Scattering of Light by Raindrops with Single-Mode Oscillations

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ABSTRACT

Light scattering by oscillating raindrops is studied theoretically in the ray optics approximation. The effects of oscillation mode, amplitude, time dependence, drop size, and size distribution on the light scattering are studied. The oscillations are modeled as individual spherical harmonics modes with sinusoidal time dependence, and an example size-dependent oscillation scheme is created for size distribution simulations. Marshall–Palmer (M–P) and gamma size distributions are used to represent continuous frontal rain and convective showers, respectively. Lognormal distribution is used in studying the effect of the width of the distribution on light scattering. It is shown that the introduction of studied oscillation modes (2, 0), (2, 1), and (3, 1) change the scattering properties of equilibrium raindrops differently and are thus, in principle, recognizable by light-scattering measurements. Further, individual oscillation modes introduce new features in the scattering pattern, unlike the Gaussian random oscillations that tend to smooth it. Even a population of differently oscillating drops causes relatively little smoothing as long as each drop has only one oscillation mode. The M–P and gamma size distributions, although remarkably different, smooth the scattering patterns in a very similar manner, making the derivation of raindrop size distribution as an inverse light-scattering problem very difficult. The time dependence of scattering is found to be quite strong. The location of the so-called 90° rainbow depends strongly on the drop size. As a result, realistic M–P and gamma size distributions effectively smooth away the bow, whereas it is clearly seen in narrow lognormal distributions with drop size centered around \( d = 2.0 \) mm. As instantaneous size distributions are observed to be more monodisperse than the averaged distributions, it is thus possible that in some rare conditions this novel feature could be seen in nature.

1. Introduction

The exact shape of falling raindrops has been a subject for many studies lately. Contrary to popular belief, raindrops do not have a teardrop-like shape. Nor are they spheres or spheroids, although these approximations are widely used (e.g., Glantschnig and Chen 1981; Aydin and Lure 1991). The raindrop shape is a result of nonlinear interaction of external aerodynamic forces and internal forces within the drop. Thus, raindrop shape is complex and clearly size dependent (e.g., Chuang and Beard 1990; Beard et al. 1989).

The interest toward raindrop shape is strongly connected to advances in radar technology and better evaluation of rainfall rate, and thus the subject is mostly considered in radar applications’ point of view. The scattering of visible light is, however, similarly dependent on the geometrical factors of raindrops, and therefore, the increased information about drop shapes, orientations, and size distributions can be readily adapted to the studies in optical regime. As the amount of optical applications is increasing, there is clearly a need for studies on the scattering of visible light by realistically shaped raindrops.

Raindrops are homogeneous, irregular, but fairly round scatterers, whose light-scattering properties depend mainly on the wavelength-dependent relative refractive index and geometrical factors. They are also so much larger than the wavelength of visible light that their scattering properties can be derived by the so-called ray optics approximation with high accuracy. This approximation, consisting of geometric optics and diffraction parts that are solved separately, is adapted in this study. This study is a continuation to Nousiainen and Muinonen (1999, hereafter referred to as NM) in which light scattering by single, randomly oscillating raindrops was studied.

In NM, the raindrop shape was modeled as a product of an equilibrium part and an oscillation part, the latter described by a so-called Gaussian random sphere model (Muinonen 1996). It was found that most features introduced in the scattering pattern were due to the equilibrium part, whereas the oscillation part mainly tended to smooth away the features.

The Gaussian random sphere model is a convenient first approximation for the raindrop oscillations, but it
has several shortcomings. First, the oscillation cannot be restricted into individual spherical harmonic oscillation modes, which often appears to be the case in natural raindrops. Second, the spherical harmonics expansion is in the exponent, resulting in slightly different oscillations compared to a direct spherical harmonics expansion, especially in the case of large amplitude oscillations. Third, real oscillations have sinusoidal time dependence, which cannot be taken into account by the Gaussian random sphere model. Here it is studied whether these shortcomings are significant or if the Gaussian random sphere model is indeed a good model for oscillating raindrops. One important question is whether the effect of oscillations is still that of smoothing when individual oscillation modes are used, or do different oscillation modes introduce different scattering patterns, thus possibly allowing the derivation of oscillation modes from light-scattering observations from raindrops.

The dependence of light scattering on the oscillation amplitude is also studied here, both in the case of single drop sizes and size distributions. Along with the light-scattering simulations by non-oscillating equilibrium raindrops, three different oscillation amplitudes are studied for each mode. The oscillation amplitudes are also size dependent.

In NM, the effect of size distributions on light scattering was not included. In the present study, this is studied by simulating the scattering properties with two considerably different sample size distributions: continuous frontal rain with rainfall rate of 5 mm h\(^{-1}\), described by the so-called Marshall–Palmer size distribution (Marshall and Palmer 1948), and a convective rain shower with rainfall rate of 50 mm h\(^{-1}\), described by gamma distribution (Willis and Tattelman 1989). One of the main points of interest is to find out whether the so-called 90° rainbow, discovered in the simulations of NM, is still visible when a size distribution is included.

In section 2, light-scattering theory is reviewed in parts that are not discussed in NM and that are relevant for this study. The changes in the light-scattering model since the version used in NM are described in section 3. The shape of model raindrops is defined in section 4, and size distributions and raindrop size dependence of oscillations, in section 5. The results of simulations are discussed in section 6, and the conclusions follow in section 7.

2. Light-scattering theory

The light-scattering properties of oscillating equilibrium raindrops are studied using the ray optics approximation. This approximation was already described in NM for most aspects relevant for this study, and it is not repeated here. The introduction of a size distribution, however, requires some changes.

In NM, the scattering phase matrix \( \mathbf{P} \), which is just one of the forms for the \( 4 \times 4 \) scattering matrix describing the relation between incident and scattered Stokes vectors, was used. This form is used in radiative transfer theory, for example, as it is normalized, and it can readily be used to describe the probability density for the scattering direction. As it does not include the scattering cross section, however, it is not additive for scatterers with different cross sections and thus cannot be used to describe the scattering properties of a population consisting of statistically different raindrops. Thus, in this study we use the scattering matrix \( \mathbf{S} \), related to \( \mathbf{P} \) matrix by

\[
\mathbf{S} = \frac{k^2 \sigma_{\text{sca}}}{4\pi} \mathbf{P},
\]

where \( k \) is the wavenumber and \( \sigma_{\text{sca}} \) the scattering cross section. Here, the matrix \( \mathbf{P} \) is a sum of two parts,

\[
\mathbf{P} = \frac{1}{\sigma_{\text{sca}}}(\sigma_{\text{diff}}^D \mathbf{P}^D + \sigma_{\text{geo}}^G \mathbf{P}^G),
\]

where superscripts \( D \) and \( G \) stand for the diffraction and geometric optics parts, respectively (Muinonen et al. 1996). Taking into account the fact that in the case of visible light and millimeter-sized drops, the diffraction pattern is practically a delta spike in the forward direction, we can write

\[
\mathbf{S} = \frac{k^2 \sigma_{\text{sca}}^G}{4\pi} \mathbf{P}^G
\]

outside the forward direction.

The \( \mathbf{S} \) matrices for statistically different populations can be added to form a sum matrix representing the scattering properties of a sum population. Statistically different populations are, for example, drops with different sizes, different oscillation modes or amplitudes, or different orientations. For analysis, the resulting matrices are normalized back to \( \mathbf{P} \) for simplicity. Further, for brevity, the symbol \( \mathbf{P} \) is used instead of \( \mathbf{P}^G \), although all the plots presented are for the geometric optics part, as the matrices coincide outside the forward direction.

3. Computational methods

Introduction of size distributions forces some significant changes in the light-scattering model. The general algorithm of the Monte Carlo geometric optics ray-tracing model used is explained in Muinonen et al. (1996) and NM. The modifications due to the introduction of size distributions are described below.

As mentioned above, the relative scattering cross sections must be taken into account in computing the scattering matrix, as model drops do not necessarily belong into a statistically identical population. In the model, the computations for a single drop are done using a mean radius normalized to unity, so some weighting must take place before the scattering by drops with different cross-sectional surface area can be added together. Also, to keep the model efficient, the observation volume into
which the simulated drops are put in the model, hereafter referred to as an observation window, should be as little larger than the drop as possible, but still larger than any of the drops studied. Thus, we need to find an observation window for every drop, normalize it for computations, and counterbalance the change in the surface area of an observation window by a weighting coefficient.

The optimal size for the computational observation window is searched by measuring how much the radius of a drop, with a mean radius normalized to unity, can deviate from unity. For this, the largest and thus the most nonspherical raindrop with all possible oscillation modes with maximum oscillation amplitudes is generated. The maximum radius \([\text{max}(r)]\) of such a drop is found and used to define a coefficient

\[
\Delta_r = \frac{(1 + \epsilon) \text{max}(r)}{r_c},
\]

where \(r_c\) is the mean radius of the largest equilibrium raindrop included in the simulation, and \(\epsilon\) is a small positive number. Then, the radius of the observation window for any raindrop with mean equilibrium radius \(r\) (i.e., original, unnormalized drop) is given by

\[
a_{\text{orig}} = r \Delta_r.
\]

To get the radius of the computational observation window, both the drop and the window are divided by the mean drop radius,

\[
a_{\text{com}} = \frac{a_{\text{orig}}}{r} = \Delta_r.
\]

Thus, the radius of the computational observation window is simply \(\Delta_r\). To counterbalance the normalization of drop size, the absorption coefficient must be changed accordingly, and light scattered by the drop must then be multiplied by a weighting coefficient \(w\), defined by

\[
w = \frac{a_{\text{com}}^2}{a_{\text{orig}}^2} = \frac{r^2}{r_c^2},
\]

the ratio of original and computational observation windows. At the end of the simulation, the results are then divided by the sum of weighting coefficients used.

The average scattering cross section for drops is (Bohren and Huffman 1983)

\[
C_{\text{sca}} = \langle A \rangle q_{\text{sca}},
\]

where \(q_{\text{sca}}\) is the average scattering efficiency and equals to unity for nonabsorbing scatterers, and the average cross-sectional area for the population of simulated raindrops is given by

\[
\langle A \rangle = \frac{A_{\text{tot}}}{N_{\text{tot}}},
\]

where \(A_{\text{tot}}\) is the sum of surface areas of original observation windows \(a_{\text{orig}}\) for those drops the incident ray did hit, and \(N_{\text{tot}}\) the total number of incident rays (missed and hit).

The oscillation part of the model is changed completely, as a size-dependent oscillation scheme is added. The scheme is implemented in such a way that it would be flexible and easily modifiable if necessary, as it is realized that no last word about size-dependent raindrop oscillations has been said. In the scheme, the size range has been divided into four subranges (\(d = 1.0–1.35\) mm, \(1.35–1.75\) mm, \(1.75–2.5\) mm, and \(>2.5\) mm), where relative probabilities for different modes and weighting coefficients for their amplitudes can be given. These are the only user-configurable values. Only the three most commonly encountered modes, \((2, 0), (2, 1),\) and \((3, 1)\) are included in the scheme, as data about other modes are considered insufficient. The size-dependent oscillation amplitudes are hard coded in the model, as described in section 4.

4. Raindrop shape

As in NM, the raindrop geometry is defined as a product of the equilibrium part \(F(\theta)\) and the oscillation part \(G(\theta, \varphi)\) in spherical polar coordinates \((\theta, \varphi)\). The definition of the oscillation part is changed, however, following

\[
r(\theta, \varphi) = aF(\theta)G(\theta, \varphi),
\]

\[
F(\theta) = 1 + \sum_{n=0}^{N} F_n \cos n \theta,
\]

\[
G(\theta, \varphi) = 1 + \sum_{j=0}^{l} \sum_{m=0}^{l} P_{j}^{m}(\cos \theta) a_{\text{in}} \cos m \varphi,
\]

where \(r\) is the radius of a raindrop and \(N = 10\) is used for the equilibrium part. The size-dependent equilibrium shape coefficients \(F_n\) taken from Chuang and Beard (1990), are given in Table 1, modified with the \((-1)^n\) coefficient to make their \(\theta\) angle coincide with the \(\theta\) angle of the spherical coordinate system. The coefficients given in Chuang and Beard (1990) are practically the same as those given in Beard and Chuang (1987) that were used in NM, but they are available for a wider size range. The maximum \(l\) in spherical harmonics expansion depends on the maximum degree of oscillations in the drop.

Partial derivatives and the surface normal of a raindrop needed in ray tracing are derived in the same way, as in NM. A new function,

\[
H(\theta, \varphi) = r - aF(\theta)G(\theta, \varphi),
\]

is introduced. Unit normal vector of a raindrop surface is then given by

\[
n(\theta, \varphi) = \frac{\nabla H}{|\nabla H|},
\]

where (Arfken 1970)

\[
\nabla H = r_0 + \theta_0 \frac{1}{r} \frac{\partial H}{\partial \theta} + \phi_0 \frac{1}{r \sin \theta} \frac{\partial H}{\partial \varphi}.
\]
For size distribution simulations, the equilibrium shape coefficients were linearly interpolated. This method was considered safe and sound, as the terms of cosine series are orthogonal, and thus they are not interconnected. At the lower limit, the raindrops with size up to \(d = 0.5\) mm were considered spherical with \(F_a = 0\). At the large drop size limit, the size distributions used were cut at \(d = 6.0\) mm.

The coefficients \(a_{lm}\) describe the amplitudes of real-valued spherical harmonic modes. The sinusoidal time dependence of the oscillations was taken into account by defining the oscillation amplitude by

\[
a_{lm} = A_{lm} \sin x,
\]

where \(A_{lm}\) is the maximum oscillation amplitude for given mode \((l, m);\) and \(x\) is a random variable with uniform probability distribution between 0 and \(2\pi\), uniquely defined for each drop generated. In NM, the raindrop oscillations were described by exponential spherical harmonics expansion. The change into the direct spherical harmonics expansion allows, in principle, the occurrence of negative radii for generated raindrops. However, this is not a problem here because, unlike in the case of Gaussian random sphere oscillations of NM, we now specify the maximum amplitude of oscillations \(A_{lm}\), and the time dependence only acts to decrease the amplitude. In the case of single-mode oscillations, this allows us to control the amplitude of deformation of the equilibrium shape. In practice, realistic oscillation amplitudes are too small to cause negative radii.

The raindrop oscillations are usually studied by measuring the scatter of the axis ratio and the oscillation frequencies of observed raindrops. Oscillations with spherical harmonics modes \((l, m)\) with degree \(l = 2\) and \(l = 3\) are the most pronounced in natural raindrops, and among them, only modes \((2, 0)\) and \((2, 2)\) produce axis ratio scatter that is symmetric about the axis ratio for equilibrium raindrops (Beard and Kubesh 1991). All other modes have asymmetric scatter above \(\{modes (2, 1)\text{ and } (3, 1)\}\) or below \(\{modes (3, 0), (3, 2), \text{ and } (3, 3)\}\) the equilibrium ratio.

Figure 1 in Tokay and Beard (1996) summarizes most of the available results of axis ratio measurements. It can be seen that the average axis ratios are mostly above equilibrium and that the scatter of the axis ratio generally increases with drop size. It can also be seen that there appears not to be any clear systematic difference between laboratory and real rain measurements, indicating the dominance of eddy shedding as a cause of the oscillations. It is not a simple task to deduce the oscillation modes responsible for such axis ratio variations, however. For example, observed scatter of axis ratio may be due to multiple-mode oscillations in individual drops, or different oscillation modes in different drops of the measured population. This problem is further enhanced by the fact that many different modes cause similar scatter in the axis ratio. More information about oscillation modes, or at least the degree of oscillations, can be deduced if the oscillation frequencies are also considered, but for oscillation amplitudes this information is useless.

To derive the oscillation amplitudes for model raindrops, a relation between observed axis ratio behavior and mode-dependent oscillation amplitudes needed to be established. This was accomplished by using the oscillation energy as a middle step. It was assumed that the observed scatter in the axis ratio was caused only by one oscillation mode, and thus Fig. 13 of Beard and Kubesh (1991) and the equations within could be used to compute the oscillation energies for observed raindrops. On the other hand, the oscillation energy is directly proportional to the change in surface area of the drop due to an oscillation, so the relation between the oscillation amplitude of given mode and its oscillation energy could be established by deriving the change in the surface area of the drop. Thus, in accordance with the derivation of Fig. 13 of Beard and Kubesh (1991), model raindrops with spherical equilibrium shape were generated and the changes in surface area due to given oscillations were computed. The oscillation amplitude was varied until the change in surface area was equal to the desired oscillation energy. At this point, the time dependence of oscillations was only a nuisance, so the time-independent maximum amplitude \(A_{lm}\) was used in computing the change in surface area, and accordingly, the observed extreme axis ratios in derivation of the change in surface energy. Thus, the relation between axis ratio observations and oscillation amplitudes was established. Nonspherical base shapes were also tested, but it was found that for them, the oscillations could actually lower the surface energy. It was also found that the introduction of sinusoidal time dependence approximately halved the average change in surface energy.

It turned out that currently there is not sufficient data available to identify the oscillation modes and amplitudes accurately as a function of size. Thus, in this study, the focus was to study the effect of different oscillation modes and amplitudes individually without size depen-
dence. Some size distribution simulations were done, however, and for this a following sample oscillation scheme was created. It is emphasized that this is an example scheme rather than a solution to the problem of size-dependent oscillations.

It is well established that raindrops smaller than \( d = 1.0 \) mm do not oscillate, whereas those larger generally do. For larger, oscillating drops, the scheme is simplified by assuming that each drop can only have one oscillation mode. The problem was then to derive how probable the different oscillation modes are and what are the amplitudes from available data. For simplicity, the focus was on the laboratory measurements by Beard and Kubesh (Beard and Kubesh 1991; Kubesh and Beard 1993), covering most of the size range of interest. It must be emphasized that oscillations forced by collisions are not included in these laboratory measurements. This may affect the results somewhat, as most collisions are off-center collisions and thus probably tend to produce transverse-mode oscillations (Andsager et al. 1999). As the intention was to study realistic natural oscillations, only the data from the lowest observation level (C) of Kubesh and Beard (1993) were used, as they were clearest from the initial oscillations produced by the drop generator. For raindrops with \( d > 2.5 \) mm, results by Andsager et al. (1999) were used. It should be pointed out that, unlike in radar applications and Rayleigh scattering, in the visible light regime the large drops are not that dominating, but instead, their relative role is minor due to their sparseness (see Fig. 4, e.g.). This and the sparseness of large-raindrop data resulted in relatively little effort being put into modeling the large-drop oscillations.

From the data shown in Fig. 10 of Beard and Kubesh (1991) and Fig. 2 of Kubesh and Beard (1993), it is obvious that for \( d = 1.108, d = 1.196, d = 1.294, d = 2.0, \) and \( d = 2.5 \) mm raindrops, the oscillations are practically pure transverse modes. Assuming that the mode is purely \((2, 1)\) or \((3, 1)\), the oscillation energies \((\Delta E)\) were derived applying the method described above. The results are shown in Fig. 1 for modes \((2, 1)\) and \((3, 1)\). It seems that the oscillation energy is very

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### Table 1. Coefficients for the cosine series for generating equilibrium raindrop shapes with diameters from 1.0 to 6.0 mm (modified from Chuang and Beard 1990).

<table>
<thead>
<tr>
<th>(d) (mm)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
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<tbody>
<tr>
<td>1.0</td>
<td>-0.0028</td>
<td>0.0030</td>
<td>-0.0083</td>
<td>0.0022</td>
<td>-0.0003</td>
<td>-0.0002</td>
<td>0.0001</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>1.5</td>
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<td>0.0070</td>
<td>-0.0210</td>
<td>0.0057</td>
<td>-0.0006</td>
<td>-0.0007</td>
<td>0.0003</td>
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<td>0.0000</td>
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<tr>
<td>2.0</td>
<td>-0.0134</td>
<td>0.0118</td>
<td>-0.0385</td>
<td>0.0100</td>
<td>-0.0005</td>
<td>-0.0017</td>
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<tr>
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<td>-0.0592</td>
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<td>0.0004</td>
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<td>-0.0020</td>
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**Fig. 1.** Changes in surface energy \((\Delta E)\) due to oscillations observed by Beard and Kubesh (1991) and Kubesh and Beard (1993) for \( d = 1.108, d = 1.196, d = 1.294, d = 2.0, \) and \( d = 2.5 \) mm raindrops, assuming that the oscillations are purely mode \((2, 1)\) (○) or \((3, 1)\) (*). Solid lines are linear least squares fits.

**Fig. 2.** Comparison of axis ratio statistics for model-generated and observed laboratory-generated drops. The gray area represents the range of variation of axis ratio, and the solid line the average axis ratio for model raindrops. The dashed line is the axis ratio for nonoscillating equilibrium raindrops. Stars and their error bars represent the mean axis ratios and the detected range of variation for laboratory-generated drops studied by Beard and Kubesh (1991), Kubesh and Beard (1993), and Andsager et al. (1999).
closely linearly related to the drop size in this size range; thus a linear least squares fit was applied.

The dependence of $\Delta E$ on the oscillation amplitude was then studied for both (2, 1) and (3, 1) modes. It was found that it is quite closely linearly proportional to the square of the oscillation amplitude. Thus, applying a linear least squares fit again, the relation between oscillation energy and oscillation amplitude was derived.

Thus, the relations between drop size and oscillation energy and between amplitude and oscillation energy were established. This allowed resolving the relation between oscillation amplitude and drop size, leading to

$$A_{2,1} = \left[ \frac{4.489d - 4.13}{236.5} \right]^{\frac{1}{2}}, \quad 1.0 \leq d \leq 2.5 \text{ mm} \quad (19)$$

$$A_{3,1} = \left[ \frac{3.268d - 3.13}{589.0} \right]^{\frac{1}{2}}, \quad 1.0 \leq d \leq 2.5 \text{ mm} \quad (20)$$

where $d$ is in millimeters. In the light-scattering simulations, these empirical relations were used to compute the maximum oscillation amplitudes for model raindrops in the size range from 1.0 to 2.5 mm, oscillating with modes (2, 1) or (3, 1).

Drop sizes $d = 1.398$ mm and $d = 1.538$ mm were not included in Fig. 1; as for them the oscillations are clearly not transverse modes. Beard and Kubesh (1991) found the correlation between changes in vertical and horizontal axes to be strongly negative ($-0.4$ to $-0.6$) for them, consistent with (2, 0) oscillations. Thus, for these sizes the transverse modes were considered a minor constituent and the (2, 0) mode the predominant one. Only a few drop sizes with a predominant (2, 0) mode do not allow studying the size dependence of this mode confidently; thus a constant maximum amplitude of $A_{2,0} = 0.5$ was chosen for all sizes, consistent with oscillations detected in $d = 1.398$ mm and $d = 1.538$ mm drops.

Laboratory measurements of Andsager et al. (1999) were used to derive the oscillations for raindrops larger than $d = 2.5$ mm. The data indicate that large raindrop oscillations have a strong sinusoidal component about the equilibrium in axis ratio, but the mean axis ratio is mostly slightly above the equilibrium. This indicates the presence of a strong (2, 0) oscillation together with a weaker transverse component.

The measurements of $d = 2.5$ mm raindrops were included both in Andsager et al. (1999) and Kubesh and Beard (1993) studies, and there appears to be a significant conflict in the results. Whereas the results by Kubesh and Beard indicate oscillations in transverse modes, the results by Andsager et al. indicate the presence of an almost pure axisymmetric mode. This difference probably results from different drop generators used, giving the drops different initial oscillations (Andsager et al. 1999; K. Beard 1998, personal communication). It thus seems that initial oscillations can affect the resulting steady-state oscillations. If so, the off-center collisions in natural rain should enhance the probability of transverse modes compared to that observed in the laboratory in the absence of collisions.

Before simulations, the size-dependent oscillation scheme needed to be configured; that is, relative probabilities and weighting coefficients for subranges needed to be set. For $1.0 \leq d \leq 1.35$ mm raindrops, only transverse modes (2, 1) and (3, 1) were considered possible and equally probable, with maximum amplitudes those given by Eqs. (19) and (20). In the size range of $1.35 \leq d \leq 1.75$ mm, mode (2, 0) oscillations were considered the most probable with an 80% chance, with maximum amplitude being 70% of preset $A_{2,0} = 0.05$ or 0.035. The transverse modes were set to have equal probabilities of 10%, maximum amplitudes being 90% of those given by Eqs. (19) and (20). For $1.75 \leq d \leq 2.5$ mm drops, probabilities were 20% for mode (2, 0) and 40% for both transverse modes. Maximum amplitudes were 20% of preset $A_{2,0} = 0.05$, or 0.01 for (2, 0), and 90% of those given by Eqs. (19) and (20) for transverse modes. For drops larger than $d = 2.5$ mm, mode (2, 0) had a probability of 70% and an amplitude of preset $A_{2,0} = 0.05$, whereas transverse modes had equal probabilities of 15% and amplitudes of preset $A_{2,1} = 0.13$ and $A_{3,1} = 0.07$.

This scheme was tested by generating model raindrops, computing their silhouettes as seen from $\theta = 90^\circ$ and random $\varphi$ angle, computing their axis ratio statistics, and comparing the results with observations. This comparison is shown in Fig. 2. For $d = 2.5$ mm raindrops the observed values are those given by Kubesh and Beard (1993), although this size range was also studied by Andsager et al. (1999). The reason for this selection is simply that the latter data were not yet published when writing this manuscript.

It can be seen that the axis ratio statistics of model raindrops and laboratory-generated drops agree well. For $d < 1.35$ mm raindrops, the agreement is excellent. For drops around $d = 1.5$ mm, the variations produced by the model appear too small, even though the standard deviations (not shown) of the axis ratio are close to the observed ones. Thus, it appears that in the observations, there have been only a few drops causing large variation, whereas most of the drops have weaker oscillations. This kind of oscillation amplitude distribution is also proposed by Chandrasekar et al. (1988) to explain their axis ratio observations. In the $1.75 \leq d \leq 2.5$ mm size range the minimum axis ratios generated appear to be generally below the observed values, but otherwise the statistics agree. This is because for $d = 2.5$ mm raindrops, the axisymmetric (2, 0) oscillations appear to be present, and thus in the sample distribution, they were allowed in the whole subrange. The model agrees well with observations also for the largest drops, although this subrange was incorporated into the sample distribution only approximately.

It must be emphasized that it would have been easy to improve this agreement by adjusting the model pa-
rameters. Considering the sparseness of the size-dependent oscillation data and the fact that the model does not include the change in oscillation amplitude except that caused by time dependence, it was considered sufficient that the average axis ratios agree well with the observations and that the axis ratio scatter is similar with observations. It is not, after all, the purpose of this study to find a best-fit size-dependent oscillation scheme with observations, but, to create a sample size-dependent scheme that could be used in studying the effect of size distributions on the other factors affecting light-scattering properties of oscillating raindrops. Thus, the agreement was considered quite satisfying. It also provides an indication that, in general, the raindrop geometry used in the model is quite realistic.

5. Size distributions

A raindrop size distribution is the end product of all the cloud microphysical processes, cloud dynamical processes, and interactions that affect the formation and growth of liquid precipitation. Ordinarily, water drops with diameters smaller than \( d = 0.2 \text{ mm} \) are called cloud drops, and larger drops are raindrops, although these definitions appear slightly vague.

Natural size distributions vary strongly both in time and space (e.g., Stow et al. 1991; Cataneo and Stout 1968), but there are usually some common factors: the number of drops decreases rapidly with increasing size; the number of large drops tends to increase with increasing rainfall rate; and there are usually few very small raindrops. It follows that there are a number of different models for raindrop size distributions.

The classical model is that of Marshall and Palmer (1948). They proposed a single-parameter negative exponential size distribution,

\[
N(d) = N_0 e^{-\Lambda d},
\]

where \( N_0 = 0.08 \text{ cm}^{-4} \) is constant; and \( \Lambda = 41 R^{-0.21} \text{ cm}^{-1} \) depends on the rainfall rate \( R \), given in mm h\(^{-1}\). Here, \( N(d) \) describes the number of raindrops in unit volume per size range. This raindrop size distribution is usually called the Marshall–Palmer (M–P) distribution.

It appears that negative exponential size distribution is a good approximation for raindrops, when sufficient averaging in time or space is carried out (Ulbrich 1983; Joss and Gori 1978). As shown by Joss and Gori (1978), however, “instantaneous” (measured during 1 min or less) distributions differ significantly from exponential, generally toward monodisperse distributions. It seems that, however, even after averaging, fewer small and large drops are observed compared to the M–P distribution (Willis 1984; Sauvageot and Lacaux 1995).

The lack of small drops has led to propositions of other size distribution functions, typically three-parameter distributions. For example, gamma and lognormal distributions are used commonly. Naturally, as the number of parameters increases, so does the flexibility of the model, thus allowing better fits to observed distributions.

The gamma distribution has a form,

\[
N(d) = N_\mu d^\mu e^{-\Delta d},
\]

where the exponent \( \mu \) can have any positive or negative value (Ulbrich 1983). According to Willis (1984), gamma distribution provides the best compromise between fitting error and realistic characterization of coalescence growth and drop evaporation for, at least, hurricane clouds, but likely for any warm-based convective cloud system. Willis and Tattelman (1989) note that the gamma distribution fits well to observations from several different geographical locations measured from airplanes in heavy tropical rain.

Feingold and Levin (1986), on the other hand, conclude that a lognormal distribution fits best to the observations of raindrop size distributions for rains in Israel with rainfall rates mostly below about 5 mm h\(^{-1}\). The general three-parameter lognormal distribution is given by

\[
N(d) = \frac{N_F}{d \sqrt{2 \pi \ln \sigma}} \exp \left[ -\frac{\ln^2(d/d_s)}{2 \ln^2 \sigma} \right],
\]

where \( d_s \) is the mean geometrical diameter, \( \sigma \) the standard geometrical deviation, and \( N_F \) the number of drops (Sauvageot and Lacaux 1995).

Here, two sample raindrop size distributions are used to model natural size distributions. The M–P distribution is used to represent a typical size distribution in continuous frontal rain. A rainfall rate of 5 mm h\(^{-1}\) is chosen, and parameters \( N_0 \) and \( \Lambda \) are as given by Marshall and Palmer (1948). The gamma distribution with parameters \( N_\mu = 44.7 \text{ cm}^{-4}, \mu = 2.16, \) and \( \Lambda = 31.6 \text{ cm}^{-1} \) is used to represent a typical convective rain shower. These values are linearly interpolated for the rainfall rate of 50 mm h\(^{-1}\) from the values given by Willis and Tattelman (1989). These size distributions are shown in Fig. 3. Since in the geometrical optics regime the scattering cross sections are proportional to the square of the diameter, approximated (probability densities multiplied by the square of diameter) contributions of drops with different size to the total scattering cross section could be computed. These are shown in Fig. 4. It can be seen that, in general, small non-oscillating drops are important in the M–P distribution, whereas in the gamma distribution the main contribution comes from the \( d = 1.0–2.0 \text{ mm} \) raindrops. Very large raindrops are insignificant in both distributions.

The lognormal size distribution is used to study the dependence of light scattering on the width of the size distribution. This distribution was chosen because its parameters have simple geometrical interpretations, and thus the desired form is easily given for the distribution. As the size distribution functions were normalized to probability density functions, the parameter \( N_F \) was re-
6. Results and discussion

The coordinate systems and angles relevant for the scattering problem are described in Fig. 6. The raindrop geometry is described in spherical coordinates \((\theta, \varphi)\), with the origin defined by the equilibrium shape. The scattering angles \((\theta_s, \phi_s)\) are defined relative to the direction of incident light. The Cartesian coordinates are mainly used in rotation operations.

All the light-scattering simulations were carried out assuming the refractive index of 1.33 + i0.0, representative for clean water in the visible region of the electromagnetic spectrum. In the single-size simulations, a million incident rays were used, whereas the size distribution simulations were run with 10 million rays. Up to 11 surface crossings were allowed for both the internal and the external parts of the ray tree, and the cutoff value for the ray intensity was 10^{-4} times the incoming intensity. With these settings, the energy lost by cutoff or problems with bracketing or with impossible normals was usually much less than a percent and always below 2%. The \(\varphi_i\) angle was randomized for each incoming ray, as natural raindrops cannot be considered azimuthally oriented.

The dependence of light-scattering properties of raindrops on the oscillation modes and amplitudes were studied using a single drop size with different oscillations. The drop size of \(d = 2.0\) mm was chosen for this investigation.
than the lower-degree modes. These findings are consistent with results of NM and Muinonen et al. (1996). It can also be seen that the asymmetry parameters are systematically higher when the light is incident from above the drop ($\theta = 180^\circ$), compared to the incidence below the drop ($\theta = 0^\circ$), and that the asymmetry parameter tends to be highest for side incidence ($\theta = 90^\circ$), except in the case of strong transverse oscillations. It appears that the (2, 0) oscillations of simulated amplitude do not change the asymmetry parameter significantly.

Computed average cross-sectional surface areas ($A$) are shown in Table 3. Except for the $A_{3,1} = 0.077$ case, the maximum values are found at ($\theta = 0^\circ$) and ($\theta = 180^\circ$), being equal within the statistical accuracy. The minimum values are found at ($\theta = 90^\circ$). It can be seen that oscillations, in general, increase the cross-sectional surface area. Thus the asymmetry parameter and the cross-sectional surface area appear to be anticorrelated. Also, for mode (2, 1) oscillations, the range of variation tends to be larger than that for the equilibrium case, whereas for mode (3, 1) oscillations it is smaller. It is expected that further increase in the degree of oscillation would continue to decrease the range of variation.

In light of these results, it is hard to determine the relative importance of orientation and oscillations on the scattering properties, but both factors are clearly important.

The geometric optics scattering phase matrix elements $P_{11}$, $P_{21}$, $P_{31}$, and $P_{41}$—that is, those elements that affect the scattering of unpolarized light—were further analyzed. For the analysis, scattering matrix elements were normalized and plotted similarly as in NM— that is, $P_{11}$ elements were normalized with respect to its maximum and minimum values—while other elements are automatically in the range of $[-1, 1]$. Due to nor-

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Table 2: The dependence of the ray optics asymmetry parameter $g$ on the angle of incidence ($\theta$), oscillation mode ($l, m$), and the maximum oscillation amplitude ($A_{l,m}$) for $d = 2.0$ mm raindrops. Values for plain equilibrium shape (ES) are included for comparison.
malization, any differences in absolute values among $P_{ij}$ elements are not visible.

The analysis revealed that, for the amplitudes used here, the (2, 0) oscillations do not change the scattering pattern strongly. Even for the strongest amplitude case, the differences between the (2, 0) oscillations case and the equilibrium drops were small. For the cases with $\phi_i < 90^\circ$, most differences were seen in the $\theta_i = 90^\circ$--$120^\circ$ region near the $\phi_i = 180^\circ$ azimuth angle, whereas for $\theta_i > 90^\circ$, more changes took place near the $\phi_i = 0^\circ$ azimuth angle. The $90^\circ$ rainbow region, for orientations where it was visible, showed significant changes. The $90^\circ$ rainbow itself was a lot more diffuse, and the deformation of ordinary rainbows was more complex in the case of (2, 0) oscillations than for equilibrium drops. In general, these differences were more obvious in the polarization elements than in the phase function. Halving the amplitude was sufficient to remove most of these visible differences in scattering patterns, but the $90^\circ$ rainbow was still a lot more diffuse than that for equilibrium raindrops. For one-quarter maximum amplitude, the results were practically identical with equilibrium cases.

In the strongest amplitude case, the (2, 1) oscillation and the equilibrium cases generally agreed only in the forward scattering region, where polarization is weak and intensity high. In general, both cases also produced a strongly polarized weak-intensity size-scattering region and a weakly polarized weak-intensity backscattering region, separated by rainbows, but in these regions the differences were generally large. Deformations due to oscillations were occasionally very strong, to a degree where even the primary rainbow was barely recognizable and the secondary rainbow indistinguishable. Naturally, for both half and one-quarter maximum amplitudes, the differences between equilibrium and oscillation cases were a lot smaller but still clearly present. The $90^\circ$ rainbow was clearly visible only in the one-quarter amplitude case.

For (3, 1) oscillations, the differences from the equilibrium cases were even larger. Although, in general, the side-scattering region tended to have weak intensity and strong polarization, the angle dependence in this region was very strong. Most changes appeared to occur near the $\phi_i = 0^\circ$ or $\phi_i = 180^\circ$ angle, depending on the angle of incidence. The primary rainbow was deformed into practically separate arcs, and the secondary rainbow was undetectable. The backscattering region was highly variable both in intensity and polarization, depending on the angle of incidence. As for (2, 1) oscillations, the decrease in maximum oscillation amplitude decreased the differences between oscillation cases and equilibrium cases. For both half and one-quarter maximum amplitudes, the differences between equilibrium and oscillation cases were a lot smaller but clear.

The transverse oscillations (2, 1) and (3, 1) were mutually different. For example, side- and backscattering regions were more angle dependent and rainbows more deformed in the (3, 1) case. The dependence on the angle of incidence appeared simpler in the (2, 1) oscillation case. In general, the (2, 1) case appeared to be closer to the (2, 0) and the equilibrium cases, but this is obvious only for weaker-amplitude cases.

Comparing the scattering simulations of NM and simulations of this study, it is obvious that individual oscillation modes change the scattering in a different manner compared to the Gaussian random oscillations. This is quite a reasonable result, as in Gaussian random oscillations there are many different oscillation modes simultaneously present, and single-mode simulations performed here prove that different modes result in different scattering properties. It follows that for Gaussian

### Table 3. Same as Table 2 except for average cross-sectional surface area $\langle A \rangle$ (units in mm$^2$).

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random oscillations, features introduced by individual modes are more or less canceled out by the coexistence of different modes in a same drop. In a population consisting of drops oscillating with different modes but each in only one mode, such canceling does not take place, but instead, the scattering pattern is a superposition of different patterns. This explains why the Gaussian random oscillations basically tend to smooth away the features introduced by the equilibrium shape, whereas individual oscillation modes add new features. This also indicates that, in the presence of differently oscillating raindrops, the Gaussian oscillation assumption is a reasonable first approximation for the drop shape, but when scattering from single raindrops is studied, it should not be used unless the drop undergoes oscillations with different modes simultaneously. To be on the safe side, Gaussian random oscillations should probably be used only for drops oscillating in multiple modes regardless of the size distribution. These results also indicate that as long as only a single-oscillation mode is present, light-scattering measurements could be used to study the oscillations.

The time (or phase) dependence of light scattering by an oscillating single raindrop was studied briefly. The $d = 2.0 \text{ mm}$ equilibrium raindrop was used, with an imposed $A_{i j} = 0.145$ oscillation. The orientation of $\theta_i = 135^\circ$ was chosen, as it is normal to the oscillation direction of mode (2, 1). Following Eq. (18), the oscillation amplitude $a_{ij}$ was changed by varying $x$ systematically. It was found that the time dependence is quite significant. For example, the cross-sectional surface area varied in the range of [3.19, 3.32] mm$^2$. The asymmetry parameter, on the other hand, showed little time dependence. The range of variation for the $\phi_s$-averaged geometric optics phase function $P_{ij}$ is shown in Fig. 7. It can be seen that, for most angles with $\theta_i > 90^\circ$, the $\phi_s$-averaged data show strong variations. For unaveraged plots over $4\pi$ (not shown), the variations are even stronger. It is somewhat surprising that the time dependence is stronger in the forward direction than in any other angles with $\theta_i < 90^\circ$. The higher sensitivity in the $\theta_i > 90^\circ$ region is quite expected. The $90^\circ$ rainbow region seems also quite time dependent.

The possibility of seeing the $90^\circ$ rainbow in nature was studied in detail. The simulations were carried out with $\theta_i = 130^\circ$ and nonoscillating equilibrium drops. The simulations with different single-size cases showed that the $90^\circ$ rainbow was visible in some form for all nonspherical raindrop sizes. For $d = 1.0 \text{ mm}$ drops, it was a barely visible arc around $\theta_i = 120^\circ$ (around azimuth angle $\phi_s = 180^\circ$) next to a secondary rainbow, moving toward the forward direction and gaining brightness with increasing drop size, reaching $\theta_i = 90^\circ$ at $d = 2.0 \text{ mm}$. At $d = 3.0 \text{ it had reached } \theta_i = 60^\circ$ and started to diminish. Increasing drop size any further did not move it in the $\theta_i$ direction, but decreased its brightness and deformed the bow into two separate spots, starting to move azimuthally, away from $\phi_s = 180^\circ$. These spots were still visible at $d = 6.0 \text{ mm}$. In the size range of [1.0, 2.5] mm, a weak “secondary $90^\circ$ rainbow” was visible at smaller $\theta_i$ angles, moving similarly with increasing drop size.

The primary rainbow was also deformed toward the forward direction around azimuth angle $\phi_s = 180^\circ$ with increasing drop size, reaching about $\theta_i = 75^\circ$ by $d = 6.0 \text{ mm}$. This also proved the conclusion of NM that in the $d = 6.0 \text{ mm}$ case the bright deformed bow near the $\theta_i = 90^\circ$ angle is not the $90^\circ$ rainbow but the primary rainbow. This is, however, mostly of academic interest, as drops this large are very rare in nature and they always oscillate.

The size-dependent behavior of the $90^\circ$ rainbow suggested that any distribution with moderate width would cause these bows to disappear. Simulations for continuous frontal rain and convective shower cases with equilibrium raindrops proved this to be the case. When the M−P size distribution was introduced, the $90^\circ$ rainbow indeed practically disappeared. No distinct bow was visible, but the whole zone of about $\phi_s > 135^\circ$ and $\phi_s < -135^\circ$ was clearly brighter. The introduction of the gamma distribution caused similar results, only the zone $\phi_s > 135^\circ$ and $\phi_s < -135^\circ$ was brighter and had a little more detail. However, the $90^\circ$ rainbow was indistinguishable. In general, the M−P distribution and the gamma distribution cases differed significantly little, considering the remarkable difference between the distributions. This is consistent with findings by Macke and Großklaus (1998).

As the $90^\circ$ rainbow was not clearly visible in M−P or gamma size distribution simulations, it was studied if a narrower size distribution centered around $d = 2.0 \text{ mm}$ could show the bow better. Thus, the simulations were repeated with lognormal size distributions shown in Fig. 5. For the $\sigma = 1.1$ case, the bow was almost as well defined as in the single-size simulations for $d = 2.0 \text{ mm}$ equilibrium raindrops, and even “the secondary $90^\circ$ rainbow” was weakly visible. For the $\sigma = 1.2$ case, the $90^\circ$ rainbow was more diffuse, filling a wider solid
angle and “the secondary 90° rainbow” was indistinguishable. In the $\sigma = 1.3$ case, the clearly visible bow had disappeared and been replaced by the luminous region, resembling closely the gamma distribution cases. It thus appears that relatively wide size distributions can result in a visible 90° rainbow, at least as long as most of the drops are near $d = 2.0$ mm in size.

When the size-dependent oscillation scheme was included in the simulation, the light-scattering simulation results changed significantly. Features introduced by transverse modes showed up clearly despite the smoothing effect of the size distribution. The side-scattering region changed significantly around the $\phi_s = 0^\circ$ region. Also, the oscillations slightly increased differences between the M–P and the gamma size distribution cases. Scattered intensities for M–P and gamma size distributions are shown in Figs. 8 and 9 for the $\theta_i = 130^\circ$ case. Because of the higher number of small spherical drops, the primary rainbow was a lot more distinct and less deformed in the M–P distribution case, with a faint secondary rainbow also visible. The features caused by transverse modes were, on the other hand, clearly stron-
light in the gamma distribution case, a clear contribution of the higher number of larger oscillating drops. As for equilibrium cases, the region of the $90^\circ$ rainbow was brighter in the gamma distribution case. The lognormal cases were similarly changed by the oscillations, but interestingly, the $90^\circ$ rainbow was clearly visible, although strongly smoothed, in the $\sigma = 1.1$ case. It must be emphasized that, in nature, most oscillating raindrops probably have weaker amplitudes than those given by the sample oscillation scheme, while a few drops may have stronger amplitudes. Thus, it is expected that in nature the effect of oscillations may not be just that strong.

Finally, the size-dependent oscillation scheme was used with a fixed size to study how a population of differently oscillating drops scatters light. A drop size of $d = 2.0$ mm was selected for this simulation, as all the modes are possible for it in the size-dependent oscillation scheme, and there are also Gaussian random oscillation results (NM) available for it. Simulations indicate that the scattering pattern of such a population is not as smoothed as for Gaussian oscillations but, instead, a relatively unsmoothed superposition of features by differently oscillating drops. It thus appears that, as long as there is only one oscillation mode in any one drop of the population, the effect of oscillations on scattering patterns is not that of smoothing, but instead, the signature of different modes is, at least in principle, visible. In theory, then, light-scattering measurements might be used to detect if there are multiple modes present in drops. In practice, however, this may be too difficult to be measured.

7. Conclusions

Light scattering by oscillating equilibrium raindrops was studied with and without size distributions. Single-size simulations were used to study the dependence of light-scattering properties on the orientation, the oscillation modes, and the oscillation amplitudes. The effect of size distributions on light scattering was studied with Marshall–Palmer, gamma, and lognormal size distributions, including a size-dependent oscillation scheme.

The light-scattering simulations with different oscillation modes revealed that, in general, different modes affect the scattering properties of oscillating equilibrium raindrops differently. The difference was especially notable between transverse and axisymmetric modes studied, but the transverse modes were also mutually different. Modes $(2, 1)$ and $(3, 1)$ changed the scattering of equilibrium drops strongly, and thus their presence should be relatively easy to detect by light-scattering measurements. The $(2, 0)$ mode, on the other hand, changed the scattering patterns of equilibrium drops only slightly. It appears that very sophisticated light-scattering measurements would be necessary to study the properties of $(2, 0)$ oscillations.

Unlike the Gaussian random oscillations, individual oscillation modes introduced new features in the scattering patterns. Thus, their effect was not that of smoothing. It appears that unless more than one oscillation mode is present in a drop, the introduction of new features in scattering patterns is the dominating phenomenon, rather than the smoothing. In the case where there are multiple modes present in drops, Gaussian random oscillations should approximate the oscillations well from a light-scattering point of view.

The time dependence of light scattering by a single oscillating raindrop is clearly strong enough to be taken into consideration in scattering measurement. It is recommended that, if possible, sufficiently long integration times be used in measurements so that any drop in the measurement volume can undergo a full oscillation cycle. On the other hand, if the measurement equipment allows, the time dependence may be used to identify the oscillation modes. For this, the use of different angles of incidence should be beneficial. Especially, the time-dependent measurements could be used more effectively in studying the $(2, 0)$ oscillations.

The scattering patterns for M–P and gamma size distribution simulations differed surprisingly little, considering the remarkable difference between these distributions. The introduction of oscillations increased the differences somewhat, but, in general, it seems that size distributions cannot be easily derived by solving an inverse light-scattering problem. The only exception to this may be the very narrow distributions, which resemble closely the appropriate single-size cases.

The results of the size distribution simulations do not look promising for the $90^\circ$ rainbow to be seen in nature. The introductions of both M–P and gamma distributions effectively removed the bow, leaving the zone of its appearance diffusively bright. The simulations with individual drop sizes, on the other hand, show that this bow is not a property of only a narrow size range, but instead it can be seen over a wide range of sizes. Thus, its disappearance is due to the movement of its location with size. The simulations with lognormal size distributions of varying width revealed that the bow is definitely visible in size distribution simulations if the distribution is not too wide and if the distribution is centered near $d = 2.0$ mm raindrops. Interestingly, the “instantaneous” size distributions observed by Joss and Gori (1978) are toward the monodisperse forms; thus special circumstances might allow one to see this novel feature.

The scattering of visible light by raindrops can be described with great accuracy by the ray optics approximation. Thus, if there are differences between the model simulations conducted here and light scattering by real raindrops, these very likely result from the inaccuracy of the geometric model of the drops. Apart from the variation of oscillation amplitudes between drops, a factor not included, the raindrop geometry model used here is probably almost as good a model as possible, considering the current knowledge. It is likely
that the drop geometry cannot be greatly improved without considering the coupling between the equilibrium part and the oscillation part, that is, without deriving the geometry by a hydrodynamic model including the interconnection between drop shape and aerodynamic forces.

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