A Numerical Study of the Effects of Differential Cloud Cover on Cold Frontal Structure and Dynamics

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ABSTRACT

The effects of sensible heating and momentum mixing on the low-level structure and dynamics of a two-dimensional cold front are studied with a hydrostatic primitive equation model. Effects of inhomogeneous heating arising from a contrast in low-level cloud cover across the front are emphasized. The relative importance of grid resolution and the choice of method for parameterizing planetary boundary layer (PBL) processes in the model are also examined. Frontal updraft dynamics are studied in terms of the following inquiries: (a) the relative importance of turbulent momentum transport, differential sensible heating, and the reduction in static stability in the heated region ahead of the front; (b) the nature of the interaction between the adiabatic, semigeostrophic frontal circulation and the thermally forced circulation; and (c) possible roles played by dry symmetric instability and density current dynamics. The terms in the frontogenesis and divergence budget equations are computed to determine the relative roles played by the various physical and dynamical processes in generating the frontal secondary circulation system.

A strong, narrow updraft jet forms in the presence of uniform sensible heating across the front. Although the greatest impact on frontogenesis occurs as a response to the reduction in static stability resulting from uniform sensible heating, additional forcing results from the nonlinear interaction between the adiabatic frontal circulation and the thermally forced circulation arising from a cross-front gradient in heating (due to the introduction of an overcast low cloud deck behind the front). The relative importance of inhomogeneous heating, however, increases with the grid resolution and the use of a multilevel treatment in place of bulk mixed-layer PBL models.

Numerical experiments reveal that symmetric instability does not create the updraft jet, despite the development of negative potential vorticity ahead of the surface cold front. Highly unbalanced dynamics and a density current--like "feeder flow"--behind the cold front are strongly indicated in the presence of sensible heating effects. Budget analyses show that the frontogenetical effect of sensible heating is only indirectly important through its strengthening of the confluence (convergence) field. The nonlinear and unbalanced ageostrophic vorticity terms in the divergence budget equation exert the strongest controls on the development of the updraft jet when sensible heating is nonuniform.

These results suggest that differential cloud cover across cold fronts may promote the development of frontal squall lines. Nonhydrostatic models that include explicit prognostic equations for microphysics and use improved parameterization of boundary layer fluxes in the presence of clouds are needed to more fully address this possibility.

1. Introduction

The relative roles played by adiabatic frontogenesis, friction, and diabatic effects in frontal scale contraction are not well understood. Two-dimensional (2D) models of adiabatic frontogenesis predict that in the absence of friction and diffusion processes, the cross-front temperature gradient and absolute vorticity should become infinite in the final stages of frontogenesis (leading to frontal collapse). Scale contraction is accelerated by the self-sharpening process related to the development of the ageostrophic secondary circulation transverse to the front (Eliassen 1962). Two-dimensional primitive equation models of frontogenesis have indicated that extreme scale contraction leading to a density current--like feature at the surface front can develop in the ab-
sence of precipitation when sensible heating occurs ahead of an idealized front. This development appears to be caused by the acceleration of cold air under the influence of the pressure gradient that has been enhanced by the heating (Reeder 1986; Garratt and Physick 1986; Garratt 1988; Howells and Kuo 1988). Likewise, analysis of observations has shown that a narrow (<5 km wide) band of shallow but vigorous, essentially two-dimensional line convection can develop at nonprecipitating fronts and is oftentimes accompanied by intense (1–5 m s⁻¹) updrafts within a microscale region resembling a density current (Berson 1958; Brundidge 1965; Koch 1984; Shapiro 1984; Young and Johnson 1984; Shapiro et al. 1985; Garratt 1988; Dorian et al. 1988). Fair agreement has been found between the observed frontal speed and the theoretical speed of propagation of steady density currents, but Smith and Reeder (1988) point out that there is considerable uncertainty in the choice of predictor parameter values, that frontal systems are often unsteady, and that observed airflow patterns often do not show the cold air being advected toward its leading edge by a “feeder flow” extending back to the source of the cold air. A fundamental issue then is whether and how a density current balance can evolve in dry cold fronts.

Resolution of this issue requires a priori knowledge of the frontal motion in the absence of any density current dynamics. The only frontogenesis theory that provides a prediction for the speed of movement of an idealized cold front is the semigeostrophic Eady wave model, in which the front moves with the steering level of the baroclinic wave (Smith and Reeder 1988). Semigeostrophic (SG) theory provides a basis for understanding how frontogenesis is forced by geostrophic deformation in an adiabatic and inviscid atmosphere (e.g., Hoskins 1971; Hoskins and Bretherton 1972; Blumen 1980). The Hoskins–Bretherton finite amplitude, nonlinear extension to the Eady (1949) linear baroclinic wave problem describes the frontogenetical effects of horizontal shear deformation acting upon the alongfront temperature gradient [(∂υ/∂x)(∂θ/∂y)]. Implicit to this theory is the geostrophic momentum approximation, according to which the parcel acceleration is replaced by dV_p/dt; that is, a balanced flow (small Rossby number) is assumed (Hoskins 1975).

Although certain features predicted by SG theory are qualitatively similar to those seen in analyses of actual cold fronts, the theory contains major shortcomings. These include its inability to account for the existence of an intense, narrow updraft jet near the top of the planetary boundary layer (PBL) and differing PBL structures across the front. Blumen (1980) emphasized the frontogenetical importance of the missing PBL updraft jet, since it acts to tilt isentropes into the crossfront plane and ageostrophic convergence is associated with the jet. The secondary circulation predicted by SG theory is quite broad (the separation between the centers of rising and sinking motion being nearly 1000 km) and weak, unless the models are run out for longer than a few days. Other serious limitations of SG theory are that it implicitly assumes that the timescale over which the flow accelerates is large compared to the inertial timescale and that the accompanying transverse circulation preserves cross-front thermal wind balance.

Largely for these reasons, modelers have attempted to introduce various physical processes not contained in SG theory by using the SG solutions as initial conditions for 2D primitive equation models. For example, the inclusion of diffusion processes prevents infinite cross-frontal gradients from occurring as they do in SG analytical models (Williams 1974). Although a weak updraft jet develops when using simplified boundary layer treatments like the Ekman PBL and bulk aerodynamic formula (Blumen 1980), the introduction of frictional and turbulent momentum fluxes in a high-resolution PBL is required to produce an acceptable updraft jet structure, PBL stability patterns across the front, and the vertical structure of the frictionally induced ageostrophic inflow feeding the updraft jet (Keyser and Anthes 1982). An updraft of 7 cm s⁻¹ was attained after 84 h in their 40-km resolution hydrostatic model initialized with the Hoskins and Bretherton (1972) horizontal shear model (or Eady wave model) of frontogenesis. An identical updraft strength has appeared in simulations with the same model that included explicit precipitation processes (Hsie and Anthes 1984; Hsie et al. 1984; Knight and Hobbs 1988), using grid meshes of 5–40 km. Benard et al. (1992) employed a 5-km resolution nonhydrostatic model that was initialized with the Eady wave solutions. This model produced a narrow cold-frontal rainband associated with an updraft jet of 35 cm s⁻¹ forced by the combined actions of condensational heating and PBL friction. Numerical studies that have investigated the role of latent heating on frontogenesis have shown the temperature gradient at low levels to be relatively unaffected by latent heating (Ross and Orlanski 1978; Baldwin et al. 1984), since horizontal deformation at the surface remains the dominant frontoigenetic factor in 2D studies of frontogenesis, in agreement with observations (Sanders 1955; Ogura and Portis 1982).

By contrast, sensible heating can intensify the crossfrontal temperature gradient at low levels when the fluxes are strongest in the warm air ahead of the front (Pinkerton 1978). The basic dynamics of the thermally forced circulation resulting from diurnal variation of temperature contrast across a front was elucidated by Sun and Ogura (1979). They showed that differential development of the mixed layer across the front (such as results from inhomogeneous cloud cover) drives a solenoidal circulation, whose location and intensity are rather sensitive to the direction of the background wind. Their model atmosphere was characterized by a crossfront geostrophic wind shear (∂υ/∂z) without fronto- genetical forcing by geostrophic deformation. A concentrated updraft with a maximum intensity of 30
cm s\(^{-1}\) developed near the deepest part of the mixed layer, and a compensating downdraft appeared about 60 km behind the updraft in the inversion capping the shallower mixed layer in the cool air. Segal et al. (1986) showed in another nonfrontal 2D study that the imposition of differential cloud cover can result in an updraft similar to that typifying sea breezes. Segal et al. (1993) found that cloud shading in the cold air behind a front will enhance frontogenesis only if the shading is prolonged, the cloud cover is sufficiently overcast, and the ground moisture availability is low. On the other hand, the thermal contrast across the front can be weakened when the strongest sensible heating effects occur behind the front, as demonstrated in a simulation of an Arctic front by Thompson and Burk (1991). Reeder (1986) made use of a zonal shear flow in thermal wind balance to study the change in structure of a shallow front as it passed from cool ocean waters to a heated landmass. This model produced an updraft jet of 24 cm s\(^{-1}\) at 1.2 km above the ground. The front also developed a density current—like feeder flow where the air moved faster than the front in a limited region at low levels behind the front (also see Physick 1988). The phenomenon seen in these simulations, however, was not a true steady density current, since it did not consist of a continuous feeder flow extending through the cold air.

The general purpose of the present paper is to understand the effects of an overcast low cloud deck existing behind an idealized cold front as it passes over a landmass and experiences sensible heating in the warm air ahead of the front. A motivation for this study is the increasing observational evidence that cross-frontal cloud differences can intensify and contract the scale of continental cold fronts (Koch 1984; Dorian et al. 1988; Businger et al. 1991). Our experimental approach provides a systematic investigation of the sensitivity of the results to modeling approaches, the relative importance of static destabilization versus sensible heating gradients in producing the frontal updraft, and the role of density current dynamics and symmetric instability. For example, the sensitivity of the results to grid resolution has not been systematically tested previously. Also, the treatment of the PBL has varied from the bulk aerodynamic method with a simple convective adjustment scheme and neglect of momentum mixing effects (Reeder 1986), to a multilevel PBL model (Pinkerton 1978; Segal et al. 1986; Physick 1988; Segal et al. 1993), to the level 2.5 approximations of Mellor and Yamada (1974), for representation of the turbulent transport of heat and momentum (Sun and Ogura 1979). The method of initializing the model has also varied widely—Physick (1988) relied on an unbalanced initial state, whereas the Sawyer (1956)–Eliassen (1962) equation was employed by Reeder (1986) to specify the frontal secondary circulation. In addition, some studies have arbitrarily specified either a sensible heating or a ground temperature contrast across the front, whereas others have more realistically allowed the fluxes and temperatures to evolve in response to imposed differences in solar irradiance through the surface energy balance. The study by Reeder (1986) was the only one to use budget equations to provide quantitative insight into the relative roles of various processes in the frontogenesis and updraft jet evolution. Our study also attempts to further examine the question of whether the leading edge of cold fronts under the influence of differential sensible heating can develop the local characteristics of a density current, which is a characteristic of thermally forced circulations like the sea breeze (Atkinson 1981; Simpson 1987).

A description of the numerical model, PBL parameterizations, and surface energy budget equation approach is presented in section 2. The frontal initialization and results of sensitivity tests are discussed in section 3. The relative importance of bulk versus high-resolution PBL treatments, grid resolution, and nonhomogeneity in the sensible heating are described in section 4. The dynamics of the prefrontal updraft jet forms the subject matter of section 5, and the results of the budget diagnoses appear in section 6.

2. The numerical model

The two-dimensional version of the hydrostatic, primitive equation Mesoscale Atmospheric Simulation System (MASS) model is used here. Kaplan et al. (1982) describe an earlier version of the 3D model, and recent changes to the model are mentioned in Manobianco et al. (1994). The model has been comprehensively evaluated (Koch 1985; Koch et al. 1985) and has been used in studies ranging from the dynamics of snowstorms and the preconvective environment to impact assessments of satellite and other remote sensing data (e.g., Kaplan et al. 1984; Kocin et al. 1985; Uccellini et al. 1987; Zack and Kaplan 1987; Kaplan and Karyampudi 1992a,b).

a. Model numerics, surface energy budget, and parameter values

A vertically staggered, terrain-following vertical coordinate, unequally spaced to provide sufficient resolution within the PBL, for estimating the vertical distribution of fluxes, is used in the current version of the model. The vertical coordinate is a terrain-following sigma system defined as \( \sigma_p = (p - p_{top})/(p_{dfc} - p_{top}) = (p - p_{top})/\pi \), where the top of the model \( p_{top} = 213 \) mb. Thirty vertical levels are used in all experiments, unless otherwise noted. The effective depths of the four lowest layers in the variable \( \Delta \sigma \) version of the model used herein are 21, 62, 124, and 250 m, whereas in the constant \( \Delta \sigma \) version used in the bulk PBL experiments they are, respectively, 250, 442, 670, and 890 m. The vertical velocity is assumed to vanish (\( \hat{\sigma} = 0 \)) at the
Table 1. MASS model and Hoskins–Bretherton Eady wave parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Mathematical expression</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_f$</td>
<td>top of model</td>
<td></td>
<td>10 km</td>
</tr>
<tr>
<td>$p_0$</td>
<td>background surface pressure</td>
<td></td>
<td>1000 mb</td>
</tr>
<tr>
<td>$z_0$</td>
<td>pseudoheight of the top of the atmosphere</td>
<td>$c_f \theta_0 / g$</td>
<td>28 km</td>
</tr>
<tr>
<td>$p_r$</td>
<td>pressure at top of model</td>
<td>$p_0(1 - h_f/z_0)^{\alpha}$</td>
<td>213 mb</td>
</tr>
<tr>
<td>$\frac{\partial \theta}{\partial y}$</td>
<td>alongfront temperature gradient</td>
<td>$-1 \cdot 10^{-7}$ $s^{-1}$</td>
<td>3.59 $\times$ $10^{-7}$ $s^{-2}$</td>
</tr>
<tr>
<td>$s^2$</td>
<td>constant related to vertical shear of cross-front geostrophic wind</td>
<td>$\frac{g}{\theta_0 \partial_y}$</td>
<td>$1 \times 10^{-4}$ $s^{-1}$</td>
</tr>
<tr>
<td>$f$</td>
<td>Coriolis parameter</td>
<td>$\frac{g}{\theta_0 \partial_y}$</td>
<td>9.92 $\times$ $10^{-3}$ $s^{-2}$</td>
</tr>
<tr>
<td>$N^2$</td>
<td>square of Brunt–Väisälä frequency</td>
<td>$\frac{\pi}{0.8}$ $\frac{\partial f}{f}$</td>
<td>3900 km</td>
</tr>
<tr>
<td>$L_c$</td>
<td>wavelength of maximum growth rate in analytic frontal model</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>$-17.9$ m $s^{-1}$</td>
</tr>
<tr>
<td>$u_{sp}$</td>
<td>surface value of cross-front geostrophic wind relative to analytic front</td>
<td>$s^2 f^{-1}(h_f/2)$</td>
<td>17.9 m $s^{-1}$</td>
</tr>
<tr>
<td>$C$</td>
<td>Eady wave phase speed</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>60 h</td>
</tr>
<tr>
<td>$\tau$</td>
<td>initialization time (HB hours) for inviscid GMASS</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>77 h (1400 UTC 16 April)</td>
</tr>
<tr>
<td>$\tau^d$</td>
<td>initialization time for diabatic GMASS simulations</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>45°N</td>
</tr>
<tr>
<td>$\delta$</td>
<td>latitude</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>40 km or 10 km</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>horizontal grid mesh size</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>60 s or 15 s</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time increment</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>0.01 (40 km run)</td>
</tr>
<tr>
<td>$\tilde{\sigma}$</td>
<td>coefficient for Pepper filter</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>0.03 (10 km run)</td>
</tr>
<tr>
<td>$K_{ib}$</td>
<td>background diffusion coefficient</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>0</td>
</tr>
<tr>
<td>$GW_c$</td>
<td>ground wetness</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>{ }</td>
</tr>
<tr>
<td>$\text{RH}$</td>
<td>atmospheric relative humidity</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>{ }</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>albedo</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>0.40</td>
</tr>
<tr>
<td>$z_0$</td>
<td>roughness length</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>40 cm</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient for bulk PBL</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$l$</td>
<td>Blackadar PBL mixing length</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>100 m</td>
</tr>
<tr>
<td>$R_l$</td>
<td>critical Richardson number in Blackadar scheme</td>
<td>$\frac{s^2 f^{-1}(h_f/2)}{0.8}$</td>
<td>1.00</td>
</tr>
</tbody>
</table>

A surface energy budget formulation based on the Blackadar (1976, 1979) force–restore model is used to compute the equilibrium surface temperature (see Kaplan et al. 1982). The parameterization of incoming solar radiation, which is particularly important to the present study, includes the effects of ground albedo, gaseous scattering and absorption, forward Rayleigh scattering, water vapor absorptivity, and the transmissivity due to cloudiness in each of three (low, middle, and high) model layers, but it ignores diffuse transmission and multiple scattering effects. We employ a dry version of the MASS model that allows for surface sensible heat and momentum fluxes but senses the presence of clouds only through their effects on the surface energy budget. Completely overcast low cloud cover (RH = 89%) is specified throughout a region 1600-km wide behind (west of) the surface position of the front, with clear skies ahead of the front, separated by a 2Δx transition zone (increasing the width of the transition zone to 4Δx had no noticeable impact on the results). This cloud distribution seems justifiable, since high uniformity and persistence typically characterize areas in which the surface sensible heat flux is suppressed due to cloud cover (Segal and Arritt 1992). The empirical cloud fraction enters into the calculation of the cloud radiative effects. The effective shortwave transmissivity in the cloudy region is 0.21, which is the minimum value that can occur for overcast low clouds in the model; that is, our simulations concern only the most extreme contrast of low cloud cover across a cold front. This study makes no attempt to explore a wide

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1 The squared ratio of the depth of the largest vertical disturbance in the model to the horizontal scale of this disturbance (DL) is very small (0.006), assuming the heated boundary layer just ahead of the cold front is that disturbance. The ratio of the mean vertical grid spacing to horizontal grid spacing in the PBL is $\ll 1$ (Pielke 1984).

2 Most mesoscale models represent the fact that significant cloud cover can exist in a grid cell even when the average relative humidity (RH) in the cell is less than 100% by using a cloud fraction–RH relationship to specify the existence of subgrid-scale clouds when the RH exceeds some threshold value below 100%.
range of cloud conditions that can affect the surface energy balance and the resultant temperature contrast across idealized cold fronts; the interested reader should refer to Segal et al. (1993) for the results of such sensitivity tests.

The albedo, roughness length, latitude, and initialization time are set to values representative of conditions in eastern Kansas in mid-April (Table 1), because an original motivation for this study was to test the hypothesis of Koch (1984) that a cold frontal squall line on 16 April 1982 in eastern Kansas was generated by the interaction between the deformation-forced transverse frontal circulation and a thermally forced circulation created by the cross-frontal cloud distribution. The albedo is given a value ($\alpha = 0.40$) that is meant to be representative of typical conditions in the grasslands of eastern Kansas, while at the same time producing in a simple manner a rate of warming ahead of the front that corresponded to the observations on that day without explicitly attempting to model the observed meteorological situation (see Koch and McQueen 1991).

b. Treatment of the planetary boundary layer and surface fluxes

Generally speaking, the parameterization of subgrid boundary layer processes in numerical models is handled either by treating the PBL as a single layer or by resolving it into a number of discrete levels. Approaches for the parameterization of subgrid-scale PBL fluxes use either drag coefficient representations, local exchange coefficients, exchange coefficients derived from profile functions, or explicit equations for the fluxes (Pielke 1984). The first two types of treatments were tested in the MASS model simulations reported here.

1) Generalized similarity theory bulk PBL
(BLMIX)

Bulk formulations assume that surface fluxes of heat and momentum decrease linearly from the surface to zero at the top of the lowest model layer. Hence, this method is only appropriate for models with coarse vertical resolution. Above this level, a local exchange coefficient can be used if there is sufficient vertical resolution within the PBL. The approach used in MASS prior to the late 1980s when a high-resolution PBL scheme became available (see below) is termed “generalized similarity theory” because it matches the surface-layer solution from Monin–Obukhov similarity theory to an outer-layer solution, using the empirical functions from Yamada (1976) for calculating the momentum and heat fluxes (Arya 1977). The outer-layer flow employs similarity theory to calculate the deficits of velocity and potential temperature in terms of the surface friction velocity $u_*$, flux temperature $T_*$, and universal functions. The latter depend upon the non-dimensional height $z/h$ (where $h$ is the height of the boundary layer) and stability parameter $\xi = h/L$ (where $L$ is the Obukhov length), according to whether stable, unstable, or free convection conditions exist. These functions are used with the non-dimensional height $h_*/z_0$, where $z_0$ is the roughness length, to determine the geostrophic drag and heat transfer coefficients $C_U$ and $C_H$, respectively, that in turn determine the sensible heat flux $H$ and momentum flux (related to shearing stress $\tau$) as

$$H = \rho c_p w^* \theta^* = \rho c_p C_U C_H |V_m| (\theta_m - \theta_s),$$

and

$$\tau = -\rho u^* w^* = \rho u^* \theta^* = \rho C_U |V^2_m|.$$

The mean wind velocity and potential temperature in the PBL are given by $V_m$ and $\theta_m$, $\theta_s$ is the potential temperature at the surface, and $w^* \theta^*$ and $u^* w^*$ are proportional to the subgrid-scale vertical fluxes of sensible heat and horizontal momentum. Our approach for treating the generalized similarity PBL differs in two important respects from the description provided by Kaplan et al. (1982). First, dependent variables were adjusted within the unstable PBL using a weighted averaging formula that conserves the vertically integrated variables below the predicted PBL height. The second difference is in the case of free convection—the turbulent structure no longer depends on $u_*$ (Wyggaard et al. 1971), since it was discovered that the free convection heat flux formulations used in Kaplan et al. (1982) produced unreasonably large fluxes. Instead, the expression for the convective scaling velocity $u_{conv}$ was reevaluated using a polynomial curve fit to the universal functions in Businger (1973). After all of the PBL variables, surface fluxes, and height of the PBL (following Stull 1976) are predicted, the boundary layer tendencies are calculated in MASS. The tendency for the potential temperature is expressible in terms of the relationship between the sensible heating rate $Q$ and the sensible heat flux as

$$\frac{d\theta}{dt} = \frac{Q}{c_p} = \frac{\partial H/\partial z}{\rho c_p} = \frac{\partial (w^* \theta^*)}{\partial z}.$$

The winds within the PBL are calculated from the vertical divergence of the shearing stress:

$$\rho \frac{du}{dt} = \frac{\partial \tau}{\partial z} = -\rho \frac{\partial (u^* w^*)}{\partial z}.$$

We discovered that this generalized similarity theory model for the PBL did not yield the expected well-mixed boundary layer under unstable conditions. The MASS model attempts to correct for this deficiency by invoking an explicit mixing routine (BLMIX) that adjusts the profiles of $u$, $v$, and $\theta$ to constant values equaling the mass-weighted mean PBL values after the boundary layer tendencies have been determined. The
resultant profile of the sensible heating rate produced by the BLMIX scheme (the dashed line in Fig. 1) is invariant with height. This behavior is characteristic of observations and second-order closure models of turbulence showing a linear decrease of sensible heat flux with height in the PBL (Deardorff 1974). The generalized similarity theory bulk PBL with BLMIX activated assumes a height-independent shearing stress, thereby prohibiting any low-level jet from developing in the boundary layer.

2) **Bulk PBL with a linear sensible heating profile (LINMX)**

After much experimentation in a fruitless attempt to improve upon this improper heating distribution and arbitrariness, which resulted in some very undesirable side effects as discussed later, a method was arrived at that produced the desired low-level warming adequate to maintain a well-mixed PBL (Pielke 1984) without the problems associated with BLMIX. This procedure specifies a *linear decrease of the rate of sensible heating* with height to zero values at the inversion level $h_i$ and to negative values above $h_i$. A number of experiments were made with this linear weighting approach (LINMX), in which the weighting function was varied and applied either separately or similarly to the sensible heating and shearing stress divergence. The best results were obtained when a constant shearing stress profile was maintained, the surface value of the linear weighting function for the sensible heating was set to a value of 2.5 (solid line in Fig. 1), and a small region of negative flux near the top of the PBL was added to simulate entrainment effects neglected by the BLMIX method. This profile maintains the vertical integral of heating produced by the bulk similarity method, produces a subjectively diagnosed boundary layer depth that agrees best with that predicted from the Stull (1976) method used in MASS, and improves the thermodynamic structure (as demonstrated below). This same weighting scheme, however, could not be applied to the wind stress profile without causing the PBL ahead of the front to become unrealistically stable and an extremely strong vertical wind shear and updraft jet to develop in response to the stronger low-level ageostrophic advection of cool, stable air from the east.

3) **Blackadar high-resolution PBL (BLACK)**

The multilevel Blackadar scheme (Blackadar 1976, 1978, 1979; Zhang and Anthes 1982) was adapted to MASS during this research study in order to address the above deficiencies. When the PBL is stably stratified, this scheme uses first-order closure ($K$ theory) to compute the local exchange coefficients $K = K_m = K_h$ as functions of the vertical wind shear, the bulk Richardson number, and the mixing length. Methods employing such local exchange coefficients are only appropriate when there is sufficient vertical resolution within the PBL to accurately approximate the vertical gradients. The mixing depth and the surface layer in which Monin–Obukhov similarity theory is assumed to apply were both specified to be 10 m deep, and 12 levels were employed in just the lowest 2.1 km of the modeled atmosphere in this study for use with this scheme.

When the atmosphere is statically unstable, an entraining thermal plume model is called (Blackadar 1976, 1978). This condition is defined to exist when $h_i/L < 0$ and a superadiabatic lapse rate occurs in the model surface layer. Countergradient heat fluxes can occur naturally away from the heated surface in this scheme. Furthermore, mixing can occur even when the gradient Richardson number exceeds the critical value for the onset of free convection, because the vertical exchanges take place between the lowest layer and each level of the mixed layer, not just between adjacent layers as in $K$ theory. The heat flux is assumed to decrease linearly with height within the mixed layer, to vanish at the top of this layer, and to become slightly negative in the entrainment zone above the level of zero buoyancy. A weighting function that accounts for the linear decrease of exchange rate with height is used for momentum, but not for the potential temperature distribution (Blackadar 1978). The depth of the PBL is not determined by a predictive equation such as that used in the MASS model similarity theory PBL. Rather, the PBL depth is *diagnosed* based on analysis of the level of zero buoyancy for thermals (Zhang and Anthes 1982).

3. **Frontal initialization and results of sensitivity tests**

The Hoskins and Bretherton (1972, hereafter HB) shear deformation model of frontogenesis is used as the
initial condition for the MASS model, following a procedure similar to that used by Keyser and Anthes (1982) for the initialization of the 2D version of the Pennsylvania State University mesoscale model. This section briefly summarizes the methods used to initialize the model with this baroclinic atmosphere and the effects of differing numerics and background states on the resultant frontal evolution.

a. Initial and boundary conditions from the analytical frontal model

The reduction of the three-dimensional system of model equations to two dimensions \((x, \sigma)\) is accomplished by specifying the \(y\) variation of the horizontal wind components \((u, v)\) and potential temperature \((\theta)\) in a manner consistent with the HB analytic model as

\[
\frac{\partial u}{\partial y} = \frac{\partial v}{\partial z} = 0; \quad \frac{\partial \theta}{\partial y} = -\frac{\theta_s \sigma^2}{g}, \tag{5}
\]

where \(s^2\) is a constant related to the vertical shear of the cross-front geostrophic wind \((\partial u_T/\partial z)\) through the thermal wind relation (Table 1). In effect, these formulations require the addition of longitudinal boundary condition terms to the primitive equations to define the \(y\) variation on \(\sigma\) surfaces of the relevant terms in the predictive equations for \(u, v, T\), and \(\sigma\). The north–south pressure gradient depends on the surface value of the zonal geostrophic wind \((u_T)\) and the constant phase speed \(C\) of the analytic wave (Table 1). In order to minimize motion of the frontal system in the model domain, \((u_T + C)\) is evaluated relative to the motion of the analytic Eady wave; the boundary layer terms involving \(u\) momentum are modified to take this into account.

Eady wave lateral boundary conditions are periodic over the model’s domain of 3900 km, the latter being chosen to match the wavelength of maximum disturbance growth rate \((L_c\) in Table 1). Shortwave transmissivity was gradually blended to its clear sky value well west of the surface front \((x < -1600 \text{ km})\) so as to be consistent with the periodic boundary conditions. This necessarily produced a strong gradient of sensible heat flux near the western boundary in the inhomogeneous heating runs, but the interior solution was unaffected.

The MASS model was initialized with the 60-h \((t' = \text{HB}^{60h})\) HB analytic frontal expressions, similar to the approach of Keyser (1981). All experiments consist of a 24-h simulation beginning from this time. Experiments that include frictional effects consist of a 17-h adiabatic, inviscid simulation (until HB\(^{77b}\)) to produce a balanced initial state, after which time surface and PBL processes are activated in the model for the remaining 7 h of the forecast. The HB\(^{60h}\) and HB\(^{77b}\) initial states are pictured in Fig. 2 (these patterns are grid resolution independent). The intensification during this 17-h time period of the cross-front temperature gradient \(\partial \theta / \partial x\), vorticity of the alongfront flow \(\partial v / \partial x\), and the direct secondary circulation revealed in part by the vertical motion field \(w\) are all a response to the frontogenetical process whereby horizontal shearing deformation acts upon the alongfront temperature gradient. The notation used for the initial time of the diabatic/frictional experiments (HB\(^{77b}\)) is \(t'^{id}\). At this instant solar radiative forcing representative of conditions at 45\(^\circ\)N latitude at 1400 UTC in mid-April was activated. We found this approach to be superior to one wherein PBL processes are activated at an earlier time, because of problems encountered with the transition from stable nocturnal conditions to unstable daytime conditions in the Blackadar scheme. The initial cross-front wind at \(t = t'^{id}\) was adjusted to remove the vertically integrated mass divergence. The initial value of the surface potential temperature for the homogeneous heating runs was determined using an iterative procedure to produce a nearly homogeneous surface sensible heat flux of +85 W m\(^{-2}\) (Fig. 3) to allow for the fact that \(t'^{id}\) occurs nearly 2 h after sunrise. This method was not entirely successful just ahead of the front because of the dependence of heat fluxes on the wind speed [see (1)], which was a minimum in the earth-relative sense at the leading edge of the front. Since the initial inhomogeneity was no more than 20 W m\(^{-2}\), however, and the surface temperature underwent an inconsequential and short (~30 min) adjustment due to this initialization procedure, we believe the method to be quite satisfactory.

b. Summary of experiments

Many numerical experiments were conducted in the course of this investigation to examine the effects of various formulations for the bulk PBL parameterizations, grid resolutions, diffusion, and other factors, but only a small segment is shown in this paper (Table 2). Furthermore, although both 10- and 40-km resolution experiments were performed with a variety of boundary layer physics packages, only the 10-km results will be illustrated, with reference made to the coarser grid results where it is appropriate to stress the importance of grid resolution. The first experiment to be discussed in this paper is the no-physics run (NOPHY), an adiabatic, inviscid run made to provide the initial conditions for the model and to compare with experiments that contained diabatic and/or frictional physics. The FRICT experiments refer to runs in which surface friction and momentum mixing effects were allowed in the absence of sensible heating, so as to make possible a systematic assessment of the relative importance of frictional and diabatic forcing to frontal evolution. Homogeneous sensible heating (HOMHT) and inhomogeneous (differential) sensible heating (DIFHT) experiments were also made with the various PBL parameterization methods. The DIFHT runs using the cloud contrast setup explained earlier produced strong con-
trasts in sensible heating amounting to a transition from 450 W m$^{-2}$ ahead of the front to <50 W m$^{-2}$ immediately behind the front by midday (Fig. 3). By contrast, the HOMHT cases display essentially equal values of sensible heating across the front. Finally, two specialized types of experiments were conducted to assist in the understanding of the frontal updraft dynamics. The first of these, denoted SEABR, was a simulation made to generate a sea breeze in the same kind of environment as that characterizing the Eady wave simulations, in order to isolate the nature of the interaction between the adiabatic frontal circulation and the thermally forced circulation arising from the cloud contrast. Secondly, the possibility that dry symmetric baroclinic instability (SBI) might have contributed to the formation of the prefrontal updraft jet was investigated in a series of experiments elaborated upon in later sections.

c. Sensitivity to numerics and background state

Experiments were performed that compared the effectiveness of explicit diffusion to that of the Pepper filter (Long et al. 1978; Alpert 1981) in controlling noise in the fine-grid ($\Delta x = 10$ km) runs employing PBL physics. The choice of smoothing parameter $\alpha$ used (Table 1) gave a low-pass filter that completely eliminates the $2\Delta x$ wave but entirely passes the $4\Delta x$ wave. We found that unrealistically high values for diffusion would be required otherwise with this grid to achieve similar filtering properties. Another sensitivity experiment tested whether 30 vertical levels were sufficient to resolve the frontal dynamics in the presence of differential heating without numerically generating gravity waves (e.g., Pecnick and Keyser 1989; Persson and Warner 1991). An increase to 60 levels did not produce any discernible improvement over the use of 30 levels, despite the prediction that 60 were required to eliminate gravity waves due to improper vertical discretization on a 10-km mesh, probably because of the wave suppression effects of the Euler-backward implicit diffusion and the Pepper filter. We also examined the effect of increasing the static stability $N$ in order to keep the growth of the PBL to an absolute minimum in the cloudy air behind the front. Although this was achieved to a certain extent, a doubling of $N$ in the Eady wave framework resulted in a drastic loss of secondary
efficient in the LINMX scheme. In summary, the results presented here are those that produced the most realistic and noiseless thermal and low-level structures.

4. The role of diabatic heating in frontogenesis and its dependence upon boundary layer formulations

In this section, we examine the effects of sensible heating on the structure of the low-level frontal thermal and momentum fields, and the sensitivity of these results to the level of sophistication of the PBL physics and to the grid resolution. The relative importance of friction, uniform sensible heating, and cross-frontal difference in heating is evaluated.

a. Sensible heating effects on the low-level frontal fields

The effects of differential sensible heating on the structure and intensity of the low-level front are best demonstrated in the DIFHT/BLACK run. Examination of the model fields at 2100 UTC (HB84) reveals that heating of the PBL ahead of the front has resulted in a deep and narrow PBL plume (Fig. 4b). The depth of the neutral to unstably stratified PBL varies from over 4 km ahead of the front to only approximately 1 km in the cloudy air behind the front. The exaggerated depth of the prefrontal PBL is primarily the result of the weak prefrontal static stability, whereas its narrowness owes to effects of the periodic boundary conditions. Other thermal structures in the DIFHT run that have no counterpart in the NOPHY run (Fig. 4a) are the superadiabatic layer near the surface both behind and ahead of the front, and the restraining inversion at the top of the PBL. These features are generated primarily by the viscous effects related to the action of the vertical shear of the cross-front ageostrophic wind upon the cross-front temperature gradient \( \partial u_{avg}/\partial x \partial \theta/\partial x \) > 0, as explained by Keyser and Anthes (1982). The \( \partial \theta/\partial x \)

![Fig. 3. Space–time evolution of surface sensible heat flux \( H_0 \) (W m\(^{-2}\)) in homogeneous and inhomogeneous heating 10-km resolution experiments. The initial state for the homogeneous case attempts to produce a uniform sensible heating rate of 85 W m\(^{-2}\) across the front. The initial state for the inhomogeneous case has the same value for the heat flux ahead of the front but negative heat fluxes behind the front (\( x < 200 \) km) resulting from the imposed cloud cover. The cross-front contrast in sensible heating increases dramatically with time for the inhomogeneous case.](image)

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<tr>
<th>Experiment designator</th>
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<td>Homogeneous</td>
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<td>SEABR (sea-breeze study)</td>
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field is strongly enhanced in the presence of differential sensible heating (compare Figs. 4g and 4h).

The rising branch of the secondary circulation is also much stronger in the DIFHT run and consists of a narrow 22 cm s⁻¹ frontal updraft jet, compared to a broad updraft of only 5 cm s⁻¹ in the NOPHY case (compare Figs. 4c and 4d). This intense updraft is quite similar in strength and appearance to the one simulated by Reeder (1986) in his study of the effects of differential sensible heating on a shallow front, though the core of his updraft did not extend quite as high as in DIFHT. This difference is largely explainable as being due to the considerably smaller value of sensible heating (272 W m⁻²) used by him.

Yet another effect of PBL physics is the creation of low-level jets (LLJs) on both sides of the front (Fig. 4f). Notice that the alongfront wind speed within and above the jet core is similar in strength to the speed at the corresponding height in NOPHY but that friction has retarded the winds below this level, resulting in the formation of the LLJ. Diabatic heating has no effect on the strength of this LLJ, since the strength of the LLJ in DIFHT is identical to that in the FRICT/BLACK run.

On the other hand, differential sensible heating has profound effects on the alongfront wind within the core of the updraft plume (x = 400 km). Here a small-scale increase in the alongfront geostrophic wind 𝑣ₓ (Fig. 5a) results in a negative anomaly in the ageostrophic wind 𝑣ₘ (Fig. 5c). These features are forced by noneageostrophic dynamics associated with an unbalanced cross-front pressure gradient force ∂𝜙/∂x. Consider the SG u-momentum equation written in the form

$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \left(f - \frac{\partial u_x}{\partial y}\right) v_{ag} - u_{ag} \frac{\partial u_x}{\partial x} - w \frac{\partial u_x}{\partial z}. \quad (6)$$

This equation reduces to the following simple form in the case of the two-dimensional Eady wave [in which 𝑣ₓ = 𝑣ₓ(𝑧) and ∂𝑢ₓ/∂𝑡 = ∂𝑢ₓ/∂x = ∂𝑢ₓ/∂y = 0]:

$$fv_{ag} = f_v - \frac{\partial \phi}{\partial x} = w \frac{\partial u_x}{\partial z}. \quad (7)$$

Nonageostrophic parcel accelerations in the cross-front direction within the 2D Eady wave system are forced by several mechanisms, including the neglected ∂𝜙/∂x term, as follows:

$$\frac{du_{ag}}{dt} = \frac{du}{dt} - \frac{du}{dx} = f_v' - \frac{\partial \phi'}{\partial x} - w' \frac{\partial u_x}{\partial z}. \quad (8)$$

Accordingly, ageostrophic cross-frontal divergence is produced in the region of unbalanced ageostrophic vorticity (f∂v'ₘ/∂x), since

$$\frac{d}{dt} \left(\frac{\partial u_{ag}}{\partial x}\right) = f \frac{\partial v'_{ag}}{\partial x} - \frac{\partial w'}{\partial x} \frac{\partial u_x}{\partial z}. \quad (9)$$

and the last term can only contribute to the production of convergence in this same region, which appears to the left of the updraft jet (Fig. 4d). Thus, there are three forcing mechanisms for the 𝑣ₘ patterns in Figs. 5c and 5d. First, the pattern of near-surface 𝑣ₘ > 0 behind the front and 𝑣ₘ < 0 ahead of the front represents subageostrophic winds caused by surface frictional retardation (equally evident in the FRICT and DIFHT runs). Second, semageostrophic forcing produces the positive 𝑣ₘ anomaly at low levels within the frontal updraft in both the DIFHT and FRICT runs according to Eq. (7), since ∂𝑢ₓ/∂z is a positive constant (s² in Table 1). A similar feature is apparent in the frontal model of Xu (1990), who emphasizes that an alongfront ageostrophic jet must develop in association with a frontal updraft to maintain cross-front geostrophy. Importantly, the third mechanism, which is clearly noneageostrophic, appears only in the DIFHT run as the negative 𝑣ₘ anomaly (thus, f∂v'ₘ/∂x > 0). This feature forces strong divergence in the 1.5–4.0-km layer directly above the frictionally induced low-level convergence, so it strongly contributes to the development of the updraft jet. Since 𝑓v'ₘ = 𝑓_v' - ∂𝜙/∂x, the negative anomaly is related to a combination of an unbalanced alongfront wind and low pressure region (the latter being a hydrostatic consequence of the diabatic sensible heating) within the updraft plume at x = 400 km. A similar ageostrophic feature was mentioned by Reeder (1986), though he did not discuss the relative roles of friction, heating, and differential heating, nor their vertical structures. Actually, this unbalanced ageostrophic vorticity forcing explains only part of the updraft dynamics, as discussed in section 6.

b. Importance of PBL formulation and grid resolution

A comparison between the generalized similarity theory, with and without the sensible heating profile

---

Fig. 4. Comparison of 10-km resolution model 7-h predictions for 2100 UTC (HB₄₄) from NOPHY (left) and DIFHT/BLACK runs (right): (a, b) ageostrophic front-relative flow [note scale of arrows differs slightly between left and right panels (see bottom of figure) and vertical motions are exaggerated] and θ (contour interval 2 K), with diagnosed PBL height h depicted by thick curve in (b); (c, d) vertical velocity w (contour interval 2 cm s⁻¹); (e, f) alongfront wind 𝑣ₓ [contour interval 5 m s⁻¹, with southerly (northerly) winds solid (dashed)]; and (g, h) cross-front temperature gradient ∂𝓔/∂x (contour interval of 1 K/100 km). Surface position of cold front for NOPHY case is at x = 550 km and at x = 400 km for DIFHT/BLACK case. Note that only the lowest 6 km and the centermost 1000-km horizontal region of model domain is shown.
modification (HOMHT/BLMIX and DIFHT/LINMX, respectively) and the high-resolution Blackadar treatments of the PBL is made in Fig. 6. It is readily apparent that all three simulations produce a rather similar updraft jet at the nose of the low-level front (x = 400 km), but that this feature is somewhat weaker in the HOMHT/BLMIX case. It is shown in Table 3 that the differential heating run using the Blackadar PBL and a 10-km grid produces an updraft jet significantly stronger than that for the homogeneous heating case but that the LINMX and BLMIX runs produce nearly identical updrafts for these different sensible heating configurations. Thus, the weaker updraft seen for HOMHT/BLMIX is the result of the choice for the PBL parameterization, rather than the sensible heating being homogeneous.

Anthes et al. (1980) showed that mixed-layer models are not well suited to modeling situations involving differential sensible heating. Multilevel treatments (like the Blackadar scheme) are needed because sig-
FIG. 6. Comparison of sensible heating simulations at 2100 UTC using three different treatments of the planetary boundary layer in the MASS model: HOMHT/BLMX (left), DIFHT/LINMX (middle), and DIFHT/BLACK (right). Top row depicts vertical velocity w (contour interval 2 cm s⁻¹); middle row depicts ageostrophic front-relative flow, PBL height h, and θ (contour interval 2 K); and bottom row depicts alongfront wind v [contour interval 5 m s⁻¹, with southerly (northerly) winds solid (dashed)l and locations of jets shown by J. Broad sense of circulation is depicted by bold arrows. Surface position of cold front in all runs is x = 400 km. Centermost 2000 km of model domain is presented.

Significant vertical gradients of mass and momentum exist within the PBL; small errors in the temperature structure can create large errors in the mixed-layer wind due to accelerations resulting from pressure gradient forces. They suggest that the erroneous accelerations develop because bulk, mixed-layer models are unable to represent perturbations associated with the tilted nature of the potential temperature pattern just above the mixed layer when patterns of divergence and convergence exist within the PBL. Likewise, our results suggest that specified mixing of the temperature and winds is the source of considerable noise, probably because it disrupts any kind of mass–momentum balance in the model. Notice that multiple updrafts appear on both sides of the primary updraft in BLMX, and that these noisy features are a quarter-wavelength out of phase with wiggles in the isentropes defining the top of the PBL, highly suggestive of internal gravity waves. This noise is not present in the LINMX and BLACK results. Clearly, the bulk-type treatment is not the real culprit.
since the gravity waves are not present in LINMX. The noise disappeared when we prevented calling the BLMIX mixing scheme in the similarity theory code. Unfortunately, this yielded a stable boundary layer in the diabatically heated air ahead of the front. Forced mixing of temperature but not wind variables reduced the noise but did not eliminate it. Many different experiments were conducted in order to arrive at an acceptable result, but without success. Thus, the LINMX technique was developed with the objective of devising a scheme for use in the bulk similarity theory framework that would provide acceptably small noise, yield a smoother transition at the top of the PBL while maintaining a well-mixed profile within the PBL, and still produce realistic vertical wind shear. LINMX offers distinct advantages over BLMIX, but it fails to produce a realistic structure for the low-level jet (Fig. 6). By contrast, the Blackadar PBL scheme produces realistic LLJ features.

The relative influences of PBL physics, PBL parameterization, and grid resolution on the intensity of the prefrontal updraft jet are summarized in Table 3. The grid size has no effect on the strength of the updraft and associated secondary frontal circulation when adiabatic, inviscid conditions exist (NOPY). A slightly stronger updraft results from increasing the grid resolution when friction and sensible heating are included, except in the DIFHT/BLACK case, where a substantial increase from 14 to 22 cm s\(^{-1}\) is realized (a 57% increase). These results suggest that, just as in the well-known case of sea-breeze modeling (Atkinson 1981), the intensity of the thermally forced circulation is quite sensitive to the grid resolution. It follows that further reduction of the grid size should result in an even stronger frontal circulation in the case of differential sensible heating. Unfortunately, there is no way to test this hypothesis at grid sizes smaller than 10 km without using a nonhydrostatic model. Despite the fact that the introduction of differential heating results in the strongest frontal circulation, the greatest relative impact is felt when uniform sensible heating is introduced. Nevertheless, we emphasize that the differences between the homogeneous and inhomogeneous heating situations would be expected to be much larger at grid resolutions finer than 10 km. In addition, it is quite plausible that oversmoothing from the Euler-backward scheme and the Pepper filter, and perhaps other limitations inherent to the Blackadar PBL scheme, can help to explain the similarity between the updraft intensities in the homogeneous and differential heating runs.

5. The dynamics of the prefrontal updraft

In order to understand why sensible heating has such a strong effect on the frontal updraft, it is helpful to partition the total circulation into its components and examine the important forcing mechanisms. Several important questions are considered in this context. First, we consider whether this resultant circulation is any stronger than one describing a sea breeze with the same kind of atmospheric stability and wind shear conditions as those used in the Eady wave experiments. We then seek to understand whether the interaction between the adiabatic, inviscid frontal circulation and the thermally forced circulation is a linear or nonlinear one. Last, we address whether dry symmetric instability plays an important role in the development of the frontal updraft jet and whether the secondary circulation in any of the cases resembles a density current.

The sea-breeze experiment had the same cloud contrast as before for the thermal forcing function, but zero-gradient conditions were specified on the lateral boundaries, the horizontal model domain was reduced to 1000 km, and the top of the model was lowered to \(p_{\text{top}} = 418\) mb for economic reasons. Frontogenetical forcing \((\partial \theta/\partial x)(\partial \theta/\partial y)\), the associated secondary circulation, and cross-frontal gradients were eliminated with use of the following set of values for the HB\(^{77\text{h}}\) initialization:

\[
\begin{align*}
\partial \theta/\partial x &= \partial \theta/\partial y = 0, \\
\partial \phi/\partial x &= \partial \phi/\partial y = 0, \\
u(z) &= u_0(z). \\
\end{align*}
\]
Since \( u_0(z) \) in the low levels is zero and develops an increasingly westerly component with height in the earth-relative sense, it is directed from the cloudy area to the heated area throughout the greater depth of the boundary layer. Thus, this situation falls into the category of "onshore-positive shear" (Atkinson 1981). Onshore winds weaken the horizontal temperature and pressure gradients (Estoque 1962), and positive shear further decreases the strength of the sea breeze (Magata 1965). Therefore, it is anticipated that a rather weak sea-breeze circulation should develop in our simulations. The strong stratification and rather coarse (10 km) grid resolution used in our experiments also are not conducive to obtaining a strong sea breeze. Absence of a jet in the \( v \) field has the effect of further weakening the sea-breeze circulation, because sensible heating (which depends on wind speed) is reduced relative to the Eady wave simulations. Hence, we lowered the surface albedo to produce the correct amount of sensible heating. As expected, the SEABR run (Fig. 7) exhibits a weak updraft (6 cm s\(^{-1}\)) centered about the cloud transition region. We deduce from this experiment that the strong circulation seen in the differentially heated frontal simulation is not fully explained by a simple thermally direct circulation resulting from the cloud contrast.

a. The nature of the interaction between the adiabatic frontal circulation and the thermally forced circulation

Time series of the domain maximum relative vorticity \( \partial \theta / \partial x \), cross-frontal temperature gradient \( \partial \theta / \partial x \), and convergence in the frontal plane \( \partial u / \partial x \) are compared for the adiabatic, inviscid (NOPHY); adiabatic, viscous (FRICHT/BLACK); homogeneous heating (HOMHT/BLACK); differential heating (DIFHT/BLACK); and sea-breeze (SEABR) simulations in Fig. 8. The maxima in all cases occur in the lowest levels of the model where the frontogenetical process is strongest. Notice first of all that the viscous simulation produces a vorticity field weaker than the NOPHY case. This behavior is consistent with that observed by Keyser and Anthes (1982), who explain that frictional dissipation exceeds vorticity generation as the \( v \) momentum and corresponding gradient \( \partial v / \partial x \) are gradually reduced near the surface by friction. The HOMHT and DIFHT cases display considerably larger vorticity during the afternoon hours, when they are each roughly equal to the sum of the vorticity values from the SEABR and NOPHY runs. Thus, the frontal vorticity in the presence of sensible heating is apparently the result of a linear interaction between the adiabatic, inviscid and the thermally forced vorticity fields. A similar comparison between the \( \partial \theta / \partial x \) time series shows that the temperature gradient in the DIFHT case equals the sum of \( \partial \theta / \partial x \) from the NOPHY and SEABR cases through 2000 UTC, again suggesting an essentially linear interaction between the adiabatic, inviscid and the thermally forced solutions. The differential heating case consistently exhibits the strongest thermal contrast of all the experiments but primarily in the first few hours of the simulation, after which it differs surprisingly little from that of the homogeneous heating case.

By contrast, the convergence time series indicate that a highly nonlinear interaction between the NOPHY and the SEABR solutions appears to explain the development of the frontal circulation in the differential heating case after 1630 UTC.\(^4\) That frontal vorticity in DIFHT can be explained as a linear combination of the adiabatic, inviscid and the thermally forced solutions might seem inconsistent with the finding that frontal convergence in DIFHT appears to be the result of a nonlinear interaction between these same solutions.

---

\(^4\) The reason why HOMHT and DIFHT exhibit remarkably similar convergence time series when the updraft intensities are so dissimilar (Table 3) is related to effects of the strong cross-frontal pressure gradient that develops in DIFHT (section 4a). This mesoscale forced by differential sensible heating causes the vertical motion \( \epsilon \) to be stronger, even though \( \partial w / \partial x \) may be the same, due to the underlined terms in the \( \epsilon \) equation:

\[
\epsilon = -\frac{1}{\rho} \left[ (1 - \sigma) \partial \theta / \partial x - \int \left( \partial \Pi / \partial x + \partial \Pi / \partial y \right) dy \right].
\]
\[
\frac{d}{dt} (f + \zeta) = (f + \zeta) \frac{\partial u}{\partial x} + \eta \frac{\partial w}{\partial x} + K_w \frac{\partial^2 \zeta}{\partial z^2},
\]

where \( \zeta = \partial v / \partial x, \eta = -\partial u / \partial z, \) and \( K_w \) is the turbulent exchange coefficient for horizontal momentum. This expression shows that the Lagrangian rate of change of absolute vorticity about a vertical axis is governed by a balance between vortex stretching, tilting, and frictional torques. When only the stretching term is considered, we see that

\[
\log(f + \zeta) = -\int \frac{\partial u}{\partial x} \, dt.
\]

If this Lagrangian time integral of convergence were the only operative vorticity dynamics, our explanation of the behavior of the vorticity and convergence fields in DIFHT would indeed be confounding. Tilting and frictional effects are, however, quite important in the environs of the frontal updraft and low-level jet (vorticity is strongest at the surface cold front). Consideration of the \( \eta \) vorticity equation reveals that the dynamics of the \( \zeta \) and \( \eta \) vorticity components are highly coupled. It is for this reason that convergence and vorticity dynamics are so different.

Thus far, we have seen that the development of the strong updraft within the heated PBL plume is a nonlinear product of the interaction between the adiabatic frontal circulation and the inland sea-breeze circulation produced by the cloud contrast across the front. Data shown in Table 3 suggest that the circulation in DIFHT is due to static destabilization and is also a response to the direct forcing provided by the differential diabatic heating. The first of these two factors appears as the static stability coefficient multiplying the first term on the left-hand (response) side of the Sawyer–Eliassen (S–E) equation, whereas the second factor appears as the diabatic heating gradient term on the right-hand (forcing) side of the equation (the last term):

\[
\left( -\frac{\gamma}{\partial p} \frac{\partial^2 \psi}{\partial x^2} + \frac{2}{\partial p} \frac{\partial^2 \psi}{\partial x \partial p} - \frac{\partial m}{\partial x} \frac{\partial^2 \psi}{\partial p^2} \right) = -2 J_s (u_s, v_s) \frac{\partial F}{\partial p} - \frac{\gamma}{C_p} \frac{\partial Q}{\partial x}.
\]

The ageostrophic streamfunction is denoted by \( \psi \), the absolute momentum \( m = v_s - fx \) is related to the cross-front temperature gradient through the thermal wind relation \( \partial m / \partial p = \gamma \partial \theta / \partial x \), \( \gamma \) is a nondimensionalized inverse pressure, \( J_s \) is the Jacobian expression for the geostrophic deformation frontogenetical forcing terms, and \( \partial F / \partial p \) is the frictional forcing term. The relative magnitudes and distributions of the coefficients on the response side of the equation, which represent the effects of static stability, baroclinicity, and inertial stability, determine the eccentricity and tilt of the cir-
culation (Keyser and Shapiro 1986). In particular, small static stability relative to the inertial stability, or, equivalently, low values of geostrophic potential vorticity,

$$GPV = \frac{\partial m}{\partial x} \frac{\partial \theta}{\partial p} - \gamma^{-1} \left( \frac{\partial m}{\partial p} \right)^2,$$  \hspace{1cm} (14)

favor highly elliptical circulations with strong vertical motions. Since the static stability is nearly zero in the DIFHT run, the S–E equation predicts that a cross-frontal gradient of sensible heating should produce a strong ageostrophic circulation, just as observed in the DIFHT simulation.

b. The possible role of symmetric instability in forcing the prefrontal updraft jet

The S–E equation can only be solved under an ellipticity condition that effectively requires positive GPV everywhere in the domain. The following expression for the potential vorticity was evaluated in determining whether this condition was satisfied here:

$$q = \pi^{-1} \left[ \frac{\partial u_s}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial u_s}{\partial x} \frac{\partial \theta}{\partial y} - \left( f + \frac{\partial u_s}{\partial x} \right) \frac{\partial \theta}{\partial \sigma} \right].$$

(15)

This expression includes a term that accounts for the alongfront temperature gradient ($\partial \theta/\partial y$) in the Eady model that does not appear in (14). Analogous expressions appear in Reeder (1986) and Keyser and Anthes (1982), though the actual wind was used in place of its geostrophic component in the latter study.

Highly negative potential vorticity appears within the updraft jet (Fig. 9) and is due mostly to term I, where the cross-front temperature gradient (Fig. 4h) and vertical shear of the alongfront geostrophic wind (Fig. 5a) are strongest. The existence of generally negative potential vorticity throughout the domain near the surface, and particularly within the updraft plume at the leading edge of the front, does not allow us to obtain a solution for the S–E equation. Negative GPV also suggests the possibility that symmetric baroclinic instability (SBI) should be considered (Hoskins 1974). Semigeostrophic frontal theory and the S–E framework do not allow SBI to be present because of the constraint of positive GPV. On the other hand, SBI occurs only in the absence of frontal circulations and geostrophic deformation forcing (the Jacobian term in (13)). Xu (1989a,b) extended the S–E equation for the transverse circulation to include eddy viscosity, so as to remain valid in the presence of small negative moist GPV and weak frontogenetic forcing. More recently, Xu (1992) has treated the problem of diagnosing frontal circulations within the S–E framework in the presence of moderately negative PV and strong

---

5 Although the basic assumptions of two-dimensionality with unidirectional wind shear in thermal wind balance are violated here, we will assume (e.g., Reeder 1986; Knight and Hobbs 1988) that the theory can still be used to investigate the possible role played by symmetric instability in creating the strong frontal updraft if we modify the definition of GPV to be that expressed by (15).
frontogenetic forcing. Thus, our qualitative application of the Sawyer–Eliassen equation to understand the nature of the frontal circulation is believed to have validity.

Symmetric instability is manifested as slanted roll circulations with their axes oriented along the thermal wind vector, and with slopes somewhere between the isentropic and absolute momentum surfaces. We examined the possible role of symmetric instability by reinitializing the MASS model with the results from the DIFHT run at HB94th, which exhibited the necessary condition for symmetric instability (GPV < 0) above and ahead of the surface front. In the control SBI experiment, the alongfront wind \( v = u_x(x, \sigma) \) and potential temperature field \( \theta(x, \sigma) \) were specified by the DIFHT solution, but frontogenetical forcing \( [(\partial u_x/\partial x)(\partial \theta/\partial y)] \), the ageostrophic transverse circulation, and PBL friction and sensible heating were switched off at the initial time. A random perturbation of 0.1 m s\(^{-1}\) was then added to the alongfront wind, and the model was run to see whether a strong updraft would re-form within a 12-h period within the symmetrically unstable region. This procedure is similar to that used by Orlanski and Ross (1977) and Reeder (1986) in their assessment of the role of SBI in generating strong frontal updrafts. Only a weak, transient circulation developed in this experiment, with the 2 cm s\(^{-1}\) updraft located at the surface front. Another experiment in which \( u_x \) given by the DIFHT solution was kept in the initial state rapidly developed an 11 cm s\(^{-1}\) updraft, but it was not maintained beyond a couple of hours: the depth of the cold air mass collapsed, and the frontal circulation entirely disintegrated. A similarly unrealistic behavior was displayed in a run in which both \( u_x \) and friction were allowed. Linear theory predicts that perturbation motions should grow exponentially in a symmetrically unstable atmosphere (Bennetts and Hoskins 1979); since an intense and prolonged updraft did not reform, we conclude that symmetric baroclinic instability did not play any role despite the occurrence of the necessary conditions for its existence. Reeder (1986) reached an identical conclusion in his study of the effects of differential sensible heating on low-level frontal structure.

c. Density current characteristics

The issue of whether the local structure of the simulated front in any of the simulations exhibited similarity to a density current is now addressed. A universal and salient feature of a density current is that the flow immediately behind the head of the current and above a shallow friction layer is directed toward the head in a frame of reference moving with it, that is, the so-called feeder flow extending back to the source of cold air (Simpson 1987). Most investigators consider the existence of a feeder flow to be an important factor in assessing the relevance of density current theory to their observations or model results (Smith and Reeder 1988). Although in principle we could also compare the observed frontal motion with a theoretical prediction for the speed of a steady-state density current, ambiguities in assigning meaningful values for the required variables (as discussed in the introduction) make such an effort of little value. The front-relative flow in the differential heating run (Fig. 10) shows a region immediately behind the front where the flow is positive (directed toward the front), with values as large as 3.5 m s\(^{-1}\). Furthermore, the mean depth of this feederlike flow is 800 m, which is a typical value for a density current. Thus, we conclude that the simulated front in DIFHT displays the character of a density current insofar as there is a region at low levels extending back to the core of the cold air moving faster than the front, and the depth of this region is one that typifies most atmospheric density currents. These findings are similar to those reported by Reeder (1986) and Segal et al. (1993).

Since thermally forced circulations arising from a horizontal gradient in sensible heating are known to display the character of density currents (Simpson 1987), it is reasonable to expect one in the DIFHT run. Yet, a region of positive front-relative flow also existed in HOMHT (not shown) similar in shape and size to that in DIFHT, though with only half the magnitude. This is not to say that a true density current can exist under homogeneous heating conditions, since density currents owe their existence to density contrasts resulting from diabatic processes (e.g., evaporatively cooled thunderstorm outflows and sea breezes generated by land–sea sensible heating contrasts). Rather, this finding provides an additional warning about drawing too strong a conclusion about the appearance of a density current structure at the leading edge of a front based purely on the existence of a region of positive front-relative flow. Furthermore, our results do not unequivocally imply that the frontal speed is governed by density current dynamics rather than by the frontal-scale deformation processes. That issue remains unresolved (Smith and Reeder 1988).

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**Fig. 11.** Cross-front temperature gradient \( \partial \theta/\partial x \) budget, averaged over 1 h (240 \( \Delta x \)) and 80 km (8\( \Delta x \)) at 2100 UTC (HB\( ^{94} \)) for DIFHT/BLACK experiment: (a) \( \partial \theta/\partial x \) (contour interval 1 K (100 km\(^{-1}\))\(^{-1}\); (b) \( \partial \theta/\partial x \partial \theta/\partial x \partial \theta/\partial y \); (c) \( \partial u_x/\partial x \partial \theta/\partial x \partial \theta/\partial y \); (d) \( \partial u_x/\partial x \partial \theta/\partial y \partial \theta/\partial y \); (e) \( \partial \theta/\partial x \partial \theta/\partial y \); and (f) the sum of terms V, VI, and VII in (16). Contour interval is 2.5 K (100 km\(^{-1}\))\(^{-1}\) (day\(^{-1}\))\(^{-1}\) in all panels except (a), with positive (negative) values solid (dashed).
6. Budget studies

Isolation of the various physical processes contributing to the development of the transverse frontal circulation and the frontogenesis process is achieved by analyzing the budget equations for divergence \(d(\partial u/\partial x)/dt\), the cross-frontal temperature gradient \(d(\partial \theta/\partial x)/dt\), and other quantities. An approach similar to that employed by Keyser and Anthes (1982) and Baldwin et al. (1984) is used here. Horizontal gradients are computed along \(\sigma\) surfaces using centered finite differences to avoid interpolation errors, whereas vertical gradients are computed at vertical half-\(\sigma\) levels. All alongfront terms are neglected, with the exception of the shearing deformation term in the \(d(\partial \theta/\partial x)/dt\) budget. Each term is averaged over 80 km \((8 \Delta x)\) and 1-h \((240 \Delta t)\) time intervals. Slight filtering of the fields was performed to reduce truncation errors in the vicinity of intense, small-scale features. The parcel tendencies are estimated using a frame of reference moving eastward with constant speed \(C\) (Table 1); thus, they represent quasi-Lagrangian tendencies. The prognostic equation for the cross-frontal temperature gradient is given as

![Figure 12](image-url)

Fig. 12. Sum of terms in cross-front temperature gradient budget \(d(\partial \theta/\partial x)/dt\), using same averaging and contouring as in Fig. 11a, but for HOMHT, SEABR, FRICIT, and NOPHY experiments.
\[
\frac{d}{dt} \left( \frac{\partial u}{\partial x} \right) = - \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} - \frac{\partial \theta}{\partial x} \frac{\partial \theta}{\partial y} - \frac{\partial \theta}{\partial x} \frac{\partial \theta}{\partial \sigma} + \frac{1}{c_p} \frac{\partial Q}{\partial x} + \frac{\partial}{\partial x} \left( \frac{\partial \theta}{\partial t} \right)_{\text{diff}} + \frac{\partial}{\partial x} \left( \frac{\partial \theta}{\partial t} \right)_{\text{CA}} .
\]

(16)

The terms in this equation are (I) the quasi-Lagrangian tendency of cross-front temperature gradient, (II) confluence deformation acting upon the cross-front temperature gradient, (III) shearing deformation acting upon the alongfront temperature gradient, (IV) tilting effects, and (V, VI, and VII) the horizontal gradients of diabatic heating, horizontal diffusion (Pepper filter), and convective adjustment, respectively. Similarly, for the wind divergence budget, we have

\[
\frac{d}{dt} \left( \frac{\partial u}{\partial x} \right) = - \left( \frac{\partial u}{\partial x} \right)^2 + f \frac{\partial u}{\partial x} \frac{\partial \sigma}{\partial x} + \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial t} \right)_{\text{diff}} + \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial t} \right)_{\text{PHY}} .
\]

(17)

where the respective terms are (I) the quasi-Lagrangian divergence tendency, (II) nonlinear divergence term, (III) unbalanced ageostrophic vorticity, (IV) tilting, (V) horizontal gradient of horizontal diffusion, and (VI) boundary layer friction.

a. Horizontal potential temperature gradient budget

The cross-front temperature gradient budget terms for the DIFHT/BLACK run at HB\(^{44}\) (2100 UTC) are given in Figs. 11b–f, along with the cross-front temperature gradient \(\partial \theta/\partial x\) value at that time (Fig. 11a). Frontogenesis is strongest near the ground at the leading edge of the front, attaining a value of 70 K (100 km\(^{-1}\) (day\(^{-1}\))\(^{-1}\)), which is within the range estimated in the observational line convection study by Koch (1984). This favorable agreement suggests that the present idealized study of frontogenesis can be used to aid in the understanding of the complex processes that occurred in that line convection frontal situation. The quasi-Lagrangian frontogenesis maximum at low levels is strongly dominated by the confluence term (Fig. 11c). The predominance of this frontogenetical agent conforms with the results of other studies (e.g., Sanders 1955; Keyser and Anthes 1982; Ogura and Portis 1982; Baldwin et al. 1984). An interesting feature of the confluence forcing not apparent in previous studies is frontolysis at midlevels above the frontogenesis max-

mum—a property that is clearly related to the updraft plume dynamics, namely, the diffuseness at the top of the plume (Fig. 10). Examination of the quasi-Lagrangian frontogenesis fields for the other simulations (Fig. 12) and the component fields (not shown) reveals that a similar feature was present in the HOMHT run and with a somewhat lower magnitude and altitude in the SEABR run, but it was not present in the FRICT or NPHY runs. Shearing deformation effects in DIFHT (Fig. 11d) contribute to the frontogenesis near the base of the updraft plume, where the absolute vorticity is maximized. Tilting effects are confined to a narrow region of strong frontolysis at \(z = 1\) km and a second frontolysis maximum at the top of the updraft plume where moderate vertical motion resides with strong stratification. Both of these features are related to the intense secondary circulation. The sensible heating term (Fig. 11f) produces only secondary frontogenetical or frontotylytical effects [terms VI and VII in (16) are negligible] —the maximum ahead of the front at low levels results from \(\partial Q/\partial x\) in the PBL being strongest there, whereas the deep region of coupled frontogenesis/frontolysis in the layer \(1.0 < z < 4.0\) km is related to the entraining plume dynamics. Thus, sensible heating produces strong frontogenesis primarily as an indirect consequence of its effects upon the cross-front confluence field, and only secondarily as a direct forcing term in the budget equation.

b. Horizontal wind divergence budget

The divergence equation budget terms and the value of divergence for the DIFHT/BLACK run at HB\(^{44}\) are given in Fig. 13. The strongest quasi-Lagrangian divergence tendency occurs at the leading edge of the front, directly underneath a region of equally strong divergence tendency. The two largest contributors to the low-level convergence feeding the updraft jet are the nonlinear (Fig. 13c) and unbalanced ageostrophic vorticity (Fig. 13d) terms, with the nonlinear effects prevailing. Tilting and ageostrophic vorticity forcing effects (Figs. 13d and 13e) are responsible for the divergence feature aloft. The boundary layer term (Fig. 13f) causes divergence to be created at low levels ahead of the updraft jet. This unexpected effect was found to be due to the turbulent upward transport of easterly momentum within the updraft plume, as stronger westerlies and upward motions are found at higher levels within the plume. It can be seen from (4) that air parcels must decelerate \([du/dt = -\partial(u^2w^3)/\partial z < 0]\) within the plume, effectuating the divergence buildup at low levels just ahead of the plume.

Our study provides new understanding of the role of friction and mixing processes in frontogenesis dynamics when a pronounced updraft exists in the presence of a strong horizontal gradient in static stability. The direct effects of friction and mixing processes are to weaken the updraft. This statement does not contradict
the conclusion of Keyser and Anthes (1982), who show that frictionally induced ageostrophic inflow provides for a stronger updraft (compare the net parcel divergence tendencies for the FRICT and NOPHY runs in Figs. 14c and 14d). In particular, nonlinear and ageostrophic vorticity effects are augmented by friction (Fig. 15), thereby contributing to the stronger updraft in the FRICT run. On the other hand, the direct effects of mixing processes (term VI) do not help to drive the updraft, since the outcome is the creation of low-level divergence (Fig. 15f). Thus, even though turbulent mixing and friction weaken the updraft in a direct

**Fig. 14.** Sum of terms in divergence equation budget $\frac{d(\partial u/\partial x)}{dt}$, using same averaging and contouring as in Fig. 13b but for HOMHT, SEABR, FRICT, and NOPHY experiments.
sense, these effects are more than compensated by the indirect effects related to frictional retardation of the alongfront flow and nonlinear convergence intensification. The retardation effect appears through the ageostrophic vorticity term, since as discussed earlier, subgeostrophic alongfront flow develops at low levels under the influence of friction, resulting in the development of strong low-level convergence [see (9)], which then also contributes to the nonlinear term.

Earlier it was suggested that frictional and diabatic effects forced the low-level convergence, whereas diabatic heating effects forced the midlevel divergence at the top of the updraft plume (section 4a). The budget analyses in Figs. 13b and 14c reveal that friction contributes roughly 25% to the total low-level convergence tendency, leaving sensible heating effects to contribute the remaining 75%. Clearly the main reason for this difference lies in the nonlinear effect (compare the respective panels in Fig. 13 to those in Fig. 15). In other words, sensible heating has a much more nonlinear dynamical effect upon the frontogenesis process at the lowest levels than does friction. The unbalanced ageostrophic vorticity and tilting terms are both very important in midlevels only in the presence of sensible heating as well and are important contributors to the vertically integrated mass flux divergence profile within the strong front updraft. Time series of the individual terms in the divergence equation budget (not shown) suggest that an increasingly unbalanced frontal circulation develops in the differential heating run as the nonlinear effects become increasingly dominant after the time of maximum diurnal heating.

7. Summary and concluding remarks

This numerical study has investigated the frontogenetical and dynamical effects of a cross-frontal gradient in sensible heating resulting from the contrast between clear skies ahead of an idealized midlatitude, dry, continental cold front and an overcast low cloud deck existing behind the front. A systematic investigation of various numerical and dynamical factors that exert an influence on modeled frontogenesis also has been conducted. A dry, hydrostatic, two-dimensional model was initialized with the Hoskins–Bretherton shear deformation model of frontogenesis. The simulated frontal structure and dynamics were sensitive to the type of parameterized boundary layer treatment. A bulk mixed-layer approach was found to be ill suited to modeling frontogenesis in the presence of differential sensible heating, whereas the Blackadar multilevel PBL treatment produced realistic low-level jet and thermodynamic structures.

Introduction of uniform sensible heating (no clouds) produced a deep and narrow updraft jet (plume) at the leading edge of the modeled cold front, whose magnitude using a 10-km grid resolution was 16 cm s⁻¹. The strength of this updraft reached 22 cm s⁻¹ in the differential sensible heating (DIFHT) run, wherein completely overcast low cloud cover was specified west of the front, and clear skies to the east, resulting in a sensible heat flux of 450 W m⁻² ahead of (and nearly zero behind) the front. Drastic reduction of the alongfront wind component u within the plume resulted in a strong ageostrophic component vₑ and associated maximum in the unbalanced ageostrophic vorticity ($f\partial v'_ₑ/\partial x$), which acted as a significant source for ageostrophic cross-frontal wind divergence in midlevels. The combination of this divergence with low-level convergence produced by frictional and highly nonlinear diabatic effects explains the development of the frontal updraft jet. Although the greatest strengthening of the updraft in our simulations ensued from the introduction of sensible heating, rather than the introduction of an inhomogeneity in sensible heating, analysis indicates that the differences would be expected to be much larger at grid resolutions Δx < 10 km (requiring the use of a nonhydrostatic model), since a sizable increase in frontal updraft intensity was realized only in the DIFHT case by decreasing the grid size.

Highly negative geostrophic potential vorticity developed within the updraft plume from the interaction of the low-level southerly geostrophic jet with the strong cross-frontal temperature gradient in the DIFHT case. Numerical experiments showed, curiously enough, that dry symmetric instability could not have played a significant role in driving the strong frontal updraft that characterized the secondary circulation.

The simulated front in the DIFHT case displayed similarity to a density current inasmuch as there was a feeder flow of postfrontal air at low levels extending back to the core of the cold air mass moving faster than the front, and the depth of this region was typical of atmospheric density currents. Such an extensive feeder flow has not been seen in previous numerical models of frontogenesis. A similar (though weaker) region of positive front-relative flow existed in the homogeneous heating run. Since density currents are usually generated by fields of nonuniform diabatic heating (or cooling), this result is surprising. The positive front-relative flow in the homogeneous heating run developed as a consequence of the acceleration of the cross-frontal ageostrophic flow due to the destabilizing effects of sensible heating.

Koch (1984) and Dorian et al. (1988) have reported cases of strong frontogenesis in which a frontal line convection developed in the presence of a strong cold cloud contrast across cold fronts. The findings from this idealized study of frontogenesis lend strong support to

Fig. 15. As in Fig. 13 except for FRICT/BLACK experiment.
their hypothesis that a highly nonlinear interaction between the adiabatic, large-scale frontal circulation and the thermally forced circulation produced by the cloud contrast produced the updraft that triggered the frontal squall lines. Those case studies were limited by their reliance upon synoptic surface and upper-air data, preventing any understanding of the three-dimensional mesoscale dynamics. Our model results show that frontogenesis is dominated by confluence forcing near the surface. This lends support to the usefulness of surface frontogenesis diagnostics in observational studies lacking mesoscale data aloft. Diffuseness-forced frontolysis in midlevels appeared above the frontogenesis maximum in the model. The diffuseness is a feature related to the updraft plume dynamics. Direct forcing of frontogenesis by the diabatic gradient term in the frontogenesis budget equation produced only secondary effects; the primary frontogenetical forcing was due to an indirect effect of heating on enhancing the cross-front confluence (convergence). The strongest contributors to the low-level convergence feeding the updraft jet were the nonlinear \((\partial u/\partial x)^2\) and unbalanced ageostrophic vorticity terms, with the nonlinear effects dominating. Interestingly, the boundary layer effects were highly frontolytical at low levels, being related to the turbulent upward transport of easterly momentum within the updraft plume. Nonlinear effects became stronger than the ageostrophic residue effect in the later stages of frontogenesis in the DIFHT case. This suggests that an increasingly unbalanced frontal circulation developed after the time of maximum diurnal heating. Our results also showed that friction and mixing processes weakened the updraft due to the creation of low-level divergence but that the nonlinear divergence and ageostrophic vorticity terms were augmented as an indirect consequence of these viscous processes.

One might wonder what limitations are imposed by the two-dimensional assumption used in our modeling study. Reeder et al. (1991) showed that the vertical motions associated with a 3D frontal zone characterized by a vertically sheared mean zonal wind and sensible heating were quite similar to those in the corresponding 2D model. Perhaps a more critical limitation is the Hoskins–Bretherton frontal model used in the present study, which necessitated that periodic boundary conditions and a deep front be specified. The boundary conditions created several problems, such as the unrealistic narrowness of the frontal updraft and the difficulty of handling the ageostrophic inflow of cooler air from ahead of the front. Another limitation is that none of our PBL approaches considered the effects of cumulus clouds on the boundary layer structure and turbulent fluxes of mass, momentum, and heat. The relative effects of turbulence, radiation, and subgrid-scale cloud condensation were considered in the case of a stratocumulus-capped mixed layer by Chen and Cotton (1983). It would be of great interest to parameterize these effects (e.g., cumulus-induced subsidence and cloud-top entrainment) into the boundary layer physics, since none of the simulations was successful at producing the strong microscale downdraft that Koch (1984) hypothesized was the cause for the observed postfrontal clear zone behind the cumulus line convection. Higher-order treatments of the momentum and heat fluxes in the PBL may be required to improve simulations of the turbulent structure during rapid frontogenesis. Finally, it is important to revisit this frontogenesis problem with a high-resolution, nonhydrostatic model, and perhaps to include moisture physics to see whether a frontal squall line can actually be produced due to the forcing provided by the intense frontal updraft.

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APPENDIX

Partial List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition of symbol</th>
<th>Additional information</th>
</tr>
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<tbody>
<tr>
<td>(h_i)</td>
<td>boundary layer height</td>
<td>[see (3)]</td>
</tr>
<tr>
<td>(H)</td>
<td>sensible heat flux</td>
<td></td>
</tr>
<tr>
<td>(L)</td>
<td>Monin–Obukhov length</td>
<td></td>
</tr>
<tr>
<td>(m)</td>
<td>absolute momentum</td>
<td>(m = v_g - f x) (see Table 1)</td>
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<tr>
<td>(N)</td>
<td>static stability (Brunt–Väisälä frequency)</td>
<td>[see (5)]</td>
</tr>
<tr>
<td>(\dot{Q})</td>
<td>sensible heating rate</td>
<td>((dV/dt))/(fN)</td>
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<tr>
<td>(\text{Ro})</td>
<td>Rossby number</td>
<td></td>
</tr>
<tr>
<td>(u_g, u_{ag})</td>
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<td>alongfront geostrophic (ageostrophic) wind</td>
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</tr>
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<tr>
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<td>sigma-p coordinate</td>
<td>[see (1)]</td>
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<td>(\sigma)</td>
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<tr>
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REFERENCES


