Stratospheric Vacillations in a General Circulation Model

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ABSTRACT

The variability in the Northern Hemisphere winter stratosphere is studied with a general circulation model run in perpetual January mode. The variability is of oscillatory nature with a timescale of approximately 100 days. Warmings appear in the upper stratosphere and descend through the stratosphere until they dissipate close to the tropopause. Warming of the upper stratosphere is accompanied by cooling of the lower stratosphere and vice versa. Experiments with time-independent tropospheres show that the vacillations originate from a stratospheric instability when driven by a constant wave forcing at the tropopause. The general circulation model experiments are discussed in the light of low-dimensional models.

1. Introduction

Occasionally, the westerly zonal flow in the Northern Hemisphere winter stratosphere is disrupted by spectacular events weakening or even changing the direction of the mean flow. Accompanying the change in the mean flow is a rapid increase in the polar temperatures resulting in a positive latitudinal temperature gradient. Observations show that major warmings—usually defined as a reversal of the zonal mean flow reaching below 10 hPa north of 60°N—appear on average every second year and typically in January. Since the appearance of the paper by Matsuno (1971), forced planetary waves propagating from the troposphere have been attributed a key role in the explanation of the warmings. However, it is still an open question whether the warmings are driven by transient waves penetrating from the troposphere or originate from an internal stratospheric instability. Previously, quasigeostrophic models have shown the existence of stratospheric vacillations driven by a constant forcing on the lower boundary (Holton and Mass 1976; Schoeberl 1983). In this paper we report a similar periodic behavior in a general circulation model (GCM) run in perpetual January mode. The vacillation is investigated by changing the forcing from the troposphere either by changing the height of the mountains or by keeping the state of the troposphere constant in time. We note that the perpetual January mode is a natural starting point for a study of dynamics of the stratospheric variability for two reasons. First, the strongest stratospheric variability is found in the Northern Hemisphere winter. This variability is believed to be driven by large-scale waves that are resolved by the model and described from first principles. This is in contrast to the Southern Hemisphere winter where unresolved gravity waves are thought to be relatively more important (Garcia and Boville 1994). Second, although obviously unrealistic, especially regarding the timescales of more than 50 days studied in this work, this approach isolates the intrinsic variability in the model atmosphere from the most important external periodic forcing. The only remaining external frequency is the daily cycle.

The paper is organized as follows. In section 2 we describe the stratospheric variability in the control experiment. The analysis is performed in terms of the transformed Eulerian-mean equations emphasizing the importance of the wave–mean interactions. In section 3 a series of experiments, including experiments with time-independent states in the troposphere, is presented. It is demonstrated that vacillations are present even with a constant wave forcing at the tropopause if the strength of the forcing exceeds a critical threshold. In section 4 these results are discussed in the light of previous studies of low-dimensional models and a conceptual picture of the mechanism of the vacillations is offered. This mechanism involves the competing effects of critical layer absorption and radiative relaxation. The paper is closed by a summary and discussion in section 5.

The general circulation model used in this study is the French community climate model Arpege cycle 14 (Déqué et al. 1994). The resolution is T42 with 41 levels in the vertical. The parameterizations active in the stratosphere are the Morcrette radiation scheme (Mor-
and 2. The control experiments

The variability on the winter hemisphere has a vacillatory nature as seen from Fig. 1 where the temperature anomaly at 60°N in the control experiment is shown as a function of pressure and time. As noted in Christiansen et al. (1997) the temperature anomalies develop in the mesosphere and slowly descend through the stratosphere until they dissipate in the lower stratosphere. The vacillations have a timescale of about 80–130 days. The warmings are accompanied through the thermal wind relation with weak zonal wind (Fig. 2a). Occasionally warmings are accompanied by interactions between the zonal mean and the resolved eddies. Figures 2c and 2d show the divergence of the horizontal and vertical components of the EP flux, respectively. Negative values of the total EP flux divergence are composed of large negative values of the vertical component and large, but slightly smaller, positive values of the horizontal component; that is, accelerations of the zonal wind correlate positively with the vertical component and negatively with the horizontal component.

Figure 3 shows the three explicit terms in Eq. (1) along with the temperature tendency as a function of time and latitude at 10 hPa. Note that the scaling depends on latitude. It is seen that the vacillations dominate the variability on the Northern Hemisphere even in the equatorial region. In general the Coriolis force is strongly anticorrelated with the wave forcing. The Coriolis force is weaker than the wave forcing when the wave forcing is large so the sum \( f \vec{V} \) will, through the continuity constraint, be felt in the dynamical heating \( \vec{V}^2 (HN^2/R) \). A characteristic feature of Fig. 3d is the phase difference of \( T \) between the high latitudes and
Fig. 2. (a) The anomaly of the zonal mean zonal wind (m s$^{-1}$), (b) the divergence of the Eliassen–Palm flux (m s$^{-1}$ day$^{-1}$), (c) the divergence of the horizontal component of the Eliassen–Palm flux (m s$^{-1}$ day$^{-1}$), and (d) the divergence of the vertical component of the Eliassen–Palm flux (m s$^{-1}$ day$^{-1}$) at 60°N as function of pressure and time.
low latitudes. In general the global mean of the dynamical heating will vanish when averaged over a surface of constant pressure due to the continuity constraint. Therefore, in the absence of diabatic effects a heating at high latitudes must be accompanied by a cooling at low latitudes and vice versa. The divergence of the EP flux drives a residual mean meridional circulation with poleward flow over the entire Northern Hemisphere. This flow forces upwelling and dynamical cooling in the equatorial region and subsidence and dynamical heating at higher latitudes. It is the modulation of this suction pump that is depicted in Fig. 3. From the information in Figs. 2 and 3 it is reasonable to infer that the vacillations are driven by the vertical EP flux, while the modulations of the horizontal EP flux are slaved to the vertical flux.

The temporal standard deviations of the zonal mean temperature $\sigma(T)$ and the zonal mean zonal wind $\sigma(u)$ are shown in Fig. 4. For all altitudes the temperature variability increases with latitude to reach the maximum at the North Pole. The variability increases with altitude below 1 hPa. The decay of the variability above 1 hPa is probably a consequence of the model’s mesospheric relaxation—a linear relaxation toward prescribed wind and temperature fields. On the Northern Hemisphere the standard deviation of the temperature is everywhere larger than 0.5 K. The maximum is 15 K, which is found at 1 hPa over the North Pole. On the Southern Hemi-

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**Fig. 3.** (a) The Coriolis acceleration $-f\mathbf{v}$ (s$^{-1}$), (b) the wave forcing $[1/(\rho_0 \cos \phi)] \mathbf{\nabla} \cdot \mathbf{F}$ (s$^{-1}$), (c) the zonal mean wind tendency $\Pi$ (s$^{-1}$), and (d) the zonal mean temperature anomaly (dimensionless) at the 10-hPa pressure surface as a function of latitude and time. The three first expressions have been normalized with the standard deviation $\sigma(u)$ of the zonal mean velocity while the zonal mean temperature is normalized with its standard deviation, $\sigma(T)$. 
sphere the values are smaller than 0.75 K. The standard deviation for the zonal mean zonal wind peaks at 23 m s\(^{-1}\) at 1 hPa, 68\(^\circ\)N. For lower altitudes the maximum variability is found at higher latitudes. This variability is up to a factor of 2 weaker than the variability in the Community Climate Model (CCM0) reported by Boville and Randel (1986), when daily and monthly variability are summed. Compared to the Berlin GCM (Pawson et al. 1995) the variability in the present model is around 40\% too weak.

When the lower polar stratosphere is in the cold phase the deviations from the zonal mean are small, as seen in Fig. 5. As the warming starts the amplitudes of the zonal waves grow corresponding to a deformation of the vortex. It is clear from the figure that the time evolution of the different zonal wavenumbers is phase locked. For most warmings wavenumber 1 dominates but some warmings have a large contribution from wavenumber 2. During the following coolings the vortex recombines and resumes its original nearly zonal structure.

From the above discussion it is evident that the vacillations are driven by the wave–mean interactions. Whether the instability has its origin in the stratosphere or troposphere remains to be seen. As a first attempt to address this question Fig. 6 shows the vertical component at the EP flux from the control experiment as function of time on the Northern Hemisphere for three different pressure levels. The variability on long timescales (larger than 50 days) is not observed in the troposphere but is dominant at 10 hPa. We take this as evidence that the variability on these timescales is inherent in the stratospheric dynamics and that it is not a consequence of transient forcing from the troposphere. However, the possibility exists that a low-frequency wave component in the troposphere—unrecognizable without filtering—is amplified while propagating through the troposphere. Figure 6b can be directly compared to Fig. 5b in Pawson et al. (1995) except for a factor 0.093 due to different choices of units. We note that the strength of the wave forcing on the stratosphere is almost the same in the two models.

The stationary wave forcing on the tropopause level has a maximum at 60\(^\circ\)N and consists mainly of zonal wavenumbers 1 and 2 as shown in Fig. 7, where the amplitude of the stationary eddies in the geopotential height \(h\) is a function of latitude. Whereas the amplitudes of wavenumbers 1 and 2 are comparable in size on high northern latitudes, wavenumbers larger than 2 contribute much less to the total forcing. For low northern latitudes wavenumber 1 dominates. The average over the Northern Hemisphere is 48, 36, and 22 m for
wavenumbers 1, 2 and larger than 2, respectively. The hemispheric mean of the total zonal standard deviation of $[h]$ is 65 m.

3. The perturbation experiments

To probe the nature of the vacillations we have performed a series of experiments designed mainly to manipulate the strength and nature of the wave forcing that the troposphere exerts on the stratosphere.

As noted in Christiansen et al. (1997) the vacillations seem robust to changes in the parameterizations. In particular, elimination of the gravity-wave drag parameterization affects the mean flow drastically but leaves the variability almost unchanged. In uniform ozone reduction experiments of 50% and 75% the nature of the oscillations are the same but the amplitude has decreased.

A straightforward way to manipulate the strength of the stationary waves is to change the height of the mountains in the model by changing the surface geopotential height. We have performed one such experiment, NM (no mountains), where the surface geopotential height was set to zero globally and the mountains therefore

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**Fig. 5.** The amplitudes of the zonal wavenumber 1, 2, and 3 components of temperature (K) at 60°N as a function of time and pressure.
Fig. 6. The vertical component $F^2/\rho_0$ (m$^2$ s$^{-2}$) of the Eliassen–Palm flux as a function of latitude and time for the pressure levels 500, 100, and 10 hPa.

Fig. 7. The stationary waves of the geopotential height (m) at 100 hPa expanded in zonal wavenumbers. The dashed and dotted curves show wavenumbers 1 and 2, respectively. The dotted–dashed curve shows wavenumbers larger or equal to 3.

completely leveled. We note that in this experiment the gravity-wave drag has not been changed so the change is purely in the resolved waves. To further investigate the nature of the stratospheric variability we have performed a series of experiments in which the troposphere is independent of time. The values of the temperature $T$, zonal wind $u$, meridional wind $v$, and humidity $q$ are kept constant in the lowest 15 model layers, that is, below approximately 200 hPa. The basis for the constant troposphere is calculated as the mean over 5 days (days 451–455) in the control run. Tropospheres with different strength of the stationary waves have been constructed by amplifying the eddy components by a factor of $\lambda$. Denoting the experiments with constant tropospheres by $CT\times\lambda$ we have for, for example, the temperature field $T_{CT}\times\lambda = T_{CT} + \lambda(T_{CT+1} - T_{CT})$. For the experiments presented in this paper we have chosen $\lambda = 0.75, 1, \text{ and } 1.5$. Finally, two of the experiments have been repeated with changed horizontal diffusion. In the standard version of Arpege the horizontal diffusion is of sixth order and increases inverse proportionally to the
Table 1. The stationary and transient forcing at the tropopause level, the amplitude of the stationary waves of the geopotential height at the tropopause level, the timescale of the vacillation, and the temporal mean of the zonal mean zonal wind and its standard deviation at 60°N and 10 hPa.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$F/p$ at 100 hPa (m$^2$ s$^{-2}$) ($\times$ 1000)</th>
<th>$\sqrt{h_-^2}$ (m)</th>
<th>Period (day)</th>
<th>$[\mu]$ (m s$^{-1}$)</th>
<th>$\sqrt{[\mu^-]^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>17.1 (10.5)</td>
<td>14.9 (1.8)</td>
<td>65.2 (18.5)</td>
<td>80–130</td>
<td>20</td>
</tr>
<tr>
<td>NM</td>
<td>23.7 (10.8)</td>
<td>4.7 (2.4)</td>
<td>42.5 (4.8)</td>
<td>Irregular</td>
<td>25</td>
</tr>
<tr>
<td>CT $\times$ 0.75</td>
<td>$-1.0$ (0.0)</td>
<td>22.5 (6.1)</td>
<td>89.4 (29.1)</td>
<td>Steady</td>
<td>40</td>
</tr>
<tr>
<td>CT $\times$ 1.0</td>
<td>$-1.5$ (0.0)</td>
<td>27.2 (6.6)</td>
<td>108.7 (36.5)</td>
<td>150</td>
<td>30</td>
</tr>
<tr>
<td>CT $\times$ 1.5</td>
<td>$-1.7$ (0.0)</td>
<td>70.8 (6)</td>
<td>149.3 (52.2)</td>
<td>85</td>
<td>20</td>
</tr>
<tr>
<td>ControlLRD</td>
<td>19.7 (10.0)</td>
<td>16.1 (2.0)</td>
<td>66.4 (21.0)</td>
<td>80–120</td>
<td>22</td>
</tr>
<tr>
<td>CT $\times$ 1.5,RD</td>
<td>$-2.5$ (0)</td>
<td>81.3 (5.6)</td>
<td>151.8 (52.4)</td>
<td>85</td>
<td>20</td>
</tr>
</tbody>
</table>

The main conclusion to be drawn from these experiments is that vacillations can be sustained even with a constant troposphere. However, a qualitatively different regime without vacillations and of much lower variability exists for weak stationary waves.

In conclusion, the control experiment the average over the Northern Hemisphere of the vertical EP flux is $32 \times 10^{-3}$ m$^2$ s$^{-2}$ almost equally divided between the transients ($17 \times 10^{-3}$ m$^2$ s$^{-2}$) and the stationary ($15 \times 10^{-3}$ m$^2$ s$^{-2}$) parts. The vacillations are somewhat irregular with an average period of around 100 days. In the series of experiments with time-independent tropospheres the stationary EP flux increases from $22.5 \times 10^{-3}$ in CT $\times$ 0.75, over $27 \times 10^{-3}$ in CT $\times$ 1, to $70.8 \times 10^{-3}$ m$^2$ s$^{-2}$ in CT $\times$ 1.5. The transient EP flux is slightly negative in all three experiments showing a small downward penetration of the stratospheric variability (note the existence of a zone between the tropopause at 100 hPa and the top level at 200 hPa of the constant part of the atmosphere). In CT $\times$ 1 and CT $\times$ 1.5 regular vacillations exist with periods of 150 and 85 days, respectively. In CT $\times$ 0.75 no vacillation is apparent in Fig. 8, and the stratospheric variability is considerably weaker. This series of experiments agrees well with the notion that the vacillations are due to an instability in the stratosphere but that a constant wave forcing of finite strength from the troposphere is needed. The value of the threshold forcing is between $22.5$ and $27 \times 10^{-3}$ m$^2$ s$^{-2}$, corresponding to an amplitude of 90–110 m of the stationary eddies of the geopotential height. The stationary wave forcing in the control experiment is below this threshold while the sum of the transient and stationary forcing is well above. It is not surprising that the existence of noise might drive a system into a neighboring dynamical regime. The decrease of the timescale of the vacillations with the strength of the wave forcing is in agreement with the Holton–Mass model and fits the general behavior of a Hopf bifurcation.

As expected the absence of mountains in experiment NM has a strong influence on the stationary waves. The penetration of the stationary EP flux into the stratosphere has decreased by a factor of 3 in the Northern Hemisphere while the stationary flux in the Southern Hemisphere is almost unchanged as compared to the control experiment. A large increase in the transient flux in the Northern Hemisphere compensates for the decrease in the stationary flux and the total flux is almost unchanged. The vacillations have vanished as seen from Fig. 8 and have been replaced with a weaker irregular variability.

In both experiments with reduced horizontal diffusion the vacillations persist. In the experiment with constant troposphere, CT $\times$ 1.5, the only discernible changes are
Fig. 8. The temperature anomaly of the zonal mean temperature \( T \) at 60°N as a function of pressure and time for (a) NM, (b) CT \( \times 0.75 \), (c) CT \( \times 1 \), (d) CT \( \times 1.5 \), (e) the control experiment with reduced horizontal diffusion, and (f) CT \( \times 1.5 \) with reduced horizontal diffusion.
Fig. 9. (a)-(f) The time average of the zonal mean wind $u$ (m s$^{-1}$) for the same set of experiments as in Fig. 8 and (g) the control experiment.
FIG. 10. The standard deviation of the zonal mean zonal wind $\sigma(\mathbf{u})$ (m s$^{-1}$) for the same set of experiments as in Fig. 8.
a small transport of variability from wavenumber 1 to wavenumbers 2 and 3 as depicted in figures similar to Fig. 5 (not shown), and an increase of 15% in the stationary wave forcing at the tropopause. The strong diffusion in the stratosphere is thus of minor importance, a fact that is consistent with the notion that only the lowest wavenumbers survive the propagation from the troposphere into the stratosphere. The increase of the stationary wave forcing is obviously due to changes in the buffer zone between the tropopause and the uppermost level of the constant part of the troposphere. In the control experiment with reduced diffusion both the stationary and transient wave forcing at the tropopause have increased by 7% and 15%, respectively. The amplitudes of the zonal wavenumber 1, 2, and 3 components of the geopotential heights are now 45.5, 40.3, and 20.6 m. Compared to the control run the wavenumber 2 forcing has increased at the expense of wave number 1. The vacillations have the same irregular behavior as in the control experiment.

In Fig. 9 the time average \( \overline{\mathbf{u}} \) of the zonal mean flow is shown for all experiments. As described in Christiansen et al. (1997) the winter stratospheric jet in the control experiment is positioned too close to the equator as compared to the climatology. In the experiment \( \text{CT} \times 1.5 \) the zonal flow has the same structure as in the control experiment only with the jet maximum 10 m s\(^{-1}\) stronger. With the decreased EP flux in \( \text{CT} \times 1 \) the jet maximum weakens and a secondary maximum appears at 70°N and 10 hPa. This development is continued further in \( \text{CT} \times 0.75 \) where the maximum at 70°N is now the dominant one. From comparison with Fig. 5 in Christiansen et al. (1997), which shows \( \overline{\mathbf{u}} \) and \( \overline{\mathbf{r}} \) for the two extreme phases of the vacillation, it is obvious that the maximum at 70°N is related to the cold phase (in the lower stratosphere) of the vacillation with the weakest wave forcing and largest latitudinal temperature gradient. The maximum near 40°N, which dominates the control run, is on the other hand related to the warm phase of the vacillation. In the experiment without mountains both the meridional temperature gradient and the strength of the zonal wind have increased as compared to the control run. The reduction of the horizontal diffusion has only a minor influence on the zonal mean flow.

From the temporal standard deviation of the zonal mean flow presented in Fig. 10 we find that the experiments NM and \( \text{CT} \times 0.75 \) have a considerably weaker variability than the rest of the experiments. Comparing \( \text{CT} \times 1 \) and \( \text{CT} \times 1.5 \) we find only a small increase in the variability. Reducing the horizontal diffusion in the control experiment leads to larger variability at low latitudes. A similar effect is not found in \( \text{CT} \times 1.5 \text{RD} \), indicating that the variability at low latitudes is of tropospheric origin.

### 4. Low-dimensional models

The vacillatory behavior has previously been predicted by quasigeostrophic models. In this section a discussion of these models will be followed by a qualitative description of the mechanism of the vacillations.

A simple model of the stratospheric dynamics was introduced by Holton and Mass (1976). It is a quasigeostrophic, beta-plane model truncated to include only one zonal wavenumber and one meridional mode. The independent variables are time and the vertical coordinate. The model includes three vertical fields of dependent variables; one is the zonal mean state and the two others describe the amplitude and longitudinal phase of the eddies. The parameters are the vertical radiative equilibrium profile and damping coefficients and the strength of the wave forcing at the lower boundary. This model was analyzed with a steady wave forcing at the lower boundary by Yoden (1987a,b, 1990). For low forcing the only solution is a cold stable steady state close to the radiative equilibrium. For an intermediate range of forcings this state coexists with two steady states: a warm stable state and an unstable steady state. For a critical forcing an oscillating state branches off the warm stable steady state through a Hopf bifurcation. Immediately after the bifurcation the period of the oscillating state is approximately 100 days. The period decreases as the forcing increases. A similar model without meridional truncation was studied by Schoeberl (1983). This model also sustains stratospheric vacillations. The main difference to the Holton–Mass model is a reduced sensitivity to the wave forcing. Recently, Pierce and Fairlie (1993) found observational evidence for preferred flow regimes in the Northern Hemisphere winter similar to the three states found in the Holton–Mass model for the intermediate range of forcings.

The mechanism of the vacillations can be understood by simple qualitative arguments. With the Newtonian cooling approximation the prognostic equation for the zonal mean velocity \( \overline{\mathbf{u}} \) can be written

\[
\frac{\partial^2 \overline{\mathbf{u}}}{\partial y^2} + \frac{f^2}{\rho N^2} \frac{\partial}{\partial z} \left( \rho_0 \frac{\partial \overline{\mathbf{u}}}{\partial z} \right) + \frac{f^2}{\rho N^2} \frac{\partial}{\partial z} \left[ \frac{\rho_0}{\tau} \frac{\partial}{\partial z} (\overline{\mathbf{u}} - u_r) \right] = \frac{\partial^2 \mathbf{F}}{\partial y^2} \frac{1}{\rho_0}.
\]

(3)

The second term on the left-hand side describes the relaxation toward the radiative equilibrium profile \( u_r \). Typically the radiative timescales \( \tau \) vary from 40 days in the lower stratosphere to 5 days at the mesosphere. The term on the right-hand side is the eddy forcing on the mean flow in the form of the divergence of the EP flux \( \mathbf{F} \). It is the eddy forcing that drives \( \overline{\mathbf{u}} \) away from the radiative equilibrium \( u_r \). The propagation of the EP flux is, in turn, determined by the structure of the zonal mean flow \( \overline{\mathbf{u}} \). In the quasigeostrophic models discussed above, the eddies are obtained by considering the potential vorticity equation. Here, we take a more qualitative point of view. In the “optical limit” one can define a refractive index \( n^2 \) that guides the forced planetary waves in the sense that they will tend to avoid regions
with low or negative values of the refractive index. Mathematically this behavior is captured by the time-independent wave equation \(\nabla^2 \mathbf{F} + n_0^2 \mathbf{F} = 0\). The refractive index can be written

\[
n_0^2 = \frac{\bar{q}_a}{a \beta} - \frac{\beta^2}{4N^2 H^2},
\]

where

\[
\bar{q}_a = 2\Omega \cos \phi - \frac{(\pi \cos \phi \beta)}{a \cos \phi} - \frac{\alpha}{\rho_0} \left( \frac{\beta \rho_f^2}{N^2} \right).
\]

is the latitudinal gradient of the quasigeostrophic potential vorticity (Andrews et al. 1987). Noting that the refractive index is negative in regions with easterlies, it follows that regions above easterlies are shielded from the wave flux propagating from the troposphere. The zonal wind in this region will therefore relax toward the radiative equilibrium profile. This shielding offers a qualitative explanation of the mechanism that drives the vacillations. Starting from the radiative equilibrium profile the eddies will decelerate the zonal flow as the wave forcing is negative. The strength of the deceleration \(\nabla \cdot \mathbf{F}/\rho_0\) increases with height and easterlies will first occur at the top of the atmosphere (Fig. 11a). The lowest level with easterlies is called the critical line. The region above the critical line will be shielded from the wave forcing and the mean zonal flow starts to relax toward the radiative equilibrium profile. If the wave forcing is strong enough the critical line continues to move downward until it is disrupted by the lower boundary. When this happens the total stratospheric column is again open to the penetration of the waves and the process can start over. For weaker wave forcing the system evolves into a steady state as the downward movement of the critical line will stop when it reaches a region where the wave forcing is too weak to produce easterlies against the relaxation.

From the mechanism described above we can get an estimate of the timescale \(\tau_{vac}\) of the vacillation as the time it takes the waves to decelerate the zonal mean flow from the radiative equilibrium to zero in the lowest part of the stratosphere. Balancing the inertial term with the radiative damping and the wave forcing we get from Eq. (3)

\[
\frac{\pi \beta - u_c}{a \tau_c} = \frac{\nabla \cdot \mathbf{F}}{\rho_0},
\]

where \(\alpha = (NH/L)^2 \) and \(L\) is the meridional length scale. Integrating this expression we find that oscillations are sustained if the strength of the (negative) wave forcing exceeds the threshold \(\nabla \cdot \mathbf{F}_{thres}/\rho_0 = -u_c / a \tau_c\). The period is

\[
\tau_{vac} = -\alpha \tau_c \ln \left( 1 - \frac{\nabla \cdot \mathbf{F}_{thres}}{\nabla \cdot \mathbf{F}} \right).
\]

The order of magnitude of the divergence of the EP flux is obtained as the ratio of the EP flux imposed on the tropopause \(r^{-a}\) to the scale height \(H\), that is, \(\nabla \cdot \mathbf{F} \sim -r^{-a} / H\). We then have \(r^{-a}_{thres}/\rho_0 = u_c H / a \tau_c\). With \(\alpha = 1\), \(H = 7000\) m, and typical values for the lower troposphere \(u_c = 100\) m s\(^{-1}\) and \(\tau_c = 25\) days, we arrive at the estimate \(r^{-a}_{thres}/\rho_0 = -0.032\) m\(^2\) s\(^{-2}\) close to the observed threshold in the GCM. For a forcing 10% above the threshold we find \(\tau_{vac} = 60\) days, which is of the same order of magnitude as the timescales found in the GCM.

5. Discussion

In this paper we have demonstrated the existence of stratospheric vacillations in general circulation model simulations of the Northern Hemisphere winter. We have seen that vacillations are possible even when the stratosphere is driven by a time-independent forcing from the troposphere. For weak time-independent forcing the
stratosphere evolves into a quiescent state. If the constant forcing at the lower boundary is increased above a finite threshold the system bifurcates from the quiescent state to a vacillating state. This transition has previously been predicted by quasigeostrophic models and can be understood in terms of the competing effects of radiative relaxation and critical line absorption. The shielding effect of the critical line makes the zonal wind decelerate below it and accelerate above it, resulting in a downward propagation of the critical line. We believe that the same mechanism is responsible for the vacillations in the control experiment, although the stationary part of the forcing alone is below the critical threshold found in the experiments with time-independent forcing. A perfect agreement should not be expected as the stationary forcing differs in its wavenumber composition. Also, the possibility exists that the transient “noise” in the control experiment drives the system into the nearby vacillating regime.

We note that the vacillations are robust to changes in the parameterizations of the model. The vacillations were first reported by Christiansen et al. (1997) in an earlier version of Arpege with fewer vertical levels and a less elaborate surface scheme. In this study it was noted that the vacillations persisted in experiments with 50% and 75% uniform ozone reductions, as well as in an experiment without the gravity-wave drag parameterization. In particular in the latter experiment the climatology of the stratosphere changed dramatically; for example, the strength of the winter stratospheric jet increased from 40 to 110 m s$^{-1}$. In the present paper it was demonstrated that the vacillations also are robust to changes in the formulation of the horizontal diffusion.

The critical threshold depends on the relative strengths and phases of the wavenumber components included in the forcing at the lower boundary. In this paper we have presented experiments where the constant troposphere has been deduced from a particular 5-day average. A thorough investigation of the parameter space is not possible with a comprehensive GCM but is most meaningful when accomplished with simpler models. However, there seems to be nothing special with the particular 5-day average used here, as vacillations similar to those presented in CT $\times$ 1 are found using a constant troposphere deduced from other 5-day averages.

We close the paper with a brief discussion of other general circulation model experiments. Pawson et al. (1995) observed variability on a broad spectrum of timescales in a 1200-day perpetual January experiment with the Berlin GCM. In particular, they studied the spectral power at 60°N of both the vertical component of the EP flux at 100 hPa and the mean zonal wind at 1 hPa. The power of $F^z$ showed two peaks at periods close to 40 days and 80–100 days. However, only the latter exceeded the 95% significance level and the forcing could hardly be distinguished from random noise. The response, $\pi$ at 1 hPa, showed significant power on rather longer timescales, 100–300 days. On that basis they concluded that the nature of the stratospheric variability is episodic rather than periodic and that the stratospheric variability is essentially forced by transients from the troposphere. Boville (1986) observed a transition between two qualitatively different states of the winter stratosphere in a perpetual January experiment with the National Center for Atmospheric Research Community Climate Model. Schoeberl and Strobel (1980) found some evidence for an 80-day cycle in the occurrence of major warmings in a global, quasigeostrophic, semispectral model, although their integration covered less than two full periods.

Obviously the model experiments presented in this paper resemble much closer the patterns of the quasigeostrophic models than the red-noise variability observed by Pawson et al. (1995). There are several possible explanations for this difference between the general circulation models. Compared to Arpege the Berlin GCM might have (i) stronger transients from troposphere obscuring the vacillations, (ii) stronger stationary wave forcing driving the system into a chaotic state, or (iii) more active horizontal modes with incommensurable frequencies or that interact chaotically. Although i and ii cannot be ruled out entirely these explanations are not supported by a visual comparison of Figs. 6b and 5b in Pawson et al. (1995). In this context it is also interesting to note the absence of aperiodic states in the quasigeostrophic models even for high wave forcing. Explanation iii is supported by recent results from an extended version of the Holton–Mass model including both zonal wavenumbers 1 and 2 (Chen et al. 1996). With this extension the Holton–Mass model does allow for aperiodic behavior. More meridional modes do not seem to have the same effect as no aperiodic states were observed in the meridional untruncated model (Schoeberl 1983). The number of horizontal modes in the stratosphere could either be a direct result of the forcing at the tropopause or be related to the refractive properties of the stratosphere itself. We note from Fig. 2 that although the mechanism of the oscillations can be explained by one-dimensional arguments alone, the horizontal propagation of the waves is not negligible.

The final comment is to the relevance of the vacillations for sudden warmings. The existence of the instability in a full three-dimensional model with comprehensive physical parameterizations makes it credible that the instability is of relevance to the real atmosphere. Of special interest is the model’s ability to reproduce the temporal statistics of major warmings. A natural way to proceed from a physical point of view is to investigate the parameter space spanned by the frequency and amplitude of an external forcing. The period of the vacillations is long enough that nonlinear effects such as phase locking and chaos might occur when the system is forced with the annual cycle. In the Holton–Mass model irregular behavior was only found for forcing frequencies much faster than the annual cycle (Yoden
1990). When forced by the annual cycle the memory of the system is washed out by radiative damping as the radiative damping time is much smaller than 1 yr. However, the radiative damping time in the lower stratosphere chosen by Yoden was 23 days, a factor of 4 smaller than the damping times calculated by Kiehl and Solomon (1986). In a full three-dimensional model the situation may be somewhat different because the competing horizontal modes might drive even the perpetual January model into a chaotic region.

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REFERENCES


