The Role of Equatorial Waves Forced by Convection in the Tropical Semiannual Oscillation

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ABSTRACT

Recent satellite observations suggest that convection over the tropical continents is capable of exciting wave motions over a wide range of spatial and temporal scales. An equatorial beta-plane model was used to investigate the forcing by convective heating of equatorial waves with zonal wavenumbers from 1 to 15 and a wide range of periods, including diurnal oscillations. Also studied are the propagation of these waves in the equatorial middle atmosphere and their role in driving the tropical semiannual oscillation (SAO). Specification of the heating distribution used to force the model is guided by observations and analyses of tropical convection. It was found that intermediate-scale Kelvin and inertia-gravity waves provide between 25% and 50% of the forcing necessary to drive the westerly phase of the SAO near the stratopause, while the remainder is supplied by planetary-scale Kelvin waves. In the mesosphere, intermediate-scale waves account for an even larger fraction of the force required to drive the westerly phase and they are solely responsible for driving the easterly phase. The resulting SAO agrees well with ground-based and satellite observations in both the stratosphere and mesosphere. The dependence of the simulated SAO on various model parameters has also been explored. A simulation wherein only planetary-scale waves (k = 1±3) are included yields a weaker than observed stratopause oscillation and fails to produce a mesospheric oscillation. If the full range of zonal wavenumbers (k = 1±15) is included but the diurnal component of the forcing is omitted, the stratopause oscillation is again weaker than observed, while the amplitude of the mesospheric oscillation is greatly diminished. These results suggest that strong excitation of intermediate-scale equatorial waves depends on the diurnal cycle of convection and that the waves thus excited play an important role in the forcing of the tropical semiannual oscillation.

1. Introduction

The circulation of the equatorial upper stratosphere and mesosphere is characterized by a semiannual oscillation (SAO) of the zonal-mean zonal wind (e.g., Hiraoka 1978, 1980, and references therein), the period being set by the seasonal cycle at the stratopause level. The semiannual amplitude of the SAO is about 30 m s$^{-1}$ at the stratopause and mesopause; the phase is such that the mesospheric oscillation is 3 months out of phase with the stratospheric one. Dunkerton (1979) showed that the westerly phase of the stratopause oscillation can be generated and maintained by planetary-scale Kelvin waves forced with sufficient amplitude at a lower boundary, while Holton and Wehrbein (1980) suggested that easterly winds are the result of easterly momentum transport by the mean meridional circulation from the summer to the winter hemisphere. Dunkerton (1982) hypothesized that the zonal-mean zonal winds in the upper stratosphere act as a filter on a spectrum of gravity waves with westerly and easterly zonal phase propagation; therefore, the wave spectrum in the mesosphere will consist of waves with zonal phase velocities predominantly opposite to the stratospheric mean winds. From these considerations, the out-of-phase relationship between the stratospheric and mesospheric semiannual oscillations is to be expected. Contrary to the stratospheric oscillation, both phases of the mesospheric SAO descend with time. Several studies (e.g., Hamilton 1982; Hitchman and Leovy 1986; Vincent 1993; Eluszkiewicz et al. 1996) have confirmed that the stratospheric and mesospheric oscillations of the zonal mean zonal wind are a climatological feature of the middle atmosphere.

Numerical simulations of the SAO have been carried out with only partial success. While simpler (one- or two-dimensional) models have produced equatorial zonal-mean winds that resemble the observations (e.g., Gray and Pyle 1986; Garcia et al. 1992), general circulation models (GCMs) have not fared quite as well. Examples of the latter are the simulations of the stratopause SAO by Sassi et al. (1993), with the National Center for Atmospheric Research (NCAR) Community Climate Model (CCM2), and Hamilton et al. (1995),

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who used the Geophysical Fluid Dynamics Laboratory (GFDL) SKYHI model. Both simulations are unsatisfactory when compared to observations, although for different reasons. In the NCAR CCM2 simulation, westerly winds do not descend below the stratosphere at equinox; Sassi et al. (1993) showed that the westerly momentum forcing at equinox drops off rapidly at wavenumbers beyond those corresponding to planetary scales. On the other hand, the GFDL SKYHI simulation has westerly winds penetrating into the stratosphere, but their magnitude is rather small. Hamilton et al. (1995) did not present a momentum budget but, if the analysis of previous simulations (Hamilton and Mahlman 1988) still applies, the momentum forcing in the SKYHI model appears to be substantial at high frequencies and at zonal scales much smaller than the planetary. Regardless of the differences between the two GCMs, neither model produces sufficient westerly forcing in the upper stratosphere.

It is commonly believed that the discrepancy between the numerical simulations and the observations occurs because the planetary scales are not the sole contributors to the momentum budget at stratosphere level. This was originally pointed out by Hitchman and Leovy (1988), who used Limb Interferometer Monitor of the Stratosphere (LIMS) data to compute the total forcing exerted by all waves at the equator as a residual between the actual zonal-mean zonal wind tendency and the advection of momentum by the mean circulation (see their Fig. 8a). Then, they estimated the forcing due to planetary-scale Kelvin waves \( k = 1\pm3 \) (see their Fig. 8b) and showed that the two differ considerably at stratosphere level and above. More quantitatively, Hitchman and Leovy suggested that planetary-scale Kelvin waves account for about 30\%–70\% of the total forcing around the stratosphere level (the actual fraction depending on time and location). Below the stratosphere, the planetary scales seem to dominate the momentum budget. Although the satellite observations could resolve up to wavenumber 6, the authors noted that it was not possible to estimate the momentum forcing exerted by waves beyond the planetary scales because the temperature amplitudes were below the instrument noise threshold.

Hitchman and Leovy then argued that the missing forcing at stratosphere level could be due to small-scale gravity waves, which need to be parameterized in models. Without reliable synoptic information at small zonal scales, it appears impossible to ascertain the role played in the middle atmosphere by waves that cannot be resolved by polar orbiting satellites. However, vertically propagating equatorial waves in the Tropics are believed to be generated by convection and it is possible to establish a relationship between the underlying convection and the wave activity emerging at tropopause level. Salby et al. (1991) used radiances from the International Satellite Cloud Climatology Project (ISCCP) to infer the spectral characteristics of tropical convection. They showed that the frequency spectrum of brightness temperature (averaged over broad areas in the Tropics) manifests the “redness” typical of geophysical spectra; however, over the continents there is substantial power at high frequencies. Bergman and Salby (1994) used the spectrum of brightness temperature derived by Salby et al. (1991) as a proxy for tropical heating, determining the vertical scale of the forced waves from the assumed depth of the convecting layer in the troposphere and the horizontal wave structure by projecting the spatial distribution of convection on the horizontal normal modes on the sphere. Assuming fixed and uniform background properties for the troposphere, they were able to compute the wave activity emerging at tropopause level. Their analysis shows that, while the spectrum of temperature fluctuations is very red in space and time, momentum fluxes are significant over a wide range of temporal and spatial scales. The spectrum of the Eliassen–Palm (EP) flux at tropopause level (see their Fig. 13) has power concentrated along the two lobes corresponding to the first and second vertical projections of the convective heating (see Salby and Garcia 1987): the first projection contains most of the power and includes waves with zonal phase velocities in the range of 30–50 m s\(^{-1}\) and zonal wavenumbers between 1 and 20. Corresponding periods range from a few days to less than 1 day. The second vertical projection is centered at slower phase velocities (\( \sim 15 \) m s\(^{-1}\)) with substantial power stretching up to \( k = 100 \). Bergman and Salby (1994) also showed that high-frequency wave activity in the Tropics is topographically locked to the three regions of strongest convection (South America, Indonesia, and Africa) and that over one-half of the power occurs at periods shorter than 2 days. While it has been known since Ramage (1968) that tropical convection is concentrated in those regions, the novel feature identified by Bergman and Salby is the high-frequency content of the emerging momentum fluxes.

These results suggest that high-frequency gravity waves of zonal scales smaller than the planetary are possible contributors to the momentum budget of the equatorial middle atmosphere. In this paper we pursue the idea that tropical convection produces a rich spectrum of waves, and we study their propagation in the middle atmosphere and their interaction with the zonal mean zonal wind. For this purpose, we use a numerical model in which the forcing is specified following the analysis of Bergman and Salby (1994). In section 2, the model is described; in section 3, we present results from simulations of the SAO at the stratosphere and mesopause levels with different tropospheric sources. Section 4 gives some concluding remarks.

### 2. Model description

The model used in this study is a three-dimensional, quasi-nonlinear equatorial \( \beta \)-plane model. It solves the (zonal and meridional) momentum and temperature equations for both the eddies and the zonal-mean state.
There is no wave–wave interaction, but the eddies interact with the zonal-mean state through the convections/divergences of heat and momentum fluxes; the governing equations are based on Andrews et al. (1987; section 4.7). In this approach, the dynamics is assumed to take place on a plane centered on the equator, and the Coriolis parameter is linearized as

\[ f = \beta y, \]  

(2.1)

where \( \beta = 2\Omega/a \). \( \Omega \) is the earth’s angular frequency, and \( a \) is the earth’s radius. Wave modes in the equatorial \( \beta \)-plane are trapped in a waveguide in the neighborhood of the equator (see Gill 1982, chapter 11). Therefore, meridional motions decay away from the equator and, with proper treatment of the lateral boundaries, an accurate description of the dynamics is possible.

A drawback of the \( \beta \)-plane framework is the impossibility of modeling the global meridional circulation from the summer to the winter hemisphere. As noted earlier, the easterly phase of the stratopause SAO is controlled by advection of easterly momentum by the meridional circulation, which is in turn maintained by the seasonal cycle of heating and the extratropical wave-momentum forcing. Therefore, it is necessary to impose a momentum source at \(~50\) km to produce easterlies at solstice and establish the clock for the oscillation. The imposed easterly momentum is given a meridional profile that makes the resulting easterlies resemble observations: during northern winter the momentum source is centered at about \(10^\circ S\) with a maximum amplitude of \(~8\) m s\(^{-1}\) day\(^{-1}\); during southern winter the momentum source is at \(10^\circ N\) and has a maximum of \(~6\) m s\(^{-1}\) day\(^{-1}\). The reason for this configuration is to introduce a seasonal asymmetry at the equatorial stratopause and at the same time to avoid excessive inertial instability that would occur were the easterlies centered right on the equator.

The model’s meridional resolution is 300 km, levels are spaced in the vertical by 1 km, and a number of zonal waves can be modeled. We used 15 zonal wavenumbers in most of the simulations presented here. The model extends from the ground up to 120 km. A kinematic boundary condition is imposed at the ground and an absorbing layer is specified in the uppermost 20 km; at the top level, the vertical velocity is set to zero. Lateral dissipation poleward of 2000 km is also used to prevent reflection from the lateral boundaries. A stable simulation was obtained with a time step of 5 min.

**Forcing**

An important aspect of this model is the forcing of tropospheric waves that may then propagate upward into the middle atmosphere. The generation of waves from convection amounts to unsteady heating and the ensuing deformations of the isentropic surfaces, which may in turn generate internal waves in the fluid. Therefore, it is only necessary to specify a suitable heating rate in the eddy thermodynamic equations:

\[ \frac{\partial T'}{\partial t} + \cdots = H(x, y, t)F(z), \]  

(2.2)

where \( T' \) is the eddy temperature; \( H \) is a heating rate to be determined; \( F \) is a height profile; and \( x, y, z, \) and \( t \) are longitude, latitude, height, and time.

Salby et al. (1991) showed that the spectra of tropical convection over the ocean and continental landmasses are quite dissimilar at high frequencies. The different behavior of these heat sources needs to be included in order to ascertain its importance. We thus assume that the heating rate can be separated into two components, a contribution from the ocean and one from the continents:

\[ H(x, y, t) = H_o(x, y, t) + H_t(x, y, t). \]  

(2.3)

A further assumption is that \( H_o \) and \( H_t \) are separable in space and time:

\[ H_o(x, y, t) = S_o(x)T_o(t)Y_o(y), \]  

(2.4a)

\[ H_t(x, y, t) = S_t(x)T_t(t)Y_t(y). \]  

(2.4b)

The spatial structures \( (S_o, S_t) \) are chosen to be Gaussian in longitude; since the model is spectral in the zonal direction, this simplifies the algebra as well, making it straightforward to obtain the Fourier components of the heating. We let

\[ S_o(x) = A_o \exp \left( \frac{-x^2}{2 \lambda_o^2} \right), \]  

(2.5)

where \( A_o \) is the peak amplitude of the localized oceanic heating (to be determined later) and \( \lambda_o \) the forcing half-width in longitude. If \( \lambda_o \ll 2\pi a \), then the corresponding Fourier amplitude at zonal wavenumber \( k \) will be given approximately by

\[ \hat{S}_o(k) \approx \frac{A_o \lambda_o \sqrt{2\pi}}{2\pi a} \exp \left( \frac{-k^2 \lambda_o^2}{2} \right). \]  

(2.6)

Similarly, for the continental forcing,

\[ S_t(x) = A_t \exp \left( \frac{-x^2}{2 \lambda_t^2} \right), \]  

(2.7)

where \( A_t \) is the peak amplitude of the localized continental heating and \( \lambda_t \) the forcing half-width in longitude. As for (2.6), we obtain

\[ \hat{S}_t(k) \approx \frac{A_t \lambda_t \sqrt{2\pi}}{2\pi a} \exp \left( \frac{-k^2 \lambda_t^2}{2} \right). \]  

(2.8)

The temporal evolution \( (T_o, T_t) \) of the heating must have frequency spectra that resemble Salby et al.’s (1991; see their Fig. 9). In order to reproduce the redness of those spectra, a Markov autoregressive process of second order is used. Since the correlation coefficient determines the amount of power at low frequency, we
conducted some tests with different values and compared the results to Salby et al.’s spectra; we found that, for a correlation coefficient at lag 1 day of about 0.5, we could reproduce the observed spectrum of brightness temperature. Besides this variability, the observed spectrum of brightness temperature over the continents has a strong diurnal cycle. This aspect of the heating has been reproduced by introducing a deterministic oscillation of period 1 day. More specifically,

\[ T_v(t) = rT_v(t - \Delta t) + (1 - r)\mu + \epsilon, \]

\[ T_c(t) = rT_c(t - \Delta t) + (1 - r)\mu + \epsilon + \sqrt{2}\cos\Omega t, \]

where \( r \) is the correlation coefficient (\( r = 0.5 \) at lag 1 day), \( \Delta t \) is the time step, \( \mu \) is the mean of the stochastic process (\( \mu = 0 \)), \( \epsilon \) is a random process with variance \( \langle \epsilon^2 \rangle = 1 - r^2 \), and \( \Omega \) is the diurnal frequency (\( \Omega = 2\pi / 86400 \text{ rad s}^{-1} \)). With these choices of parameters, the variances of both the autoregressive and the deterministic processes equal unity. The spectrum from (2.10) is shown in Fig. 1; it resembles the spectrum shown in Fig. 8 of Salby et al. (1991).

The meridional structures \((Y_v, Y_c)\) consist of a Gaussian envelope of meridional half-width \( \sim 850 \text{ km} \) for both components. Sensitivity tests conducted with broader forcing suggested that the structure and response of the waves and of the zonal-mean state are quite similar for comparable latitudinally integrated forcing. The position of the heating in the north–south plane changes according to the season, drifting from 10°S during the northern winter to the equator at equinoxes and to 10°N during southern winter. The heating is modulated in height by a simple sinusoidal function of height,

\[ F(z) = \sin \left( \frac{\pi}{D}(z - z_0) \right) \text{ for } z \leq 12 \text{ km}, \]

where \( D \) is the depth of the forcing set to 12 km and \( z_0 \) is the lowest level of heating, chosen to coincide with the ground.

In order to determine the amplitudes \((A_v, A_c)\) and zonal scales \((L_v, L_c)\) of the forcing, we used some guidance from observations. Since Bergman and Salby showed that, for the continental heating source, about one-half of the total power is contained in the high-frequency components, it may be expected that the oceanic component is mainly responsible for the generation of the planetary scales. We also know from satellite observations (Salby et al. 1984; Hitchman and Leovy 1988; Canziani et al. 1994) that time-mean wave amplitudes in the tropical upper stratosphere are about 1 K at zonal wavenumber 1 and substantially less at zonal wavenumbers \( \geq 4 \). Several authors (Hitchman and Leovy 1988; Canziani et al. 1994) have noted that, for time-mean amplitudes much below 1 K, the wave structures are difficult to detect in the presence of measurement noise. With these facts in mind, we ran the model with only the oceanic forcing (2.4a) and we tuned the parameters \((A_v, L_v)\) to obtain stratospheric wave amplitudes at \( k = 1 \) of about 1 K and substantially less at \( k = 4 \). We found that planetary wave amplitudes consistent with observations could be obtained with \( A_v = 13 \text{ K day}^{-1} \) and \( L_v = 2000 \text{ km} \). Nevertheless, it will be shown in the following discussion that, if the strong diurnal component of the continental forcing is omitted, the model does not produce a realistic SAO. It turns out that it is necessary to force the continental component (2.4b) with \( A_c = 20 \text{ K day}^{-1} \) and \( L_c = 200 \text{ km} \) to obtain a satisfactory simulation.

Although it is not a simple task to determine the heating amplitude in the atmosphere on the scales that are of interest to this study, we can try to compare our choices for \( A_v \) and \( A_c \) to observations. Houze (1982) estimated that the peak amplitude of the total heating associated with convective towers (occupying an area of about \( 10^8 \text{ km}^2 \)) could be as large as 28 K day\(^{-1} \) (see his Fig 12). However this estimate corresponds to a single squall line over the ocean, whereas the forcings in (2.4a)–(2.4b) are meant to model the large-scale “envelopes” of convective activity. It is more instructive to compare the precipitation rates implied by (2.4a) and (2.4b) to known climatologies. If we estimate the root-mean-square heating in the volume of atmosphere between the ground and 12 km, 10°N and 10°S, and average in the zonal direction, the total expected precipitation is \( \sim 280 \text{ mm yr}^{-1} \) from the continental forcing and \( \sim 920 \text{ mm yr}^{-1} \) from the oceanic. Since we do not include a time-mean heating, these values represent the contribution from the variable component only; however, they do not exceed the values inferred from climatology [e.g., Fig. 7.25 in Peixoto and Oort (1992)], which include both time-mean and variable contributions.

In the following we present first a simulation with all the components of the forcing included; we discuss the zonal-mean simulation and the properties (in space and

![Fig. 1. Frequency spectrum of continental heating. The frequency is in units of cycles per day (cpd). The power is normalized using the value at 1 cpd.](Image)
Fig. 2. (a) Equatorial zonal-mean wind during the first year of the simulation, as a function of height (in km). Time is expressed as the month of the simulation, with the tags denoting the middle of each month. Contour interval is 10 m s$^{-1}$. (b) The EP flux divergence; contours are ±1, ±2, ±4, ±8, ±10 m s$^{-1}$ day$^{-1}$.

Fig. 3. Amplitude (continuous line) and phase (dashed line) of the Fourier semiannual component of the equatorial zonal-mean wind as a function of height. The scale for the amplitude is shown in the lower axis, while the upper axis shows the scale for the phase.

3. Results

a. Case 1: Simulation with the complete wavenumber spectrum

Figure 2a shows the equatorial zonal-mean zonal wind as a function of height and time, during the first year of a simulation (the model was run for a second year but there are no remarkable interannual differences) with the complete spectral forcing of oceanic and continental components [Eqs. (2.4a) and (2.4b)]. Figure 2b shows the corresponding EP flux divergence. The zonal-mean simulation compares well to observations (e.g., Hirota 1978; Hitchman and Leovy 1986; Garcia et al. 1997, manuscript submitted to *J. Atmos. Sci.*, hereafter GVLD): there is an oscillation in the zonal-mean zonal wind around the stratopause with westerlies descending at equinoctial in the upper stratosphere; at any given time, the oscillation at about 85 km has the opposite sign of the one at 50 km. The amplitude of the semiannual component of the zonal-mean zonal wind (Fig. 3) is
about 33 m s\(^{-1}\) at both levels (peaks are located at \(\approx 55\) and 82 km) and the phase difference is about 180° between them. A node at \(\approx 68\) km separates the two peaks.

The EP flux divergence (Fig. 2b) shows westerly forcing in the upper stratosphere at the time of the descending westerlies at equinox. At the 50-km level, the maximum westerly force is about 2 m s\(^{-1}\) day\(^{-1}\). This increases to 4 m s\(^{-1}\) day\(^{-1}\) in the lower mesosphere and falls to less than 1 m s\(^{-1}\) day\(^{-1}\) below 45 km; these values are in good agreement with Hitchman and Leovy’s (1988) findings (see their Fig. 8a). In the mesosphere, westerly and easterly forces are present at different times. The easterly force attains larger intensity (up to \(\approx 8\) m s\(^{-1}\) day\(^{-1}\)) than the westerly force (4 m s\(^{-1}\) day\(^{-1}\)). The more intense easterly forces in the mesosphere are associated with easterly waves. It will be shown below that the easterly modes are high-frequency inertia–gravity waves, which propagate vertically more rapidly than the planetary-scale westerly waves; therefore, the easterly forces are substantial only at the upper levels where the amplitude of the corresponding waves is sufficiently large. In Fig. 2, there is also some asymmetry in the two cycles of the SAO, although not as pronounced as in observations (e.g., Delisi and Dunkerton 1988; GVLD).

In order to obtain information on the nature of the waves that contribute to the momentum budget, we separate the total EP flux divergences into planetary scales (wavenumbers 1–3) and intermediate scales (wavenumbers 4–15); the results are shown in Fig. 4 for levels between 30 and 60 km. Planetary scales are as important as the intermediate scales around the stratopause, both producing westerly forces of about 1 m s\(^{-1}\) day\(^{-1}\) in the late equinox. While both components of the EP flux divergence decrease below 50 km, the planetary scales (Fig. 4a) contribute a larger share of the EP flux divergence than the intermediate scales (Fig. 4b) in the upper stratosphere. The opposite occurs in the lower mesosphere, where forcing due to intermediate-scale waves is larger (\(\geq 2\) m s\(^{-1}\) day\(^{-1}\)) than that produced by planetary-scale waves (\(\leq 1.5\) m s\(^{-1}\) day\(^{-1}\)). This is summarized in Fig. 5, which shows the planetary-scale forcing as a fraction of the total (see figure caption for details): the lighter the shade the more important the planetary scales are. Figure 5 indicates that, in the upper stratosphere at equinox (between 40 and 50 km), the planetary-scale EP flux divergence accounts for over to 75% of the total; at stratopause level and above, the EP flux divergence contains a rapidly increasing contribution from intermediate-scale waves, as indicated by the darker shades.

As a spectrum of waves propagates through time- and height-varying background winds, it undergoes selective absorption. Therefore, it is expected that the spectral composition of the EP flux divergence will change accordingly. In order to investigate this point, we now examine the forcing exerted on the zonal-mean zonal wind by each zonal component. The contribution to the EP flux divergence by each zonal wavenumber at stratopause level is shown in Fig. 6 as a function of time.
The forcing is positive (westerly) at all times, with the largest magnitudes at zonal wavenumber 2 and 3 (≈0.9 m s\(^{-1}\) day\(^{-1}\) in late April). At higher zonal wavenumbers (\(k > 10\)), the EP flux divergence is smaller (≈0.05 m s\(^{-1}\) day\(^{-1}\) during equinox); however, it should be noted that EP flux divergence of this magnitude extend to about \(k = 15\), suggesting that, had we run the model with more zonal waves, the total contribution of the intermediate-scales forcing probably would have been larger. At 79.5 km (Fig. 7), the EP flux divergence shows a different structure: at wavenumbers \(k > 6\), the forces are mostly easterly (up to ≈1.5 m s\(^{-1}\) day\(^{-1}\)), while at planetary scales they are still westerly but of comparable magnitude (about 1 m s\(^{-1}\) day\(^{-1}\)).

The consistent westerly forcing at planetary scales at all levels is a strong indication that Kelvin waves are predominant at these scales throughout the stratosphere and mesosphere. On the other hand, the intermediate scales contribute both to westerly and easterly momentum forcing at different levels and times, suggesting that inertia–gravity waves are involved. In order to ascertain the modal composition of the wave field, we need to look at the actual wave components. It is known (see Andrews et al. 1987) that the solutions of the equatorial \(\beta\)-plane equations in uniform background conditions are symmetric and antisymmetric Hermite polynomials, these terms referring to symmetry with respect to the equator.\(^1\) Salby et al. (1984) showed that in LIMS data planetary-scale eddy temperature is prevalently symmetric in the stratosphere. In our simulations both symmetric and antisymmetric modes are present; the latter are most prominent during the solstices (when the heating is off the equator) and disappear at the equinoxes (when the heating is centered on the equator). In the interest of brevity, the following discussion concentrates on the symmetric modes (which are present year-round and that dominate the behavior in the stratosphere and lower mesosphere).

Figure 8 shows the magnitude of the symmetric temperature and antisymmetric meridional velocity at zonal wavenumber 1, for westerly and easterly waves (see figure caption for details), as functions of latitude and height. These fields represent a Fourier synthesis for 1 month centered about the second solstice of Fig. 2a: the computed fields were Fourier-analyzed in time, the westerly and easterly components separated according to the method of Hayashi (1971), and the modulus of each component plotted as a function of latitude and height. Clearly, the westerly components of the temperature field (Figs. 8a, b) dominate over the easterly ones (Figs. 8c, d), and the \(v\) field is very small for both components. The temperature amplitude of the westerly waves (Fig. 8a) attains a maximum at the equator in the upper stratosphere (≈1.2 K), but other local maxima are observed in the mesosphere; the amplitude grows with height substantially more slowly than \(e^{\theta/2H}\) because the waves are damped strongly by Newtonian cooling when they encounter the time-mean easterlies in the lower mesosphere. Closer inspection of the latitude–height profiles of the zonal-mean zonal wind (not shown) indicates that the largest amplitudes of \(T\) in Fig. 8a are associated with weakening westerly mean winds at the same levels. It appears from Figs. 8a, b that equatorially trapped Kelvin waves are the largest modal contributions at planetary scales. Our results are in acceptable agreement with observations at stratosphere level (Salby et al. 1984; Canziani et al. 1994). However, contrary to observations, the modeled temperature in the lower stratosphere is about twice as large as that observed. Canziani et al. (1994) report a temperature amplitude at 20 mb (≈30 km) of about 0.3 K for \(k = 1\), while we have 0.5–0.6 K in Fig. 8a. Interestingly, the temperature field due to planetary-scale waves in the lower stratosphere is also overestimated in the GCM simulation of Sassi et al. (1993).

It is possible to estimate the Rossby radius of deformation \(R_R\) from the width of the Gaussian envelope in Fig. 8a. In the equatorial \(\beta\)-plane the Rossby radius is defined as

\[ R_R = \frac{1}{\beta} \sqrt{\frac{1}{2 \pi} \int \left( T - T_0 \right)^2 \, d\theta} \]

\(^1\) In the following we associate the symmetry of wave modes with that of the temperature field, remembering that the meridional velocity always has the opposite symmetry.
$R_d = \sqrt{\frac{N}{\beta m^2}}$  \hspace{1cm} (3.1)

where $N$ is the buoyancy frequency and $m$ is the vertical wavenumber. At 45 km $R_d \approx 1550$ km, corresponding to a vertical wavelength of 17 km; at 85 km, $R_d \approx 1750$ km and the vertical wavelength is 22 km. Above 90 km, $R_d$ is over 2000 km and the vertical wavelength is about 30 km. The value of $R_d$ is a measure of the distance from the equator where the trapping takes effect (see Gill 1982); via the dispersion relationship (longer vertical wavelengths $\Rightarrow$ higher frequencies), the increase
of $R_d$ with height implies that the phase velocity of the waves also increases with increasing altitude.

In Fig. 9 we show the spectra of symmetric temperature at zonal wavenumber 1 as a function of frequency and height for westerly (Fig. 9a) and easterly (Fig. 9b) waves. Significant power is present only in the westerly component, in agreement with the findings in Fig. 8. There is a clear tilt with height of the spectrum, consistent with the increase of $R_d$ with altitude noted above. The dominant period is about 10 days in the lower strat-
osphere, while in the upper mesosphere it can be as small as 3 days. The frequency shift is more marked in the lower mesosphere, where Kelvin waves encounter time-mean westerlies and the slowest waves are dissipated. The retrogressive components have negligible power below the upper mesosphere. These spectra compare well with Canziani et al.’s (1994) and Salby et al.’s (1984) findings from Upper Atmosphere Research Satellite (UARS) and LIMS observations, respectively; they are also consistent with the notion that selective absorption of low-frequency components shifts the spectral variance to higher frequencies with increasing altitude (Garcia and Salby 1987).

In Fig. 10 we show the height–latitude profiles of temperature and meridional velocity at zonal wavenumber 4. The westerly component (Figs. 10a,b) has a maximum temperature amplitude of 0.4 K at the stratopause, with negligible meridional velocity. As in the case of zonal wavenumber 1, we conclude that the stratospheric response is characterized by a Kelvin wave signature. In accord with observations (Hitchman and Leovy 1988), the temperature at $k = 4$ is considerably smaller than at $k = 1$; however, the zonal mean forcing (Fig. 6) associated with the former is not negligible compared to the latter. At higher levels, both $T^r$ and $v^r$ attain larger amplitudes: temperatures are in excess of 1 K and meridional velocities are about 1 m s$^{-1}$ between 80 and 90 km. The presence of a sizable meridional velocity is an indication of inertia–gravity modes at these heights. The easterly component (Figs. 10c,d) is negligible in the stratosphere and most of the mesosphere; it attains significant amplitude only at the uppermost levels, with temperature $\sim$ 1 K and meridional velocity $\sim$ 1.2 m s$^{-1}$. In the light of the EP flux divergence results (Fig. 7), it appears that a mixture of inertia–gravity and Kelvin modes is present in the mesosphere; the latter account for the positive (westerly) zonal mean forces, the former for positive and negative (easterly) forces depending on the height and the time of the year.

In Fig. 11, we show spectra of symmetric temperature at zonal wavenumber 4. The westerly modes (Fig. 13a) have a broad frequency spectrum throughout the stratosphere; the latter account for the positive (westerly) zonal mean forces, the former for positive and negative (easterly) forces depending on the height and the time of the year. However, when they encounter mesospheric westerlies above about 60 km (cf. Fig. 2), a shift to higher frequencies occurs because power associated with longer period waves is dissipated. At higher altitudes ($\sim$ 90 km), there is a second shift to even higher frequencies, with most of the power now appearing at periods between 2 days and 1 day. The easterly modes (Fig. 11b) show a strong response at 1 cpd, clearly due to the presence of the diurnal oscillation in the tropospheric heating; however, the power at other frequencies is negligible until waves reach the upper mesosphere, where the spectrum broadens, with substantial power at frequencies in excess of 1 cpd.

In Fig. 12, we show the symmetric temperature and antisymmetric meridional velocity at zonal wavenumber 11. Temperature of the westerly waves (Fig. 12a) reach a maximum of 0.5 K at about 70 km; meridional velocities (Fig. 12b) at that level are about 0.5 m s$^{-1}$ but they attain the largest amplitudes (1.3 m s$^{-1}$) at 90 km. The easterly waves have larger temperature amplitudes in the upper mesosphere ($\sim$ 1.5 K at 95 km, Fig. 12c),
and $v'$ is about 1 m s$^{-1}$ at the same level (Fig. 12d). The height–latitude profiles indicate the presence of inertia–gravity waves (easterly and westerly modes) in the mesosphere and Kelvin waves in the stratosphere.

Figure 13 shows spectra of symmetric temperature for $k = 11$ as a function of height and frequency. In both directions of zonal propagation, the diurnal signal is evident; at this zonal scale, waves are present predominantly at very high frequencies (larger than 1 cpd) and they are little affected by thermal dissipation. In accord with Fig. 12, the largest power is attained by the easterly modes. The large wave amplitudes between 90 and 100 km are not an artifact due to reflection off the sponge layer that becomes effective just above the 100 km level; in a simulation in which the sponge layer does not start until 110 km, the amplitudes remain nearly the same.

**b. Case 2: Simulation with planetary-scale waves only**

In our model it is possible to separate the role played by the planetary-scale waves in driving the zonal-mean circulation in the middle atmosphere from the effect of all other waves. In order to do so, we ran the same model as in Case 1 but with the number of zonal waves restricted to three; all the other parameters in the tropospheric forcing are unchanged. In Fig. 14 we show the equatorial zonal-mean zonal wind from this simulation as a function of time and height. The following is evident when comparing Case 2 to Case 1.

- The stratopause SAO is weak in Fig. 14; the westerly winds barely reach the 55 km level and the zero-wind line is not able to break through the time mean easterlies at 50 km. Morphologically, Case 2 is very similar to the results obtained by Sassi et al. (1993) using the NCAR CCM2.
  - The time-mean westerlies at 70 km are not very different from the ones of the earlier case.
  - Time-mean westerlies are present throughout the mesosphere; the semiannual oscillation at 85 km of Case 1 is completely lost in Case 2.

The reason for the weak descent of the westerlies at stratopause level is the insufficient forcing exerted by the planetary-scale waves. According to the results for Case 1, the descent of the equinoctial westerlies in the stratosphere is due to the combined presence of the planetary and intermediate scales. Figure 14 supports Hitchman and Leovy's (1988) contention that the planetary scales alone cannot maintain an SAO at 50 km. Had the amplitude of the planetary scales been larger, the westerly phase of the stratopause SAO would have been more intense; however, Sassi et al. (1993) showed that, even when the calculated amplitudes are $\sim 60\%$ larger than observed, the westerly phase is still weak. Therefore, with acceptable amplitudes for planetary-scale waves ($k = 1–3$), it is not possible to obtain a realistic SAO at the stratopause.

The reason for the disappearance of the mesospheric oscillation in Fig. 14 is obvious from the modal composition of the planetary-scale waves. It was shown in Case 1 (see Fig. 8) that the westerly waves at planetary scales consist of Kelvin waves while the easterly component is negligible. Since Kelvin waves transport only
westerly momentum, easterlies should not be expected at any level. Without a source of easterly momentum, it is not possible to obtain a mesospheric SAO from the planetary-scale waves only.

c. Case 3: Simulation without diurnal forcing

Sassi and Garcia (1994) were able to obtain a reasonable SAO in the stratosphere and mesosphere in a one-dimensional model wherein a single intermediate-scale wave \((k = 12)\) was forced by convective heating having a frequency spectrum similar to the one used here; the depth of the heating distribution was 12 km, the same as in the present calculations. This heating depth projects preferentially onto waves with vertical wavelength of about 24 km. The choice of zonal wavenumber 12 thus ensured a strong response to diurnal forcing, since the dispersion relation (in the absence of

![Fig. 12. As in Fig. 8, but for zonal wavenumber 11.](image)

![Fig. 13. As in Fig. 10, but for zonal wavenumber 11.](image)
rotation) predicts a period of about 1 day at $k = 12$ and a vertical wavelength of 24 km. Not surprisingly, the presence of a strong diurnal component was then necessary to produce a realistic SAO. However, we have shown above that, in the present model, the response in the middle atmosphere includes a broad range of frequencies and wavenumbers, so strong diurnal forcing may not be necessary to produce the SAO.

To test the role of the diurnal component of the heating, we repeated the calculation of Case 1 without diurnal forcing [i.e., without the last term in Eq. (2.10)]. Figure 15 shows the equatorial zonal-mean zonal wind and the corresponding EP flux divergence. The SAO at stratopause level is able to break through the solsticial easterlies, although the descent of the equinoctial winds is not as deep as in Case 1. The wave forcing at stratopause level is about 1 m s$^{-1}$ day$^{-1}$, becoming negligible below the 50-km level. There are time-mean westerlies in the mesosphere associated with positive (westerly) accelerations of about 1 m s$^{-1}$ day$^{-1}$ throughout. Easterly forcing is present only in the uppermost level (up to 2 m s$^{-1}$ day$^{-1}$) but is not sufficient to produce and maintain a strong SAO in the upper mesosphere. The EP flux divergence due to planetary-scale waves is not different from the one in Case 1 and is not shown for this simulation. Instead, in Fig. 16 we show the ratio of the wave forcing due to planetary scales to the total wave forcing (this can be compared to Fig. 5). Lighter shadings predominate at most times and levels, with darker ones only near the 60-km level, indicating that planetary waves are responsible for most of the forcing in the stratosphere and lower mesosphere. Evidently, without diurnal forcing, intermediate-scale inertia–gravity modes do not attain large enough amplitudes to play a role at the stratopause level. This is clearly seen in Fig. 17, which shows the EP flux divergence at stratopause level as a function of time and zonal wavenumber. At the planetary scales, the wave forcing is essentially the same as in Case 1 (magnitudes are somewhat smaller and a time lag is present, both effects resulting from the weaker descent of the westerlies); the strongest forcing is about 0.3 m s$^{-1}$ day$^{-1}$ at zonal wavenumber

![Fig. 14. As in Fig. 2a, but for Case 2.](image1.png)

![Fig. 15. As in Fig. 2, but for Case 3.](image2.png)
2. Even at zonal wavenumber 4, there seems to be little difference between the two simulations; it is the lack of any momentum forcing during equinox at the intermediate scales that constitutes the largest difference. Higher up, at 79.5 km (Fig. 18), only westerly forcing is present at the planetary scales but, in contrast to Fig. 7, no significant easterly momentum source is present at any time and zonal scale. Another substantial difference from Case 1 is the presence of westerly forcing at all times, rather than only at solstices.

The absence of accelerations at the intermediate scales in Fig. 18 results from a deficit of diurnal forcing. This point is illustrated further in Fig. 19, which shows the dispersion relationship curves for several equatorially trapped modes (see figure caption for details). Insofar as the spectrum of the forcing has most of the power at the lowest frequencies, a large part of the projection will occur at the planetary scales where the Kelvin mode is dominant. This is in accord with the findings in Fig. 8. On the other hand, intermediate scales are forced more efficiently at higher frequencies; the modal response at $k > 6$ occurs at periods of 2 days or shorter. Therefore, the lack of strong forcing at the diurnal frequency prevents these waves from attaining large amplitudes in the middle atmosphere.

d. Effect of other model parameters

As noted above, the semiannual periodicity of the zonal wind oscillation in the stratosphere and mesosphere arises from the seasonal cycle of the global mean meridional circulation, which advection zonal mean easterlies across the equator at the solstices. In our $\beta$-plane model, this behavior must be introduced by specifying a seasonally varying drag that produces easterly winds at the solstices. Thus, the role of easterly winds in the upper stratosphere and lower mesosphere during the solstices can be tested by removing from the model the specified easterly drag.

We conducted an experiment in which the seasonal
In Case 3, the easterlies at stratopause were absent. In this case, the zonal-mean zonal wind response is shown in Fig. 20. It consists of time-mean westerlies of about 20 m s⁻¹ in the upper stratosphere and lower mesosphere; in the upper mesosphere, easterly and westerly zonal-mean zonal winds alternate with a period of 2–3 months. This behavior can be understood in terms of the dispersion relation for vertically propagating equatorial waves (Fig. 19), which show that, at frequencies lower than about

![Dispersion Curves](image1.png)

**Fig. 19.** Dispersion curves for equatorially waves with vertical wavelength of 24 km plotted as a function of (dimensionless) zonal wavenumber and frequency (cpd). The labels identify the modes corresponding to each curve: $K$ is the Kelvin mode; MRG is the mixed Rossby–gravity mode; and $n = 1, 2, 3$ are the meridional indices of the inertia-gravity modes. Note that odd indexes correspond to the symmetric modes and even indexes to the antisymmetric modes.

![Equatorial Wave-Driving](image2.png)

**Fig. 18.** As in Fig. 7, but for Case 3. Contour interval is 0.1 m s⁻¹ day⁻¹.

![Equatorial Zonal-Mean Zonal Wind](image3.png)

**Fig. 20.** Equatorial zonal-mean zonal wind during the second year of a simulation with no seasonal cycle at the stratopause. Contour interval is 10 m s⁻¹.
0.2 cpd (periods longer than 5 days) only the Kelvin mode is excited efficiently. Since, apart from the diurnal signal, forcing power is concentrated at low frequencies (cf. Fig. 1), the vertical flux of eddy momentum in the model must be biased toward positive (westerly) values. The larger westerly momentum flux associated with the Kelvin waves should then produce predominantly westerly forcing at lower altitudes. This expectation is borne out by Fig. 6, which shows that the wave forcing at stratopause level is in fact always westerly. At higher altitudes, on the other hand, the contribution of the inertia–gravity waves becomes important, and there is both westerly and easterly forcing (Fig. 7).

These considerations imply that the westerly wind layer below about 70 km in Fig. 20, as well as the westerly layer seen in Case 1 (Fig. 2) between about 70 km and the top of the solstitial easterlies, are the result of the preponderance of westerly forcing below 70 km. The interpretation is consistent with the modeled (and observed) node in the amplitude of the SAO near 65 km (cf. Fig. 3). This altitude lies above the region influenced by the semiannual varying stratopause easterlies but below the range of altitudes where easterly and westerly forcing by inertia–gravity waves becomes important. Comparison of Figs. 2 and 20 also indicates that the stratopause SAO is the result of the interaction between the seasonal cycle (which advects easterlies across the equator at the solstices) and westerly forces due to (planetary- and intermediate-scale) Kelvin waves, which tend to produce westerly winds throughout a broad layer between about 40 and 70 km. However, the latter are able to produce westerlies at and below the stratopause only near the equinoxes, when the cross-equatorial advection of mean easterly momentum vanishes.

It is also evident that the presence of easterlies near the stratopause at the solstices has a profound influence on the behavior of the mesopause oscillation. As noted previously, without the stratopause easterlies, the direction of the mesospheric zonal wind varies with a period of less than 3 months. Clearly, the filtering effect of the stratopause easterlies produces a semianurnal oscillation at the mesopause by removing easterly waves during the solstices.

We have also tested the effect of the meridional excursion of the convective heating as a function of season. Recall that the center of the heating distribution moves from 10°N in northern summer to 10°S in southern summer, to mimic the observed seasonal migration of the region of deep convection. In a model run wherein we kept the heating distribution centered on the equator throughout the entire year, the seasonal behavior of the zonal wind in the stratosphere and mesosphere (not shown) was qualitatively the same as that reported in section 3a. Since the antisymmetric inertia–gravity modes are not forced in this case, the result implies that the momentum flux due to the symmetric modes alone is sufficient to produce a realistic SAO.

4. Summary and conclusions

It has been known for some time that tropical convection displays complex behavior in space and time (Ramage 1968; Gruber 1974; Gray and Jacobson 1977; Salby and Garcia 1987; Salby et al. 1991; Bergman and Salby 1994). The study of Salby and Garcia (1987) showed that the near-field response to convective heating is determined by the projection of the forcing upon a broad range of scales and periods, while the far-field response (Garcia and Salby 1987) is to a large extent ascribable, for a given input spectrum, to the properties of the background atmosphere through which the waves propagate. The almost complete lack of data on wave scales other than the planetary has led to a substantial underestimation of the momentum forcing exerted by waves in the equatorial middle atmosphere. In this paper we have argued that the “missing” momentum can be attributed, at least in part, to waves whose zonal wave-length lies between those of planetary-scale waves and small-scale gravity waves. Our investigation has been motivated by evidence, derived from analyses of satellite cloud imagery, that the wave activity at tropopause level has a strong response for waves with zonal phase speeds in the range 30–50 m s⁻¹ (Bergman and Salby 1994). At these speeds, waves can reach high altitudes with relatively little dissipation.

The equatorial trapping of vertically propagating modes allows for the study of these waves using a β-plane channel model, assuming that we have a good knowledge of the spectrum generated in the lowest levels near the equator. We used a quasi-linear equatorial β-plane model that extends from the ground up to 120 km and is forced by specifying an unsteady convective heating distribution in the model’s troposphere. We projected the heating onto a spectrum of zonal wave components, with the Fourier zonal decomposition determined by the imposed spatial structure of the forcing. Because of the different characteristics of the spectrum of convection over land and ocean, two components were included in the heating distribution, one characteristic of oceanic regions and the other of continents. The spatial and temporal behavior of each component was specified in accordance with observations. Although it is not possible to determine a priori the actual amount of heating, guidance was used from studies of mesoscale convection in the tropics and satellite evidence of the wave amplitudes in the middle atmosphere. The values of heating used in this study are consistent with climatology (e.g., Peixoto and Oort 1992).

The results of a model simulation (Case 1) indicate that a realistic SAO can be obtained throughout the stratosphere and mesosphere. The analysis of the spatial and temporal structures of wave modes contributing to the zonal-mean momentum budget in the middle atmosphere show that planetary (k = 1–3) and intermediate (k = 4–15) scales contribute equally to the stratopause SAO; however, at higher levels the latter be-
come increasingly important. The modal composition is such that in the stratosphere planetary-scale Kelvin waves are dominant. In the mesosphere, the westerly accelerations are associated with planetary- and intermediate-scale Kelvin waves, while the easterly accelerations are due mostly to inertia–gravity waves at intermediate scales. It is worth noting that the latter occur at zonal scales ($k > 6$) and frequencies ($>1$ cpd) not resolvable by single polar-orbiting satellites. If these waves are actually present in the atmosphere with sufficient amplitudes, aliasing of the observed fields should be expected from satellite observations in the upper mesosphere.

A simulation (Case 2) was also presented in which only the planetary scales were included. The resulting stratopause SAO is much weaker and the mesopause oscillation is completely missing in this case. The weaker stratopause oscillation is consistent with observational evidence (Hitchman and Leovy 1988) of insufficient eastward momentum forcing near 50 km. The lack of a mesopause SAO points to the intermediate-scale waves as a source of westward momentum at higher levels. Finally, a simulation (Case 3) in which the diurnal component was removed from the continental heating spectrum shows a somewhat weaker SAO at the stratopause and a substantial deficit of easterly forcing at higher levels. It was shown that intermediate-scale waves are solely responsible for the easterly forcing in the mesosphere and that the heating must have a strong diurnal component in order to produce the amplitudes necessary to generate and maintain the mesospheric SAO.

We also investigated the role of the seasonal cycle in the generation of the SAO. The seasonally varying mean meridional circulation advects easterly momentum during the solstices and produces easterly winds in the vicinity of the stratopause. In our β-plane model, we simulate this mechanism by specifying an easterly drag that maximizes at the solstices (see section 2). An experiment in which we removed the easterly drag produced a layer of time-mean easterlies between 40 and 70 km, and alternating easterly and westerly wind regimes above 70 km (Fig. 20).

As noted in section 3d, these results may be understood in terms of the relative intensity with which easterly and westerly wave components are forced. In particular, strong forcing at periods longer than 5 days excited only Kelvin waves (cf. Fig. 19) and thus biases the vertical eddy momentum flux toward positive (westerly) values. Thus, forcing in the upper stratosphere and lower mesosphere is predominantly westerly, whereas both easterly and westerly forces are present in the upper mesosphere.

The implications of these findings may be summarized to give the following picture for the SAO. The basic period of the oscillation is set by the seasonal easterlies near the stratopause at the solstices. This easterly forcing counteracts the effect of westerly forcing by Kelvin waves, which would otherwise produce a westerly wind layer throughout the upper stratosphere and lower mesosphere. Nonetheless, westerly forcing predominates at the equinoxes, and generates absolute westerly flow down to about 40–45 km. In the upper mesosphere, easterly and westerly forcing by inertia–gravity waves drives zonal wind regimes of alternating sign. However, the solstitial easterlies at the stratopause are also an essential ingredient of the mesopause oscillation in that they set its semiannual period by preventing easterly waves from propagating to higher altitudes during the solstices.

It may be asked why the waves documented in this study do not appear to play an important role in the semiannual oscillation produced by most GCMs, even though these models are capable of resolving the zonal scales and periods examined here. (One exception is the GFDL SKYHI model, wherein intermediate-scale waves do provide a significant fraction of the acceleration associated with the SAO, although the latter is still weaker than observed.) A possible answer is that, because of the highly parameterized nature of convection in GCMs, these waves are not forced realistically in such models.

The results of our investigation highlight the fact that, although the mechanism responsible for the tropical SAO may be understood in principle, large uncertainties remain regarding the required sources of zonal-mean momentum. We wish to emphasize that, because of the uncertainties inherent in the specification of the convective forcing, our results do not rule out a role for small-scale gravity waves in the generation of the SAO. However, they do underscore the need for further modeling and observational work on the forcing of intermediate-scale equatorial waves and their effects on the circulation of the middle atmosphere.

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