Generation Mechanisms of Convectively Forced Internal Gravity Waves and Their Propagation to the Stratosphere

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(Manuscript received 13 November 2002, in final form 18 March 2003)

ABSTRACT

Characteristics of gravity waves induced by mesoscale convective storms and the gravity wave sources are investigated using a two-dimensional cloud-resolving numerical model. In a nonlinear moist (control) simulation, the convective system reaches a quasi-steady state after 4 h in which convective cells are periodically regenerated from a gust front updraft. In the convective storms, there are two types of wave forcing: nonlinear forcing in the form of the divergences of momentum and heat flux, and diabatic forcing. The magnitude of the nonlinear source is 2 to 3 times larger than the diabatic source, especially in the upper troposphere. Three quasi-linear dry simulations forced by the wave sources obtained from the control (CTL) simulation are performed to investigate characteristics of gravity waves induced by the various wave source mechanisms. In the three dry simulations, the magnitudes of the perturbations produced in the stratosphere are comparable, yet much larger than those in the CTL simulation. However, the sum of the quasi-linear perturbations generated by the nonlinear and diabatic sources compare well with the mesoscale circulations and gravity waves in the CTL simulation.

Through the spectral analysis, it is found that the stratospheric gravity waves in all simulations have similar vertical wavelengths (6.6–9.9 km), horizontal wavelengths (10–100 km), and periods (8–80 min). Also, the magnitudes of gravity waves in the quasi-linear dry simulations are comparable with each other in spite of the differences in the magnitude of the nonlinear and diabatic sources. This is because the vertical propagation condition of linear internal gravity waves, in both the troposphere and stratosphere, restricts wave sources in the horizontal wavenumber ($k$) and frequency ($\omega$) domain, and therefore all of the forcing cannot generate gravity waves that can propagate up to the stratosphere. Compared with the diabatic sources, the nonlinear sources are inefficient in generating linear gravity waves that can propagate vertically into the stratosphere. These results suggest that wave generation mechanisms cannot be accurately understood without examining the vertical propagation condition of the gravity waves. Also, the “effective” wave sources are of comparable magnitude, yet mostly out of phase. Therefore, although the wave amplitudes produced by simulations with nonlinear forcing and diabatic forcing are about 2 to 3 times too large, their sum compares well to the control simulation.

1. Introduction

It has been recognized that vertically propagating internal gravity waves have profound effects on the large-scale circulation of the middle atmosphere (e.g., Lindzen 1981; Holton 1982; Matsuno 1982). The parameterization of orographically generated gravity wave drag can successfully alleviate excessive westerly bias appearing in the upper troposphere and lower stratosphere of the Northern Hemisphere wintertime in large-scale numerical models (e.g., Palmer et al. 1986; McFarlane 1987). In the atmosphere, there are various gravity wave sources generating nonstationary gravity waves relative to the ground other than orography generating (stationary) waves. Fritts (1984) noted that a large portion of gravity waves observed in the middle atmosphere consist of high frequencies. Bergman and Salby (1994) showed that the upward propagation of energy and momentum, in the tropical stratosphere, is mainly associated with gravity waves with frequencies higher than 0.5 cycles day$^{-1}$. Recently, cumulus convection has received a great deal of attention as a possible source of the nonstationary gravity waves, especially in the Tropics where persistent cumulus clouds exist.

There have been several numerical modeling studies of convectively generated gravity waves and their ef-
fects on the large-scale circulation of the middle at-
mosphere. Alexander et al. (1995) investigated the
spectral characteristics of internal gravity waves in-
duced by squall lines. Their spectral analysis indicated
that gravity waves with vertical wavelengths of 6–10
km, horizontal wavelengths of 10–100 km, and intrin-
sic periods of 10–60 min are dominant in the strato-
sphere. Alexander and Holton (1997) showed that con-
vectively generated gravity waves can provide about
one-fourth of the momentum forcing required for driv-
ing the quasi-biennial oscillation (QBO) in the tropical
stratosphere. Piani et al. (2000) examined an interac-
tion between the typical zonal wind of the QBO and
the gravity waves induced by three-dimensional deep
convection. They found that the gravity wave moment-
num flux can accelerate the zonal-mean zonal wind by
as much as 1.0 m s$^{-1}$ day$^{-1}$ and $-0.3$ m s$^{-1}$ day$^{-1}$ in
the QBO westerly and easterly phases, respectively,
and induces the downward propagation of shear layers
in both QBO phases.

Compared with orographic gravity waves, it is more
difficult to understand how gravity waves are generated
by convective sources. There are currently three pro-
posed generation mechanisms (thermal forcing, obsta-
cle, and mechanical oscillator mechanisms) for con-
vective gravity waves. In the first mechanism, con-
vective clouds are regarded as thermal forcings that
generate gravity waves (nonstationary or stationary waves relative to the thermal forcing) in a stably strat-
ified environment. Some analytic studies on the re-
sponse of a stably stratified atmosphere to a prescribed thermal forcing (e.g., Lin and Smith 1986; Bretherton
1988; Chun and Baik 1998; Baik et al. 1999) can be
included in this category. Based on this mechanism,
Chun and Baik (1998) proposed a parameterization of
gravity waves induced by cumulus convection (GWDC) for stationary waves relative to forcing re-
gion. Chun et al. (2001b) investigated the effects of
GWDC on large-scale circulation using the Yonsei
University atmospheric general circulation model with
the GWDC parameterization proposed by Chun and
Chun and Baik (1998)'s parameterization using more
realistic environmental conditions, including basic-
state wind shear and stability difference between the
convective region and above. In the obstacle mecha-
nism, convective clouds are considered obstacles to the
flow, and gravity waves are generated when the flow
is subsequently blocked. This mechanism was pro-
posed by Clark et al. (1986), supported by observations
above the convective boundary layer by Kuettnet et
al. (1987), and examined in numerical simulations by
this idea to time-dependent clouds, which were con-
ceptually called moving mountains. Recently, Beres et
al. (2002) used this obstacle mechanism, in their squall
line simulations with tropospheric shear, to explain the
enhancement of the momentum fluxes of gravity waves
propagating in the opposite direction of the tropo-
ospheric wind shear. In their simulations, the thermal
forcing mechanism, as well as the obstacle mechanism,
is also responsible for the generation of gravity waves
because nonsteady diabatic wave forcings related to
transient convective cells exist in their simulated squall
line. Finally, the mechanical oscillator mechanism was
first proposed by Pierce and Coroniti (1966). Fovell
et al. (1992) explained the characteristics of gravity
waves in the stratosphere induced by the midlatitude squall lines through simple numerical simulations
based on mechanical oscillator arguments. The basis
of this mechanism is that strong convective updrafts
in convective storms stimulate the stable stratosphere
and generate gravity waves above clouds. Convective
cells acting as mechanical oscillators, as well as a non-
steady diabatic forcing, can generate gravity waves
even under zero background wind relative to the con-
vective cells. Alexander et al. (1995) showed that the
dominant period of gravity waves obtained from their
spectral analysis corresponds to the regeneration period
of convective cells, and accounted for their gravity
wave spectra using this mechanical oscillator mecha-
nism. Lane et al. (2001) simulated a three-dimensional
multicellular convective system in the Tropics and an-
alyzed the sources of line forced wave equation in
order to examine the generation mechanism for con-
vectively generated gravity waves. They showed that
wave sources associated with nonlinear advection rath-
er than latent heat release are dominant in their sim-
ulated convective system and emphasized the mechan-
ical oscillator mechanism because the nonlinear wave
sources are strong near the level of neutral buoyancy,
through which strong convective updrafts overshoot.

In this study, we examine characteristics of internal
gravity waves in the stratosphere induced by mesoscale
convective storms and their generation mechanisms us-
ing a two-dimensional cloud-resolving model. Two
types of numerical simulations are performed: a non-
linear moist simulation and three quasi-linear dry sim-
ulations forced by wave sources associated with the
nonlinear and diabatic processes in the nonlinear moist
simulation. In section 2, the numerical model used in
this study is described. In section 3, results from the
nonlinear moist simulation are presented. In section 4,
the sources of linear forced internal gravity waves in
the nonlinear moist simulation are analyzed. In section
5, the three quasi-linear dry simulations forced by the
wave sources are described, and results from those dry
simulations are presented. In section 6, the charac-
teristics of the gravity waves in the stratosphere simulated
in the nonlinear moist and three quasi-linear dry sim-
ulations are analyzed using spectral analysis and the
linear internal gravity wave theory. The summary and
conclusions are presented in the last section.

2. The numerical model

The numerical model used in this study is the Ad-
vanced Regional Prediction System (ARPS; Xue et al.
1995), which is a nonhydrostatic, compressible model with explicit liquid–ice phase cloud microphysical processes. In this study, two-dimensional (x–z) simulations are performed, with the ice phase processes and the rotational effects of the earth ignored. In the ARPS, horizontal wind and thermodynamic state variables are defined as the sum of horizontally homogeneous and time-invariant basic-state variables and perturbation variables. The basic states for vertical wind, cloud water, and rainwater are assumed to be zero. The two-dimensional prognostic equations are

\[
\frac{\partial \rho u}{\partial t} = - \left( \rho u \frac{\partial u}{\partial x} + \rho w' \frac{\partial u}{\partial z} \right) - \frac{\partial \rho'}{\partial x} + D_u, \tag{1}
\]

\[
\frac{\partial \rho w'}{\partial t} = - \left( \rho u \frac{\partial w'}{\partial x} + \rho w' \frac{\partial w'}{\partial z} \right) - \frac{\partial \rho'}{\partial z} + \rho B + D_w, \tag{2}
\]

\[
\frac{\partial \rho \theta'}{\partial t} = - \left( \rho u \frac{\partial \theta'}{\partial x} + \rho w' \frac{\partial \theta'}{\partial z} \right) - \rho w' \frac{\partial \bar{\theta}}{\partial z} + Q + D_\theta, \tag{3}
\]

\[
\frac{\partial \rho'}{\partial t} = - \left( \frac{\partial \rho'}{\partial x} + w' \frac{\partial \rho'}{\partial z} \right) + \rho g w' - \rho c_s^2 \left( \frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} \right), \tag{4}
\]

\[
\frac{\partial \rho q}{\partial t} = - \left( \rho u \frac{\partial q}{\partial x} + \rho w' \frac{\partial q}{\partial z} \right) + \frac{\partial}{\partial z} (\rho V_q q) + S_q + D_q, \tag{5}
\]

where \( u = U(z) \) is the zonal state and \( u' \) is the basic-state and perturbation zonal wind; \( w', \theta' \), and \( \rho' \) are the perturbations of vertical wind, potential temperature, and pressure, respectively; and \( \bar{\rho} \) and \( \bar{\theta} \) are the basic-state density and potential temperature, respectively. In (2) the buoyancy term \( B \) is given as \(-g \rho' / \rho \), where \( g \) is the gravitational acceleration; and \( \rho' \) is the density perturbation. In (4), \( c_s = (\sqrt{\gamma R T}) \) is the speed of sound, where \( \gamma, R \), and \( T \) are the ratio of the specific heat of air at constant pressure and volume, the gas constant for dry air, and the basic-state temperature, respectively. In (5), \( q \) represents the mixing ratio of water vapor, cloud water, or rainwater. The sedimentation term, \( V_q \), is the terminal velocity of rainwater, is nonzero only when \( q \) is a rainwater mixing ratio. Here \( D_q \) represents turbulent and background mixing for any prognostic variable \( q \).

For the subgrid-scale turbulence parameterization, the first-order closure by Lilly (1962) and Smagorinsky (1963) is used. For the background mixing used to suppress numerical noise, the second- and fourth-order schemes are used in the vertical and horizontal, respectively. The vertical and horizontal background mixing coefficients are 0.002 and 0.004 s\(^{-1}\), respectively. Note that \( Q \) and \( S_q \) represent the diabatic forcing source or sink for water vapor, cloud water, or rainwater due to microphysical processes.

The model domain is 1200 km wide and 51 km deep. Horizontal and vertical grid sizes are 1 and 0.3 km, respectively. A sponge layer with a depth of 15 km is included in the uppermost part of the model domain. At the left boundary, a radiation condition based on Or-
Fig. 2. The $w' = 2 \text{ m s}^{-1}$ contour (thick) superimposed on total potential temperature field at (a) $t = 3 \text{ h}$ and (b) $t = 5 \text{ h}$ in the CTL simulation. The contour interval for total potential temperature is 7 K.

lanski (1976; with a vertically averaged outgoing phase speed) is used. At the right boundary, a sponge layer with a width of 100 km is included to prevent wave reflection detected in long-term simulations (Chun et al. 2001a). The ARPS uses the mode-splitting time integration presented by Klemp and Wilhelmson (1978) to effectively calculate acoustically active terms in the governing equations of the compressible model. In all simulations, the large and small time steps are 3 and 1 s, respectively.
3. Nonlinear moist simulation

In the nonlinear moist (control) simulation, convection is initiated by an ellipsoidal warm bubble with a maximum potential temperature perturbation of 2 K centered at \( x = 800 \text{ km} \) and \( z = 1.5 \text{ km} \). The time integration for the control (CTL) simulation is carried out for 12 h. The analytic sounding of Weisman and Klemp (1982) is used for the thermodynamic basic state. This sounding has a convective available potential energy (CAPE) of 2436 J kg\(^{-1}\). The Brunt–Väisälä frequency calculated from this sounding is about 0.01 s\(^{-1}\) in the troposphere and 0.021 s\(^{-1}\) in the stratosphere (Fig. 1a). The basic-state wind is assumed to have a constant shear below \( z = 6 \text{ km} \) and be uniform above it (Fig. 1b). This basic-state wind is relative to a gust front propagating eastward with a speed of 18 m s\(^{-1}\). Thus, the results shown hereafter are in the reference frame moving with the gust front, and therefore the convective system is confined to the model domain for the long time integration.

Figure 2 shows the contours of vertical velocity of 2 m s\(^{-1}\) and total potential temperature fields at \( t = 3 \text{ h} \) and \( t = 5 \text{ h} \). In the CTL simulation, the convective system reaches a quasi-steady state after \( t = 4 \text{ h} \) with a periodic cell regeneration. Convective cells are tilted downshear before \( t = 3 \text{ h} \), become upright at \( t = 3 \text{ h} \), and are tilted upshear after \( t = 4 \text{ h} \). This can be explained by vorticity balance between the vertical shear of the basic-state wind and the cold pool induced by the evaporation of rainwater (Rotunno et al. 1988). Before \( t = 3 \text{ h} \), the positive vorticity induced by the basic-state wind with a positive shear dominates the negative vorticity induced by the cold pool, because rainwater is not sufficient to produce a strong enough cold pool. In the quasi-steady state after \( t = 4 \text{ h} \), the evaporative cooling gradually strengthens due to the increase of rainwater mixing ratio and dry inflow from the rear side of the convective system. As a result, the vorticity generated by the cold pool begins to overcome the positive vorticity of the basic-state wind shear, and convective cells are tilted upshear. As the simulated convective system reaches the quasi-steady state, the structure of convectively generated internal gravity waves in the stratosphere also begins to change. At \( t = 3 \text{ h} \), upright convective cells induce gravity waves propagating westward and eastward with comparable magnitude. On the other hand, at \( t = 5 \text{ h} \), the westward-propagating gravity waves are dominant in the stratosphere due to the convective cells propagating rearward (westward in the present case) relative to the gust front located near \( x = 770 \text{ km} \) (Fig. 2b). This result is consistent with Fovell et al. (1992).

Figure 3 shows the time series of domain maximum vertical velocity and its power spectral density (PSD) calculated after \( t = 6 \text{ h} \). In the quasi-steady state, the average value of domain maximum vertical velocity is about 12 m s\(^{-1}\), and its fluctuation is quite regular after \( t = 7 \text{ h} \). Before calculating the PSD, a trend with timescale larger than the maximum period resolvable in the time series is removed by fitting a second-order polynomial. In addition, a Welch window (Press et al. 1992) is applied to the time series to prevent energy leakage at each frequency to neighboring frequencies in the PSD. Using the time series modified by the conditioning processes, the PSD is calculated as follows:

\[
\text{PSD}(f) = \frac{2\Delta t}{\omega_{ss}}|\text{FFT}(w'(f > 0))|^2, \tag{6}
\]

where \( \Delta t (=60 \text{ s}) \) is the time interval in the time series; FFT is the fast Fourier transform; \( \omega \) is the Welch window function given by \( 1 - [(j - N/2)/N/2]^2, j = 0, \ldots, N - 1 \), where \( N \) is the number of data; and \( \omega_{ss} = \Sigma_{j=1}^{N} \omega^2 \). In the PSD (Fig. 3b), there is a primary peak at 18.1 min and a secondary peak at 45.1 min. From \( t = 4 \) to 12 h, most of the domain maximum vertical velocities coincide with the maximum updrafts of convective cells being separated from the gust front, and therefore the primary periodicity of 18.1 min is asso-
associated with the convective cell regeneration. However, in the period from \( t = 6 \) h to 6 h, 45 min, the cell regeneration cannot be seen in the domain maximum vertical velocity, although periodic cell regenerations still exist (not shown). This is because the periodically generated convective updrafts are weaker than the gust front updraft during this period.

In the next section, the wave sources that generate the stratospheric internal gravity waves in the CTL simulation, shown in Fig. 2, are analyzed. As mentioned by Raymond (1975) and Lane et al. (2001), there are generally two types of wave sources in a convective system. These sources are diabatic forcing due to latent heating and cooling and nonlinear forcing associated with strong perturbations in the convective system. Because the mechanisms for generating the convectively generated gravity waves are closely related to these wave sources, it is necessary to understand their characteristics before analyzing the gravity waves in the stratosphere.

4. Gravity wave sources

For the calculation of the gravity wave forcings, it is assumed that the compressible flow simulated in the ARPS can be approximated by the anelastic flow (e.g., Ogura and Phillips 1962; Wilhelmson and Ogura 1972). The validity of such an approximation can be examined through a scale analysis of the prognostic equation for \( p' \) given in (4). For the scale analysis, (4) can be rewritten as

\[
\frac{1}{c_s^2} \left( \frac{\partial p'}{\partial t} + U \frac{\partial p'}{\partial x} \right) - \frac{p'}{c_s^2} \left( \frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} \right) - \frac{\rho g}{c_s^2} w' + \rho \left( \frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} \right) = -\frac{1}{c_s^2} \left[ \frac{\partial}{\partial x} (p'u') + \frac{\partial}{\partial z} (p'w') \right].
\]
When the terms in (7) are calculated in the CTL simulation, the terms A and B are from 10 to 100 times larger than the other terms. Note that the term A can be rewritten as \( \frac{\partial}{\partial x}(\overline{p}u'') + \frac{\partial}{\partial z}(\overline{p}w') = 0 \) (8).

Under the anelastic approximation and by separating linear and nonlinear terms, (1)–(3) in the dry and inviscid atmosphere can be rewritten as

\[
\frac{\overline{D}u'}{Dt} + \overline{p}w'\frac{dU}{dz} + \overline{p} \frac{\partial \phi'}{\partial x} = F_u, \tag{9}
\]

\[
\frac{\overline{D}w'}{Dt} - \overline{g} \frac{\theta'}{\theta} + \overline{p} \frac{\partial \phi'}{\partial z} = F_w, \tag{10}
\]

\[
\frac{\overline{D}u'}{Dt} + \overline{p}w' \frac{d\theta'}{dz} = F_\theta + Q. \tag{11}
\]

where \( \phi' = p'/\overline{p} \) and \( \overline{D}/Dt = \partial/Dt + \bar{U}/\partial \bar{x} \). In (9)–(11), the forcing terms \( F_u, F_w, F_\theta, \) and \( Q \) are expressed as

\[
F_u = -\left[ \frac{\partial}{\partial x}(\overline{p}u'') + \frac{\partial}{\partial z}(\overline{p}w') \right], \tag{12}
\]

\[
F_w = -\left[ \frac{\partial}{\partial x}(\overline{p}w'') + \frac{\partial}{\partial z}(\overline{p}w') \right], \tag{13}
\]

\[
F_\theta = -\left[ \frac{\partial}{\partial x}(\overline{p}\theta'') + \frac{\partial}{\partial z}(\overline{p}\theta') \right], \tag{14}
\]

\[
Q = \overline{p}[H + C], \tag{15}
\]

where \( H \) and \( C \) are diabatic heating and cooling rates due to microphysical processes, respectively.

In (12)–(15), \( F_u, F_w, \) and \( F_\theta \) are the divergences of perturbation Reynolds stress, \( F_u \) is the divergence of perturbation heat flux, and \( Q \) is the diabatic forcing. These forcing terms are generally confined to being important in small regions of strong convective activity and act as forcing for the linear momentum and thermodynamic energy equations. The forcing terms \( F_u, F_w, \) and \( F_\theta \) are calculated using perturbation variables saved at a 1-min interval in the CTL simulation, and the diabatic forcing term \( Q \) is calculated using diabatic heating and cooling rates obtained at a 1-min interval from the saturation adjustment and rain evaporation process in the CTL simulation. These forcing terms at \( t = 6 \) h are shown in Fig. 4. The magnitude of the forcing terms above \( z = 15 \) km is much smaller than below \( z = 15 \) km. Below \( z = 15 \) km, strong forcing is concentrated in the convectively active region (720 km ≤ \( x \) ≤ 770 km), behind the gust front updraft located at \( x = 770 \) km. In the stratiform cloud region (\( x \leq 720 \) km), the forcing is much weaker than in the convectively active region and appears to be related to weak convective cells propagating rearward in the upper troposphere. For a quantitative analysis of the spatial and temporal structures of the forcing terms, their PSDs as functions of vertical
and horizontal wavelengths and period are calculated (Fig. 5). The domain used for the spectral analysis is 1100 km wide from \( x = 0 \) to 1100 km and 15 km deep from \( z = 0 \) to 15 km. The forcing terms calculated every 1 min from \( t = 4 \) to 12 h are used. The PSDs as functions of vertical and horizontal wavelengths and period are averaged over \( x-t, z-t, \) and \( x-z \) surfaces, respectively. The PSDs are plotted in area-preserving forms. All forcing terms have spectral peaks at periods of 9.25–9.62 and 16.58–18.5 min. However, the dominant spatial scales of the forcings are different from each other: \( F_u, F_w, \) and \( Q \) have peaks at the vertical wavelength of 7.5 km, whereas \( F_s \) has a peak at 5.0 km. In the horizontal wavelength spectra, \( F_u \) is dominant only in the region near 12 km; \( F_u, F_w, \) and \( Q \) have peaks at the horizontal wavelengths larger than 25 km as well as near 12 km. Particularly, the peak of \( Q \) at a horizontal wavelength of 47.8 km is much larger than that near 12 km. In this study, the forcing terms above \( z = 15 \) km are ignored because the objective of this study is to investigate the gravity waves generated by the tropospheric convection, which is mostly confined below \( z = 15 \) km. Even though the potential temperature (Fig. 2) showed that the boundary between the troposphere and stratosphere is near \( z = 12 \) km, the tropospheric wave forcings are calculated below \( z = 15 \) km since convective cells penetrate above \( z = 12 \) km frequently during the integration.

Following Lane et al. (2001), (8)–(11) can be written as a single equation for \( w^r \):

\[
\begin{aligned}
\frac{D^2}{Dt^2} \left[ \frac{\partial^2 w^r}{\partial x^2} + \frac{\partial}{\partial z} \left( \frac{1}{\rho} \frac{\partial w^r}{\partial z} \right) \right] - \frac{D^2}{Dt^2} \left( \frac{\partial w^r}{\partial x} \right) + \frac{dU}{dz} \frac{\partial w^r}{\partial x} + \frac{dU}{dz} \left( \frac{\partial w^r}{\partial z} \right) + \frac{N^2}{\partial z} \frac{\partial^2 w^r}{\partial x^2} \\
= \frac{\partial^2}{\partial z^2} \left( \frac{\bar{F}_r}{\bar{\rho}} \right) - \frac{\partial^2}{\partial z^2} \left( \frac{\bar{F}_w}{\bar{\rho}} \right) + \frac{g}{\partial x} \frac{\partial^2}{\partial z^2} \left( \frac{F_s}{\bar{\rho}} \right) + \frac{g}{\partial x} \frac{\partial^2}{\partial z^2} \left( \frac{Q}{\bar{\rho}} \right) + \frac{dU}{dz} \frac{\partial^2}{\partial z^2} \left( \frac{F_s}{\bar{\rho}} \right),
\end{aligned}
\]

where \( N \) is the Brunt–Väisälä frequency. Equation (16) implies that the generation of linear gravity waves is related to the nonlinear and diabatic processes within convection. These nonlinear terms will be large within the cloud, and negligible some distance away from the cloud, and therefore form a compact source of gravity waves. This wave generation is analogous to Lighthill’s (1952) radiation tensor. Lane et al. (2001) grouped those wave sources into three terms—A, B, and C—according to the related physical processes (nonlinear advection, diabatic heating and cooling, and the effects of environmental wind shear), respectively. Consistent with Lane et al., the term involved in shear generation C is about an order of magnitude smaller than the terms involved in A and B in the region of nonzero vertical shear below \( z = 6 \) km. Nevertheless, the low-level shear is important in maintaining the convective system and affecting the propagation of lower-tropospheric waves. The effects of the wind shear on the vertically propagating gravity waves will be examined in section 6.

5. Quasi-linear dry simulations

In order to investigate characteristics of linear internal gravity waves induced by the wave sources given in (16), three quasi-linear dry simulations (DRYMH, DRYM, and DRYQ) are performed. The methodology of this quasi-linear dry simulation is almost the same as that in Pandya and Alexander (1999). The magnitudes of \( F_u, F_w, F_s, \) and \( Q \), calculated from the CTL simulation, are reduced by factor of 10 000. These reduced forcing terms are applied to the governing equations to elicit a quasi-linear response in the nonlinear numerical model, and they will induce small-amplitude perturbations that will behave similarly to those in a forced linear model. In these quasi-linear dry simulations, the model numerics, configurations related to physical processes, and basic-state wind and thermodynamic structures are the same as those in the CTL simulation except that cloud microphysical processes are ignored. Thus, the comparison between the nonlinear simulation and the forced quasi-linear simulations is probably more appropriate than the same comparisons with a forced linear model. In the quasi-linear dry simulations, those forcings are added in the right-hand sides of the prognostic equations given in (1)–(3). The DRYMH simulation is forced by \( F_u, F_w, \) and \( F_s \); the DRYM simulation forced by \( F_u \) and \( F_s \); and the DRYQ simulation forced by \( Q \). In these quasi-linear dry simulations, the forcings below \( z = 15 \) km are imposed, and the analysis of gravity waves is carried out above \( z = 15 \) km. All quasi-linear dry simulations are carried out for 12 h. For time integration, the nonlinear and diabatic forcings at each large time step (3 s) are linearly interpolated from the forcings calculated at a 1-min interval.

Figure 6 shows the wave sources in the DRYMH, DRYM, and DRYQ simulations at \( t = 6 \) h. The wave sources in Figs. 6a–c are...
Fig. 6. Wave sources in the (a) DRYMH, (b) DRYM, and (c) DRYQ simulations at $t = 6$ h. Contour interval is $0.1 \times 10^{-3}$ m s$^{-2}$ for each panel. Negative values are plotted with dotted lines.

\[
\frac{\partial^2}{\partial x^2} \left[ \frac{\overline{D} \left( \frac{F_u}{p} \right)}{\partial t} \right] - \frac{\partial^2}{\partial x \partial z} \left[ \frac{\overline{D} \left( \frac{F_u}{p} \right)}{\partial t} \right] + \frac{dU}{dz} \frac{\partial^2}{\partial x^2} \left( \frac{F_u}{p} \right) = \frac{g}{\theta} \frac{\partial^2}{\partial x^2} \left( \frac{Q}{p} \right), \tag{17}
\]

\[
\frac{\partial^2}{\partial x^2} \left[ \frac{\overline{D} \left( \frac{F_u}{p} \right)}{\partial t} \right] - \frac{\partial^2}{\partial x \partial z} \left[ \frac{\overline{D} \left( \frac{F_u}{p} \right)}{\partial t} \right] + \frac{dU}{dz} \frac{\partial^2}{\partial x^2} \left( \frac{F_u}{p} \right), \tag{18}
\]

and

\[
\frac{\partial^2}{\partial x^2} \left[ \frac{\overline{D} \left( \frac{F_u}{p} \right)}{\partial t} \right] - \frac{\partial^2}{\partial x \partial z} \left[ \frac{\overline{D} \left( \frac{F_u}{p} \right)}{\partial t} \right] + \frac{dU}{dz} \frac{\partial^2}{\partial x^2} \left( \frac{F_u}{p} \right), \tag{19}
\]

respectively. Strong wave sources are concentrated in the convectively active region ($720 \text{ km} \leq x \leq 770 \text{ km}$). The horizontal and vertical scales of each source are less than 10 and 6 km, respectively. The horizontal scales of these sources are considerably reduced compared with that of the forcings shown in Fig. 5b. This is due to the second- or higher-order horizontal deriv-
Fig. 7. Perturbation vertical velocity fields in the (a) CTL, (b) DRYMH, (c) DRYM, and (d) DRYQ simulations at $t = 6$ h. Contour interval for the CTL simulation is 1 m s$^{-1}$ and for the other simulations is 2.0 m s$^{-1}$, with negative values shaded. The perturbation vertical velocities in the DRYMH, DRYM, and DRYQ simulations are multiplied by factor of 10 000 for comparison with the CTL simulation.

Figure 7 shows the perturbation vertical velocity field in the CTL, DRYMH, DRYM, and DRYQ simulations at $t = 6$ h. The perturbation vertical velocities in the quasi-linear dry simulations are multiplied by factor of 10 000 for comparison with the CTL simulation. As already mentioned in some previous studies (e.g., Pandya and Alexander 1999; Lane and Reeder 2001), the magnitude of perturbations in the quasi-linear dry simulations is about 2 to 3 times larger than that in the CTL simulation. Also, as shown in Fig. 7, the spatial structures of perturbation vertical velocities in the three
quasi-linear simulations are also different from those in the CTL simulation. In the troposphere, none of the quasi-linear simulations reproduce the perturbation vertical velocity field associated with the mesoscale circulation simulated in the CTL simulation. In the convectively active region, the perturbation vertical velocities in the DRYQ and CTL simulations are mostly in phase with each other, while the perturbation vertical velocities in the DRYMH and DRYM simulations are roughly out of phase with those in the CTL simulation. Particularly, in the DRYMH and DRYM simulations, there is strong downdraft near $x = 765$ km, that is, where the gust front updraft is located in the CTL and DRYQ simulations. Unlike in the troposphere, the gravity waves in the stratosphere in the DRYMH simulation are in phase with those in the CTL simulation, while the waves in the DRYQ simulation are almost out of phase with those in the CTL simulation. In the stratosphere, it is very interesting that the magnitudes of the gravity waves in all of the quasi-linear dry simulations are almost identical, although the magnitudes of the nonlinear wave sources are 2 to 3 times larger than that of the diabatic source (as shown in Fig. 6). In order to explain this result, factors that control the spatial and temporal structures and magnitude of quasi-linear perturbations in the troposphere as well as in the stratosphere will be examined in section 6.

Figure 8 shows the sum of the perturbation vertical velocities in the DRYMH and DRYQ simulations (hereafter, DRYMH + DRYQ). The perturbations in DRYMH + DRYQ are believed to be directly comparable to perturbations in a single quasi-linear simulation forced by a combination of the nonlinear and diabatic forcings used in the DRYMH and DRYQ simulations, because the response of the atmosphere to those forcings is quasi-linear. As can be seen in Fig. 8, the perturbations in the DRYMH + DRYQ compares well to the stratospheric gravity waves, as well as the detailed mesoscale and convective cell-scale circulations in the CTL simulation. The differences between vertical velocities shown in Fig. 8c can be regarded as insignificant, considering that the wave forcings are derived under the anelastic assumption, and that the time variation of the forcing terms on a subminute scale is assumed to be linear. There are, however, relatively large differences near $z = 30$ km in the region of $650$ km $< x < 700$ km. In this region, strong wave perturbations occur after $t = 4$ h in the CTL simulation, and these may give rise to nonlinear wave–wave interactions in the stratosphere. Thus, the differences near $z = 30$ km, shown in Fig. 8c, are probably caused by nonlinear processes such as wave–wave interaction that do not occur in the quasi-linear simulations. Pandya and Durran (1996) discussed the differences between linear and finite amplitude perturbations forced by the diabatic forcing. They concluded that linear models fail to accurately represent the mesoscale circulations in the lower troposphere because they cannot support nonlinear processes that modify the basic-state stability associated with the development of a cold pool. In the present study, the perturbations in the nonlinear moist simulation are reasonably reproduced using a quasi-linear simulation forced by both the time-varying nonlinear and diabatic forcings in the troposphere. Thus, including the nonlinear wave forcings given in (12), (13), and (14), in addition to the diabatic forcing, is a crucial factor for successful quasi-linear simulations of the circulations and gravity waves.

It is interesting to note that the magnitude of the
gravity waves in the DRYMH + DRYQ is close to that in the CTL simulation, which is 2 to 3 times smaller than that in either the DRYMH or DRYQ simulation. One can speculate that it is due to some cancellation between the forcing terms in the DRYMH and DRYQ simulations. However, considering that the nonlinear forcing terms are much stronger than the diabatic forcing (Fig. 6), the magnitude of the combined forcing is close to the forcing in the DRYMH simulation, even with a complete cancellation of the diabatic forcing by some part of nonlinear forcing. This result, along with Fig. 7, implies that the magnitude of the linear gravity waves in the stratosphere cannot be explained simply by the magnitude of the imposed tropospheric forcing shown in Fig. 6. In section 6, we will show why the gravity waves in the DRYMH + DRYQ resemble those in the CTL simulation.

6. Characteristics of gravity waves in the stratosphere

a. Spectral analysis of gravity waves

A spectral analysis of the stratospheric gravity waves simulated in the CTL, DRYMH, DRYM, and DRYQ simulations is carried out to characterize convectively generated gravity waves and understand the influence of the different generation mechanisms. The quasi-steady convective system in the CTL simulation induces the westward- and eastward-propagating gravity waves relative to the (almost stationary) gust front updraft, located near \( x = 770 \) km. In all simulations, the vertical velocity fields saved at a 1-min interval from \( t = 6 \) to 12 h are used for the spectral analysis. The domain used for the spectral analysis is 900 km wide from \( x = 170 \) to 1070 km and 19.8 km deep above \( z = 15 \) km. As shown in Fig. 2, the westward-propagating waves are much stronger than the eastward-propagating waves in the CTL simulation. As pointed out by Lane et al. (2001), it is, however, possible that the energy of the eastward-propagating waves is underestimated when the vertical velocities are used in the gravity wave analysis. This is because the intrinsic frequencies of the eastward-propagating waves are smaller than the westward-propagating waves on account of the Doppler shifting by the basic-state wind (2 m s\(^{-1}\) in the stratosphere). Therefore, the PSDs of the stratospheric gravity waves are calculated separately in the western and eastern regions of \( x = 770 \) km. In the eastern region of \( x = 770 \) km, westward-propagating waves are negligibly weak because there is no significant convective activity in the eastern region of gust front updraft in the quasi-steady state after \( t = 4 \) h. However, in the western region of \( x = 770 \) km, there may be some eastward-propagating waves generated by westward-moving convective cells in the convective system. In order to examine the magnitude of eastward-propagating waves in the western region of \( x = 770 \) km, phase speed spectra are calculated using the perturbation vertical velocities of the stratospheric gravity waves in the region of \( 170 \) km < \( x < 770 \) km (not shown). The eastward-propagating waves in the western region of \( x = 770 \) km are found to be negligibly weak compared with the westward-propagating waves. Thus, the gravity waves in the western and eastern region of \( x = 770 \) km can be regarded as the westward- and eastward-propagating gravity waves in this study.

For the PSD of the gravity waves in the stratosphere, the preprocesses applied to the PSD in Fig. 3b are used, except a linear trend is removed from the vertical velocity field. In the quasi-linear dry simulations, the vertical velocities in the stratosphere are multiplied by a factor of 10 000 before the PSD calculation. The PSDs of the westward- and eastward-propagating waves, as functions of vertical and horizontal wavelengths and period, are averaged for \( x - t, z - t, \) and \( x - z \) surfaces, respectively. Because \( \bar{\rho} \) exponentially decreases with height, and the magnitude of upward-propagating gravity waves increases with height in proportion to \( \bar{\rho}^{-1/2} \), the vertical velocity perturbations are multiplied by \( \sqrt{\bar{\rho}(z)/\bar{\rho}(z_0)} \) \( (z_0 = 15 \) km) before calculating the PSDs in order to prevent the bias of the PSDs toward the spectral wave characteristics at higher altitudes.

Figure 9 shows PSDs of the gravity waves as functions of vertical wavelength (\( \lambda_z \)); horizontal wavelength (\( \lambda_x \)); and period in the CTL, DRYMH, DRYM, and DRYQ simulations and in DRYMH + DRYQ. Resolvable vertical wavelengths and periods range from 0.6 to 19.8 km and 2 min to 6 h, respectively. Resolvable horizontal wavelengths range from 2 to 600 km (300 km) for the westward- (eastward) propagating waves. The dominant vertical wavelengths range roughly from 6.6 to 9.9 km in all simulations regardless of the horizontal propagation direction. For the westward- (eastward) propagating waves, the dominant horizontal wavelengths and periods are 20–100 (30–80) km and 8–80 (15–50) min, respectively. In all quasi-linear dry simulations, the peak at 17.1 min associated with the convective cell regeneration is dominant. The PSDs in the DRYMH + DRYQ are also almost identical to those in the CTL simulation, even though the gravity waves in all quasi-linear dry simulations are generally much stronger than in the CTL simulation. This will be explained in section 6b.

Salby and Garcia (1987) showed that the vertical wavelength of large-scale waves induced by thermal forcing depends on the depth of thermal forcing rather than a detailed vertical structure of thermal forcing. Alexander et al. (1995) related the dominant peaks at \( \lambda_x = 6–10 \) km to the vertical scale of diabatic forcing in simulated convective storms. Recently, however, Holton et al. (2002) showed that the horizontal wavenumber spectrum of wave forcing with a given frequency is crucial for determining dominant vertical wavelengths of gravity waves, and noted that the relation of the dominant vertical wavelengths of waves to the depth of the wave forcings should be regarded as a “rule of thumb.”
The dependence of the vertical wavelengths of gravity waves on their horizontal wavenumbers and frequencies will be examined in section 6c.

Figure 10 shows the storm-relative horizontal phase speed \( c \) spectra in all simulations and in DRYMH + DRYQ. To obtain the storm-relative phase speed spectrum, a two-dimensional spectrum is calculated using the vertical velocity field over the horizontal area of 900 km from \( x = 170 \) to 1070 km and the period of 6 h from \( t = 6 \) to 12 h in all simulations. The two-dimensional spectrum is calculated as follows:

\[
\text{PSD}(k, \omega) = \frac{2\Delta x \Delta \tau}{N_z N_t} |\text{FFT2D}[w'(x, t)](k > 0)|^2. \tag{20}
\]
Fig. 10. Storm-relative phase speed spectra of the (a) westward- and (b) eastward-propagating gravity wave perturbations \( \omega' \) in the stratosphere in CTL (thick solid), DRYMH (thin solid), DRYM (dotted), DRYQ (dashed), and DRYMH + DRYQ (thick gray solid).

Here, \( \Delta x \) is the horizontal grid size; FFT2D is the two-dimensional FFT; and \( N_x \) and \( N_t \) are the number of data points in the horizontal direction and time, respectively. The vertical velocity is multiplied by \( r(z)/\rho(z_0) \) as in the case of the one-dimensional PSD. The phase speed spectrum is calculated using 2 m s\(^{-1}\) phase speed bins. The spectral power at each bin \( c_0 \) represents the integration of the two-dimensional PSD with respect to \( k \) and \( \omega \) over the area where \( c_0 - 1.0 < c_0/k < c_0 + 1.0 \). The phase speed spectrum is then averaged for 19.8-km depth above \( z = 15 \) km. In Fig. 10, PSDs of westward- and eastward-propagating waves are separately plotted because the magnitude of vertical velocity perturbations of the eastward-propagating waves is about 50 times smaller than that of the westward-propagating waves. Westward-propagating waves in all simulations have phase speeds between \(-52\) and \(-6\) m s\(^{-1}\) relative to the convective system. The strong spectral peaks exist at \(-20\) and \(-24\) m s\(^{-1}\) in the CTL simulation. The phase speed spectra in the quasi-linear dry simulations are quite different from the CTL simulation in their respective shapes and the location of strong peaks.

As can be seen in Fig. 10a, the waves generated by nonlinear forcing in the DRYM simulation tend to propagate slowly westward from the gust front compared with the waves induced by the nonlinear forcing (including the heat flux) in the DRYMH simulation. This implies that the effect of the heat flux forcing on the characteristics of gravity waves can be significant, although the heat flux forcing is not likely to greatly modify the overall structure and magnitude of nonlinear wave sources when Figs. 6a and 6b are compared. Despite the large differences between the phase speed spectra in the CTL and quasi-linear dry simulations, the spectrum in DRYMH + DRYQ is quite similar to that in the CTL simulation except in the phase speed region between \(-24\) and \(-20\) m s\(^{-1}\), in which the magnitude of gravity waves is about twice as large as those in CTL. As mentioned in Fig. 8, gravity waves in the CTL simulation can be affected by the nonlinear wave–wave interaction caused by strong gravity wave perturbations. Thus, in the present study, it can be also seen that the nonlinear processes in the gravity wave field in the CTL simulation reduce the magnitude of gravity waves with phase speeds of \(-24\) to \(-20\) m s\(^{-1}\). In the phase speed spectrum for the eastward-propagating waves in the CTL simulation, the gravity waves with storm-relative phase speeds of 20–44 m s\(^{-1}\) are dominant. The magnitude of phase speed spectra in the quasi-linear dry simulations is too large, and their dominant phase speeds are shifted by about \(-5\) m s\(^{-1}\) compared with the CTL simulation. In the eastward-propagating waves, the PSD in DRYMH + DRYQ compares well to the CTL simulation for most phase speeds.

b. Wave propagation

In this section, a more detailed analysis of the individual wave sources are performed using a combination of spectral analysis of the wave source, and the vertical propagation condition for linear gravity waves. This analysis attempts to explain why the amplitudes of the gravity waves in all of the quasi-linear simulations are similar, even though the magnitudes of the respective sources are very different. Also, the reason why DRYMH + DRYQ compares well to CTL, even though both DRYMH and DRYQ produce gravity waves whose amplitudes are too large, is examined.

Taking Fourier transform \([(x, t) \rightarrow (k, \omega)] \) of (16) gives

\[
(U - c)^2 \frac{\partial^2 \tilde{\omega}}{\partial z^2} + (U - c)^2 m^2 \tilde{\omega} = S, \tag{21}
\]

where
In (21)–(23), \( k \) is the horizontal wavenumber, \( m \) is the vertical wavenumber; \( c (= \omega/k, \text{where } \omega \text{ is a storm-relative frequency}) \) is the storm-relative horizontal phase speed; and \( \hat{\psi} = \hat{\psi} \exp[z/(2H)] \), where \( \hat{\psi} \) is the Fourier transform of \( \psi' \); \( H \) is the scale height of basic-state density, which is 9.6 km in the troposphere and 6.3 km in the stratosphere. As already shown in several theoretical studies (e.g., Smith and Lin 1982; Chun and Lee 1994; Chun and Baik 1998), the magnitude of \( \hat{\psi} \) is proportional to the magnitude of \( S \) in the troposphere.

Figure 11 shows the two-dimensional \((k-\omega)\) PSDs of the wave source \( S \) for each DRYMH, DRYM, and DRYQ. (The shown spectra are constructed by averaging the PSD of \( S \) from the surface to 15 km.) This figure shows a number of differences between the character of the respective sources. For example, the source in DRYQ (Fig. 11c) has negligible power at small wavenumbers \((k < 0.25 \times 10^{-4} \text{ cycles m}^{-1})\), whereas the sources in DRYMH (Fig. 11a) and DRYM (Fig. 11b) possess significant power at those wavenumbers for all frequencies. Also, the source in DRYQ does not possess significant power at \( k > 1.0 \times 10^{-4} \text{ cycles m}^{-1} \) and \( -\omega > 2.0 \times 10^{-3} \text{ cycles s}^{-1} \), whereas the sources in DRYM and DRYMH do. One similarity, however, is that the sources in all of the quasi-linear simulations possess strong power at \( \omega \approx 0 \), for a wide range of horizontal wavenumbers. These quasi-stationary (relative to the gust front) sources are mainly confined below \( z = 4.5 \text{ km} \) (not shown).

Thus, based on the forcing spectra in Fig. 11, it is reasonable to expect that stratospheric gravity waves with \( c \approx -26.7 \text{ m s}^{-1} \) [i.e., along the line connecting
(k = 0, ω = 0) and (k = 1.5 × 10^{-4}, -ω = 4.0 × 10^{-3}) should be much stronger in DRYMH and DRYM than in the DRYQ. Also, the quasi-stationary waves should be important in the stratosphere in all quasi-linear simulations. However, the two-dimensional (k−ω) PSDs of gravity waves at z = 18 km in the quasi-linear simulations (Fig. 12) are different from their respective source spectra (Fig. 11). The spectra of gravity waves in the stratosphere occupies a small region in the k−ω domain, and their PSDs are all qualitatively similar regardless of the imposed forcing. Compared with Fig. 11, it is clear that the nonlinear wave sources with k < 1.0 × 10^{-4} cycles m^{-1} and -ω > 2.0 × 10^{-3} cycles s^{-1} are not contributing to the gravity waves that reach the stratosphere. Also, the quasi-stationary gravity waves are negligibly weak at z = 18 km because of the absorption of those gravity waves near the critical level located at z = 5.4 km (see Fig. 1b).

A linear gravity wave generated in the troposphere can propagate vertically to the stratosphere, for each value of k−ω, if m^2 in (21) is larger than 0 in the forcing region and aloft (provided U ≠ c). Therefore, in the absence of critical levels, a linear gravity wave that reaches the stratosphere satisfies the condition that m^2 > 0 at every level between the wave source and the stratosphere. Thus, the vertical propagation condition defines a region in k−ω space, within which vertically propagating gravity waves can be generated. Using this propagation condition, we define the effective wave forcing as the forcing able to generate waves that propagate vertically.

Figure 13 shows horizontal wavenumbers and frequencies at which the vertical propagation condition at every level from z = 12, 6, 3, and 0 km to z = 15 km is satisfied. For convenience, those horizontal wavenumbers and frequencies included in the shaded region are denoted as k_p and ω_p, respectively. From the vertical propagation condition, it can be seen that for wave sources in each layer of z = 12±15, 6±15, 3±15, and 0±15 km, there is some region in k−ω space that can support linear gravity waves that can propagate above z = 15 km.

The area defined by k_p−ω_p is determined by a combination of stability and the basic-state wind through the dispersion relation. In comparison to Fig. 13a, the reduced area in Fig. 13b is due to the lower stability in
the troposphere. With its lower stability, the troposphere filters the higher-frequency waves, reducing the effectiveness of high-frequency wave sources (in DRYMH and DRYM). The area difference between Figs. 13b and 13c is mostly due to changes in the basic-state wind. The wind shear can significantly restrict vertical propagation, and hence the effectiveness of wave sources between $z = 3$ and $6$ km. In particular, the short wavelength part of the wave sources can be mostly filtered by the wind shear in the troposphere. This can be clearly seen by comparing the PSDs of the wave forcing (Fig. 5b) with those of the linear gravity waves in the stratosphere (Fig. 9b). This suggests that the basic-state wind shear is important not only for determining the mesoscale convective structure (Weisman and Klemp 1982; Fovell and Dailey 1995) but also for controlling the wave propagation condition, especially in the major convective region. Recently, Beres et al. (2002) investigated the effect of tropospheric wind shear on convectively forced gravity waves and showed that wind shear increases (reduces) momentum flux of waves propagating in the opposite (same) direction of basic-state wind. Because the wave sources in the troposphere (Fig. 6) are concentrated between $z = 3$ and $10$ km (except near the gust front), the wave sources within the area defined by $k_p$ and $\omega_p$ (within which the vertical propagation condition at every level between $z = z_0$ and $15$ km are satisfied) are removed. The remaining coefficients are multiplied by $-k_p^2 \exp[z_0/(2H)]$ so that the effective sources in Figs. 14a and 14b can be expressed as (17) and (19). Figure 14 shows the effective wave sources in the DRYMH and DRYQ simulations and in DRYMH + DRYQ at $t = 6$ h. The shading in Fig. 14 represents regions where the source in DRYMH is out of phase with the source in DRYQ. Compared with Fig. 6, the magnitude of the nonlinear and diabatic sources shown in Fig. 14 is about 10 to 20 times smaller than their original value. A significant part of the original source fields are removed because they do not project to a part of the spectrum where the propagation condition is satisfied. Also, the reduction in magnitude of the nonlinear sources is more than the reduction of the diabatic source. The result is that the magnitudes of the effective sources are comparable to the effective diabatic source. This explains why the magnitude of the gravity waves in all of the quasi-linear simulations are similar, even though the diabatic source is much weaker than the nonlinear sources.

Fig. 13. Two-dimensional ($k-\omega$) spectral region where the vertical propagation conditions at every level from (a) $z = 12$, (b) $6$, (c) $3$, and (d) $0$ km to $z = 15$ km are satisfied. 
Fig. 14. Wave sources in the DRYMH and DRYQ simulations, and in DRYMH + DRYQ at $t = 6$ h that can generate the linear internal gravity waves propagating to the stratosphere above $z = 15$ km. Contour interval is $0.2 \times 10^{-10}$ m$^{-1}$ s$^{-3}$. The shading represents the region where the nonlinear source in the DRYMH simulation is out of phase with the diabatic source in the DRYQ simulation.

DRYMH + DRYQ resemble those in CTL, even though the waves in DRYMH and DRYQ are 2 to 3 times larger.

Two important points arise from Fig. 14. First, because of the restrictions placed on the wave source by the vertical propagation condition, the nonlinear and diabatic sources derived from the convective system are inefficient in generating gravity waves. The effective nonlinear and diabatic sources are significantly different from and smaller in magnitude than the original sources. Therefore, the propagation condition must be considered when analyzing gravity wave sources. Second, there is significant cancellation between the nonlinear and the diabatic sources, resulting in a reduced net gravity wave source. Therefore, both nonlinear and diabatic sources are important in generating the gravity waves that reach the stratosphere.
c. Dominant vertical wavelength and group velocity

Figure 15 shows the contours of vertical wavelength and zonal and vertical components of group velocities superimposed on the two-dimensional \((k-\omega)\) spectrum of gravity waves at \(z = 18\) km in the CTL simulation. The vertical wavelengths and group velocities are calculated from (22) using the basic-state wind and stability in the stratosphere. For the westward-propagating waves (Fig. 15a), the vertical wavelengths of 4–20 km are dominant. Gravity waves with vertical wavelengths of 6–14 km have horizontal wavenumbers less than \(1.25 \times 10^{-4}\) cycles m\(^{-1}\) \((\lambda_z = 8\) km\), while gravity waves with vertical wavelengths of 4 and 20 km have horizontal wavenumbers less than \(1.0 \times 10^{-4}\) and \(0.5 \times 10^{-4}\) cycles m\(^{-1}\), respectively. For the eastward-propagating waves, the vertical wavelengths of 6–14 km are dominant, and those waves have horizontal wavenumbers less than \(0.5 \times 10^{-4}\) cycles m\(^{-1}\). The shape of this two-dimensional spectrum in the CTL simulation is almost the same as those in the quasi-linear dry simulations. Figure 15 clearly shows that the dominant vertical wavelengths depend on the horizontal wavenumbers and frequencies via the dispersion relation. The vertical wavelengths of gravity waves are determined by the basic-state wind and stability, horizontal wavenumber, and frequency, not only by the depth of diabatic forcing, as suggested by Salby and Garcia (1987) and Alexander et al. (1995).

The westward- and eastward-propagating gravity waves have zonal group velocities \((c_{gx})\) of \(-50\) to \(-10\) and \(20\) to \(50\) m s\(^{-1}\) and vertical group velocities \((c_{gz})\) of \(2\) to \(20\) and \(4\) to \(10\) m s\(^{-1}\), respectively. The fastest westward- and eastward-propagating waves have \(c_{gx}\) of \(10\) to \(14\) and \(6\) to \(10\) m s\(^{-1}\), respectively. A ray composed of the fastest westward- (eastward) propagating waves can propagate as far as 85 (142) km in the horizontal, and 17 km from \(z = 18\) km to the model top in 28.3 (47.2) min. Given that the gust front updraft is almost stationary near \(x = 770\) km, and that the model domain (used in phase speed analysis) is 900 km wide from 170 to 1070 km, the vertically averaged phase spectra shown in Fig. 10 are not biased toward high intrinsic frequencies. This is because the fastest horizontally propagating waves generated in the CTL simulation reach the phys-
ical model top before propagating out the lateral boundaries of the model domain, and there are no significant wave sources from \( x = 170 \) to 255 km and \( x = 928 \) to 1070 km.

7. Summary and conclusions

The characteristics of internal gravity waves in the stratosphere generated by mesoscale convective storms, and the generation mechanisms for those waves were investigated using a two-dimensional version of ARPS. In the nonlinear moist (control) simulation, a typical multicellular-type convective system was developed under a basic-state wind and thermodynamic condition appropriate for midlatitude squall lines. The simulated convective system reaches a quasi-steady state after \( t = 4 \) h, in which a gust front is quasi stationary in the model domain and convective cells are periodically regenerated from the gust front at an interval of about 18.1 min. The gravity waves in the stratosphere generated by convective storms are almost isotropic before the quasi-steady state, but the westward propagation relative to the gust front becomes dominant in the quasi-steady state.

In a convective system, there are the two types of wave forcings for linear internal gravity waves: nonlinear forcing in the form of momentum and heat flux divergences and diabatic forcing. In the control (CTL) simulation, the nonlinear forcing is much stronger than the diabatic forcing, especially in the upper troposphere. In order to examine differences in the responses of a stably stratified atmosphere to the different forcings, three quasi-linear dry simulations forced by the momentum flux forcings (DRYMH), momentum and heat flux forcings (DRYMY), and diabatic forcings (DRYQ) are conducted. The magnitude of the stratospheric gravity waves in these quasi-linear dry simulations are comparable with each other but are about 2 times larger than those in the CTL simulation. However, the stratospheric gravity waves in the stratosphere in the CTL simulation are reproduced reasonably well by DRYMH + DRYQ.

For a quantitative understanding of the gravity waves, spectral analysis in the vertical, horizontal, and time domains were carried out. In all simulations, gravity waves with vertical wavelengths of 6.6–9.9 km, horizontal wavelengths of 10–100 km, periods of 8–80 min are dominant for both westward- and eastward-propagating waves. In the CTL simulation, westward- and eastward-propagating gravity waves with storm-relative phase speeds of \(-25\) to \(-20\) and \(20\)–44 m s\(^{-1}\) are dominant, respectively. The shape and the location of peaks of phase speed spectra in the quasi-linear dry simulations are different from each other, and also quite different from those in the CTL simulation. However, again, the phase spectrum in the CTL simulation is reproduced by DRYMH + DRYQ.

The results raise two interesting questions. 1) Why is the magnitude of the gravity waves in all quasi-linear dry simulations comparable, even though the magnitude of the nonlinear forcing is 2 to 3 times larger than that of the diabatic forcing? 2) Why do the linear gravity waves in DRYMH + DRYQ compare well, in both the magnitude and spectral characteristics, to waves in the CTL simulation? In order to answer these questions, two-dimensional \((k-\omega)\) spectra of the wave sources and induced gravity waves in the stratosphere were examined. From these spectra, it was found that a large portion of the nonlinear wave sources cannot generate gravity waves that can reach the stratosphere, due to the vertical propagation condition of the linear internal gravity waves. Using the vertical propagation condition, the following can be found.

1) The effective nonlinear and diabatic sources that can generate gravity waves that reach the stratosphere are strongly confined by the vertical propagation condition. This condition is determined by the basic-state wind, vertical wind shear, and the stability in the troposphere and stratosphere.

2) A large part of the nonlinear wave source cannot generate linear gravity waves that reach the stratosphere.

3) The effective nonlinear and diabatic forcings have comparable magnitudes, and they are largely out of phase in major source regions.

4) Consequently, the magnitude of gravity waves in the DRYMH + DRYQ is comparable to that in the CTL simulation, which is 2 to 3 times less than either the DRYMH or DRYQ simulation, due to cancellation of the effective sources. This implies that both the nonlinear and diabatic forcings are important for convective wave generation.

The above points have interesting implications for gravity wave source parameterizations. First, 1) and 2) suggest that the wave propagation condition and the resultant filtering play a crucial role in shaping the spectrum of stratospheric waves. Therefore, in a parameterization, realistic filtering by the background flow, which determines the effectiveness of wave forcing and the vertical propagation of gravity waves above the forcing, should be considered. Second, 3) and 4) imply that a source parameterization based on diabatic forcing alone will not include the cancellation due to nonlinear forcing. Therefore, such a parameterization may give a poor representation of wave characteristics in parts of the spectrum. In summary, the results of this study suggest the convective systems, like the one modeled here, generate gravity waves through a complicated relationship between diabatic and nonlinear forcing and the wave propagation condition. This relationship should be a topic of continuing research.

Acknowledgments. The authors thank two anonymous reviewers for their careful reading the manuscript and helpful comments. This research was supported by the Climate Environment System Research Center spon-
sored by the SRC Program of the Korea Science and Engineering Foundation.

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