Planetary-Scale Circulations in the Presence of Climatological and Wave-Induced Heating

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ABSTRACT

Interaction between the large-scale circulation and the convective pattern is investigated in a coupled system governed by the linearized primitive equations. Convection is represented in terms of two components of heating: A "climatological component" is prescribed stochastically to represent convection that is maintained by fixed distributions of land and sea and SST. An "induced component" is defined in terms of the column-integrated moisture flux convergence to represent convection that is produced through feedback with the circulation. Each component describes the envelope organizing mesoscale convective activity.

As SST on the equator is increased, induced heating amplifies in the gravest zonal wavenumbers at eastward frequencies, where positive feedback offsets dissipation. Under barotropic stratification, a critical SST of 29.5°C results in positive feedback exactly cancelling dissipation in wavenumber 1 for an eastward phase speed of 5 m s⁻¹. The neutral disturbance is dominated by Kelvin structure along the equator and Rossby gyres in the subtropics of each hemisphere. Heating induced by the neutral disturbance is magnified in a neighborhood of the equator, where nearly geostrophic balance in the boundary layer gives way to frictional balance. Moisture convergence induced by the Kelvin and Rossby structures fuels heating that is positively correlated with the temperature anomaly. Induced heating then generates eddy available potential energy, which offsets dissipation in the neutral disturbance. This sympathetic interaction between the circulation and the induced heating is the basis for "fractional wave-CISK," which is distinguished from classical wave-CISK by rendering the gravest zonal dimensions most unstable. Under baroclinic stratification, the coupled system exhibits similar behavior. The critical SST is only 26.5°C for conditions representative of equinox, but in excess of 30°C for conditions representative of solstice. However, the neutral disturbance is then no longer confined to the tropical troposphere. Forced by the induced heating, wave activity radiates poleward into extratropical westerlies and vertically into the stratosphere.

Having the form of an unsteady Walker circulation, the disturbance produced by fractional wave-CISK compares favorably with the observed life cycle of the Madden–Julian oscillation (MJO). SST above the critical value produces an amplifying disturbance in which enhanced convection coincides with upper-tropospheric westerlies and is positively correlated with temperature and surface convergence. Conversely, SST below the critical value produces a decaying disturbance in which enhanced convection coincides with upper-tropospheric easterlies and is nearly in quadrature with temperature and surface convergence. The observed convective anomaly, which propagates across the Eastern Hemisphere at some 5 m s⁻¹, undergoes a similar shift between amplifying and decaying stages of the MJO. During the same transition, enhanced convection remains phase-locked to inviscid convergence above the boundary layer, as does induced heating in the calculations. Fractional wave-CISK also predicts seasonality in accord with that observed. The coupled system is most unstable under equinoctial conditions, for which climatological convection and maximum SST neighbor the equator. While sharing essential features with the MJO in the Eastern Hemisphere, fractional wave-CISK does not explain observed behavior in the Western Hemisphere, where the convective signal is largely absent. Comprised of Kelvin structure with the same frequency, observed behavior in the Western Hemisphere can be understood as a propagating response that is excited in and radiates away from the fluctuation of convection in the Eastern Hemisphere.

1. Introduction

In efforts to explain the so-called Madden–Julian oscillation (MJO: Madden and Julian 1972, 1994), a number of studies have addressed the relationship between convection and fluctuations of the tropical circulation. The MJO is a discrete eastward-propagating disturbance of global dimension that is observed in dynamical properties across most of the Tropics. Similar
behavior has been suggested of the convective pattern, by observational analyses and by large-scale numerical integrations (e.g., Lorenc 1984; Weickmann et al. 1985; Madden 1986; Hayashi and Sumi 1986; Lau et al. 1988; Rui and Wang 1990). While they exhibit coherence between convection and the circulation and share certain features in common with the MJO, these studies differ from one another and from observed dynamical behavior in several key respects. As a result, feedback between the tropical circulation and convection and the mechanism underlying the MJO remain unclear.

A recent observational study (Salby and Hendon 1994; Hendon and Salby 1994; hereafter SH and HS) shows that a discrete signal in convection is confined along the equator to centers of mean or "climatological convection" in the Eastern Hemisphere. A measure of feedback between the circulation and the convective pattern, that discrete signal modulates climatological convection over the equatorial Indian Ocean and western Pacific. Covariance analyses of outgoing longwave radiation (OLR), tropospheric mean temperature, and horizontal motion suggest that frictional convergence onto the equator in the planetary boundary layer plays a key role in the amplification of the disturbance, a hypothesis which has been advanced previously as an explanation for the MJO (Wang and Rui 1990; Salby 1987). Further, the seasonal cycle of the convective signal mirrors the semiannual migration of the intertropical convergence zone (ITCZ) across the equator, amplifying to a maximum shortly before vernal equinox.

The structure and seasonality of the convective signal point to a close relationship between the MJO and climatological convection, which is shaped in large part by SST. We investigate feedback between the circulation and the convective pattern in a coupled system that involves two components of heating. A climatological component, which represents convection that is maintained by fixed properties of the general circulation, is imposed as a second-order stochastic process. A wave-induced component, which represents feedback between the convective pattern and the circulation, is defined in terms of the column-integrated moisture convergence. Section 2 describes the mathematical framework, which is based on the linearized primitive equations on the sphere. Details of the formulation are provided in appendix A. In section 3, feedback between the circulation and convection is examined in a barotropically stratified atmosphere, wherein we explore the roles of SST, boundary-layer friction, and static stability. A simple analytical model, which illustrates salient features of the observed feedback, is developed in appendix B. Influences that mean winds exert on the tropical and global response of the coupled system are then investigated in section 4 in baroclinic basic states containing vertical and latitudinal shear. In section 5, insights drawn from the numerical solutions are tied together with the observational picture to interpret the structure and seasonality of the MJO.

2. Formulation

Disturbances to the large-scale circulation are described in the linearized primitive equations; for a detailed discussion see Salby and Garcia (1987, hereafter SG). The heating field used to force the linearized primitive equations is defined in terms of two components:

\[ q = q_c + q_r, \]  

(1)
a climatological component \( q_c \), which is maintained by external influences like land–sea contrast and SST that are fixed on intraseasonal timescales, and an induced component \( q_r \), which operates sympathetically with the tropical circulation. The climatological component is imposed, at least statistically, so it functions independently of fluctuations in the circulation. In contrast, the induced component is organized entirely by fluctuations in the circulation, so it represents feedback between the motion and the convective pattern. Since fluctuations of climatological heating stimulate dynamical fluctuations, which in turn introduce anomalous moisture convergence, the induced component of heating is coupled indirectly to the climatological component.

a. Climatological heating

The climatological component of heating is represented by the space–time stochastic process

\[ q_c(x, t) = \hat{q}(x, t), \]  

(2)
where \( x = (\lambda, \phi, \xi) \) denotes coordinates of longitude, latitude, and log pressure, and the superscript refers to the \( \delta \)th realization in the stochastic ensemble. A formal treatment of the stochastic equations is provided in appendix A. The random process (2) is specified by its second-order statistics, which define the characteristic space and time scales of heating fluctuations, and by a variance envelope, which confines those fluctuations horizontally (SG). Climatological heating is characterized by the space–time power spectrum

\[ S_q(m, l, \sigma, \xi) = \frac{1}{(2\pi)^2} e^{-\Lambda m^2 + \Phi \left( \frac{\xi}{T} \right)^2 + \Gamma \frac{\sigma^2}{T^2}} q_r^2(\xi), \]  

(3)
where \( m, l, \) and \( \sigma \) denote zonal and meridional wavenumbers and frequency, respectively, and the heating profile \( q_r(\xi) \) is a simple sinusoid that extends from the surface to 2.0 scale heights (15 km). The Gaussian spectrum (3) describes heating variability that is red, with characteristic wavenumber and frequency scales \( 1/\Lambda, 1/\Phi, \) and \( 1/T \) and autocorrelation scales \( 2\Lambda, 2\Phi, \) and \( T/\pi \).

We consider zonally symmetric basic states. Longitudinally uniform, climatological heating statistics then
describe a simple zonally symmetric ITCZ, in which mesoscale convective fluctuations occur. Heating fluctuations are confined meridionally by a Gaussian variance envelope, which may be absorbed into the spectrum (3). Although it differs from observed convection, the idealization of longitudinally uniform mean conditions provides a simple framework for studying feedback and leads to a direct interpretation of observed behavior associated with the MJO.

b. Wave-induced heating

The wave-induced component of heating is represented in terms of the column-integrated moisture flux convergence. If \( q_t \) is the specific latent heating rate for an individual air parcel,

\[
q_t = -l \frac{d\mu}{dt},
\]

where \( l \) is the latent heat of evaporation and \( \mu \) is the water vapor mixing ratio (which is used interchangeably with the specific humidity). In pressure coordinates, the local budget of water vapor may be expressed

\[
\frac{\partial \mu}{\partial t} + \nabla \cdot (\mu \mathbf{v}) = \dot{e} - \dot{c},
\]

with \( \dot{e} \) and \( \dot{c} \) denoting the specific evaporation and condensation rates, respectively. Averaging over the column and over times long compared to the characteristic lifetime of convective complexes leads to

\[
\nabla \cdot (\langle \mu \mathbf{v} \rangle_h) = \frac{8}{p_0} [\dot{P}],
\]

where

\[
\langle \ast \rangle = \frac{1}{p_0} \int_0^{p_0} \ast \, dp
\]

is the column average,

\[
[\ast] = \frac{1}{T} \int_{-T/2}^{T/2} \ast \, dt
\]

is a time average, \( \mathbf{v}_h \) is the anomalous horizontal velocity, \( \dot{P} = (p_0/g) \langle \dot{e} - \dot{c} \rangle \) is proportional to the precipitation rate, and the small tendency in \( \langle \mu \rangle \) is neglected relative to the moisture flux convergence.

Time-averaged latent heating that follows from (6.3) is equivalent to an average over convective complexes, so it describes the envelope organizing mesoscale convective activity. That average is used to define the large-scale induced heating

\[
q_t = Q(p) \cdot \langle q_t \rangle(\lambda, \phi, t),
\]

where the heating profile \( Q(p) \) corresponds to \( q_\xi(\xi) \) in (3) and is normalized,

\[
\langle Q \rangle = 1,
\]

and changes of \( t \) are understood to be much longer than the characteristic lifetime of individual convective complexes. Then

\[
q_t = -lQ(p) \nabla \cdot (\langle \mu \mathbf{v} \rangle_h)
\]

\[
= -lQ(p) \nabla \cdot \int_0^\infty [\mu \mathbf{v}_h] e^{-\xi d\xi}.
\]

The induced heating (8) is fueled by the column-integrated moisture convergence, which is determined by the anomalous moisture \( \mathbf{v}_h \) and the distribution of water vapor mixing ratio

\[
\mu(\phi, \xi) = \mu_0(\phi) \exp\left(-\frac{\xi}{\Delta \xi_{ML}}\right),
\]

where \( \Delta \xi_{ML} \) is the characteristic depth of the moisture layer. The surface mixing ratio in (9) is fixed by the surface relative humidity and the saturation mixing ratio

\[
\mu_0 = RH_0 \mu_s(T_0),
\]

the latter following from the SST, \( T_0 \), through the Claussius–Clapeyron equation. Most of the calculations below rely on a surface relative humidity of \( RH_0 = 0.90 \), a surface temperature distribution

\[
T_0(\phi) = \begin{cases} T_\infty \cos \left[ \frac{\pi (\phi - \phi_0)}{2\phi} \right], & |\phi| < \phi \\ 0, & \text{otherwise} \end{cases}
\]

with \( \phi = 75^\circ \), and a characteristic moisture depth of \( \Delta \xi_{ML} = 0.25 \), which corresponds to an e-folding of \( \mu \) at 775 mb (e.g., Oort 1983). Figure 1 shows the surface temperature and mixing ratio as functions of latitude for \( T_\infty = 30^\circ \). Because of the exponential dependence on temperature of \( \mu_s \), surface moisture decreases sharply with latitude, from 25 g kg\(^{-1}\) over the equator to 8 g kg\(^{-1}\) by 40\(^\circ\).

c. Numerical framework

The numerical scheme used to integrate the linearized primitive equations with random heating relies upon the wavenumber–frequency transform

\[
\mathcal{F}[\ast] = \int_{-\infty}^{\infty} dt e^{i\omega t} \frac{1}{2\pi} \int_{-\pi}^{\pi} d\lambda e^{-i\lambda \ast}[\ast]
\]

to calculate covariance properties of the large-scale response (SG). For each wavenumber–frequency \((m, \sigma)\) component, a boundary value problem in latitude and altitude is solved. In the present application, the resolution has been increased to 3.6\(^\circ\) in latitude and 0.2 scale heights (~1.4 km) in altitude, with the useful domain being global and extending upward from the surface to about six scale heights (~45 km). Above that altitude, an additional four scale heights in com-
combination with strong dissipation serve to absorb any wave activity reflected from the lid at ten scale heights before it can influence behavior below in the useful domain. In addition to convective feedback, the calculation also includes a planetary boundary layer in which Rayleigh friction increases exponentially from a background rate of 1/15 days$^{-1}$ to a maximum at the surface.

Covariance properties of the response, like power spectra and correlation functions, are calculated for planetary wavenumbers between frequencies of $\sigma = \pm 0.10$ cpd/2 with a spectral bandwidth of $\Delta \sigma = 0.002$ cpd/2. If the frequency in (12) is interpreted as complex, simple disturbances correspond to a pole in the complex $\sigma$ plane; a formal development is provided in appendix A. For neutral disturbances (i.e., those of constant amplitude), the pole coincides with the real axis, whereas it lies in the negative (positive) imaginary half-plane for decaying (amplifying) disturbances. Simple disturbances are also marked by a peak in the power spectrum, whose spectral width is inversely proportional to their $\epsilon$-folding time in physical space (e.g., Båth 1974).

Induced heating that feeds back positively onto a frictionally damped disturbance reduces its decay rate by offsetting dissipation. As positive feedback is increased, the $\epsilon$-folding time increases, so the pole in the negative imaginary half-plane approaches the real axis and the peak in spectral power becomes narrower. When positive feedback cancels dissipation exactly, the disturbance is neutral and the $\epsilon$-folding time is infinite. The pole then coincides with the real axis and spectral power is fully concentrated in a delta function. A further increase of positive feedback results in amplification because it more than offsets dissipation. The pole then moves into the positive imaginary half-plane and the power spectral peak again assumes a finite width, now inversely proportional to the $\epsilon$-folding time of amplification.

3. Feedback in a barotropically stratified atmosphere

We consider first the dynamical response to (1) in a resting atmosphere that is barotropically stratified according to the profiles of temperature and Brunt–Väisälä frequency shown in Fig. 2. Below two scale heights, temperature decreases vertically at a constant lapse rate of 6 K/km, which corresponds to weak static stability that is typical of the Tropics. At higher altitudes, the atmosphere approaches isothermal conditions with strong static stability typical of the lower stratosphere.

An envelope of climatological heating, 20° wide in latitude, is imposed along the equator. Heating fluctuations inside this variance envelope are assigned characteristic scales of $\Lambda = 20^\circ, \Phi = 10^\circ$, and $T = 30$ days, which corresponds to a red spectrum with most of the variance at periods longer than $T/2\pi \sim 5$ days. The boundary layer has a depth of 0.12 scale heights ($\sim 1$ km) with friction at the surface having a rate of 1.0 day$^{-1}$.

a. Dependence on SST

We consider first SST that maximizes on the equator ($\phi_0 = 0$), as is representative of equinox. Among properties of the response, the heating $q = q_c + q_i$ best reflects feedback between the circulation and the con-

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1 Disturbances with frequencies greater than 0.10 cpd/2 exert only a limited influence in the troposphere because they propagate quickly in the vertical (e.g., SG, Fig. 26).

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Fig. 1. Distributions of surface temperature (solid) and surface mixing ratio (dashed) for $T_\infty = 30^\circ$C.

Fig. 2. Temperature and Brunt–Väisälä frequency squared for barotropically stratified atmosphere.
vective pattern. For equatorial temperatures $T_{e0}$ below 15°C, the heating spectrum does not differ appreciably from that of climatological heating (3). However, as $T_{e0}$ and surface moisture are increased, so too is large-scale heating power at eastward frequencies. Figure 3 shows the power spectrum of wavenumber-1 heating over the equator for three values of equatorial SST. For $T_{e0} = 26°C$ (dotted), heating power is magnified at eastward periods of 25 days and longer. For $T_{e0} = 28.5°C$ (dashed), heating power has become concentrated about an eastward period of 80 days, where the spectral density has increased by an order of magnitude. At an equatorial SST of 29.505°C (solid), the wavenumber-1 heating spectrum is fully concentrated at an eastward period of about 80 days (a phase speed of approximately 6 m s$^{-1}$), where heating power is now four orders of magnitude greater than that for SST below 15°C.

A “cardinal realization” of the heating process (1) is displayed in Fig. 4 for wavenumber 1. Analogous to the autocovariance of $q$, the cardinal realization provides a snapshot of the random response and is a counterpart of the composite life cycle derived from observations in HS; see SG and appendix A for details. For $T_{e0} = 26°C$ (Fig. 4a), the wavenumber-1 component of heating undergoes a pulse at $t = 0$, which reflects an episode of climatological heating, but little induced heating. For an equatorial SST of 28.5°C (Fig. 4b), the wavenumber-1 component of heating again undergoes a pulse associated with $q$, but now includes an induced component that drifts eastward for about half a cycle before disappearing. The eastward drift and magnified power in Fig. 3 reflect positive feedback that offsets dissipation at eastward frequencies. Only a hint of such feedback is evident for $T_{e0} = 26°C$. At $T_{e0} = 29.505°C$ (Fig. 4c), feedback cancels dissipation completely, so the disturbance propagates eastward without loss of amplitude.

The neutral behavior in Figs. 3 and 4c describes a disturbance that sustains itself through interaction with the convective pattern. Moist static energy near the surface is swept up by the anomalous circulation and converted into eddy available potential energy (EAPE), which is then converted into eddy kinetic energy (EKE) to maintain the disturbance against dissipation. A peak also appears in spectra of dynamical quantities. However, below the critical SST, dynamical spectra are characterized by the damped responses of the Kelvin and Rossby modes (e.g., SG), rather than the positive feedback that prevails in eastward components of the heating spectrum (Fig. 3).

The amplitude and phase structure of the neutral disturbance is shown in the geopotential in Fig. 5. Confined to the tropical troposphere, the disturbance has simple baroclinic structure with a node and nearly 180° phase shift in the middle troposphere. Beneath the node, the neutral disturbance is dominated by Kelvin structure, with a much smaller Rossby component that is accompanied by a continuous meridional phase tilt. Above the node, Kelvin and Rossby structures are equally prominent and are separated by a sharp phase shift of nearly 180°. Beyond the phase shift, a continuous meridional tilt reappears. That phase tilt describes poleward propagation, albeit highly damped, which gives the Rossby component a leaky modal structure. The disturbance also propagates vertically above the heating, but decays sharply in the stratosphere due to thermal dissipation.

The combined Kelvin–Rossby structure, which also appears in observations (Weickmann et al. 1985; Madden 1986; SH), has been attributed to nonlinearity associated with positive-only heating and to CIK feedback that couples these components of the response (Lau and Peng 1987; Hendon 1988). However, observations of OLR reveal negative as well as positive anomalous convection in the Eastern Hemisphere, where the convective signal associated with the MJO is concentrated (HS). In the present calculations, the combined structure results from friction, which causes the Kelvin and Rossby responses to overlap at low frequency (SG; see also Wang and Rui 1990). It is noteworthy that similar structure appears at individual frequencies in the absence of convective feedback, for example, with RH$_{0} = 0$.

The heating structure of the neutral disturbance is shown in Fig. 6. Wave-induced heating, which dominates $q$ at this frequency, extends some 20° to either

2. Owing to its finite spectral bandwidth, (12) requires the behavior to be periodic, so the disturbance in Fig. 4c reappears at negative times (appendix A).
Fig. 4. Cardinal realizations of wavenumber-1 heating over the equator, as a function of longitude and time, for equatorial surface temperatures of (a) $T_\infty = 26^\circ C$, (b) $T_\infty = 28.5^\circ C$, and (c) $T_\infty = 29.505^\circ C$.

Fig. 5. Geopotential height structure for wavenumber $m = 1$ and frequency $\sigma = 0.006$ (cpd/2), corresponding to the neutral disturbance in Figs. 3 and 4c.

constant phase, the magnified heating near the equator projects strongly onto the Kelvin structure. The narrow meridional scale of that heating discriminates to slower components of the Kelvin response, which pushes the most unstable disturbance to longer period (appendix B). The induced heating also overlies the Rossby structure, especially poleward of 5°. However, the sharp meridional phase shift places $q_i$ at higher latitudes nearly in quadrature with the temperature anomaly, as will be seen below.

Induced heating in Fig. 6 is related closely to the structure of the disturbance, which is instrumental in establishing positive feedback between $q_i$ and the circulation. Figure 7 shows the structure on day 80 of the cardinal realization, after decaying components of the

Fig. 6. As for Fig. 5 but for the induced heating.
response have dissipated. At 200 mb (Fig. 7a), the geopotential disturbance is dominated by Kelvin structure over the equator and nearly out-of-phase Rossby structure poleward of 10°. The motion is strong and zonal in a neighborhood of the equator and is in approximate geostrophic balance throughout. A similar picture exists at 850 mb (Fig. 7b), except nearly out of phase with that at 200 mb and dominated by Kelvin structure. Much weaker in the lower troposphere, the Rossby component is associated with a comparatively small meridional phase gradient (cf. Fig. 5) that gives it the appearance of a continuous tilt of the Kelvin structure. However, because it is displaced longitudinally relative to the Kelvin component, the Rossby component leads to zonal convergence along the equator that is shifted eastward of its position in an inviscid Kelvin wave into the low and, as will become clear below, into phase with the temperature anomaly.

The surface pressure anomaly (Fig. 7c) mirrors the structure at 850 mb, as does the motion away from the
equator. However, equatorward of 20°, nearly geostrophic motion is augmented by a strong meridional component that converges onto the equator into low pressure. Introduced by friction, this cross-isobaric component prevails at low latitude where $f$ becomes small and nearly geostrophic balance in the boundary layer gives way to frictional balance. Friction also shifts the zonal component toward low pressure, but much of that displacement results from the Rossby structure, which is present above the boundary layer (Fig. 7b).

Through meridional tilt, the Rossby structure advances the latitudinal gradient and accompanying zonal motion eastward of where it would be for inviscid Kelvin structure alone. That eastward shift follows from Rossby structure with inviscid convergence overlying frictional convergence in the boundary layer structure, which is forced by the corresponding contribution to induced heating (8). In that position, Rossby structure also deepens the surface pressure trough along the equator by steepening the surrounding gradient, which in turn reinforces frictional convergence in the boundary layer, and so forth.

Figure 8 shows a vertical section over the equator on the same day. Except for a slight westward tilt, the geopotential anomaly is simple baroclinic, which implies a nearly barotropic temperature anomaly that maximizes near 150°. Zonal convergence at 850 mb and zonal divergence at 200 mb (Fig. 7) are positioned to the west of but positively correlated with the temperature anomaly. Zonal motion deviates from the simple baroclinic structure near the surface, where friction advances the circulation relative to the geopotential anomaly. Together with rising motion, which is likewise shifted into positive temperature, these components form an unsteady Walker circulation that propagates eastward at about 6 m s⁻¹.

The heating distribution is shown in Fig. 9, along with the moisture flux convergence at the surface and above the boundary layer. Meridional motion in Fig. 7c sharply increases the convergence of surface moisture (Fig. 9c) in a narrow neighborhood of the equator—the same region where $q_i$ is magnified in Fig. 6. An abrupt meridional phase shift, which follows from the Kelvin structure (appendix B), places surface convergence poleward of 5° nearly out of phase with that over the equator. Dominated by meridional convergence, frictional convergence inside the boundary layer is advanced some 30° eastward of nearly inviscid and slightly broader zonal convergence at 850 mb (Fig. 9b). The induced heating (Fig. 9a) resembles its contribution from 850 mb, but positioned slightly to the west. Hence, the column-integrated moisture convergence in (8) is dominated by inviscid convergence above the boundary layer. Coincident with rising motion over the equator, $q_i$ lags surface convergence by some 50°, placing it just west of the temperature anomaly.

![Diagram](image)

**Fig. 8.** Vertical section over the equator on day 80 of the cardinal realization, with geopotential contoured and motion shown as vectors.

Positively correlated with temperature, induced heating along the equator feeds back positively onto the disturbance. The generation of EAPE, $q\theta$ (Fig. 10), is concentrated within 5° of the equator, the same region where $q_i$ is magnified by frictional convergence and attending Rossby structure. Poleward of the sharp phase shift in Fig. 6, $q\theta$ is small because induced heating is nearly in quadrature with the temperature anomaly there. Similar feedback is apparent in Wang and Rui's (1990) 21/2-layer model, but heating induced in that calculation is almost in phase with frictional convergence in the boundary layer.

Even though $q_i$ is dominated by inviscid convergence above the boundary layer, frictional convergence plays a key role in establishing positive feedback by introducing Rossby structure that modifies the column-integrated moisture convergence. The importance of frictional convergence is illustrated in the same wavenumber–frequency component as the neutral disturbance, but for SST below and above the critical value. For $T_{so} = 24°C$ (Fig. 11), the system is subcritical and the response is damped. The geopotential structure (Fig. 11a) is similar to that of the neutral disturbance, but broader with less Rossby structure and meridional tilt near the equator—especially in the lower troposphere where moisture convergence occurs. That structure leads to a shallower surface pressure trough and reduced frictional convergence onto the equator. Heating fueled by that moisture convergence then forces less Rossby structure with inviscid convergence overhead. As a result, the column-integrated moisture convergence is not advanced eastward appreciably, so it lies west of its location in the neutral disturbance and nearly in quadrature with the temperature anomaly (not shown). Induced heating (Fig. 11b) then reflects inviscid convergence of the Kelvin structure alone, varying smoothly between ±20° with

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Fig. 9. Structure on day 80 of the cardinal realization of (a) induced heating, (b) moisture flux convergence at 850 mb, and (c) moisture flux convergence at the surface.

no change of phase. Whereas, in the neutral disturbance, positive heating is advanced eastward into upper-tropospheric westerlies, under these subcritical conditions, positive heating lies to the west in upper-tropospheric easterlies. Generation of EAPE (Fig. 11c) is also broader than that of the neutral disturbance and consistent with inviscid zonal convergence by the Kelvin component.

For $T_0 = 32^\circ$C (Fig. 12), the system is supercritical and the response is amplifying. The geopotential structure (Fig. 12a) is now dominated in the upper troposphere by Rossby structure. Beneath the node, Rossby
and Kelvin components are equally prominent and are separated by a sharp phase shift of about 120°, which has developed from a steepening of the meridional tilt apparent for lower SST. Nearly out of phase with behavior over the equator, that Rossby structure almost coincides with frictionally induced heating associated with the Kelvin structure; cf. Fig. 9c. It also efficiently reinforces the surface pressure trough along the equator, which in turn reinforces frictionally induced heating, and so forth. Forced by that heating, the intensified Rossby structure and meridional phase shift lead to column-integrated moisture convergence along the equator that is advanced farther east of its location in the neutral disturbance to become nearly in phase with the temperature anomaly (not shown). Induced heating (Fig. 12b) now mirrors the pattern of frictional convergence at the surface, which is out of phase between the equator and poleward of 5°. Inherent to frictional convergence associated with the Kelvin structure (appendix B), the same pattern is suggested of $q_i$ in the neutral disturbance (Fig. 6), only weaker. Positive heating is now shifted east of its location in the neutral disturbance, farther into upper-tropospheric westerlies. The meridional phase shift in Fig. 12b brings induced heating poleward of 5° into coincidence with the temperature anomaly of the Rossby structure. Consequently, generation of EAPE (Fig. 12c) is large within 5° of the equator, where it feeds back positively onto the Kelvin structure, as well as immediately poleward, where it feeds back positively onto the Rossby structure. Through mutual reinforcement, the Rossby structure and frictionally induced heating associated with the Kelvin structure result in generation of EAPE that is an order of magnitude greater than under the subcritical conditions of Fig. 11. Higher SST (as would be required to destabilize the system in the presence of stronger dissipation) gives even greater emphasis to the Rossby component and hence to generation of EAPE off the equator.

Figures 11 and 12 reflect limiting conditions under which the induced heating interacts weakly and strongly with the circulation. In the subcritical limit, $q_i$ resembles the zonal convergence of an inviscid Kelvin wave. Nearly in quadrature with the temperature anomaly, induced heating then represents a passive response.
dissipation and allows the disturbance to amplify. In an idealized analytical framework, appendix B illustrates how frictional convergence destabilizes the Kelvin structure and ultimately determines the scale and period of the most unstable disturbance.

The foregoing behavior involves a sympathetic interaction between the Rossby and Kelvin structures, one stimulated by frictional convergence in the boundary layer. Frictional convergence onto the equator, which is controlled by Kelvin structure, is advanced eastward of its location in an inviscid Kelvin wave into positive temperature (appendix B). Heating fueled by that moisture convergence then forces Rossby structure with inviscid convergence immediately overhead. Determined jointly by the Kelvin and Rossby structures, the column-integrated moisture convergence is thus shifted eastward into positive temperature. Induced heating (8) then leads to production of EAPE that offsets dissipation. Elevating the SST increases the heating fueled by frictional convergence (10), which in turn strengthens the attendant Rossby structure. The accompanying inviscid motion advances the column-integrated moisture convergence farther eastward until $q_i$ eventually overlies the pattern of surface convergence, which it mirrors in the supercritical limit shown in Fig. 12. Even though it shifts in relation to surface convergence, induced heating remains coincident with inviscid convergence above the boundary layer throughout the transition from subcritical to supercritical conditions.

This sympathetic interaction between the circulation and heating induced by it is the basis for "frictional wave–CISK." Behavior similar to that above appears in zonal wavenumbers greater than 1. However, these smaller zonal scales remain damped for $T_0 \leq 29.505^\circ C$ and are destabilized only for greater SST. Hence, the coupled system is most unstable at the greatest zonal dimensions. This property distinguishes frictional wave–CISK from classical wave–CISK, wherein the smallest dimensions are the most unstable (e.g., Hayashi 1970). In classical wave–CISK, convection becomes concentrated in an ever narrower region, eventually collapsing to mesoscales or to the smallest dimension resolved in a calculation (e.g., Lau and Peng 1987). By contrast, frictional wave–CISK, which is effective only on planetary dimensions (appendix B), describes a large-scale envelope that modulates mesoscale convection (6.3).

Frictional convergence also explains the selection of eastward frequencies. Dominated by Kelvin structure (e.g., SG), eastward components of the unsteady response have low surface pressure centered on the equator, where the boundary layer is under frictional control. Under these circumstances, frictional convergence is shifted eastward toward low pressure and in phase with the temperature anomaly. At westward frequencies, the situation is reversed. Dominated by Rossby structure, westward components of the unsteady re-

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**Fig. 12.** As in Fig. 11 but for supercritical SST ($T_0 = 32^\circ C$).
sponse have low surface pressure centered off the equator. Since \( f \) is larger there, frictional convergence in the boundary layer is weaker. Further, that pressure pattern leads to surface convergence that is \textit{out of phase} with the temperature anomaly along the equator (e.g., divergent motion in the warm anomaly), so it constitutes negative feedback. Destruction of EAPF (\( q \theta < 0 \)) there damps westward components and thus is stabilizing; see also Wang and Rui (1990). Away from the equator, \( q \theta \) is positive for Rossby structure, but generally smaller than the equatorial contribution, so net feedback tends to be negative.\(^3\)

According to the preceding discussion, frictional wave–CISK is favored by moisture at low latitude. When the SST maximum is displaced off the equator to \( \phi_0 = 10^\circ \), as is representative of summer solstice (e.g., Shea et al. 1990), the response is stable for \( T_{\text{SO}} = 29.505^\circ \text{C} \). Increasing the maximum SST to 31.85\(^\circ \text{C} \) again drives the wavenumber-1 response neutral, but now at an eastward phase speed of only 2.5 \( \text{m s}^{-1} \). Under these conditions, the neutral disturbance at 200 mb (Fig. 13a) is asymmetric and dominated by the Rossby gyre neighboring the SST maximum. An anomaly farther northward reflects meridional propagation, but again highly damped. The Kelvin component is manifested in strong zonal motion along and south of the equator. However, the intensified Rossby gyre dominates motion north of the equator. In the lower troposphere (Figs. 13b,c), the Kelvin component is more prevalent and is clearly visible in anomalies of 850-mb geopotential and surface pressure. Frictional convergence in Fig. 13c occurs both onto the equator and northward near the SST maximum.

Induced heating (Fig. 14a) now possesses two maxima. Even though moisture availability and the anomalous circulation are largest off the equator, the greatest induced heating is still found on the equator. As before, maximum \( q \) is concentrated equatorward of \( 5^\circ \), where strong frictional convergence is promoted by the Kelvin structure and small \( f \). Induced heating is also large off the equator, in a neighborhood of the SST maximum. Supported by frictional convergence, that heating projects positively onto the temperature anomaly of the Rossby structure, which leads to generation of EAPF (not shown) and pushes the neutral response to lower frequency. The enhanced Rossby structure emphasizes meridional propagation off the equator (Fig. 13), which is implied by the latitudinal phase tilt in Fig. 14a. As shown in Fig. 14b, westward-tilting structure that is accentuated from Fig. 9a produces alternating bands of \( q \) that migrate poleward as heating along the equator propagates eastward. Both regions of heating are weaker than when the SST maximum coincides with the equator and with the minimum of \( f \).

Displacing the SST maximum farther poleward results in induced heating along the equator becoming secondary to that off the equator and \( q \), that is weaker yet. The structure of \( q \), then resembles the observed convective signal of the MJO during the northern summer monsoon (Hendon and Liebmann 1994). However, the Kelvin component remains evident in fields of motion and \( q \), even for \( \phi_0 \) as large as \( 15^\circ \). This feature underscores the importance of the Kelvin component to sympathetic interaction between the circulation and the induced heating, even under asymmetric conditions that do not favor strong feedback.

\textit{b. Dependence on boundary-layer friction and static stability}

Other properties of the tropical atmosphere also influence the response. The boundary layer determines the neighborhood about the equator that is under frictional control. This bears directly on the structure of the induced heating and hence on which disturbances it reinforces. Increasing the surface friction to a rate of 2.0 \( \text{day}^{-1} \) leads to behavior similar to that in Figs. 2–10. However, stronger boundary-layer friction pushes the critical SST for wavenumber 1 to \( T_{\text{SO}} = 31.8^\circ \text{C} \). The neutral disturbance (Fig. 15a), which now has an eastward phase speed of 3.5 \( \text{m s}^{-1} \), exhibits the same general character as that in Fig. 5, only broader. In the upper troposphere, the Rossby component now maximizes near \( 30^\circ \) latitude. The accompanying phase tilt describes planetary wave activity that leaks poleward and is refracted upward into the stratosphere, where it is absorbed. In the lower troposphere, the tropical behavior is still dominated by Kelvin structure, but the extratropical behavior contains more Rossby structure.

Stronger surface friction introduces a divergent meridional component in the boundary layer that is now visible at latitudes as high as \( 40^\circ \) (not shown). In combination with the broader Kelvin structure, the attending convergence leads to magnified heating (Fig. 15b) that extends some \( 15^\circ \) to either side of the equator and is nearly \textit{in phase} with the temperature anomaly. This is analogous to the situation in Fig. 12, except \( q \) is comparatively small beyond the abrupt phase shift, which is now positioned near \( 15^\circ \). The broader meridional structure of \( q \) reinforces the Rossby component, which in turn pushes the neutral response to lower frequency. Under supercritical conditions (e.g., as would be required to destabilize the system in the presence of stronger internal damping), generation of EAPF appears off the equator, even though \( q \) remains concentrated on the equator.

Static stability also plays a key role in the convectively reinforced disturbance because the Kelvin component, which is central to feedback near the equator, depends fundamentally on the Brunt–Väisälä fre-
Reducing the lapse rate in Fig. 2 to 4 K/km increases the static stability of the troposphere, which opposes vertical motion and hence feedback between the circulation and the convective pattern. Under these conditions, the critical SST to destabilize wavenumber 1 is increased to $T_{00} = 36.5^\circ C$. Enhanced stability also increases the frequency of the Kelvin response, so the neutral disturbance now propagates eastward at 7.5 m s$^{-1}$. The structure of the neutral disturbance (not shown) resembles that in Figs. 5–10, except for a stronger Kelvin component. As illustrated in appendix B, decreasing the static stability from the typical lapse rate of 6 K/km has just the reverse effect: It destabilizes the response and reduces the frequency of the most unstable disturbance.

4. Response in a baroclinically stratified atmosphere

The response of the coupled system has also been investigated under baroclinic conditions representative of equinox, when climatological convection neighbors the equator, and summer solstice, when climatological
convection is displaced well off the equator. Figure 16 shows the mean temperature and zonal wind for March, which is based on observations of two seasonal harmonics and the QBO in its easterly phase. Under such conditions and with the SST maximum centered on the equator ($\phi_0 = 0$), the most unstable zonal scale is again wavenumber 1, which becomes neutral at an eastward phase speed of about 3.5 m s$^{-1}$. However, the critical SST to destabilize the system is now only 26.5°C.

The lower critical SST than that found under barotropic stratification is not explained by reduced static stability because the lapse rate in the tropical troposphere is virtually identical to that leading to the neutral behavior in Fig. 3. A clue to the diminished stability under equinoctial conditions follows from the structure of the neutral response, which is shown in Fig. 17. In the lower troposphere (Figs. 17b,c), the behavior is very similar to that under barotropic conditions (Figs. 7b,c), except for more pronounced Rossby structure and a more abrupt phase shift that separates it from the Kelvin structure. However, in the upper troposphere (Fig. 17a), the Rossby component now propagates well out of the Tropics. Such behavior is suggested under barotropic conditions (Figs. 5, 7a), but the Rossby component then propagates less than one oscillation before being absorbed. Under baroclinic conditions, the propagating character of the response is clearly visible in a westward-tilted wave train that radiates across both hemispheres. Equatorward of 20°, the structure resembles that under barotropic conditions, with a sharp change of phase separating the Kelvin structure from the Rossby structure. Hence, the entire pattern is again one of a leaky modal structure but now suffering less dissipation away from the heating region.

The more extensive meridional propagation in the upper troposphere follows from westerly shear, which Doppler shifts the Rossby component to higher intrinsic frequency and allows it to radiate farther before being absorbed (e.g., Garcia and Salby 1987). Westerly shear also reduces dissipation inside the source region, where positive feedback leads to generation of EAPE. Reduced damping means that less EAPE must be produced to maintain the neutral response, which in turn reduces the critical SST and the stability of the system.

Fig. 14. For the SST maximum displaced to $\phi_0 = 10^\circ$: (a) as in Fig. 6, (b) as in Fig. 9a.

Fig. 15. As in Figs. 5 and 6 but for a surface friction rate of 2.0 day$^{-1}$.
The Rossby component also propagates vertically. Figure 18 shows the EP flux (weighted by density and cosine latitude) associated with the neutral disturbance. Wave activity propagating poleward from its source region is refracted about the subtropical jets and radiates upward into the stratosphere. Convergence of meridians focuses that planetary wave activity, which magnifies the EP flux and the influence the response can exert on the mean circulation in the stratosphere. Associated with a peak in dynamical spectra (not shown), wave activity there is dissipated thermally and eventually absorbed ahead of its critical line at low latitudes, toward which it is refracted in the upper stratosphere.

Figure 19 shows the temperature and zonal wind for June. Under such conditions and with the SST maximum displaced to $\phi_0 = 10^\circ$, wavenumber 1 becomes neutral at an eastward phase speed of about 7.5 m s$^{-1}$, but not until the SST has been raised to $T_{so} = 30.2^\circ$C. Further, the induced heating is much smaller than under equinoctial conditions. The increased stability and smaller induced heating parallel similar behavior found under barotropic conditions (section 3a), when the SST maximum was displaced off the equator. At low latitude, the neutral structure and induced heating (not shown) resemble those in Figs. 13 and 14 except that, although weaker away from the SST maximum, Rossby gyres now appear on both sides of the equator. The Rossby component propagates into the extratropics of the upper troposphere, especially in the winter hemisphere where strong westerlies prevail.

Like the response under equinoctial conditions, planetary wave activity propagating poleward is refracted upward into the stratosphere. However, solstitial conditions lead to an upward EP flux (Fig. 20a) only in the winter hemisphere, where winds are westerly. In polar regions, wave activity can be seen to propagate to the highest altitude shown before being absorbed. By contrast, wave activity radiating into the summer stratosphere encounters easterlies and is quickly absorbed ahead of its critical line.

Vertical propagation is also influenced by winds in the tropical stratosphere, which are controlled by the phase of the QBO. For the westerly phase of the QBO (Fig. 20a), the EP flux is sharply terminated above the heating, where Kelvin wave activity is Doppler shifted to small intrinsic frequency and suffers strong absorption. By contrast, the easterly phase of the QBO (Fig. 20b) Doppler shifts Kelvin wave activity (downward EP flux) to large intrinsic frequency, which allows it to propagate well into the tropical stratosphere before being refracted toward and absorbed at its critical line. In conventional dynamical properties, like geopotential, this upward propagation of Kelvin wave activity is manifested by an eastward tilt with altitude.

5. Interpretation of observations

The coupled system investigated here does not contain an explicit treatment of convection. Instead, by averaging over space and time scales long compared to individual convective complexes, it focuses upon the organization of convection by the large-scale circulation—the property associated with the convective signal of the MJO (SH). In this light, the coupled system provides a simple framework for investigating interaction between the circulation and the convective pattern, one that lends itself easily to interpretation.

The coupled system reveals several key features of feedback between convection and the large-scale circulation. Frictional convergence sharply increases the organization of moisture in a neighborhood of the equator, where nearly geostrophic balance in the boundary layer gives way to frictional balance. It also advances moisture convergence eastward of its location in an inviscid Kelvin wave, into the positive temperature anomaly. Rossby structure forced by the corresponding heating introduces inviscid convergence above the boundary layer in the same region. Together, these contributions to the column-integrated moisture conver-
gence fuel heating that is positively correlated with temperature. Induced heating then results in generation of EAPE that offsets dissipation and, for sufficiently high SST, allows the disturbance to amplify.

The basic ingredients of frictional wave-CISK are embodied in Wang and Rui’s (1990) 2½-layer model on an equatorial beta plane. It is noteworthy that the frictional wave-CISK solutions derived here are not perfectly trapped, neither meridionally nor vertically. In unbounded spherical geometry, there is no reason they should be. Frictional wave-CISK also appears to be operative in Hayashi and Golder’s (1986) high-resolution GCM integration, which produces eastward propagation that is broadly consistent with observed propagation. Integrations of coarser resolution, which may be unable to resolve narrow frictional convergence about the equator, produce eastward propagation that is far too fast.

As they include the essential physics of feedback, the calculations described here provide a simple framework for interpreting observed behavior associated with the MJO. For reference, Fig. 21 shows the com-
posite structure derived from observations, shortly after anomalous convection attains a maximum. Distinguishing the calculations is the tendency for induced heating to form about the equator. Inherent to frictional wave–CISK, this feature has a direct counterpart in the observed convective signal, which is concentrated equatorward of 10°. The observed convective signal is much smaller poleward of this region (HS) and nearly out of phase with that over the equator (see also Hayashi and Miyahara 1987), which is likewise characteristic of frictional wave–CISK.

Displaced to the west of frictional convergence at the surface, the observed convective signal undergoes a clear phase shift during its life cycle (HS). Enhanced convection is in phase with surface convergence during amplification but shifts westward to become nearly in quadrature with surface convergence during decay. This shift is directly analogous to the one found in section 3a between supercritical and subcritical conditions. Even though it varies in relation to surface convergence, the observed convective signal remains phase-locked to inviscid convergence above the boundary layer, as does induced heating in the calculations.

Confined along the equator in the Eastern Hemisphere, the observed convective anomaly has a zonal scale of wavenumber 2 and a phase speed of approximately 5 m s$^{-1}$. Since the gravest zonal dimensions are the most unstable, frictional wave–CISK predicts a convective disturbance that expands to match the zonal extent of warm SST. This ratio of zonal to latitudinal scales maximizes the contribution to moisture convergence from meridional convergence onto the equator, which in turn maximizes the shift of induced heating into the temperature anomaly and the generation of EAPE. Observed temperatures of 28°C and warmer extend along the equator from the coast of Africa to the date line between December and May (Shea et al. 1990). Spanning the Eastern Hemisphere, this is about the same region and time of year that the observed convective signal is amplified (SH). Thus, frictional wave–CISK predicts a zonal scale of about wavenumber 2, which, in the calculations, is destabilized at an SST of approximately one degree higher than wavenumber 1. Under the conditions of Figs. 2--10, the phase speed of the wavenumber 2 neutral disturbance is about 5 m s$^{-1}$, which happens to correspond to a period of 50 days. Similarly, the neutral response has structure essentially identical to that of wavenumber 1.

Simple baroclinic, the neutral structure predicts behavior that is nearly out of phase between the lower and upper troposphere. The slight westward tilt in Fig. 8 is in accord with that observed (HS; see also Hayashi and Golder 1986). This property of the solutions may be compared to the vertical structure that emerges from

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$^4$ Localization of the convective signal to the eastern hemisphere would smear convective variance over adjacent wavenumbers, as is observed (SH).
other CISK formulations, which, when frictional convergence and the attending Rossby structure are not allowed for, must rely on vertical tilt of the temperature anomaly for positive feedback. Without the mechanism to emphasize narrow meridional scale, unstable behavior in such frameworks also tends to be much faster than the 2.5–7.5 m s\(^{-1}\) predicted by frictional wave–CISK.

The sharp phase shift between the Kelvin component along the equator and the Rossby gyres that flank it likewise has a close parallel in the composite structure in HS, as does the prevalence of the Rossby component in the upper troposphere and the Kelvin component in the lower troposphere (see also Hayashi and Golder 1986). Moreover, positive heating in the calculations shifts from upper-tropospheric westerlies under supercritical conditions westward into upper-tropospheric easterlies under subcritical conditions (section 3a). Following from a repositioning of the Rossby component and weakening of frictional convergence, this westward shift mirrors a similar shift observed of enhanced convection and the Rossby gyres between amplifying and decaying stages of the composite life cycle (HS). The change from supercritical to subcritical conditions in the calculations is also accompanied by a shift of the induced heating from being in phase with surface convergence and temperature to becoming nearly in quadrature with those fields. An analogous shift is observed between amplifying and decaying stages of the composite life cycle. Meridional propagation, which emerges in the calculations under baroclinic conditions, is likewise evident in the composite structure (e.g., in tropospheric-mean temperature in Fig. 21), but that behavior may be masked by other sources of extratropical wave activity.

The stability of the coupled system is minimized when the SST maximum is positioned over the equator, which leads to a critical SST under equinoctial conditions well below the 28°–29°C observed then. By contrast, SST in excess of 30°C is required to destabilize the system under conditions typical of summer solstice, when the SST maximum is displaced well off the equator. Such conditions emphasize the Rossby component and meridional propagation of induced heating, which has been suggested in connection with breaks in the Asian monsoon (e.g., see Madden and Julian 1994). But solstitial conditions lead to induced heating that is greatly reduced from that under equinoctial conditions. These characteristics mirror the seasonality of the observed convective signal, which is sharply amplified near vernal equinox and falls to a minimum near summer solstice (SH).

Asymmetry between the equinoxes can be explained on the basis of the observed seasonality of SST. Between December and May, SST of 28°C and warmer extends along the equator across the entire Eastern Hemisphere. During the other half of the year, the western Indian Ocean is cooler and the return of the SST maximum to the equatorial western Pacific (e.g., from its monsoonal position to the north) lags that in the Indian Ocean (e.g., Shea et al. 1990). Consequently, during autumnal equinox, the SST maximum is situated over the equator in the Indian Ocean but displaced north of the equator in the western Pacific. Reflecting continental influences, this difference in seasonality together with the foregoing calculations parallels the observed convective signal, which is amplified only near vernal equinox in the western Pacific but near both equinoxes in the Indian Ocean (SH).

Because they reflect a balance between positive feedback and damping, the solutions depend fundamentally upon dissipation, which cannot be specified with certainty. For example, the eastward shift of frictional convergence relative to inviscid convergence aloft and the meridional phase tilt in the lower troposphere, both of which influence positive feedback, are sensitive to boundary-layer friction. Similarly, the extratropical response is influenced importantly by background winds. Despite such uncertainty, observed behavior associated with the MJO can be understood and reproduced fairly well with representative values like those used above. The meridional extent of the observed convective sig-
nal and circulation suggests a boundary-layer friction rate of 1.0–2.0 day$^{-1}$. Increased damping implied by these values intensifies the Rossby component in the neutral disturbance and brings surface convergence into phase with the temperature anomaly (e.g., as result under supercritical conditions), as observed (HS). However, tuning the response to a particular choice of parameters for the purpose of making a detailed comparison with observations is not warranted, in view of the sensitivity the solutions have to properties that are apt to vary. Indeed, for wavenumber 2, the range of phase speed, 2.5–7.5 m s$^{-1}$, found under representative conditions corresponds to a fairly wide range of period, 30–120 days. This range may be compared to the natural variability of the observed convective signal, which is distributed over periods of 35–95 days.

The preceding calculations are adequate to explain observed behavior associated with the MJO in the Eastern Hemisphere, where the convective signal is concentrated. However, they do not explain observed behavior in the Western Hemisphere, which is characterized by considerably larger zonal scale and faster phase speed (SH, HS). East of the date line, lower SST is subcritical, so it does not support a convective response. Only the dynamical anomaly is present in the Western Hemisphere and then only the Kelvin component. Rossby gyres, which flank the convective anomaly (Fig. 21), disappear when it reaches the date line and collapses. The Kelvin response that emerges beyond the date line is also concentrated about an eastward period of 50 days, but is chiefly wavenumber 1, so it has a phase speed of approximately 10 m s$^{-1}$. Hence, this equatorial disturbance advances across the Western Hemisphere about twice as fast as the convective anomaly and accompanying Kelvin–Rossby circulation advance across the Eastern Hemisphere.

Since it has the same period as the convective signal, the disturbance in the Western Hemisphere can be un-
nderstood as a propagating response that is excited in the fluctuation of convection in the Eastern Hemisphere (e.g., SG; Hayashi and Miyahara 1987). In this light, the circulation accompanying anomalous convection in the Eastern Hemisphere has the form of a particular solution or "forced response," which is maintained by the heating anomaly, whereas the circulation in the Western Hemisphere has the form of a homogeneous solution or "propagating response," which radiates away from the heating anomaly. The propagating response generated in this manner would have similar frequency characteristics to its forcing, but, because of the localization of heating, would be red in wavenumber (SG). Consequently, the behavior anticipated outside the heating would be predominantly wavenumber 1 with a period of approximately 50 days, and therefore have a phase speed of some 10 m s⁻¹. In addition to the Kelvin response, a westward-propagating Rossby response would also be excited in this manner. While there is some evidence of such behavior in observations (HS), it is small, as would be expected for heating concentrated near the equator.

It is noteworthy that the observed dynamical signal, which prevails in the Western Hemisphere, is significantly narrower in frequency than the convective signal (SH). This suggests that some scale selection such as vertical projection (SG) plays a role in discriminating the propagating response to a narrower band of frequency than its forcing. In fact, scale selection of the response to broadband climatological heating may play a subtle but important role in stimulating the instability. In addition to the anomalous circulation that accompanies the convective signal, the dynamical signal of the MJO is present even when the convective signal is absent (SH). Much weaker, the dynamical signal then may be maintained by broadband heating associated with climatological convection. Since its period is identical to that of the convective signal, that dynamical signal can stimulate interaction between the circulation and the convective pattern when conditions over the Indian Ocean and western Pacific become favorable. In a similar fashion, the amplified Kelvin response that emanates from the fluctuation of convection in the Eastern Hemisphere can reinforce the convective signal upon its return to the Indian Ocean. However, as the observed convective signal becomes uncorrelated with itself after about one cycle (HS), such reinforcement does not appear to be essential.

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APPENDIX A

Cardinal Realization of the Stochastic Response

Formally, the linearized primitive equations forced by stochastic heating may be written

$$\frac{\partial \delta X}{\partial t} + Lx = \delta q,$$  \hspace{1cm} (A1)

where the solution vector x includes components of motion and two thermodynamic variables, L is a spatial operator that includes wave-induced (namely, autonomous) heating, q describes the climatological (i.e., imposed) heating, and the superscript denotes the 9th realization in the stochastic ensemble. Applying the space–time transform (12) and inverting leads to

$$\delta X_m = L_{m}^{-1} \delta Q_m,$$  \hspace{1cm} (A2)

where $\delta X_m$ and $\delta Q_m$ represent the wavenumber–frequency $(m, \sigma)$ components of the stochastic solution and heating, respectively, and the inverse operator $L_{m}^{-1}$ varies with m and $\sigma$. If $\delta Q_m$ is specified (e.g., by its second-order statistics), the solution $\delta X$ is uniquely determined.

A cardinal realization of the solution x is constructed from the inverse of (12)

$$S^{-1}[1] = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{-i\sigma t} \sum_{m} e^{im\eta}[1],$$  \hspace{1cm} (A3)

but with the square root of the power spectrum in place of the random amplitude spectrum $X_m^\sigma$. Analogous to the covariance function, the cardinal realization represents a snapshot of the stochastic response (SG), from which the space–time process can be constructed via random superposition (e.g., Salby 1988). The transform in (A3) admits only behavior that is bounded on the infinite interval $|\tau| < \infty$, which imposes certain properties on the cardinal realization.

Consider the temporal behavior of the complex amplitude of wavenumber m

$$X_m(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_m^\sigma e^{-i\sigma t} d\sigma = \frac{1}{2\pi} \int_{-\infty}^{\infty} L_{m}^{-1} Q_m^\sigma e^{-i\sigma t} d\sigma.$$  \hspace{1cm} (A4)

The Fourier integral in (A4) can be evaluated in the complex $\sigma$ plane. For $t > 0$, the contour can be closed downward ($\sigma_i < 0$), while for $t < 0$ it can be closed upward ($\sigma_i > 0$). Suppose the integrand $L_{m}^{-1} Q_m^\sigma$ has a single pole at $\sigma = \sigma_0$ in the negative imaginary half-plane with residue $Q_0$. Then, by the residue theorem,

$$X_m(t) = \frac{1}{2\pi} 2\pi i Q_0 e^{-i\sigma_0 t} = iQ_0 e^{i\sigma_0 t} e^{-i\sigma_0 t} (t > 0).$$  \hspace{1cm} (A5.1)
For $t < 0$ the behavior is holomorphic, so

$$X_m(t) = 0 \quad (t < 0). \quad (A5.2)$$

Since $\sigma_0 < 0$, the cardinal realization

$$X_m(t) = \begin{cases} 0, & t < 0 \\ iQ_0 e^{\sigma_0 t} e^{-i\omega t}, & t > 0 \end{cases} \quad (A6)$$

describes an oscillation of which the amplitude is zero for $t < 0$, jumps to $iQ_0$ at $t = 0$, and decays exponentially for $t > 0$.

Suppose now $L^{-1}_m Q^*_m$ has a single pole in the positive imaginary half-plane at $\sigma = \sigma_0$. Analysis similar to that above yields the cardinal realization

$$X_m(t) = \begin{cases} iQ_0 e^{\sigma_0 t} e^{-i\omega t}, & t < 0 \\ 0, & t > 0 \end{cases} \quad (A7)$$

Since $\sigma_0 > 0$, (A7) describes an oscillation that amplifies exponentially for $t < 0$, jumps to zero at $t = 0$, and remains zero for $t > 0$.

In each of the above cases, the power spectrum along the real frequency axis is marked by a peak whose width is inversely proportional to the e-folding time in physical space. The longer the e-folding time (e.g., the slower behavior decays or amplifies), the closer the pole is to the real axis and the sharper is the peak in the power spectrum. Conversely, behavior that decays rapidly at $t > 0$ or amplifies rapidly at $t < 0$ corresponds to a pole that is removed from the real axis and a peak in spectral power that is broad, in accord with the reciprocity property of the Fourier transform (see Bahl 1974). For behavior that does not decay or amplify at all (e.g., an oscillation of constant amplitude), the pole coincides with the real axis and spectral power is concentrated at $\sigma_0 = \sigma_0$, in the form of a delta function.

Behavior involving multiple poles follows as a direct extension of the above discussion. Damped oscillations correspond to a pole in the negative imaginary half-plane and appear in the cardinal realization at positive times. Amplifying oscillations correspond to a pole in the positive imaginary half-plane and appear in the cardinal realization at negative times. Each of these forms of behavior corresponds to an initial value problem that has been integrated over a semi-infinite interval, decaying behavior to one integrated forward in time and amplifying behavior to one integrated backward in time. The space–time transform (12) selects those branches of the solution where the behavior remains bounded on the infinite domain $|t| < \infty$.

In practice, the spectrum is computed only at discrete frequencies $\sigma_j = j \Delta \sigma$. As a result, the cardinal realization must be periodic with fundamental period $2\pi / \Delta \sigma$. Periodicity implies that behavior at the positive repeat time $t = +\pi / \Delta \sigma$ continues at the negative repeat time $t = -\pi / \Delta \sigma$. Thus, damped behavior that has not decayed to zero by the positive repeat time $t = +\pi / \Delta \sigma$ will continue at negative times $t > -\pi / \Delta \sigma$ and amplifying behavior that is not zero at the negative repeat time $t = -\pi / \Delta \sigma$ will appear at positive times $t < +\pi / \Delta \sigma$.

**APPENDIX B**

**Stability of Kelvin Waves in the Presence of a Frictional Boundary Layer**

The calculations discussed in this paper underscore the importance of boundary-layer friction in destabilizing equatorial Kelvin waves. Here, we consider in a simplified framework the convergence induced by a Kelvin wave inside a frictional boundary layer and how it influences the stability of the wave. Although strictly valid only in the limit of small induced heating (where the Kelvin mode uncouples from other components of the motion), the simple treatment developed below accounts for the essential physics underlying the more detailed numerical calculations and therefore provides a number of insights into the behavior discussed in the text.

Consider motions in a frictional boundary layer on an equatorial beta plane. The horizontal velocity is governed by the linearized zonal and meridional momentum equations

$$-i\omega u - \beta vy = -ik\phi \quad (B1.1)$$

$$-i\omega v + \beta uy = -i\phi, \quad (B1.2)$$

where $\omega = \sigma + i\nu$, $\sigma$ and $k$ are the frequency and zonal wavenumber, respectively, $\nu$ is the frictional damping rate, and the remaining symbols have their usual meaning. Solving (B1) for the eddy velocities gives

$$u = \frac{(k\omega \phi + \beta y\phi_y)}{(\omega^2 - \beta^2 y^2)} \quad (B2.1)$$

$$v = -\frac{(i\omega \phi_y + ik\beta y\phi)}{(\omega^2 - \beta^2 y^2)}. \quad (B2.2)$$

In the spirit of a simple analysis, we consider only the Kelvin component of the motion. Further, we assume that the geopotential inside the boundary layer is imposed by the inviscid motion aloft, so it has the latitudinal dependence

$$\phi = \exp \left(-\frac{y^2}{2y_0^2}\right), \quad (B3.1)$$

where $y_0$ is related to the inviscid Kelvin wave frequency $\sigma_0$ and the vertical wavenumber $m$ as

$$y_0 = \frac{\sigma_0}{\beta k} \quad (B3.2)$$

$$\sigma_0 = \frac{kN}{m}. \quad (B3.3)$$
Under these circumstances, the motion in the boundary layer is given by

\begin{equation}
1 - \frac{\beta}{\omega k} \left( \frac{y}{y_0} \right)^2 \frac{k \Phi}{\omega} \quad (B4.1)
\end{equation}

and the zonal and meridional components of the divergence are

\begin{equation}
u = \frac{\left( \frac{\nu}{\omega} \right) \left( \frac{y}{y_0} \right)}{1 - \left( \frac{\beta y}{\omega} \right)^2} \frac{\Phi}{\omega y_0}, \quad (B4.2)
\end{equation}

and the divergence in the frictional boundary layer is dictated mainly by the frictional scale (B6.3). For \( \nu = 1 \) day\(^{-1} \), \( Y \) is approximately 5 deg of latitude.

Several properties of the divergence (B.6) are noteworthy:

1) In a neighborhood of the equator \( (y \to 0) \), \( u, v \sim k^2 \phi / \nu \) and \( v_y \sim \phi / (\nu y_0) \), so both the zonal and meridional components of divergence are in phase with the geopotential. (Above the boundary layer \( \nu \to 0 \), and the zonal component of the divergence reverts to its nearly inviscid position, in quadrature with the geopotential field.)

2) The near-equatorial divergence is dominated by the meridional component because \( y_0^2 \gg k^2 \) for planetary-scale waves.

3) The meridional component of the divergence changes sign when \( y \approx Y \). For \( \nu = 1 \) day\(^{-1} \), this occurs approximately 5 degrees from the equator, in agreement with the behavior shown in Fig. 9c.

4) The magnitude of both components of the divergence decreases rapidly away from the equator, becoming small for \( y \gg Y \), again in agreement with Fig. 9c.

Although (B6.1) implies that \( u \), increases without bound for \( k \to \infty \), this property is irrelevant because in that limit \( \sigma \approx kN/m \to \infty \) and friction no longer dominates \( \omega \) in (B5). In fact, the divergence then approaches that of an inviscid Kelvin wave: \( u = ik^2 \phi / \sigma_0, v_y = 0 \). Thus, frictional convergence and the induced heating are effective only for large-scale waves, whose periods are long compared to the frictional damping time in the boundary layer. It is for this reason that frictional wave—CISK does not produce growth rates that increase without bound with increasing wave-number, as is characteristic of classical wave—CISK.

The frictional divergence (B6) is now used to evaluate the stability of the Kelvin wave. Above the boundary layer, where \( \nu = 0 \), the motion is governed by the thermodynamic equation

\begin{equation}
-i(\sigma + i\alpha) \phi_x + N^2 w = \left( \frac{\kappa}{H} \right) q_i, \quad (B7)
\end{equation}

where \( \alpha \) is a Newtonian cooling coefficient, and the wave-induced heating is given by

\begin{equation}
q_i = -i \Delta \zeta_{ML} \mu(y) Q(z) \nabla \cdot v_0, \quad (B8)
\end{equation}

where \( \mu(y) = \mu_0 M(y) \), which is analogous to the parameterization (10) used in the text. Although \( q_i \) is produced by frictional convergence, its vertical profile is presumed to be independent of the Kelvin wave above the boundary layer. This property allows for a phase shift between the layer of frictional convergence and the wave field aloft. For reasons discussed below, it is only through such a phase shift that positive feedback and instability can occur.

We assume the following expression for the vertical profile of \( q_i \):

\begin{equation}
Y = \nu / \beta. \quad (B6.3)
\end{equation}
\[ Q(z) = \begin{cases} \sin(m_i z), & 0 \leq z \leq \pi/m_i \\ 0, & \text{otherwise.} \end{cases} \]  \hspace{1cm} (B9)

Taking the vertical derivative of (B7) and noting that, above the boundary layer, \( w_z \approx -u_z = -ik^2 \phi/\sigma \) leads to

\[ \sigma (\sigma + i \alpha) \phi_z + (kN)^2 \phi = i \sigma \frac{\kappa}{H} \frac{\partial \phi_i}{\partial z}. \]  \hspace{1cm} (B10.1)

From (B6), (B8), and (B9),

\[ \frac{\partial \phi_i}{\partial z} = -i \Delta \xi_{\text{ML}} \mu_0 m_i \cos(m_i z) G(y) \frac{\phi_{\text{ML}}}{\nu y_0^2}, \]  \hspace{1cm} (B10.2)

where \( \phi_{\text{ML}} \) is the geopotential at the top of the boundary layer and

\[ G(y) = \frac{1 - \left( \frac{\nu}{y_0} \right)^2 - \left( \frac{\nu}{y_0} \right)^2 - \left( \frac{\nu}{y_0} \right)^2 \left( \frac{\nu}{y_0} \right)^2}{1 + \left( \frac{\nu}{y_0} \right)^2} \times M(y) \exp \left( -\frac{\nu^2}{2y_0^2} \right) \]  \hspace{1cm} (B10.3)

is the product of the latitudinal dependence of the meridional component of the divergence (B6.2) and the latitudinal dependence of the meridional component of the divergence (B6.2) and the latitudinal dependence of the water vapor mixing ratio, \( M(y) \). The zonal component of the divergence has been neglected in the derivation of (B10) because, as noted previously, the near-equatorial divergence is dominated by the meridional component.

Equation (B10) applies to the Kelvin component of the motion. For periods long compared to the dissipation timescale, the vertical structure satisfying (B10) collapses to the particular solution, which is simple baroclinic and nonzero only at altitudes within the heating (see GS). Therefore, we presume for the present analysis that the solution has the structure

\[ \phi(y, z) = \begin{cases} \phi_0 \cos(m_i z) \exp \left( -\frac{y^2}{2y_0^2} \right), & z < \pi/m_i \\ 0, & \text{elsewhere,} \end{cases} \]  \hspace{1cm} (B11)

as emerges in the numerical behavior (cf. Fig. 4).

Substituting (B11) and integrating over \( y \) reduces (B10) to

\[ \sigma^2 - i(\gamma - \alpha) \sigma - \sigma_0^2 = 0, \]  \hspace{1cm} (B12.1)

where

\[ \gamma = \frac{\kappa \Delta \xi_{\text{ML}} \mu_0 \phi_{\text{ML}}}{\nu m_i y_0^2} P_s \]  \hspace{1cm} (B12.2)

and

\[ P_s = \frac{2}{\sqrt{\pi} y_0} \int_{-\infty}^{\infty} G(y) \exp \left( -\frac{y^2}{2y_0^2} \right) dy \]  \hspace{1cm} (B12.3)

is the latitudinal projection of the induced heating onto the modal structure (B11).

Solving (B12) for the frequency \( \sigma \) gives the following expression:

\[ \sigma = \sigma_0 \left[ 1 - \left( \frac{\delta}{\sigma_0} \right)^2 \right]^{1/2} + i \delta, \]  \hspace{1cm} (B13.1)

where

\[ \delta = \frac{\gamma - \alpha}{2}. \]  \hspace{1cm} (B13.2)

According to (B13), the modified frequency \( \sigma \) has an imaginary part that is proportional to \( \gamma - \alpha \). For the case of no feedback (i.e., in the absence of moisture), \( \gamma = 0 \) and \( \sigma \) lies in the negative imaginary half-plane; namely, the solution decays with time. Increasing \( \gamma \) increases the imaginary component of \( \sigma \) and therefore reduces the decay rate, until for \( \gamma = \alpha \) the solution is neutral. Positive feedback also reduces the real part of the frequency, again in proportion to \( \gamma - \alpha \), which increases the period.

The feedback parameter \( \gamma \) depends on the moisture through the right-hand side of (B12.2); \( \gamma \) also depends on the similarity of the structure to the induced heating through the projection \( P_s \) (B12.3). This projection receives a large positive contribution from a neighborhood of the equator, where \( G(y) \) is positive and the convergence of surface moisture is in phase with the geopotential. Since the vertical structure is simple baroclinic, the induced heating is in phase with the temperature anomaly and produces EAPE. Ultimately, this positive contribution to the feedback parameter \( \gamma \) arises from the phase shift between the frictional convergence inside the boundary layer and the inviscid wave field aloft. The narrow meridional scale of the positive feedback leads to the strongest projection onto narrow components of (B11) and therefore tends to select slow components of the Kelvin wave spectrum. The feedback also depends inversely on the Brunt–Väisälä frequency \( N \), because \( y_0 \sim N \) by (B3). Thus, other things being equal, decreasing \( N \) reduces \( y_0 \), which increases \( \gamma \), destabilizing the solution and lengthening its period.

REFERENCES


