatterplot, and (b) a plot of the “orbit of the QBO” in the two-dimensional phase space of projection coefficients, formed by connecting successive points \( \langle e_1(t), e_2(t) \rangle \). In comparison to the scatter diagrams for individual levels such as the one shown in Fig. 2, the orbit is rounder and more regular, and the data points are distributed more uniformly along it. The coherence of the ring of points and the uniformity of the circular orbit is evidence of the remarkably uniform structure and nearly constant amplitude of the QBO in this 35-year period of record.

The locus of the centroid of the data points, which defines the origin of the coordinates in Figs. 5a and 5b, is located above and to the left of the visual center of the circular orbit (indicated by the cross). In order to simplify the analysis, we moved the origin of the coordinate system to the visual center of the orbit, at \((0.188, -0.107)\). The location of this point was determined numerically so as to minimize the variance of distances to the points in the scatterplot. The coordinate axes were not rotated.

Figure 6 shows how various points along the circular orbit in the shifted coordinate system can be identified with the onset of easterly and westerly wind regimes at various levels, as defined by sign changes in the smoothed, deseasonalized zonal wind. The left quadrants correspond to descending easterly regimes, and the right quadrants to descending westerly regimes. The relationship between phase angle in this coordinate system and the level of onset of the descending easterly and westerly wind regimes is not quite perfect, as indicated by the ambiguities in the labeling of the arcs in Fig. 6.

The uniformity of the orbits in Fig. 5b is least in the upper left sector, which corresponds to the phase of the cycle during which the leading edge of the easterlies propagates downward from the 10- to the 40-hPa level. The fact that 32% (as opposed to approximately 25%) of the points for individual year/months lie in this quadrant is consistent with the fact that the easterly wind regimes of the QBO are observed to descend more slowly than the westerly regimes (e.g., see Naujokat 1986).

In the new coordinates we will use the following terminology. A state of the QBO will be determined by coordinates \((a, b)\), where

\[ a = e_1 - 0.188 \quad \text{and} \quad b = e_2 + 0.107. \]

The amplitude and phase of the QBO are defined by the polar coordinates

\[ C = (a^2 + b^2)^{1/2} \quad \text{and} \quad \psi = \frac{1}{2\pi} \arctan \left( \frac{b}{a} \right). \]

The time series of \( e_1(t) \) and \( e_2(t) \) are normalized such that the mean value of \( C \), the amplitude of the QBO, is \( \sqrt{2} \). Phase angle \( \psi \) is expressed in cycles of the QBO.

The pronounced eccentricity of the orbits that motivated the coordinate transformation is a reflection of the uneven density of data points along the circular orbits in Figs. 5 and 6. This unevenness is illustrated more quantitatively in Fig. 7, which shows the frequency distribution of the individual monthly data points in Fig. 5 as a function of \( \psi \), in \( 30^\circ \) increments. Consistent with the earlier figures, it shows an approximately sinusoidal dependence on phase angle, with the highest density of points at angles of approximately \( 165^\circ \) (0.45 cycles). Referring to Fig. 6, we find that this angle corresponds to the phase of the QBO in which easterlies overlie westerlies, with a node in the vicinity of the 30-hPa level.

Figure 8a,b shows time series of \( C \) and \( \psi \) based on the same deseasonalized, 5-month running mean data. Amplitudes have ranged from 0.71 to 1.93, with a standard deviation of 0.22. By construction, \( C \) and \( \psi \) are uncorrelated. It is evident that \( \psi \) increases mono-
tonically with time at a nearly constant rate that, as determined by a linear regression, corresponds to 28.1 months for the average length of a cycle. The small irregularities in the period of the QBO are more readily apparent in the corresponding detrended time series shown in Fig. 8c, obtained by removing the linear fit
from the time series for $\psi$. Within this 35-year record, the QBO never deviates by more than 0.28 cycles from a 28.1-month "clock." In the discussion that follows, we will also refer to the rate of phase progression

$$\psi' = \frac{\Delta \psi}{\Delta t}$$

of the QBO. In estimating $\psi'$ in month $n$ of the data, we take

$$\psi'(n) = \frac{1}{2} [\psi'(n + 1) - \psi'(n - 1)].$$

The time series of $\psi'$ shown in Fig. 8d exhibits a large amount of variability, with values ranging from nearly 0.072 cycles per month, which if sustained over a full cycle of the QBO would yield a period of just over a year, to nearly zero, indicating a virtual halt (albeit brief) in the downward progression of successive wind regimes. The episodes of very small values of $\psi'$ that occurred in the second halves of 1964 and 1967 are evident in Naukojat's (1986) time–height section, and the episode in 1988 shows up clearly in our Fig. 4.

Part of the temporal variability of $\psi'$ is clearly due to the systematic tendency for westerly wind regimes to propagate downward more rapidly than do easterly regimes, as reflected in the uneven density of data

**Fig. 5.** Upper panel: Scatterplot of the normalized expansion coefficients of EOFs 1 and 2 of the 5-month running mean, deseasonalized zonal wind, as defined in Fig. 3. Lower panel: plot of the orbits in the same two-dimensional phase space, constructed by connecting the points in the left panel in chronological order. Each circuit represents an individual cycle of the QBO. The cross shows the origin of the shifted coordinate system. The arrow shows the sense of the propagation.

**Fig. 6.** As in the lower panel of Fig. 5 but in the shifted coordinate system, with labeling relating the angle $\psi$ in polar coordinates to the level of transition between westerly and easterly wind regimes of the QBO. Upper panel: onset of easterly regimes. Lower panel: onset of westerly regimes.
points along the orbit in Figs. 5 and 6. Figure 9 shows the rate of phase progression $\psi'$ as a function of phase angle. Referring again to Fig. 6 as a timetable, we see that the rate of phase progression is substantially higher when a westerly regime is descending through the 30-hPa level than when an easterly one is. In order to determine how much of the temporal variance of $\psi'$ is due to this effect, we performed a linear regression of $\psi'$ upon $\sin \psi$ and $\cos \psi$. The resulting linear fit accounts for 29% of the variance of $\psi'$.

4. Modulation by the annual cycle

Figure 10 shows a scatterplot of $\psi'$ as a function of calendar month, also based on the deseasonalized 5-month running mean data. A strong dependence is evident, with largest values (averaging 0.05 cycles per month) in April/May and smallest values (averaging 0.03 cycles per month) in December/January, in agreement with results of Dunkerton (1990) and Maruyama (1991).

Figure 11 shows the analogous plot for phase angle $\psi$ itself. The points are arranged in diagonal swaths, one of which is laid down each year. In agreement with Fig. 4 of Dunkerton (1990), transitions between easterly and westerly regimes, which correspond to phase angles of roughly 0.15 and $-0.4$, tend to be most common around midyear. The appearance of this figure would change markedly, however, if a new swath were laid down in one of the gaps. Therefore, we agree with Dunkerton (1990) that it may be premature to draw any conclusions regarding the annual modulation of the phase of the QBO. Intuitively, it seems likely that any signature of the annual cycle should be much more evident in the time series of $\psi'$ than in the time series...
of $\psi$ itself. One can easily imagine how seasonally varying environmental conditions such as the intensity of Kelvin or mixed Rossby–gravity wave forcing incident upon the stratosphere from below, or the zonally averaged vertical velocity in the lower stratosphere, could exert a direct influence upon the rate at which successive westerly and easterly wind regimes descend, as reflected in $\psi'$. It is much less clear how they could directly influence $\psi$ itself, which reflects in a complex way the history of the forcing over at least several cycles. (Note that the more periodic the phenomenon, the longer its “memory” of past states of the system.)

Amplitude $C$ also exhibits a weak but significant dependence upon the annual cycle, with largest values in August that are on average approximately 15% larger than the values observed in February.

A semiannual modulation of amplitude and rate of phase propagation of the QBO, if it existed, would be largely obscured in 5-month running means. Therefore, we performed a similar analysis on the unsmoothed time series of monthly mean data. The two leading eigenvectors explain 90.9% of the variance (as opposed to 95.5% for the smoothed, deseasonalized data). The orbit of the QBO in the two-dimensional phase space defined by EOFs 1 and 2, shown in Fig. 12, is less circular than in the case of the 5-month running mean data. For reasons that are not clear to us, the shifting of the coordinate axes does not enhance our ability to explain the temporal variability of $\psi'$ on the basis of the phase of the QBO and the annual cycle as it did in the previous case. Therefore, in the analysis of the unsmoothed data we define $\psi$ and $\psi'$ on the basis of the unshifted coordinate system. Figure 13 shows the rate of phase progression $\psi'$, as calculated from the unsmoothed data, stratified by calendar month. It is evident from a comparison of Figs. 10 and 13 that the dependence of $\psi'$ upon calendar month is substantially stronger in the unsmoothed data. For example, the mean rate of phase progression is three times as large.

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1 Analysis was performed on both raw data and deseasonalized data, and the results were found to be virtually identical. The results presented in Figs. 12, 13, and 15 are based on raw data.
from April to June as from November to February. The secondary peak in September/October reflects the existence of a semiannual cycle that, in a linear regression, exhibits a level of statistical significance comparable to that of the annual cycle.

5. A simple prediction scheme

In this section we define the rate of phase progression in terms of a one-month forward difference, based on the deseasonalized 5-month running mean data. When \( \psi' \), defined in this manner, is linearly regressed upon \( \sin 2\pi \psi, \cos 2\pi \psi, \sin \mu, \cos \mu \), where \( \mu \) is a phase angle corresponding to the calendar month (e.g., 15 December corresponds to 0 and 15 June to \( \pi \)), 46% of its variance is accounted for. The regression equation can be written

\[
\psi' = A_0 + A_1 \cos 2\pi \psi + A_2 \sin 2\pi \psi + A_3 \cos \mu + A_4 \sin \mu + e,
\]

where \( e \) is the error term to be minimized. For the whole record, the values of the \( A_i \) are (0.0371, 0.0094, -0.0060, -0.0056, -0.0073) cycles per month. Given the regression coefficients \( A_i \), a simple way to predict \( \psi \) is to integrate the difference equation

\[
\psi'_{\text{pred}} = A_0 + A_1 \cos 2\pi \psi_{\text{pred}} + A_2 \sin 2\pi \psi_{\text{pred}} + A_3 \cos \mu + A_4 \sin \mu
\]

forward in time, a scheme that should be more skillful than simply assuming a uniform rate of phase progression of 1 cycle per 28.1 months. Equation (1), in which we take \( \psi_{\text{pred}} \) to be a single month increment, can be written more simply by shifting the phase and calendar year origins to give

\[
\psi'_{\text{pred}} = A_0 + A_1 \cos 2\pi (\psi_{\text{pred}} - \psi_0) + A_2 \cos (\mu - \mu_0),
\]

where \( A_\phi = (A_1^2 + A_2^2)^{1/2} \), \( A_\mu = (A_3^2 + A_4^2)^{1/2} \), and the phase shifts \( \psi_0 = \tan^{-1}(A_2/A_1) \), and \( \mu_0 = \tan^{-1}(A_4/A_3) \) correspond to the phase of the QBO (-32.9°, when westerlies overlie easterlies) and time of year (approximately 8 April) when the phase progression is most rapid.

If both \( A_\phi \) and \( A_\mu \) are zero, (2) describes a simple periodic evolution, but nonzero coefficients can introduce very different behavior. For example, when \( A_\mu = 0 \) and \( A_\phi = A_0 \), the phase progression can be completely arrested. The case with \( A_\phi = 0 \) is a special case of a “circle map”; that is, a map of the form \( \psi' = C_0 + f(\psi) \), with periodic \( f \). Circle maps and, in particular, (2) with \( A_\phi = 0 \) have been studied in considerable depth in connection with phase locking, quasi periodicity, and chaos in systems of simple nonlinearly coupled oscillators. The different kinds of behavior depend in a delicate way on the value of \( |A_\phi/C_0| \), but for a value less than one, as is the case in our study, chaotic behavior is impossible [the reader is referred to Jensen et al. (1984) for an accessible overview and Arnol'd (1965) and Lanford (1987) for fundamental results].

The annual cycle term will reinforce or counteract the contribution of the \( A_\phi \) term, with minimum rate of phase progression occurring when \( \psi = \psi_0 + \pi \) and \( \mu = \mu_0 + \pi \). Although this model is suggestive as an explanation of the occasional hesitations in the descent of the easterly regimes, it should be noted that the amplitudes \( A_\phi \) and \( A_\mu \) calculated from the data are in fact too small to even come close to arresting the phase progression.

Figure 14a shows a time–height section generated from (2) with \( A_\phi = A_\mu = 0 \), and initial conditions corresponding to a point on a circular orbit in phase space with a radius of \( \sqrt{2} \). When the experiment is repeated with \( A_\phi \) set equal to the observed value (Fig. 14b), all the cycles are still identical, but the structure of the wind regimes more closely resembles the composite QBO cycle shown in Fig. 3a of Naujokat (1986), with a narrowing of the easterly regimes and a widening of the westerly regimes as they descend. It is barely noticeable in the sections, but in the runs with \( A_\phi \) included, the period of the simulated QBO is approximately 4% longer than in those in which it is not included. When the experiment is repeated with the observed values of \( A_\phi \) and \( A_\mu \), we begin to see some subtle variability from cycle to cycle. Figure 14c shows a short realization from this run in which the phase of the QBO relative to the annual cycle is such that a slowdown occurs in the descent of the easterlies in year 2. The variability from cycle to cycle is not nearly as large as in the observed time–height sections, but the features introduced by including the annual modulation of \( \psi' \) appear to be qualitatively realistic.

Several sets of hindcast experiments were performed. Each set comprised an ensemble of more than 350 forecasts, one starting at each month of the record and ending at the point when the error in \( \psi_{\text{pred}} \) exceeded some threshold, at which time the prediction is said to have failed. For each set of experiments, the ensemble average of the time to failure was calculated. We considered three different criteria for the failure threshold and three different versions of the prediction scheme, as summarized in Table 1. Regardless of the value of failure threshold, the predictions were improved substantially by incorporating the regression coefficients \( A_\phi \) and \( A_\mu \) into the scheme. If we assume that the mean period of the QBO is 28.1 months, a persistence forecast would be in error by 1/32 cycle at the end of 0.875 month and 1/8 cycle (45°) at the end of 3.5 months. Hence, in the dependent dataset it is possible to hindcast the phase of the QBO 7–8 times as far as persistence, based on a knowledge of the mean rate of phase progression alone, and 10–12 times as far as persistence when both \( A_\phi \) and \( A_\mu \) are included in the regression.
scheme. The hindcasts tended to be considerably more skillful during the period 1973–85, when the rate of phase progression of the QBO was rather uniform (Fig. 8c,d) than during the earlier period 1956–72, which exhibited both unusually short and unusually long cycles.

Another potentially useful predictor for very short-range forecasts is the amplitude, $C$, of the QBO, which is negatively correlated with $\psi'$ at a level of 0.4. When it is included as a fifth predictor in the linear regression scheme (1), the fraction of the variance of $\psi'$ accounted for rises to almost 60%. Unfortunately, $C$ is largely independent of $\psi'$, only weakly dependent upon the annual cycle, and has a decorrelation time of only a few months.

If the prediction strategy outlined above were to be applied in practice, one would have to initialize the model two months before the last month for which data were available in order to allow for the calculation of 5-month running means. An alternative approach, which might conceivably be adaptable to short-range forecasting, is to utilize unsmoothed monthly mean data, for which the mean and standard deviation of $\psi'$ (defined on the basis of one-month running differences) are $0.035 \pm 0.024$ cycles per month. The first two harmonics of the annual cycle account for 33% of the variance of $\psi'$, leaving a standard deviation of 0.0197 cycles per month associated with the unexplained variance. The fit to $\psi'$ is not substantially improved by including $\cos \psi$, $\sin \psi$, or $C$ as predictors.
Because of the variability in the degree of regularity of the QBO between various segments of the historical record, formal verification of statistical prediction models on the basis of an independent dataset would require more extensive calculations than we believe are warranted at this point. As an informal test of the stability of the prediction scheme, we evaluated the coefficients in (2) on the basis of segments of the record. The results, shown in Table 2, indicate that the coefficients are not highly sensitive to sampling fluctuations within the record. Hence, we expect that the forecast skill of the scheme should not be not much less than the hindcast skill.

6. Discussion and concluding remarks

Our analysis shows the importance of distinguishing clearly between the rate of phase progression of the QBO and phase itself when speaking of the influence of the annual cycle. The annual march in the rate of phase progression is so strong that it is clearly visible in time–height sections. A distinct preference for certain phases of the QBO to be observed at certain times of year has not been established at this point, and it may be several more decades before this issue can be resolved on the basis of direct statistical analysis of the observational record.

The QBO in zonal wind is believed to be driven by vertically propagating planetary (Kelvin and mixed Rossby–gravity) waves (and possibly gravity waves) in the equatorial waveguide (Holton and Lindzen 1972; Plumb 1977). In agreement with the theory, the amplitude of the planetary waves varies with the phase of the QBO (Maruyama 1968, 1969; Wallace and Kousky 1968). Maruyama (1991) has confirmed these results based on the analysis of a much longer period of record and has demonstrated that the annual cycle modulates the amplitude of mixed Rossby–gravity waves at 70, 50, and 30 hPa, and Kelvin waves at 70 hPa over Singapore. He showed that both types of waves exhibit maximum amplitude around March. The modulation by the annual cycle is more distinct for the mixed Rossby–gravity waves. Dunkerton (1991) obtained similar results for the annual modulation of the amplitude of mixed Rossby–gravity waves at Singapore, but found that the degree and timing of this modulation varies regionally. We have shown that the rate of phase progression of the QBO is largest in April, about a month later than the strongest equatorial wave activity at Singapore. As in Maruyama’s results, we find a qualitatively similar annual march in the descent rates of easterly and westerly wind regimes of the QBO, but there is some indication that the modulation may be slightly stronger during periods of descending easterlies (not shown). Maruyama’s results exhibit what might be regarded as a hint of a semiannual cycle in the amplitude of the mixed Rossby–gravity waves, but it is much less prominent than in the rate of phase progression.

Our method of analysis lends itself to statistical prediction of the gross features of the QBO. Our hindcast experiments indicate that it might be possible to achieve a modest improvement in skill, relative to a model that treats the QBO as a purely periodic phenomenon, by taking into account the annual march in the rate of phase progression. It should be noted that our prediction scheme accounts for less than half the variance of the rate of phase progression in the smoothed data. Evidently other factors modulate the rate of phase progression.

After this paper was submitted for publication, the authors became aware of a related analysis by Fraedrich et al. (1993), which also represents the QBO in terms of a 2D phase space based on the leading EOFs of the vertical structure. They have also made extended EOFs in the height–time domain, which yield a smoother representation of the time dependence.

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| Table 1. Summary of the statistical prediction experiments. Ensemble average of the time interval (months) over which the error in the predicted phase angle \( \psi \) of the QBO expressed in cycles was smaller, in absolute value, that the value given in the column heading (in cycles). Top row: predictions based on the mean rate of phase progression \( C_0 \) alone. Second row: predictions including the dependence of phase progression upon the phase of QBO but not the effect of the annual cycle. Third row: predictions based upon the full regression equation (1) or (2). |
|---|---|---|---|---|
| \( A_0 \) | \( A_4 \) | \( A_6 \) | \( \psi_0 \) |
| 5.8 | 11.3 | 29.4 |
| 7.1 | 15.3 | 40.5 |
| 8.5 | 18.5 | 43.6 |

| Table 2. Coefficients in (2) derived from segments of the data as indicated. The phase angle \( \psi_0 \) is expressed in terms of the calendar date when the phase progression is most rapid. |
|---|---|---|---|---|---|
| \( A_0 \) | \( A_4 \) | \( A_6 \) | \( \psi_0 \) | \( \mu_0 \) |
| Full dataset | 0.0371 | 0.0110 | 0.0091 | -32.9° | 7 April |
| First half | 0.0385 | 0.0108 | 0.0083 | -31.9° | 27 March |
| Second half | 0.0358 | 0.0116 | 0.0102 | -34.2° | 16 April |
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REFERENCES


