Propagation of Rossby Waves of Nonzero Frequency

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ABSTRACT

The propagation of Rossby waves of positive and negative frequency, corresponding to eastward and westward phase speeds, respectively, is investigated. The techniques used are theoretical analysis, ray tracing, and initial value problems in barotropic and baroclinic numerical models. The basic states considered are a superrotation flow and December–February climatological zonally symmetric and zonally asymmetric flows. It is found that positive and negative frequency Rossby waves can differ significantly from each other and from stationary, zero frequency Rossby waves in many aspects. Negative frequency waves tend to have larger total wavelengths and increased meridional group velocities. Enhanced meridional propagation and, indeed, cross-equatorial propagation are found for westward moving sources in both barotropic and baroclinic models. However, general deductions from studies of stationary Rossby waves, such as the existence of subtropical jet waveguides, are still found to be valid.

1. Introduction

When Rossby waves are forced by steady flow over mountains or other features of the underlying planet, it is reasonable to focus theoretical discussion on the waves of zero frequency, stationary Rossby waves. However, when one is considering the effect of a local meteorological event, Rossby waves of nonzero frequency become relevant. If there is a local eastward phase speed, as in most meteorological systems in the middle latitudes, then the frequency will be positive. A westward phase speed and negative frequency will be relevant, for example, in westward drifting blocking events. Other events, such as a fluctuating westerly flow over a mountain or fluctuating convective heating in a fixed geographical region, might have more of the nature of a standing oscillation. Such an oscillation can be decomposed into positive and negative frequency components of equal magnitude.

The nonzero frequency of a Rossby wave can be expected to influence its propagation if its zonal phase speed $c$ is comparable to the basic flow speed in the region. For example the critical line will be where the basic zonal flow is equal to $c$. For a wave of period $N$ days and zonal wavenumber $m$ at latitude $\phi$,

$$c = \frac{\omega}{k} = \frac{a \Omega \cos \phi}{mN}. \quad (1)$$

Near $\phi = 30^\circ$, $a \Omega \cos \phi \approx 400$ m s$^{-1}$, and for a 30-day, zonal wavenumber 1 oscillation, (1) implies that the oscillatory nature is important for zonal flows comparable to or weaker than 13 m s$^{-1}$. For a 30-day oscillation and wavenumber 4 the speed is 3 m s$^{-1}$, and for a 14-day oscillation and wavenumbers 1 and 4 the speeds are 29 and 7 m s$^{-1}$, respectively. Comparing these numbers with typical zonally averaged winds, it suggests that for a 14-day oscillation, and even for a 30-day oscillation, wave propagation behavior can be expected to differ from that for stationary waves, at least in the subtropics and polar regions.

In Hoskins and Karoly (1981, hereafter referred to as HK) the theory of wave propagation in a slowly varying medium was applied to stationary Rossby wave propagation in a zonally symmetric barotropic atmosphere. The results agreed well with their modeling of forced stationary waves and had similarities with the observed equivalent barotropic teleconnections detailed in Wallace and Gutzler (1981). In Hoskins and Ambrizzi (1993) the same ray theory was applied to a zonally asymmetric December–February (DJF) 300-hPa basic state using the local zonal wind. Good agreement was found with initial value integrations of a barotropic model and the observations of Hsu and Lin (1992). In particular the waveguide effect of the Northern Hemisphere jets was noted. Subsequently, Ambrizzi et al. (1995) have shown observational and barotropic modeling results for June–August (JJA). In Ambrizzi and Hoskins (1996) it is shown that similar results are obtained in a baroclinic model.

Wallace and Blackmon (1983) showed that low-frequency oscillations with timescales longer than one week make a large contribution to the total variability

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of the atmosphere and to monthly mean anomaly patterns. However, it is not immediately clear that one can deal with zero frequency waves alone to provide a theoretical framework for understanding the observed low-frequency behavior, or indeed the results from numerical integrations in which stationary forcing is switched on at an initial time.

Karoly (1983) applied ray theory to the propagation of low-frequency waves and discussed the differences between the propagation of low-frequency and stationary waves. He produced some general propagation theory and showed ray paths and propagation speeds for Rossby waves with a range of westward phase speeds on DJF and JJA zonal flows. He also considered propagation on flows with wavenumber one longitudinal variation.

Recently, Li and Nathan (1994) also used Wentzel-Kramers-Brillouin (WKB) theory to determine ray paths and wave amplitudes for meridional propagation from westward moving equatorial sources in a number of zonal flows. They showed that low wavenumber and high frequency accentuate differences from stationary wave propagation. Like Karoly (1983), they commented on the possibility of propagation of westward moving Rossby waves through tropical easterlies. Li and Nathan (1994) went on to investigate the statistically steady state achieved in a damped barotropic model using a basic zonal flow and applying equatorial forcing with temporal behavior of the form $1 + \cos(\omega t)$. For equal vorticity forcing amplitude the largest extratropical response, measured by kinetic energy, was for forcing in the period of 10–20 days.

The aim of this paper is to thoroughly investigate the propagation of Rossby waves of both positive and negative frequency, corresponding to eastward and westward phase speeds. Comparison will be made between these waves and with the behavior of stationary waves previously discussed in the literature. The ray theory with some new results will be given in section 2. Some basic information on the results to be shown and the models and flows is in section 3. Results from initial value problems with a barotropic model and superrotation, DJF zonal flow, and zonally asymmetric flow are presented in sections 4, 5, and 6, respectively. Section 7 gives results for similar experiments performed with a baroclinic model, and some concluding comments are made in section 8.

2. Wavenumber and group velocity analysis

The dispersion relation for a barotropic Rossby wave perturbation of the form $\exp\{i(kx + ly - \omega t)\}$ is

$$\omega = \bar{U}k - \frac{\beta k}{K^2},$$

where $\bar{U}$ is the basic zonal flow,

$$\beta = \frac{\partial f}{\partial y} - \frac{\partial^2 \bar{U}}{\partial y^2}$$

is the meridional gradient of absolute vorticity, and $K^2 = k^2 + l^2$ is the square of the total wavenumber.

The components of the group velocity $c_\phi$ are given by

$$u_\phi = \frac{\partial \omega}{\partial k} = c + \frac{2\beta k^2}{K^4}$$

$$v_\phi = \frac{\partial \omega}{\partial l} = \frac{2\beta kl}{K^4},$$

where $c = \omega/k$ is the $x$-phase speed.

Using (2),

$$c_\phi = (u_\phi, v_\phi) = c i + 2(\bar{U} - c) \cos \alpha K,$$

where $K$ is the unit vector normal to the crests and troughs, and $\alpha$ is the angle $K$ makes with the $x$ axis. For nonzero frequency, $c_\phi$ is no longer parallel to $K$ as it is in the stationary case. From (6), $c_\phi$ is significantly influenced by the nonzero frequency if $c$ is comparable to $\bar{U}$. This is consistent with the argument in the introduction and with the deduction by Li and Nathan (1994) that high $\omega$ and low $k$ and $\bar{U}$ favor the departure of ray paths from those for stationary waves. As indicated in Fig. 1, for positive (negative) $\omega$, (6) shows that the angle that $c_\phi$ makes with the $x$ axis is smaller (larger) than $\alpha$. Also from (4) and (5), or directly from (6) using the law of cosines, the magnitude of $c_\phi$ is given by

$$c_\phi^2 = c^2 + 4 \cos^2 \alpha \bar{U}(\bar{U} - c).$$

Thus, for westerly flow

$$c_{\phi+} < c_{\phi-} < c_{\phi-},$$

where the signs indicate the sign of the frequency $\omega$ and $s$ indicates the stationary case ($c = 0$). The larger group velocity for negative frequency is also illustrated in Fig. 1. In the special case of meridional crests and troughs, $v_\phi = 0$ and...
\[ u_g = 2\bar{U} - c. \]  

From (2) the total wavenumber is given by

\[ K^2 = \frac{\beta}{U - c} = \frac{K_s^2}{1 - c/\bar{U}}, \]  

where \( K_s = (\beta/\bar{U})^{1/2} \) is the stationary wavenumber. Assuming that \( \beta \) is positive, it is clear that

\[ K_+ > K_i > K_- . \]  

The larger total wavelength for negative frequency is depicted in Fig. 1.

For positive \( \omega \), \( u_g \) is always positive and so the wave always propagates eastward. However, from (6), for negative \( \omega \) westward propagation occurs for \( \cos \alpha \) small enough, that is, near zonal crest waves and troughs.

It is clear from (10) that for Rossby waves to be possible \( \bar{U} - c \) must be positive and less than \( \beta/k^2 \). A latitude at which the flow speed is equal to \( c = \omega/k \) is a critical latitude at which the meridional scale tends to zero (\( l \) and \( K \) tend to infinity). At such a latitude, from (6), \( e_g \) is also equal to the basic westerly flow. At a critical latitude

\[ k = k_c = \frac{\omega}{\bar{U}}. \]  

A latitude at which \( \bar{U} - c = \beta/k^2 \), that is, \( l = 0 \), is a reflection latitude. From (10) or (2), at a reflection latitude, \( k \) must satisfy the quadratic

\[ \bar{U}k^2 - \omega k - \beta = 0. \]  

This equation has two roots

\[ k_{1,2} = \frac{\omega \mp (\omega^2 + 4\beta\bar{U})^{1/2}}{2\bar{U}}. \]  

As shown schematically in Fig. 2a, for \( \beta \) and \( \omega \) positive, Rossby waves exist only for a westerly flow. The range of permissible zonal wavenumbers for \( \bar{U} \) positive is bounded on the long-wave side by a critical wavenumber, \( k_c \), and on the short-wave side by a reflection wavenumber \( k_2 \). For \( \omega \) negative (Fig. 2b), Rossby waves at sufficiently long wavelengths exist for all flow speeds. For westerly flow, there is a reflection wavenumber \( k_2 \). For easterly wind, the situation becomes more complicated. For very weak easterlies, \( U_e < \bar{U} < 0 \), where

\[ U_e = -\omega^2/4\beta \approx -58/(N^2 \cos \varphi) \]

for an \( N \)-day oscillation and planetary \( \beta \), a new short-wavelength range is permissible. At \( \bar{U} = U_e \) the two
regions coalesce, and for stronger easterlies Rossby waves exist for all wavenumbers lower than a critical wavenumber, $k_c$.

In the case that $\beta$ is negative, the schematic diagrams in Figs. 2a,b still apply but with the sense of all inequalities reversed. Thus Fig. 2b applies to $\omega$ positive and the top line to easterly flow. In each case the labels $k_1$ and $k_2$ are swapped.

For completeness we give in Figs. 2c,d the diagrams for westerly flow but with $\beta$ varying. Again, the situation for easterly flow is obtained by reversing all the signs and replacing $k_2$ with $k_1$.

It is possible to obtain general equations for the change in the direction of $e_y$ along a ray and for the radius of curvature of the ray analogous to those given in Hoskins and Ambruzzi [1993, (2.7)–(2.11)]. Again, the radius of curvature is proportional to $dK/dy$, but for sufficiently nonstationary waves the constant of proportionality can now, under certain circumstances, be negative.

3. Ray tracing techniques and model details

As in HK, it is convenient to use a Mercator projection for barotropic Rossby ray tracing on a zonal flow on the sphere. The dispersion equation (2) is then changed to

$$\omega = \frac{\overline{U}_M k - \beta_M k}{K^2},$$

(15)

where $\overline{U}_M = \overline{U}/\cos \varphi$ is the Mercator basic zonal velocity and

$$\beta_M = \frac{2\Omega}{a} \frac{\cos^2 \varphi}{\cos \varphi} \frac{d}{d\varphi} \frac{1}{\cos \varphi} \frac{d}{d\psi} (\cos^2 \varphi \overline{U}_M)$$

(16)

is $\cos \varphi$ times the meridional gradient of absolute vorticity on the sphere. As usual in this analysis, it is assumed that the basic flow is sufficiently slowly varying in $y$ that the local values of $\overline{U}_M$ and $\beta_M$ can be used throughout. Because (15) takes the same form as (2), all the equations deduced from (2) in section 2 will keep the same form in Mercator projection, with $\overline{U}$ and $\beta$ replaced by $\overline{U}_M$ and $\beta_M$.

In this paper, barotropic results will be shown for three kinds of basic flow: a superrotation flow, the 1983–89 300-hPa zonal-mean DJF flow, and the zonally asymmetric flow for the same period and level.

For each basic flow, several results will be presented. First, following the analysis of section 2, we show the
summary pictures for the propagation $k$ range and critical and reflection lines. Then ray tracing results are shown for the first two basic flows. This ray tracing is performed by integrating the differential equations,

$$ \frac{dx}{dt} = u_x, \quad \frac{dy}{dt} = v_x, \quad (17) $$

using a second-order Runge–Kutta scheme for a given initial position, wavenumber, and frequency. Since the $\beta_M$ and $\bar{U}_M$ are independent of $x$, then $k$ is constant along a ray. The variation of $l$ is most easily obtained by demanding that the local dispersion relation (15) is satisfied.

The third set of results is from initial value problems with a damped barotropic model with an oscillating forcing:

$$ \left( \frac{\partial}{\partial t} + \mathbf{V}_\phi \cdot \nabla \right) \xi = \frac{\xi}{\tau} - \lambda \nabla^2 \xi + \bar{F} + F'. \quad (18) $$

Here $\zeta$ and $\xi$ are the absolute and relative vorticities, respectively. This equation is solved using the spectral transform technique with a T21 truncation. The first term on the right-hand side is a Newtonian damping with timescale $\tau = 10$ days, and the second term is a hyperdiffusion with coefficient $\lambda = 2.338 \times 10^{16}$ m$^4$ s$^{-1}$ acting on a timescale of 4 days on the smallest length scale in the model; $\bar{F}$ is determined such that the basic state posed is an exact solution of (18) with $F' = 0$. The perturbation forcing $F'$ is switched on at $t = 0$. This is viewed purely as a wave maker rather than as representing the effect of tropical heating. Thus, it is specified in the form $fD$ with no inclusion of the divergent advection of vorticity as proposed by Sarhangsh and Hoskins (1988).

To examine the propagation of stationary Rossby waves, vorticity forcings of the form $fD$ can be used where $D$ has a local structure centered on a certain longitude, $\lambda_0$, about which it is symmetrical. Then the vorticity forcing may be written as
case with a positive vorticity center located near [20°N, 0°]. This center moves eastward to around 6°E at \( \omega t = \pi / 4 \) (Fig. 3b). It continues to move eastward and weaken (Figs. 3c,d) and is replaced by a negative center at \( \omega t = \pi \) (Fig. 3e). In the second half-period (not shown), the pictures are repeated but with the sign reversed. The source with negative frequency is the mirror image of that shown in Fig. 3, moving in the westward direction instead. The speed of movement of the centers in each case is about the basic source width, 30° per half-period. It is seen that the source acts as a local wavemaker for Rossby waves of a particular frequency. It is convenient to refer to the positive and negative frequency forcings in terms of a positive and negative period, respectively, so that terms such as “period +14 days” and “period -14 days” will be used.

Standing oscillatory sources can be obtained by adding or subtracting positive and negative frequency sources. From (20) this would give sources \( 2F_1' \cos \omega t \) and \( 2F_2' \sin \omega t \), respectively. Thus, the perturbation so-

\[
F_1' = R \sum_{0}^{M} A_m(\mu)e^{im(\lambda - \lambda_0)} = \sum_{0}^{M} A_m(\mu) \cos(\lambda - \lambda_0),
\]

because the symmetry about \( \lambda_0 \) implies that each \( A_m \) is real. In this paper we allow the vorticity forcing to have a nonzero frequency \( \omega \):

\[
F' = R \sum_{0}^{M} A_m(\mu)e^{im(\lambda - \lambda_0)e^{-i\omega t}} = F_1' \cos \omega t + F_2' \sin \omega t.
\]

Here \( F_1' \) is defined as in (19) and

\[
F_2' = \sum_{1}^{M} A_m(\mu) \sin(m(\lambda - \lambda_0)).
\]

The forcing to be used in the barotropic model corresponds to \( D \), having a cosine squared amplitude distribution in a circle of diameter 30° of latitude. The calculations are linear, but for definiteness the maximum value is chosen to be \( 5 \times 10^{-6} \text{ s}^{-1} \). Figure 3 shows half a period of the evolution of the forcing \( F' \) for \( D \) centered on [20°N, 0°] and for a positive frequency. At \( t = 0 \) (Fig. 3a), the source takes the same form as in the stationary

![Fig. 6. Barotropic model vorticity anomalies at day 10.5 for sources at [20°N, 0°] on the superrotation flow, with (a) period +14 days and (b) period -14 days. The contour interval is 3 \times 10^{-6} \text{ s}^{-1}, with negative values dashed and the zero contour not drawn. Lines of latitude and longitude are drawn every 30°, and the arrow indicates 0° longitude.](image)

![Fig. 7. A summary of the propagation behavior in terms of zonal wavenumber and latitude for a zonally symmetric DJF flow and a period of (a) +14 and (b) -14 days. The conventions are as in Fig. 4.](image)
solutions for such oscillatory forcings are obtained as the sums and differences of the solutions for positive and negative frequency forcing. The solution for $F_1' \sin \omega t$ would require separate computation, but it will show similar general behavior and will of course be equivalent to that for $F_1' \cos \omega t$, apart from a phase lag, after initial transients have died away.

In section 7 a small number of numerical results will be presented for positive and negative frequency local thermal forcing in 3D time-mean climatological basic flows using a baroclinic primitive equation model. These results are an extension of those in Ambrizzi and Hoskins (1996), which discusses stationary wave propagation in a baroclinic atmosphere, and of the tropical initial value and stationary heating investigation in Hoskins and Jin (1991) and Jin and Hoskins (1995), respectively. The model uses $\sigma$ coordinates and is run at T31, 15-layer resolution. It has Rayleigh friction in the two lowest levels, Newtonian cooling, $\nabla^6$ hyper-diffusion, and vertical diffusion. The perturbation thermal forcing, viewed here purely as a wave maker, has the same horizontal structure as $D$ in the barotropic model and a vertical profile peaking at $\sigma \approx 0.40$. For definiteness the column average is taken to have a maximum of $2.5^\circ C$ day$^{-1}$.

4. Superrotation flow

As in HK and Hoskins et al. (1977), we start by considering a superrotation basic flow $U_M = a\bar{\omega}$, with $\bar{\omega}/\Omega = 1/30.875$, implying a speed of about 15 m$^{-1}$ at the equator. Figure 4 summarizes the propagation in terms of zonal wavenumber $m$ for periods +14 and −14 days. For positive frequency, waves exist between a critical value and a reflection value at each latitude. For negative frequency, there is only the short-wave, reflection value bound, and there is a shift to larger wavelengths. In either case, waves of a certain zonal wavenumber exist only between two reflection latitudes, with longer waves able to propagate to higher latitudes. The difference between the positive and negative propagation figures increases with the frequency.

Ray tracing results for periods ±14 days with initial position [20°N, 0°] and zonal wavenumber $m = 1–6$.
are shown in Fig. 5. Both positive and negative meridional wavenumbers are used so as to allow both poleward and equatorward propagation. For period +14 days (Fig. 5b), rays for all zonal wavenumbers 1–6 can exist at 20°N, but for period +14 days (Fig. 5a), just rays for m = 3–6 can exist at that latitude (wavenumbers 7 and 8 are not shown), in agreement with Figs. 4a,b. In both cases the rays for higher m, for example, m = 5 and 6 in Fig. 5a and m = 3–6 in Fig. 5b, show almost as great circle propagation as in HK. However, rays for lower m show quite different behavior. The rays for m = 3 and 4 in Fig. 5a are nearly straight lines and more zonally oriented, and the rays for m = 1 and 2 in Fig. 5b propagate westward at some latitudes. There is generally more meridional propagation for negative frequency than for positive frequency. As shown by the cross marks in Fig. 5, which indicate the position every day, waves for negative ω propagate faster than those for positive ω. These differences are all consistent with the results derived in section 2.

For a more explicit exhibition of nonstationary Rossby wave propagation, the day 10.5 vorticity perturbation from barotropic model experiments with the same positive and negative frequency sources at [20°N, 0°] are shown in Fig. 6. The two Rossby wave responses are different and are consistent even in detail with the ray tracing discussed above and shown in Fig. 5. In particular, the period +14 days (Fig. 6a) gives a shorter wavelength, and slower and more zonally oriented propagation than −14 days (Fig. 6b). The latter clearly shows westward propagation in the largest wavelengths.

Integrating for several periods in each case gives oscillating fields with the same general character as those shown in Fig. 6.

5. DJF zonal flow

The basic flow used in this section is a climatological DJF 300-hPa zonal-mean zonal flow based on European Centre for Medium-Range Weather Forecasts (ECMWF) data for the period 1983–89. The corresponding pictures to those in section 4 will be shown.

Figure 7 is the propagating wavenumber figure for 14 day oscillations (cf. Fig. 4). For the period +14 days (Fig. 7a) all zonal wavenumbers have critical latitudes within the Tropics and m = 1 and 2 have other critical latitudes in middle and high latitudes. The range of propagating wavenumbers is large in the region of the jets near 30°N and 45°S. In the case of period −14 days (Fig. 7b), the zonal wavenumbers in the range shown have only reflection latitudes. Short waves can propagate in an equatorial band. At even shorter wavelengths than those shown, the more complicated critical line and second reflection line behavior of the kind shown in Figs. 2b,c are found in the tropical easterlies.

Ray tracing for period ±14 days with sources at [20°N, 0°] is shown in Fig. 8 (cf. Fig. 5). There are no rays for m = 1 in Fig. 8a or for m = 6 in Fig. 8b, consistent with Fig. 7. Rays for m = 2 and 3 for the period +14 days (Fig. 8a) propagate to high latitudes, but rays for higher m meet their turning points before 40°N and are then trapped near their critical latitudes, moving zonally with the local flow speed, as was shown in section 2. All rays for period −14 days (Fig. 8b) propagate across the equator and are reflected between the reflection latitudes in each hemisphere. Their propagation speed in the region of the equator is small, except for westward propagating m = 1. Because there is no critical line for this band of wavenumbers, the rays will continue to propagate around the sphere and the amount of interference will depend on the strength of the dissipation. A figure for 30°N and 30°S and −20 day sources like Fig. 8b was given previously by Karoly (1983). It was contrasted with figures for stationary and −50 day sources. The deductions made in this study are consistent with those in Karoly (1983).
significant cross-equatorial propagation with a dominant scale of wavenumbers 4–5 near 20°S and some very low wavenumber westward propagation. Again this is in excellent agreement with the ray tracing in Fig. 8b.

6. Zonally asymmetric DJF flow

In this section the full DJF time-mean 300-hPa asymmetric flow is used for the basic state. More detailed results will be shown for this more realistic flow. Full ray tracing for such a flow requires consideration of the basic flow $\vec{u}(x, y), \vec{v}(x, y)$ and the variation of both $k$ and $l$ along ray paths. Karoly (1983) performed such a ray tracing for stationary waves in idealized low wavenumber basic states. However, as recognized by Karoly (1983), there is no scale separation between the long zonal wavelength Rossby waves and a climatological basic state. Thus, ray tracing for such waves on such a flow is of doubtful validity. However, following Hoskins and Ambrizzi (1993), we look for qualitative insight by neglecting $\vec{v}$ and applying basic Rossby wave theory to the local $\vec{u}$ at each longitude.

Figure 10 shows the geographical distribution of the total wavenumber $K$ [defined as in (10) but using Mercator coordinates] for $m = 4$ and period ±14 days. The thick solid line is the critical line, and the line marked with the number equal to the zonal wavenumber $m$ is a reflection line on which $l = 0$. For the +14 day period (Fig. 10a), we can see there is an elongated tropical “no go” area enclosed by critical lines in either hemisphere. With period or $m$ increasing (not shown), the
critical area shrinks to a narrower band around the equator, and a wave propagation region appears in the equatorial east Pacific. Waves with longer positive period and shorter wavelength can propagate closer to the equator and even cross the equator in some longitudes with westerly flow.

Results for the $-14$ day period (Fig. 10b) show some quite different features. There is a much smaller no-go area in the tropical easterlies. With period and $m$ decreasing (not shown), this region disappears.

For both positive and negative periods, the reflection lines are found at higher latitudes for smaller $m$, indicating that longer waves can propagate to higher latitudes.

The three jet waveguides—the North African—Asian jet, the North Atlantic jet, and the Southern Hemisphere jet—highlighted by Hoskins and Ambrizzi (1993) for stationary Rossby waves are also found for nonzero frequency. They are evident in Fig. 10 as zonally elongated maxima in $K$. For the $+14$ day period (Fig. 10a), total wavenumber $K = 6$ and 7 dominates the North African—Asian jet. In the other two jets the typical $K$ is 5. For negative frequency (Fig. 10b) the typical $K$ is smaller, with $K = 6$ in the North African—Asian jet, $K = 4$ and 5 in the North Atlantic jet, and $K = 4$ in the Southern Hemisphere jet. For this negative frequency another possible waveguide is seen along $20^\circ$S across Australia eastward to the southeast Pacific with $K = 7$.

Another indication of wave propagation is given by wavenumber propagation figures like those shown previously in Figs. 4 and 7 but for the local zonal flows at $30^\circ$W and $150^\circ$E. The $30^\circ$W, $+14$ day propagation figure in Fig. 11a is very similar to Fig. 7a in the Southern Hemisphere. In the Northern Hemisphere the North Atlantic jet is seen as a strong waveguide for $m = 3–5$, and $m = 5–9$ are guided in the entrance of the Asian jet. The same features are apparent in the $-14$ day period (Fig. 11b), but at the larger scales $m = 1–4$ and $m = 2–8$, respectively. At $150^\circ$E (Fig. 12) the Asian

![Diagram](image_url)
for 7-day periods (Fig. 14), and the cross-equatorial propagation at various longitudes is clear in the negative period. For the −7 day period there are small vorticity centers in the two Southern Hemisphere waveguides near 20° and 45°S, consistent with Figs. 10b, 11b, and 12b. There is much more similarity in the solutions for 30 day period forcing (Fig. 15), particularly in the East Pacific. However, the zonal wavelengths in the waveguide are still noticeably different, corresponding to about zonal wavenumber 5 for the positive period and 4 for the negative period.

An interesting example of the difference in meridional propagation is provided by ±7 day forcing at [20°S, 0°], for which the day 7 solutions are shown in Fig. 16. The forcing is near the entrance of the Southern Hemisphere jet waveguide, and both cases show propagation along this guide near 45°S. The negative period again gives a response in the waveguide near 20°S. However, the striking feature is the propagation across the equator and into the North African–Asian waveguide. The resulting pattern near 30°N is remarkably similar to that shown in Fig. 14b for the −7 day period forcing at 20°N.

The amplitudes of the responses for 7 day period forcing are generally smaller than those found for 14 day and 30 day period forcings that are comparable. Li and Nathan (1994) proposed a maximum response for forcing in the 10–20 day period, which might be con-

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Fig. 13. Barotropic model vorticity anomalies at day 7 for (a) +14 day and (b) −14 day period sources at [20°N, 0°] on a zonally asymmetric DJF flow. The conventions are as in Fig. 6.

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jet waveguide is very clear with $m = 1–6$ trapped for both positive and negative frequency. In the equatorial easterlies, only long waves with negative frequency are permissible. For negative frequency, the signs of the weak waveguide near 20°S are seen.

A number of experiments have been performed with the zonally asymmetric DJF basic flow in the barotropic model, and the results from a few of these will be illustrated here. Figure 13 shows the day 7 vorticity perturbation for ±14 day period forcing at [20°N, 0°], the entrance to the North African–Asian jet waveguide. Compared with previous sections, this earlier day is chosen because the propagation is greatly enhanced in this stronger jet. The propagation along the waveguide is evident in both cases. The negative period response clearly has a larger wavelength, slightly enhanced group velocity and much enhanced meridional propagation. At day 10.5 (not shown) the positive period response has split in the East Pacific with the southern branch propagating across the equator and the northern branch propagating across North America. In contrast the negative period has less organization in the East Pacific and more general small amplitude response in the Southern Hemisphere.

To illustrate the sensitivity to the period of the forcing, the same results but for ±7 and ±30 day period forcings are given in Figs. 14 and 15, respectively. The difference in wavelength in the responses is enhanced.

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Fig. 14. Barotropic model vorticity anomalies at day 7 for (a) +7 and (b) −7 day period sources at [20°N, 0°] on a zonally asymmetric DJF flow. The conventions are as in Fig. 6.
Fig. 15. Barotropic model vorticity anomalies at day 7 for (a) +30 and (b) −30 day period sources at [20°N, 0°] on a zonally asymmetric DJF flow. The conventions are as in Fig. 6.

considered to be only in partial agreement with the results obtained here. However, their result was for a statistically steady state with equatorial forcing, which contrasts with the initial value approach and 20°N source used here.

All of the forcing experiments described so far have used sources with positive and negative $\omega$. This isolates the contrasting wave characteristics in the two cases. However, as discussed above, the solutions for oscillating stationary sources can sometimes be the most relevant, and such solutions can be obtained by addition and subtraction of the positive and negative frequency solutions. Because of the different wavelengths in, for example, the North African–Asian waveguide, there tends to be cancellation in certain regions and reinforcement elsewhere. This leads to a less obvious wave propagation picture, but a pattern that is evidently consistent with the source magnitude being zero at certain times. For example, the day 7 solution for $F'_1 \cos \omega t$ at [20°N, 0°] with a period of 14 days is given by half the sum of Figs. 13a,b. At this time there will be reinforcement near the source, cancellation near 135°, and a significant wave again eastward of this. The meridional propagation characteristics will be a mixture of the confined nature of the positive $\omega$ and the broader nature of the negative $\omega$ response. As discussed in section 3, the $F'_1 \sin \omega t$ solution may be obtained as the difference between Figs. 13a,b and will have strong reinforcement near 90°E. The solution for $F'_2 \sin \omega t$ with a period of 14 days requires separate computation and is shown in Fig. 17. The disappearance of the wave train from 0° to 50°E and the mixed characteristics are apparent.

7. Some baroclinic model results

All of the barotropic model integrations have been repeated with the baroclinic model described in section 3, and the validity of the general conclusions has been confirmed. For example, Fig. 18 shows the $\sigma \approx 0.24$, day 10.5 meridional wind for +7 day forcings at [20°N, 0°]. The meridional wind field is used for showing the Rossby wave field to be consistent with the other baroclinic model studies. The vorticity in the baroclinic model indicates the same propagation patterns but is a rougher field. Figure 18 may be compared with the corresponding day 7 vorticity fields from the barotropic model in Fig. 14. The nature of the response can be seen in Hovmöller and longitude–height plots of $v$ (not shown). For +7 day (−7 day) forcing it is found that the response in the longitudinal band 20°W–60°E is dominated by eastward (westward) moving equivalent barotropic Rossby waves. Beyond 60°E and particularly to the north of 30°N and after day 9, baroclinic instability with eastward phase speed dominates in both cases though its growth rate is larger for the eastward

Fig. 16. Barotropic model vorticity anomalies at day 7 for (a) +7 day and (b) −7 day period sources at [20°S, 0°] on a zonally asymmetric DJF flow. The conventions are as in Fig. 6.
moving source. More details of such features are given in Ambrizzi and Hoskins (1996). The major point here is that, in agreement with the barotropic theory and model results, the $-7$ day forcing produces a response with larger wavelength and enhanced meridional propagation. In fact it is seen in Fig. 18b that the $20^\circ$N forcing even produces a weak wave train response across the equatorial Atlantic southeastward and into the region of the Southern Hemisphere jet.

Figure 19 also shows the $\sigma \approx 0.24$, day 10.5 meridional wind for $\pm 7$ day forcings but this time centered at $[20^\circ S, 0^\circ]$. These results may be compared with the barotropic results given in Fig. 16. Again, the eastward moving source gives a shorter wavelength and a more meridionally confined wave train. The enhanced meridional propagation for the westward moving source again leads to a striking cross-equatorial propagation and a wave train along the North African–Asian jet.

8. Conclusions

In this paper we have considered the propagation of Rossby waves with positive or negative frequency, corresponding to eastward or westward phase propagation, respectively. In terms of the forced problem these waves can be considered to be initiated by localized forcing with eastward or westward phase propagation. A standing oscillatory source can be considered to be the sum of the positive and negative frequency sources.

The theoretical deductions made have been supported by initial value integrations with both barotropic and baroclinic models for particular source structures and positions, and damping parameters. Integrations with a range of other sources and damping parameters suggest the general validity of the deductions made.

The major results of this study are as follows.

(a) Eastward and westward moving Rossby waves share some general propagation characteristics with stationary Rossby waves. In particular the major jets still act as waveguides for them.

(b) Eastward moving (positive frequency) waves have shorter wavelengths and are more meridionally confined. Westward moving (negative frequency) waves have longer wavelengths and are less meridionally confined. In particular they exhibit enhanced cross-equatorial propagation. Standing oscillations would lead to waves with mixed characteristics.

The first of these conclusions is consistent with the fact that stationary Rossby wave theory has been generally successful in explaining the observed teleconnection patterns (Hoskins and Ambrizzi 1993; Hsu and Lin 1991; Ambrizzi et al. 1995).

Kiladis and Weickmann (1992), Hsu and Lin (1992), and Tomas and Webster (1994) find that cross-equatorial propagation is notable only in the equatorial eastern Pacific westerly wind duct and is associated with eastward moving waves in the extratropics. The second conclusion suggests that this is because of the predominance of such eastward moving systems in extratropical regions. However, more sensitive analyses might be used to look for evidence of upper-tropospheric cross-equatorial propagation of westward moving waves. The possibility of the kind of cross-equatorial propagation seen in Fig. 19, for example, certainly adds complexity to any attempt to seek the
phenomenon that acts as the source for any particular wave train.

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