Simulation of Inertia–Gravity Waves in a Poleward-Breaking Rossby Wave

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ABSTRACT

Poleward-breaking Rossby waves often induce an upper-level jet streak over northern Europe. Dominant inertia–gravity wave packets are observed downstream of this jet. The physical processes of their generation and propagation, in such a configuration, are investigated with a mesoscale model.

The study is focused on an observational campaign from 17 to 19 December 1999 over northern Germany. Different simulations with the fifth-generation Pennsylvania State University–National Center for Atmospheric Research (PSU–NCAR) Mesoscale Model (MM5) have been performed. For a high-resolution process study, three domains were set up that encompass the evolution of Rossby waves and that of inertia–gravity waves. To minimize the impact of model damping, the horizontal and vertical resolution has been adjusted appropriately.

With a novel statistical approach, the properties of inertia–gravity wave packets have been estimated. This method uses the horizontal divergence field and takes into account the spatial extension of a wave packet. It avoids the explicit treatment of the background field and works for arbitrary wavelength. Two classes of inertia–gravity waves were found: subsynoptic waves with a horizontal wavelength of about 500 km and mesoscale waves with a horizontal wavelength of about 200 km. The subsynoptic structures were also detected in radiosonde observations during this campaign. The similarity between simulated and observed wavelengths and amplitudes suggests that the simulations can be considered as near realistic.

Spontaneous radiation from unbalanced flow is an important process of inertia–gravity wave generation. Synoptic-scale imbalances in the exit region of the upper-tropospheric jet streak were identified with the smoothed cross-stream Lagrangian Rossby number. In a number of simulations with different physics, it was found that the inertia–gravity wave activity was related to the tropospheric jet, orography, and moist convection. The upward propagation of inertia–gravity waves was favored during this event of a poleward-breaking Rossby wave. The presence of the polar vortex induced background winds exceeding the critical line. Consequently, the activity of inertia–gravity waves in the lower stratosphere increased by an order of magnitude during the case study.

The successful simulation of the complex processes of generation and propagation showed the important role of poleward Rossby wave breaking for the appearance of inertia–gravity waves in the midlatitudes.

1. Introduction

Inertia–gravity waves (IGWs) are ageostrophic oscillations that are forced by rotation and buoyancy (Holton 1992). They may modify the weather at the surface and contribute to extreme weather events (Bosart and Cussen 1973; Bosart et al. 1998; Koch and Dorian 1988). In the middle atmosphere, such waves may trigger polar stratospheric clouds (Dörnbrack et al. 1999; Buss et al. 2004; Hitchman et al. 2003), local ozone reduction (Kühl et al. 2004) and noctilucent clouds (Rapp et al. 2002). They can also influence the global circulation (Fritts and Alexander 2003; Holton and Alexander 2000; Dunkerton 1984). IGWs can be generated in the troposphere and may propagate fairly high into the winterly polar vortex with the stratospheric jet in its center. A prominent source of IGWs is orography, but evidence for nonorographic IGWs above the North Atlantic Ocean is found in global maps from satellites (Tsuda and Nishida 2000; Eckermann and Preuss 1999; Wu and Zhang 2004), analyses (Plougonven and Teitelbaum 2003), and radiosondes (Plougonven et al. 2003). Such IGWs may be generated by flow imbalances in jet streaks, deep convection, frontal activity, and other situations. The detailed understanding and
quantitative description of the different processes of IGW generation and propagation is an open question. Here, we study IGWs in the context of a poleward Rossby wave breaking event—a frequently observed phenomenon over northern Europe during winter (Peters and Waugh 1996). In the climatological mean, the subtropical jet over the North Atlantic Ocean is oriented toward Europe. It establishes a waveguide for Rossby waves, which tend to break in the regions of weaker zonal winds (Swanson et al. 1997). The polar vortex may force the Rossby wave to break poleward and downstream of the subtropical jet. This situation is characterized by an anticyclonic edge of the ridge.

In this investigation, a mesoscale circulation model is used to simulate IGWs in such a configuration. It covers significant parts of the Rossby wave evolution including the upper-tropospheric jet streak. The range of simulated IGWs is controlled by the setup of the model. While global models reveal long jet-generated IGWs with an horizontal wavelength of the order $\lambda_h = 600-1000$ km (O’Sullivan and Dunkerton 1995; Kawatani et al. 2004), mesoscale models show much shorter scales ($\lambda_h = 100-400$ km; Zhang 2004; Lane et al. 2004; Buss et al. 2004; Hitchman et al. 2003). Hence, it is desirable to clarify the dependence of simulation results on the model resolution and diffusion.

The near-realistic simulation of IGWs is an aim of this study. While mesoscale models have proven their capability to simulate orographically excited IGWs (Hertzog et al. 2002; Preusse et al. 2002; Wu and Zhang 2004) few authors have performed a direct comparison of simulations with observations for jet-generated IGWs (Lane et al. 2004; Buss et al. 2004; Hitchman et al. 2003). A model simulation can be considered to be realistic if both wavelength and amplitudes are validated. A critical comparison of simulations with observations will be undertaken for an evaluation of the model’s “degree of reality.”

To study the connection between Rossby wave breaking events and the appearance of IGWs, radiosonde and radar observations were performed from 17 to 19 December 1999 at Kühlungborn (northern Germany). The analysis of 17 high-repetition radiosonde launches and continuously running very high frequency (VHF) radar showed subinertial IGWs with a 10–12-h intrinsic period. They were originated from the top of the upper-tropospheric jet streak and propagated against the background wind (Peters et al. 2003). These data will be used in this study to validate the simulations.

This study also addresses issues regarding the methods used to estimate IGW properties. According to the recommendations of Stratospheric Processes and their Role in Climate (SPARC; Hamilton and Vincent 1998) radiosonde profiles have to be filtered and then to be statistically analyzed. This method will be used for the comparison of observed and simulated profiles of wind and temperature. However, the filter procedure is sometimes problematic and leads to varying results (Zhang et al. 2004). Using the potential of spatially extended model data, another novel method for the estimation of IGW parameters based on the horizontal divergence of the wind field will be developed, tested, and discussed.

Detailed observational and numerical investigations at the regional scale have been devoted to the generation and impact of IGWs. In a synoptic study Uccellini and Koch (1987) have showed, that IGW generation is very likely in the exit region of an upper-level jet. This has been confirmed by many observations (Thomas et al. 1999; Peters et al. 2003; Buss et al. 2004; Hitchman et al. 2003; Zhang 2004). In most cases, the jet is mainly straight and departs from geostrophic balance due to linear acceleration. Plougonven et al. (2003) studied cases where the jet streak is strongly curved around an airmass tongue and is subject to a strong centrifugal acceleration. The classical picture of geostrophic adjustment predicts the emission of IGWs from an initially ageostrophic flow. It has been generalized to the concept of spontaneous radiation from unbalanced flows (McIntyre 2003). Different approaches have been applied to diagnose such situations (see Zhang et al. 2000). It remains to be clarified, how the imbalances are related to the synoptic-scale forcing and to smaller-scale IGWs.

IGWs may be generated in poleward-breaking Rossby waves by a number of processes. One is the spontaneous radiation from unbalanced jet streaks, which are formed at the boundary between poleward-propagating subtropical and polar air. If such regions of strong winds are placed over mountainous terrain, orographic IGWs can be forced. Another mechanism is convective excitation of IGWs, which becomes very likely when wet and warm air masses propagate northward. It is an open question, how relevant these different processes are and how they interact.

Linear wave theory predicts upward-propagating stationary waves in a horizontally homogeneous wind field that does not change sign with height. Once IGW packets are generated in the troposphere with west wind, their farther upward propagation is considerably favored if the lower part of the polar vortex induces high westerlies in the stratosphere too. In this study we will examine when such a situation is realized and how
much of the IGW energy is transferred upward. This completes the description of IGWs during a poleward Rossby wave breaking event.

The paper is organized as follows. In section 2, the dataset is described including the meteorological situation, radiosonde observations, model setup, and the methods of data processing. Section 3 shows the results including model validation, the statistics of wave properties such as wavelength and amplitude, the vertical propagation, and the contributions of different generation processes. The discussion in section 4 is devoted to the methods for the estimation of IGW parameters, IGW simulation in this and other studies, verification with observations, diagnosis of unbalanced flow and other sources, as well as IGW propagation into the stratosphere. Conclusions and outlook (section 5) complete this study.

2. Meteorological situation, radiosonde observations, model setup, and statistical methods

a. Meteorological situation

During the measuring campaign (17–19 December 1999) the synoptic pattern is characterized by a large anticyclone over the northeastern Atlantic Ocean (see Fig. 1a). This center of action is attributed to a poleward-downstream-breaking Rossby wave. The associated ridge was oriented from southwest to northeast, which is characteristic for the first phase of Rossby wave breaking. The southern boundary of the polar stratospheric air is visualized with the 2 PVU [1 potential vorticity unit (PVU) = $1.0 \times 10^{-6}$ m$^2$ s$^{-1}$ K kg$^{-1}$] contour in Ertel potential vorticity at the 320-K isentrope (see Fig. 1b). This boundary is over the northeast Atlantic at $\sim$55°N, which indicates a weak case of P2-type Rossby wave breaking in comparison with the events discussed in Peters and Waugh (1996). Along this airmass boundary, an intense tropospheric jet with more than 60 m s$^{-1}$ wind speed is found. It is placed over the northeast Atlantic and had its exit region over northern Europe. Thus, a pronounced generation of IGWs in this region can be expected (Uccellini and Koch 1987). In the stratosphere (Fig. 1c), a strong polar vortex established a good propagation background for IGWs.

To identify the IGW structures (Fig. 2a) we make use of the divergence of the horizontal velocity ($u, v$),

$$\delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$  \hspace{1cm} (1)

in the 300-hPa layer. Increased IGW activity was detected over northern Germany, but also over Newfoundland where another jet exit region was found. Strong IGWs over northwest Africa coincide with the equatorward-breaking part of the Rossby wave. In the stratosphere (Fig. 2b), pronounced IGW structures were found only over northern Europe. This was already expected due to the presence of an intensive wind band exceeding 40 m s$^{-1}$ in the middle stratosphere.

b. Radiosonde observations

Radiosondes were launched at the position of Kühlungsborn (54.1°N, 11.8°E) from 0600 UTC 17 December to 0600 UTC 19 December 1999 every $\sim$3 h (Peters et al. 2003). In total, 17 profiles of wind, temperature, and pressure with $\sim$10-m height resolution have been registered with sondes RS80-30 and the sounding system Digicora MW15 (Vaisala Inc, Finland), while the wind has been tracked with GPS. These raw data have been interpolated with the cubic spline method to equal-height intervals of 50 m and will be used for the validation of the model.

c. Model setup

For this process study, we used the fifth-generation Pennsylvania State University–National Center for Atmospheric Research (PSU–NCAR) Mesoscale Model (MM5; Dudhia 1993, Grell et al. 1995). This is a non-hydrostatic mesoscale circulation model involving an Arakawa–Lamb B-staggered grid on sigma surfaces.

The outer dimensions of the model cover scales of 10 000 km to follow the dynamics of significant parts of the detected Rossby wave. The model should resolve IGWs down to a horizontal wavelength of $\lambda_h \sim 100$ km and a vertical wavelength of $\lambda_z \sim 2$ km. In the following, scaling arguments are developed in order to dimension the horizontal and vertical resolution accordingly.

IGW amplitudes are damped by the model-internal diffusion according to

$$D = \exp(-\gamma t),$$  \hspace{1cm} (2)

A corresponding damping rate for a wave with horizontal wavenumber $k_h = (k_x^2 + k_y^2)^{1/2}$ and vertical wavenumber $k_z$ can be estimated with

$$\gamma = K_h k^4 + K_z k^2,$$  \hspace{1cm} (3)

where $K_h = 3 \times 10^{-3}$ is the horizontal (fourth order) hyperdiffusion, which depends on the horizontal grid size $\Delta x$ and the time step $\Delta t$, and $K_z = 1$ m$^2$ s$^{-1}$ is the vertical (second order) diffusion.

The horizontal scales are limited by horizontal diffusion. The corresponding cutoff wavelength is estimated from the condition, that the damping during one wave period $\tau_h$ is small ($K_h k_h^4 \tau_h < 1$). For a wave period of
\[
\tau_h = \frac{\lambda_h}{c} \quad \text{according to a phase speed } c = 20 \text{ m s}^{-1}
\]

this is

\[
\lambda_{h,c} = \left[\frac{(2\pi)^4 K_h}{c}\right]^{1/3}
\]

(4)

This horizontal cutoff still depends from the grid size \(s\) and the step size \(\Delta s\) through the diffusion \(K_h\). If the time step \(\Delta t\) is not known, it can be estimated from the Courant–Friedrichs–Lewy (CFL) criterion for sound waves \(\Delta t = \varepsilon \Delta s / c_s\) with the typical sound velocity of \(c_s = 300 \text{ m s}^{-1}\) and a dimensionless “security factor” \(\varepsilon = 0.2\). For this case a “rule of thumb” \(\lambda_{h,c} = 7 \Delta s\) can be found, consistent with estimates by Leutbecher and Volkert (2000). Hence, with a horizontal grid size of \(\Delta s = 8 \text{ km}\) it is possible to resolve horizontal wavelengths above \(\lambda_{h,c} = 56 \text{ km}\).

The vertical resolution should meet the condition

\[
\Delta z < \Delta z_{\text{opt}} = \max \left(\frac{f}{N} \Delta s, \frac{1}{2} \lambda_{z,c}\right)
\]

(5)

in order to resolve the relevant processes and to avoid spurious waves (Lindzen and Fox-Rabinovitz 1989; Pecnick and Keyser 1989). Here the Coriolis parameter \(f = 1.2 \times 10^{-4} \text{ s}^{-1}\) (for a latitude of \(\phi = 54^\circ \text{ N}\) corresponding to an inertial period of \(T_f = 14.5 \text{ h}\) and the Brunt–Väisälä frequency of \(N = 1.8 \times 10^{-2} \text{ s}^{-1}\) (average value for the campaign) are used. The first item in Eq. (5) is due to the proper resolution of baroclinic waves at horizontal scales of \(\lambda_{z} = N \Delta z / f\) (\(\Delta z\) is a characteristic vertical scale) and avoids the generation of spurious waves. This requirement is relaxed if the vertical diffusion of the model \(K_z\) is taken into account. For the slowest possible wave a vertical cutoff

\[
\lambda_{z,c} = 2\pi (K_z T_f)^{1/2} \approx 1 \text{ km}
\]

(6)

is estimated, which appears in the second item in Eq. (5). For dominating vertical diffusion \([\lambda_{z,c}]^2 > 500 \text{ m} > (f/N)\Delta s \approx 53 \text{ m}\) spurious waves below the cutoff \(\lambda_{z,c}\) will be damped. Hence, a vertical grid of \(\Delta z = 100 \text{ m} \ll \lambda_{z,c}\) well resolves IGWs in this case.

These findings prompted a high-resolution complex-
physics run (HC) to be set up for this process study. It encloses three interactively nested domains on an area of 9360 km × 7200 km with horizontal resolutions of $\Delta s = 72, 24$, and 8 km for the large, medium, and small domains, respectively (Table 1). The model top was set at 10 hPa (~30 km) and all levels were arranged equidistantly with $\Delta z = 100$ m spacing. In the smallest domain, the resulting horizontal cutoff wavelength was $\lambda_{hc} = 70.3$ km.

Beside this HC run a number of further simulations was conducted, which are referred to with the following abbreviations (cf. Table 2): the first letter indicates the resolution of the model—the low-resolution run (L; with one low-resolution domain), the medium-resolution run (M; with a second interactively nested medium-resolution domain), and the high-resolution run (H; with a third interactively nested high-resolution domain); the second letter relates to the physics of the model: simple physics (S; orography set to zero everywhere, moisture switched off), orographic physics (O; with orography but without moisture), moisture physics (M; without orography but with moisture), and complex physics (C; with both orography and moisture).

For the complex physics run the medium-range frequency (MRF) planetary boundary layer module, the radiative upper boundary condition (Zängl 2001), the Grell cumulus parameterization (Grell 1993), and the Dudhia scheme for microphysics (Dudhia 1993) were used. Initial and boundary conditions were constructed from the European Centre for Medium-Range Weather Forecasts (ECMWF) analyses. All simulations used two-way interacting domains without any nudging. The only forcing is provided through the surface and the lateral boundaries. The run was initialized with the low-resolution domain at 0000 UTC 16 December 1999 and continued until 0000 UTC 20 December 1999. Subsequently, the nested medium- and high-resolution domains were started with 12- and 18-h delays.

The data have been analyzed after a spinup of 1 day. This time was sufficient for initial imbalances to transform into waves and travel away during one inertial

### Table 1. Domain properties. For each of the domains with low, medium, and high resolution, the following parameters are listed: the horizontal resolution $\Delta s$, the vertical resolution $\Delta z$, the time step $\Delta t$, the effective resolution $\Delta s_{eff}$ [Eq. (16)] and the cutoff wavelength $\lambda_{hc}$ [Eq. (4)]. Based on the LC, MC, and HC runs, the empirical cutoff of the zonal wavelength $\lambda_{hc}(emp)$ = $1/\gamma_{c}$ has been determined from the damping factor $\gamma_{c}$ of the empirical autocorrelation function [Eq. (A6)]; here $\lambda_{hc}(emp)$ is the smallest value of the empirical zonal wavelength $\lambda_{c}$. For the vertical wavelength the cutoff $\lambda_{z}(emp)$ = $1/\gamma_{z}$ and the smallest empirical value of $\lambda_{z}$ [$\lambda_{z, min}(emp)$] is given.

<table>
<thead>
<tr>
<th>Domain properties</th>
<th>Low-resolution domain</th>
<th>Medium-resolution domain</th>
<th>High-resolution domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s$ (km)</td>
<td>72</td>
<td>24</td>
<td>8</td>
</tr>
<tr>
<td>$\Delta z$ (m)</td>
<td>750</td>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>$\Delta t$ (s)</td>
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<td>20</td>
<td>2.67</td>
</tr>
<tr>
<td>$\Delta s_{eff}$ (km)</td>
<td>465</td>
<td>155</td>
<td>70.3</td>
</tr>
<tr>
<td>$\lambda_{hc}$ (km)</td>
<td>347 (LC)</td>
<td>118 (MC)</td>
<td>56.2 (HC)</td>
</tr>
<tr>
<td>$\lambda_{hc}(emp)$ (km)</td>
<td>523 (LC)</td>
<td>245 (MC)</td>
<td>115 (HC)</td>
</tr>
<tr>
<td>$\lambda_{hc}$ (km)</td>
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<td>1.44</td>
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<tr>
<td>$\lambda_{hc}(emp)$ (km)</td>
<td>2.30 (LC)</td>
<td>2.22 (MC)</td>
<td>1.95 (HC)</td>
</tr>
<tr>
<td>$\lambda_{hc, min}(emp)$ (km)</td>
<td>3.8 (LC)</td>
<td>2.37 (MC)</td>
<td>2.18 (HC)</td>
</tr>
</tbody>
</table>

Fig. 2. Contours of horizontal divergence (shaded for values exceeding $|\Delta \delta| = 10^{-5}$ s$^{-1}$, displayed as a gray scale, solid line surrounds positive values, dashed line negative values, no line drawn at zero) from ECMWF analyses for 1200 UTC 17 Dec 1999. Maps are shown on the (a) 300- and (b) 50-hPa isobaric surface.
period. Orographic features in the analyses have been adapted to the model orography with a recalibration of the balanced surface pressure field. Waves shorter than ~144 km could not be transferred from analyses to the model due to the grid size of the outermost domain (Δx = 72 km). Imbalances appeared near the domain boundaries due to the nesting procedure from coarser to finer grids over five grid points. Fluctuations at this scale are subject to strong damping, which prevents their further propagation. Larger wave structures dominated in the simulations if they were forced locally; this will be verified in the following analysis. In a number of test runs with domains of different size, it was checked that the impact of the initial and boundary perturbations on the dynamics in the middle of the domain was small.

d. Statistical methods (VIN and DIV)

IGW properties such as wavelength, period, and amplitude are estimated with two different methods. The provided quantities include a certain scatter, which is natural for any estimate from a limited set of fluctuating data. One source of scatter is the uncertainty due to the particular method, which is used to find, for example, a wavelength from a profile. Another source of scatter is due to the physical variability from profile to profile. A statistical framework has been set up in order to account for these effects, which also takes care of the error propagation for derived quantities (see the appendix). These considerations are applied to two methods for the estimation of IGW parameters, which are described below.

The method recommended by SPARC (Hamilton and Vincent 1998) is based on Vincent and Fritts (1987) and Eckermann and Vincent (1989). We will refer to this method with the abbreviation “VIN.” It analyzes an observed or modeled vertical profile of wind and temperature fluctuations, which are obtained as deviations from a background profile. The intrinsic frequency is retrieved with a Stokes analysis and the vertical wavenumber from a spectral average (see the appendix for details). Once these primary parameters are found, it is assumed that an IGW packet is dominating the profile, and the horizontal wavelength is derived from the dispersion relation for hydrostatic IGWs:

\[ \omega_i = \left( f^2 + N^2 \frac{k_z^2}{h_z^2} \right)^{1/2} \]

reading

\[ \lambda_h = \frac{\lambda_z}{\left( \omega_i^2 - f^2 \right)^{1/2}}. \]

The kinematic wave energy (energy per mass) is estimated directly from the fluctuating fields

\[ e = \frac{1}{2} \left[ u'^2 + v'^2 + \left( \frac{g}{N} \frac{T'}{T_0} \right)^2 \right], \]

while the vertical velocity fluctuations are neglected (w' = 0) in hydrostatic approximation. During preprocessing of the data it is necessary to remove the background signal. This may lead to varying results: While bandpass filters often damp the wave amplitudes, polynomial fits of the background fields sometimes create overshoots. Such artifacts have been examined with test datasets (not documented here). Noisy IGW signals have been superimposed on a divergence-free background field with and without a velocity perturbation. Beside the above-mentioned filter problems, we found that noise prevented the estimation of high-frequency gravity waves (ωi/f > 5).

As an alternative measure for the IGW properties, the horizontal divergence is used. This method will be referred to as “DIV” and applies to spatially extended wind fields as they may be generated by models. Using the divergence as the basic variable effectively eliminates the geostrophic field (i.e., the divergence is zero). Hence, the balanced flow is removed in leading order.
without application of any explicit filter. The divergence still contained parts of balanced dynamics, which appeared to be much weaker than the IGWs. Such patterns of large-scale alternating divergence in the gradient wind balance have been found in the jet stream (not shown). Under the assumption, that a dominating wave packet is localized at about a certain point \((x, y, z)\), its wavelengths \((\lambda_x, \lambda_y, \lambda_z)\) can be estimated from divergence profiles in zonal, meridional, and vertical directions crossing at this point. The acceptance conditions in the harmonic analysis ensure that both vertical and horizontal wavelength can be determined with significance (details are described in the appendix). Once the wavelengths are found, the intrinsic frequency at this point is derived from the dispersion relation [Eq. (7)]. Information on the IGW amplitude is estimated from the variance of divergence \(s_D^2\). Hence, the kinematic wave energy can be derived from polarization relations:

\[
e = \frac{s_D^2}{k_x^2}
\]

The performance of this method in the above-mentioned test was very good. The balanced background perturbation did not appear in the divergence field, which allowed for the correct estimation of all wave properties. Also, high-frequency gravity waves could be treated correctly.

3. Results

a. Model validation

For the MC run, structures of the IGWs are well reflected in the divergence field over northern Europe, especially in the exit region of the tropospheric jet streak near 50°–60°N, 0°–20°E (Fig. 3a). Longer waves are also found in the HC run (Fig. 3b). Additionally, we find shorter mountain waves just above Iceland, the Scottish Highlands, the Scandinavian mountains, and the Alps. However, the mountain waves over the Alps do not appear at 50 hPa due to decreasing winds in the stratosphere (Fig. 3d). It is interesting to note, that indication for large IGWs can already be found in the ECMWF analyses (cf. Fig. 2a with Fig. 3a and Fig. 2b with Fig. 3c).

To validate the simulated IGW properties from the HC run a direct comparison with radiosonde observations was performed. Figure 4 shows profiles of wind components and temperature at the position of Kühlingsborn as modeled (full line) and as observed (dotted line). The raw data show good agreement with respect to the large-scale structures. Closer inspection shows that the simulated zonal wind was about 10 m s\(^{-1}\) weaker than observed. The consequences of this finding will be discussed later.

IGW structures can be identified in the fluctuations, which are obtained after subtraction of the background profile. It was determined either with a polynomial of fourth degree (4POL; Figs. 4d–f) or a 5-km lowpass filter (5KM; Figs. 4g–i). Radiosonde data have additionally been smoothed with a Lanczos filter over \(z_h = 1\) km to suppress fluctuations below the scale of interest (Peters et al. 2003; Vincent et al. 1996). The raw data show different background profiles—the zonal wind \(u\) (Fig. 4a) is pretty homogeneous with a slight maximum at \(\sim 10\) km, the meridional wind \(v\) (Fig. 4b) turns from south to north and back, and the temperature \(T\) (Fig. 4c) decreases from the surface to the tropopause and remains constant above. While the 4POL method reveals fluctuations at scales of 10 and 2 km for the wind components (Figs. 4d,e), the 10-km scale dominates the temperature fluctuations (Fig. 4f). These incoherent scales make it difficult to qualify the wind and temperature fluctuations as expressions of an IGW packet. The fluctuations appear more coherent with the 5KM method: For all parameters (Figs. 4g–i) scales between 1 and 4 km are found. The variability in the modeled profiles is systematically less than in the observations, although this difference is of the same order as the difference between the 4POL and 5KM methods.

Mean IGW properties over all simulated and observed profiles have been estimated with the VIN method. It was applied separately to the residuals between raw data and the polynomial (VIN4POL, see Table 3) and the output of a lowpass filter (VIN5KM, see Table 4). The particular wavelengths and amplitudes depend sensitively of the choice of background removal. The modeled and observed vertical wavelength is similar when the data are processed with the same method. Both datasets are dominated by IGWs with near-inertial period \([\tau_{(VIN)} ~ 5–10 \text{ h}]\). The modeled vertical wavelength and intrinsic period are slightly larger (25%–44%) than the observed values. The horizontal wavelength is much larger in the simulations (74%–144%).

Another result is that the modeled IGW energy is about one-third of the observed. While this ratio is similar for both methods, the absolute values for the VIN4POL method are larger.

b. IGW statistics

The impact of model resolution on the simulated IGWs is studied with a comparison of model runs with complex physics but different resolution: LC, MC, and HC (cf. Tables 1 and 2). In Fig. 5 an example is shown...
Fig. 3. Contours of the horizontal divergence for (a), (c), (e) the MC run and (b), (d), (f) the HC run at run time 36 h (1200 UTC 17 Dec 1999). The quantities are shown as in Fig. 2 at the isobaric surface of (a), (b) 300; (c), (d) 50 hPa; and (e), (f) a vertical cross section (positions of Ben Nevis and Kühlungsborn are indicated with thick vertical lines).
for the statistical analysis of a zonal section of divergence. There is an overall increase of variance with intensification of smaller-scale waves. The smallest zonal wavelength and the simulated cutoff have been estimated and compared with the theoretical cutoff (Table 1). Their proportionality implies that the smallest horizontal wavelength of the simulation is controlled by horizontal diffusion. The same holds true for

Fig. 4. Profiles of (a), (d), (g) zonal wind $u$; (b), (e), (h) meridional wind $v$; and (c), (f), (i) temperature $T$ from the HC run at the position of Kühlungsborn for time 36 h (1200 UTC 17 Dec 1999). The solid line indicates modeled data and the dotted line indicated radiosonde data. In (a)–(c) the raw data are shown, and in (d)–(f) the deviations from a fitted polynomial of fourth degree, while the radiosonde data have been smoothed over $z_c = 1$ km are shown. (g)–(i) Deviations remaining after application of a bandpass filter for $z_c = 1$–5 km.
the limitation of vertical wavelength by vertical diffusion.

Autocovariance functions (Figs. 5d–f) suggest that the point of the first zero crossing does not change with resolution. These findings are supported in resulting power spectra (Figs. 5g–i): the energy level increases but the broad peak at about 630 km remains. For the HC run (Fig. 5i) we find another smaller peak at about 350 km, which is located at the edge of the cutoff.

Table 5 contains a summary of dominant wave properties, estimated from the HC run with the DIV method for all times and heights at the position of Kühlingsborn. At first, it is verified that the detected waves can be interpreted as IGWs. To do so, the apparent period \( \tau_{\text{obs}} \approx 5.2 \) h. This estimate reasonably agrees with the tabulated value and confirms that the dominant harmonics in the divergence field are mainly caused by IGWs.

For the horizontal wavelength, a mean value of \( \lambda_h \approx 210 \) km is found. More detailed information is retrieved from the study of different distribution functions. In Fig. 6a two classes of waves are shown in the sample distribution, which separate at \( \lambda_h \approx 350 \) km. For each horizontal wavelength \( \lambda_h \), the mean energy per wave and the accumulated wave energy have been calculated. It is clearly seen, that a single long wave carries much energy (Fig. 6b). More energy is accumulated at smaller scales (Fig. 6c), because smaller waves are more numerous.

IGW properties separated by averages over large and small waves are compiled in Table 5. The large waves (\( \lambda_h \approx 500 \) km, \( \tau_i \approx 10 \) h) are classified as subsynoptic. With the term “synoptic” we refer to scales about 1000 km day\(^{-1}\) [or meso-o due to Orlanski (1975) and Fujita (1986)]. The small waves (\( \lambda_h \approx 200 \) km, \( \tau_i \approx 5 \) h) are classified as “mesoscale” at 100 km h\(^{-1}\) (or meso \( \beta \)). The vertical wavelength \( \lambda_z \) over heights \( z = 20-200 \) km was almost the same for all waves. Estimates of \( \lambda_z \) from different profiles ranged from 2.2 to 5.2 km and did not correlate with height-independent changes of the horizontal wavelength.

c. IGW propagation

The orientation of IGWs allows for an estimation of the propagation direction. The background flow is mainly oriented to the east, and the IGWs propagate at some angle into the wind (\( k_x < 0 \), \( k_y < 0 \); Fig. 3a). The analysis of the time sequence of patterns like that of Fig. 3e showed, that the vertical wave vectors point to the energy source [upward in the troposphere \( k_z(\text{tropo}) \geq 0 \) and downward in the stratosphere \( k_z(\text{strato}) < 0 \)]. Consequently, the vertical group velocity is directed downward in the troposphere \( [c_g(\text{tropo}) < 0] \) and upward in the stratosphere \( [c_g(\text{strato}) > 0] \); see Peters et al. (2003)).

The propagation conditions for IGWs are illustrated in Fig. 7a. The highest horizontal wind speed \( u_h = (u^2 + v^2)^{1/2} \approx 40 \) m s\(^{-1}\) appeared at the tropopause (\( t = 54 \) h, \( z = 10 \) km) due to intensification of the tropo-

Table 4. Modeled (MOD) and observed (SON) IGW parameters estimated from VINSKRM (see the appendix for details). Notation is the same as in Table 3.
FIG. 5. Example for the effect of resolution on the simulated zonal wavelength $\lambda$. The data are taken from the (a), (d), (g) LC run; (b), (e), (h) MC run; and (c), (f), (i) HC run at the position of Kühlungsborn for the time 60 h (1200 UTC 18 Dec 1999) and for a height of 14 km. Details of the DIV method are outlined in the appendix. Here the following quantities are shown: (a)–(c) cross sections $\mathcal{E}(x)$, (d)–(f) autocovariance functions $\mathcal{C}(\tau)$, and (g)–(i) power spectra $S_k(k)$ of the horizontal divergence. In (d)–(f) the empirical autocovariance function $[\mathcal{C}(\tau)$, thick solid line], the fitted function with the same variance [Eq. (A25), thin solid line], and the variance of the autocovariance at the first zero crossing [Eq. (A26), dashed line] are presented. The bar marks the region where the empirical covariance function is significantly different from zero. In (g)–(i) the empirical power spectrum [Eq. (A28), thick solid line], the fitted colored power spectrum [Eq. (A29), thin solid line], and the upper variance limit of a fitted red spectrum [Eq. (A30), dashed line] are shown. The bars show the region where the first peak in the empirical spectrum (indicated with a thick bar) significantly exceeds the red spectrum (range between thin bars).
spheric jet streak. It is confined to a height of 6–11 km, but the wind speed in the lower stratosphere increases, too. Afterward, 60-h wind speeds well exceed 20 m s\(^{-1}\) above the tropopause, which can be attributed to the presence of the polar vortex in the upper stratosphere.

For this background flow the IGW energy exceeds the mean value within the stratosphere at \(z = 20\) km for the time between \(t = 54\) and 78 h. Before this, a wind speed of about 20 m s\(^{-1}\) formed a critical level, preventing upward IGW propagation. Indeed, a phase speed over ground of \(c_{i,h}(z = 10\) km) \(- 22\) m s\(^{-1}\) is found for waves that were generated at the tropopause level by a background wind of \(u_{bg}(z = 10\) km) \(- 30\) m s\(^{-1}\) with an intrinsic phase speed of \(c_{i,h}(z = 10\) km) \(\lambda_b / \tau_i = 8.3\) m s\(^{-1}\).

For the further evaluation of IGW propagation, we use density-weighted quantities. The conservative propagation of waves is indicated by invariance of the wave action \(A\):

\[
A = \frac{\rho e}{\omega_i}
\]  

along the path of a wave packet (Bretherton 1966). The vertical group velocity

\[
c_{g,z} = -N^2 \frac{k_z^2}{\omega_i k_z}
\]  

has been used to illustrate the transfer activity between the different layers of the atmosphere. Fig. 7c supports the picture that an IGW packet has traveled upward during the time of increased activity of the tropopause jet and stronger stratospheric winds. An estimate of the vertical group velocity \(c_{g,z} = 0.60\) km h\(^{-1}\) suggests that a wave packet needs \(-17\) h for rising from \(z = 10\) to 20 km. Indeed, the tropospheric jet at \(z = 10\) km accelerated at the run time \(t \approx 42\) h and the wave energy level at \(z = 20\) km increased at run time \(t \sim 60\) h, just \(-18\) h later. The events with high wave activity at 54 and 60 h can be interpreted as an upward-propagating wave packet. Accordingly, the wave action in the lower stratosphere increased from \(A(24\) h, 12–20 km) \(= 0.63 \times 10^3\) kg m\(^{-1}\) s\(^{-1}\) to \(A(60\) h, 12–20 km) \(= 7.9 \times 10^3\) kg m\(^{-1}\) s\(^{-1}\)—this is one order of magnitude.

### Table 5. Mean values for all samples (second column), mesoscale samples (\(\lambda_b < 350\) km: third column) and subsynoptic samples (\(\lambda_b > 350\) km: fourth column) for zonal wavelength \(\lambda_x\), meridional wavelength \(\lambda_y\), horizontal wavelength \(\lambda_h\), vertical wavelength \(\lambda_z\), intrinsic period \(\tau_i\), and kinematic wave energy \(e\). Data from the HC run for the height range 2–20 km are analyzed with the DIV method (see the appendix for details). Notation is the same as in Table 3.

<table>
<thead>
<tr>
<th>DIV ((z = 2-20) km)</th>
<th>All</th>
<th>Mesoscale waves ((\lambda_b &lt; 350) km)</th>
<th>Subsynoptic waves ((\lambda_b &gt; 350) km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_x) (km)</td>
<td>310 ± 26 (1517)</td>
<td>294 ± 25 (1413)</td>
<td>675 ± 7 (105)</td>
</tr>
<tr>
<td>(\lambda_y) (km)</td>
<td>377 ± 35 (3045)</td>
<td>379 ± 34 (2833)</td>
<td>629 ± 9 (210)</td>
</tr>
<tr>
<td>(\lambda_h) (km)</td>
<td>211 ± 13 (1931)</td>
<td>189 ± 12 (1798)</td>
<td>508 ± 3 (133)</td>
</tr>
<tr>
<td>(\lambda_z) (km)</td>
<td>3.65 ± 0.33 (69.2)</td>
<td>3.64 ± 0.32 (64.5)</td>
<td>3.72 ± 0.09 (4.8)</td>
</tr>
<tr>
<td>(\tau_i) (h)</td>
<td>5.20 ± 0.22 (1747)</td>
<td>4.84 ± 0.21 (1627)</td>
<td>9.59 ± 0.06 (121)</td>
</tr>
<tr>
<td>(e) (m(^2) s(^{-2}))</td>
<td>2.16 ± 0.43 (2147)</td>
<td>1.72 ± 0.42 (1999)</td>
<td>8.14 ± 0.11 (148)</td>
</tr>
</tbody>
</table>

Fig. 6. Statistics for horizontal wavelength \(\lambda_h\) for the HC run: (a) frequency of a wavelength \(\lambda_h\), (b) energy per wave of wavelength \(\lambda_h\), and (c) accumulated energy of all waves of wavelength.
The wave action is used to quantify the amount of energy propagating upward. A comparison of values at the tropopause $A(30$–$60$ h, 8–10 km) $= 3.8 \times 10^3$ kg m$^{-1}$ s$^{-1}$ with the lower stratosphere $A(42$–72 h, 12–20 km) $= 2.0 \times 10^3$ kg m$^{-1}$ s$^{-1}$ suggests that 54% of the wave action propagated upward when the jet accelerated and the stratospheric wind was high.

d. Diagnosis of unbalanced flow

The exit region of the jet is an energy source for IGWs. It is highly unbalanced due to abrupt changes of the flow: While the geostrophic flow is decreasing for about $U \approx 30$ m s$^{-1}$ over a subsynoptic scale of $\delta x \approx 500$ km, the flow responds on a time scale of $\delta t = \delta x/U \approx 5$ h. This is much less than the inertial period ($T_i \approx 14.5$ h), which violates the condition of geostrophic equilibrium. This circumstance can be quantified with the Rossby number $Ro \approx U/\sqrt{\delta x f} \sim 0.3$. It is the aim of this section to diagnose those parts of unbalanced flow that are relevant for IGW generation. The basic idea is that a certain fraction of the imbalances are spontaneously radiated away.

The dimensionless cross-stream Lagrangian Rossby number has been introduced by Koch and Dorian (1988). It measures the Lagrangian acceleration of the horizontal wind speed relative to the Coriolis force:

$$\text{Ro}_L^c = -\frac{d u_h}{d t} \frac{u_a}{f u_h}$$ (14)

A threshold of 0.5 could be used to detect unbalanced flow. Here $\text{Ro}_L^c$ is zero for a rotating flow, where only the orientation of the wind velocity is changed like in gradient wind balance. Hence, it quantifies the longitudinal accelerations of the flow. It follows from the wind tendencies $(d u/d t = f v_a; d v/d t = -f u_a)$ that the cross-stream ageostrophic wind velocity

$$u_a = \frac{u v - v u}{(u^2 + v^2)^{1/2}}$$ (15)

could equally serve as a measure for the unbalanced flow field in Eq. (14).

We continue the considerations of Koch and Dorian (1988) with a discussion of the sign and scale of the cross-stream ageostrophic flow. Positive values of $\text{Ro}_L^c$ correspond to a decelerating flow. In a situation, where the jet maximum is positioned between a cyclone to the left and an anticyclone to the right, the ageostrophic flow points to the cyclonic side of the jet. Hence, the exit region of the jet should be associated with positive $\text{Ro}_L^c$. Beside such large-scale features the smaller-scale IGWs also contribute to this quantity. These oscillating fluctuations can be filtered out and the resulting $[\text{Ro}_L^c]$ only contains information on the background flow (here, the brackets [...] denote local filtering). This way the large-scale imbalances could be diagnosed, which have the potential to generate smaller-scale IGWs.

First, the performance of the unfiltered $\text{Ro}_L^c$ is demonstrated, which has been calculated wherever the wind speed was exceeding 20 m s$^{-1}$. As can be seen in Fig. 8a
and Fig. 8c (HC run), it detected a large region with positive values. Superimposed wave structures are found (cf. the divergence patterns in Fig. 3a). Values of $\Delta \mathbf{R}_L^\perp$ exceeding 0.5 were located in wave packets at about 58°N, 3°E and 52°N, 12°E—they clearly indicate unbalanced flow.

Successive smoothing of $\mathbf{R}_L^\perp$ is performed in order to eliminate the IGW components and to diagnose imbalances at synoptic scales that may serve as IGW sources. Most of the IGWs with horizontal wavelengths below 720 km (cf. Table 5) are eliminated with a Cressman filter with a horizontal cutoff $s_c = 720$ km [filter
function: $F_c(s) = (s_e^2 - s^2)/(s_e^2 + s^2)$. It leaves intense
imbalances in the tropopause region but also in the
stratosphere downstream the mountain. An estimation
of the vertical wavelength of stationary gravity waves
for a background wind of $u = 30$ m s$^{-1}$ rounds up to
$\lambda_z \sim 2\pi uN \sim 10$ km. Vertical smoothing with a cutoff
$z_e = 10$ km does not remove these mountain waves
completely. Only the combined action of horizontal
and vertical filter results in a weak, but distinct, synop-
tic-scale pattern of imbalance in the tropopause region.

In the smoothed maps at the 300-hPa layer, large-
scale imbalances are detected with [Ro$_z$]$^{720\text{km}, 10\text{km}} >
0.1$ above northern Europe in both the ECMWF analysis
(Fig. 8b) and MM5 HC run (Fig. 8d). These positive
fields are located in the exit region of the jet streak.
This means that the [Ro$_z$]$^{720\text{km}, 10\text{km}}$ does not depend on
the resolution of the data and thus represents a scale-

The smoothed Ro$_z$ is shown in the Hovmöller dia-
gram for Kühlungsborn in Fig. 7b. The highest values
appear with Ro$_z > 0.15$ at the tropopause level ($z \sim 10$
km) at run times 42 and 54 h. These events are associ-
ated with high wave activity, which may propagate up-
ward (see Fig. 7c and section 3c).

e. Separation of different IGW sources

Beside the upper-tropospheric jet streak, other pro-
cesses also may contribute to IGW generation, such as
orography and moist convection. All these processes
are likely during Rossby wave breaking. They will be
separated by variation of the model physics.

The following analysis is done with medium-
resolution (M type) runs and will be interpreted as indi-
cation of the relative importance of the different
IGW sources. In Fig. 9 vertical cross sections are shown
for the run time $t = 36$ h (1200 UTC 17 December 1999)
from different model runs with/without orography/
moisture (cf. Table 2). In the MS run (without both
orography and moisture; Fig. 9a) long waves are found
above and below the tropospheric jet at about $z = 9$
km. The phase of IGW above the jet moves upstream
downward. This supports the concept that the unbal-
anced jet streak spontaneously radiates IGWs that
propagate energy upward. Correspondingly, jet-
generated waves in the troposphere carry energy down-
ward. Some contribution of fronts in the troposphere
can also be identified as near-surface V-shaped pattern
at distances $s = 750$ and 1750 km. Mountain waves are
excited if orography is included (i.e., the MO run; Fig.
9b). These waves are relatively steep and strictly lo-

downstream of the mountain show increased amplitudes.
Adding moisture (i.e., the MM run; Fig. 9c),
three deep convection events at distances $s = 400, 900,$
and 1400 km occurred. They do not only affect the
tropospheric dynamics but also seem to amplify wave
structures in the lower stratosphere at about $z = 15$-km
height. At two occasions ($s = 550$ and 1200 km) V-
shaped patterns could be detected, which appear to be
more intense than those associated with the fronts (Fig.
9a). One structure of positive divergence extends form
the surface ($s = 1200$ km) up to the tropopause ($s =
1400$ km) with high intensity, which indicates a strong
interaction between jet and convection. In the MC run,
(Fig. 9d) contributions of all three processes are re-
tained.

A quantitative comparison of these four model con-
figurations (see Table 6) suggests the following sum-
mary: 27% of wave activity appeared with the jet
streak, 31%–51% is related to orography, and 22%–
42% is the moisture effect. The associated processes do
not simply superimpose but interact as will be discussed
in the following section.

4. Discussion

a. Estimation of IGW properties

IGW properties have been estimated from the model
data with the DIV method (Table 5). This approach
avoids variations of the results due to the method of
background removal (cf. Tables 3 and 4). While the
filter makes an a priori scale selection, it is undeter-
mined for the polynomial (Zhang et al. 2004).

The vertical wavelengths, which are determined di-
rectly in both methods, are similar for DIV and
VIN5KM while VIN4POL returns larger values. This
difference can be traced back to artificial long vertical
waves generated by the 4POL background removal.
This method tends to overshoot when applied to sharp
variations as in the presented temperature profile (see
Figs. 4c,f), while this was not as expressed for the wind
profiles (Figs. 4a,d,b,e). The modeled vertical wave-
length for each of the VIN4POL and VIN5KM is larger
than the observed. This is a joint effect of the limitation
of the model data to the cutoff at $\lambda_{zc} \sim 1$ km and
high-frequency noise in the observations.

Another advantage of the DIV method is that the
horizontal wavelengths are estimated directly from the
data and no assumption on polarization relations needs
to be made as in the VIN method. This implies a higher
accuracy the DIV estimates $[\delta\lambda_h(DIV) \sim \pm 10$ km ver-
sus $\delta\lambda_h(VIN) \sim \pm 100$ km]. A direct comparison of the
horizontal wavelengths is not as straightforward,
because the database for the VIN method are local ver-
tical profiles comparable with radiosonde observations at the position of Kühlungsborn, while the DIV method uses horizontal profiles of 2400-km length. The smaller values in the DIV method \( \lambda_{0}^{(\text{DIV})} \sim 210 \text{ km} \) versus \( \lambda_{0}^{(\text{VIN4POL})} = 490 \text{ km} \) and \( \lambda_{0}^{(\text{VIN5KM})} = 570 \text{ km} \) can be traced back to mesoscale IGWs in some distance to Kühlungsborn. This can also be seen in Fig. 5c where strong mesoscale variations take place at \( x = 800 \) and 1800 km while Kühlungsborn is situated at \( x = 1250 \) km. If only subsynoptic IGWs were considered, all three methods lead to similar values. This argumentation also holds for the comparison of

![Image](image_url)

**Fig. 9.** Vertical cross section through Mt. Ben Nevis (left thick vertical line) and Kühlungsborn (right thick vertical line). Shown is the divergence \( \delta \) (contours are shaded as in Fig. 2), the smoothed cross-stream Lagrangian Rossby number \( \left[ (\text{Ro}_{\perp}) \right] \) (values exceeding \( \pm 0.1 \) are bold lines surrounding dark shaded areas), and the precipitation mixing ratio \( q_{pr} \) (values exceeding 0.1 g kg\(^{-1}\) are long, dashed lines surrounding light shaded areas). The data are from the model runs (a) MS, (b) MO, (c) MM, and (d) MC for run time 36 h (1200 UTC 17 Dec 1999). The bold line near the bottom of (b) and (d) indicates the terrain height.

<table>
<thead>
<tr>
<th>DIV ((z = 12-20 \text{ km}))</th>
<th>Flat</th>
<th>Orographic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \left(10^{5} \text{ kg m}^{-1} \text{s}^{-1}\right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dry</td>
<td>0.194 ± 0.134 (269) [MS]</td>
<td>0.418 ± 0.241 (508) [MO]</td>
</tr>
<tr>
<td>Moist</td>
<td>0.352 ± 0.521 (368) [MM]</td>
<td>0.725 ± 0.551 (580) [MC]</td>
</tr>
</tbody>
</table>
intrinsic periods—the DIV method estimated faster IGWs in the extended profiles, which are absent in the local profiles used for the VIN method. For this parameter too a higher accuracy is found with the DIV method \[\delta \tau (\text{DIV}) \approx \pm 0.2 \text{ h} \text{ versus } \delta \tau (\text{VIN}) \approx \pm 1.0 \text{ h}\], which is due to the uncertainty in the Stokes analysis.

The similarity of the modeled DIV-derived and the observed VIN5KM-derived horizontal wavelengths should not be overinterpreted. It is a compensating effect of larger vertical wavelength (due to model damping) and shorter intrinsic period (due to distant mesoscale IGWs).

The energy estimates from the DIV method are comparable to those from VIN5KM. VIN4POL revealed much larger values that can be qualified as an artifact due to an overshooting polynomial. A disadvantage of the DIV method is that no directional information can be found (only the absolute value of wavenumbers can be determined, not the sign).

\[\text{b. Simulations of subsynoptic and mesoscale IGWs}\]

The simulated IGWs are limited by model resolution and diffusion. The general considerations in the section on “statistical methods” have related the horizontal cutoff to the resolution-dependent horizontal diffusion [Eq. (4)]. This approach has been confirmed with the simulated cutoff [cf. \(\lambda_{h,c}\) with \(\lambda_{h,c}(\text{emp})\) in Table 2]. The vertical cutoff turned out to be independent of the used model resolution due to the vertical diffusion [cf. \(\lambda_{h,c}\) with \(\lambda_{h,c}(\text{emp})\) in Table 2].

If the vertical resolution is not as high as in the present setup, the smallest vertical wavelength is given by the vertical grid size. For that, an effective horizontal resolution can be defined:

\[\Delta s_{\text{eff}} = \max \left(\frac{N}{\Delta \varphi}, \frac{N}{\Delta \varphi} \right) \times 15 \text{ km}.\]  

(16)

It accounts for the worst resolution—either directly in the horizontal or in the vertical dimension. If the resolution is vertically limited, there is also some danger of producing spurious waves. The effective resolution will be used to compare different model simulations of IGWs.

Mesoscale waves with smaller horizontal wavelengths are present more frequently and have higher amplitudes in simulations with higher resolution. In our simulations, subsynoptic waves (\(\lambda_h \approx 510 \text{ km}\)) are already well simulated in the MC run (\(\Delta s_{\text{eff}} = 112 \text{ km}\)), but mesoscale waves (\(\lambda_h \approx 190 \text{ km}\)) appeared only in the HC run (\(\Delta s_{\text{eff}} = 15 \text{ km}\)). The MM5 simulations by Zhang (2004) contain mesoscale waves with 150-km horizontal wavelength for a vertically limited effective resolution of \(\Delta s_{\text{eff}} = 54 \text{ km} (\Delta s = 10 \text{ km}, \Delta \varphi = 360 \text{ m})\).

Similar setups have been used by Leutbecher and Volkert (2000), Dönbrack et al. (1999), Hitchman et al. (2003), Buss et al. (2004), and Spichtinger et al. (2005). Mesoscale IGWs in the range of 100–400 km are dominant in these simulations. Subsynoptic waves with a horizontal wavelength of about 600–1000 km are discussed by O’Sullivan and Dunkerton (1995). Their model (T126 with \(\Delta \varphi = 159 \text{ km and } \Delta \varphi = 700 \text{ m}\)) suppressed smaller waves with a horizontally limited effective resolution of \(\Delta s_{\text{eff}} = 159 \text{ km}\). Other simulations with global circulation models (Kawatani et al. 2004) and analyses (Plougonven and Teitelbaum 2003) also bear IGWs at such scales. While the subsynoptic IGWs are sufficiently far from the cutoff, the appearance of mesoscale waves is heavily influenced by the damping. Hence, the different size of mesoscale IGWs can at least partly be explained with the different effective model resolution. From this point of view, subsynoptic waves are numerically stable while mesoscale waves have to be interpreted with care. This is in line with the review by Koch (2001).

\[\text{c. Observations of jet-generated IGWs}\]

To validate the simulations, observed profiles of wind and temperature were compared with simulations at the same time at the same location. Although the results suggest longer and slower waves in the simulation it should be underlined that the methodological variations between VIN4POL and VIN5KM are of the same order. The vertical wavelength and intrinsic period was simulated reasonably with errors below 50%. The larger differences in the horizontal wavelength are a consequence of the sensitivity of Eq. (8) to near-inertial frequencies. The simulated IGW energy was one-third of the observed, which could be attributed to the different regional situation. A closer inspection of the wind field at 300 hPa revealed that the center of the modeled tropospheric jet streak was shifted with respect to the analyses. This can be inferred from the southwestward displacement of the geopotential height of 9 km at 300 hPa for about 150 km (Figs. 8b,d). At the position of Kühlungborn, the wind speed in the simulation (~25 m s\(^{-1}\)) was 71% of the observation (~35 m s\(^{-1}\)). This is of the same order as the ratio of modeled to observed IGW amplitudes. Hence, the simulation can be considered as near realistic.

For the same observation data, subsynoptic IGWs have been reported by Peters et al. (2003). They estimated a vertical wavelength of 2–3 km and an intrinsic period of ~12.5 h, which is larger than our mean estimates. Peters et al. (2003) used Hovmöller diagrams for the radiosondes with 3-h resolution, which allows for the estimation of intrinsic periods exceeding 6 h accord-
ing to the Nyquist theorem. This corresponds to the subsynoptic IGWs, which we have identified in the DIV analysis of the simulation.

Subsynoptic jet-generated IGWs of similar scales have also been found in the Fronts and Atlantic Storm Track Experiment (FASTEX) data over the North Atlantic Ocean (Pluougouven et al. 2003) and over west Russia in December 1999 (Hitchman et al. 2003). The upper-tropospheric jet has also been identified as an energy source for IGWs over Aberystwyth, Wales, in March 1994 (Thomas et al. 1999). The smaller size of these waves possibly gives a hint on deformation of the jet-generated waves by orographically excited waves.

To compare the simulated IGW energies with climatologies derived from observations we use the result that the modeled IGW energy was one-third of the observed. Hence, the modeled mean kinematic wave energy $E_{\text{kin}} = 0.5 < u^2 + v^2 > -2 \pi^2 a^2$ corresponds to a more representative value of $\sim 6 m^2 s^{-2}$. These values fits to the data of Schöllhammer (2002), based on radiosonde launches from Lindenberg, Germany (52°N, 14°E) for 1995–98, which scatter for December between 3 and 7 m² s⁻². It is also comparable with data for April 1993 for Davis Station, Antarctica (69°S, 78°E; Allen and Vincent 1995) and for Macquarie Island (55°S, 159°E) in winter (Vincent et al. 1996). In view of this climatological information, the presented event of poleward Rossby wave breaking is associated with an increased IGW activity in the lower stratosphere of the midlatitudes.

d. Identification of synoptic-scale unbalanced flow as the IGW source

For the diagnosis of the unbalanced flow as IGW source, a dimensionless scale-independent measure is used: the smoothed cross-stream Lagrangian Rossby number $[Ro_L^L]$. It measures the longitudinal wind accelerations relative to the Coriolis force and is relatively easy to estimate and robust in comparison to other approaches such as nonlinear balance equation residuals (DNBES) or potential vorticity (PV) inversion (see Zhang et al. 2000). The cross-stream ageostrophic wind $u^*_L$ is positive in the decelerating straight jet, while higher-order quantities such as the divergence $\delta = \partial u_0/\partial x + \partial v_0/\partial y$ and its tendency $d\delta/dt \approx f_0 \omega_0 = f(\partial u_0/\partial x - \partial v_0/\partial y)$ show alternating patterns in such regions. This makes it suitable for the detection of the exit region of the jet.

The smoothing procedure allows for a separation of subsynoptic (IGW) fluctuations and synoptic (background) flow. The smoothed Rossby number $[Ro_L^L]$ characterizes the forcing for IGWs at synoptic scales. Imbalances at these scales were found in the exit region of the tropospheric jet streak, as expected. They appear in similar intensity in the low-resolution ECMWF analyses and the high-resolution MM5 runs. In our example we find scale independence of $[Ro_L^L]$, which makes it suitable as input for IGW source parameterizations.

The filtered quantities allow a description of relative imbalance carried at different scales. In Fig. 8c we found a maximum value of $Ro_L^L = 0.5$, while the smoothed value $[Ro_L^L]$ turned out to have values of about 0.1. This is consistent with the assumption that the large-scale imbalances are weak and the stronger shorter-scale part is radiated away relatively fast (over the time scale of an inertial period) in form of IGWs.

e. IGW sources and propagation into the stratosphere

During the investigated event of a poleward-breaking Rossby wave, IGWs found good propagation conditions in the polar vortex. About 54% of the wave action passed from the tropopause into the middle stratosphere. This presents a causal link of wave activity between troposphere and stratosphere. Wind speeds above 20 m s⁻¹ were necessary to avoid the formation of critical lines for IGWs. In this sense, winds in the lower stratosphere act like filters to any IGW generated below.

The reason for the damping of an upward-propagating wave packet is the model diffusion. An estimation of the corresponding reduction of the energy of a wave packet during the time $\Delta t = 17 h$ leads with Eq. (3) to $D\times (\lambda_h = 210 km, \lambda_s = 3.7 km, t = 17 h) = \exp(–2\gamma) \sim 0.46$. This is the same order as the simulated reduction in wave action. While the model has been optimized for little damping during one period $\tau \sim 5.2 h$, the slow upward propagation of IGWs implies a stronger impact of damping. Although the absolute value of wave action was not conserved, the relative numbers confirm the active role of the jets.

In a detailed investigation, it has been shown that the upper-tropospheric jet streak, orography, and moist convection significantly contribute to IGWs appearing in the lower stratosphere. Although the jet is responsible for about one-third of the wave activity, the other processes are likely important. It was found that orography not only generated (short) mountain waves but also amplified (long) jet-generated waves. Another aspect is the triggering of deep convection by downward-propagating jet-generated waves and the back effect of convection on the jet circulation. All these processes need to be taken into account for a complete description of IGW generation during a Rossby wave-breaking event.
5. Conclusions and outlook

Simulated IGWs are sensitive to model resolution and diffusion; this sensitivity was adequately described with the cutoff and damping rates. For the present model setup, IGW amplitudes were weakly damped over the time scale of one wave period. For their upward propagation into the lower stratosphere, the wave packed needed four periods and appeared to be damped stronger. Hence, it remains a matter of more refined studies to ensure the invariance of wave activity in mesoscale circulation models.

IGW properties have been estimated with the DIV method, a harmonic analysis of the horizontal divergence field. It avoids the explicit removal of the background wind and works for all wavelengths, but the required spatially extended wind field is usually available from model data only. Hence, it represents a supplement to the standard (VIN) method, which uses local vertical profiles of wind and temperature.

A validation of simulations with observation showed that structures and amplitudes of subsynoptic IGWs were simulated sufficiently. Mesoscale IGWs with a period below 6 h could not be estimated from Hovmöller diagrams based on 3-hourly samples. Consequently, radiosondes should be launched more frequently.

Synoptic-scale regions of unbalanced flow can be diagnosed with the smoothed cross-stream Lagrangian Rossby number. It is clearly related to the longitudinal acceleration of the flow, it is relatively easy to calculate and turned out to be scale independent. These findings make it advisable to use it as an input parameter for the parameterization of IGW sources in the upper-tropospheric jet streak.

The appearance of IGWs in the lower stratosphere also depends on orography and moist convection as sources and sufficiently strong wind for propagation. This underlines that all these processes must be taken into account for a quantitative description of IGW activity.

The synoptic background for the presented case study is given by a poleward-breaking Rossby wave. Both the tropospheric and the stratospheric jet are intense during such events, as indicated schematically in Fig. 10. Dominating IGWs are mainly generated through spontaneous radiation in the exit region of the upper-tropospheric jet. They propagate downward through the troposphere and upward into the stratosphere, where the stratospheric jet provides strong background winds. Hence, Rossby wave breaking is an important dynamical process for IGW activity in the midlatitudes.

It remains a matter of future research to study the problem of IGW generation and propagation on climatological scales. About 10 observational campaigns with radiosondes and VHF radar are available at our institute, which have been carried out during phases of Rossby wave breaking. They will be analyzed jointly with analysis products and model results.

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APPENDIX

Statistical Methods

a. Estimation of mean values and variances

A fluctuating time series \(x(t)\) of finite length \(t = 0 \ldots T\) is given. It is a realization of a normally distributed
ensemble with mean value \( m_x = E[x] \) and variance \( s_x^2 = E[(x - m_x)^2] \), with \( E[\ldots] \) the expectation operator. Here, we study the effects of the noise and the finite length of the time series in the time continuum because mathematical operations are relatively simple. It is straightforward to transfer the findings to spatial dimensions and discrete samples.

1) Scatter of Mean Values

The mean value, which is indicated with an asterisk, can be estimated as

\[
m^*_x = \frac{1}{T} \int_0^T x(t) \, dt.
\]

(A1)

It is a fluctuating quantity and has a mean variance of

\[
s^2[m^*_x] = E[(m^*_x - m_x)^2] = 2 \int_0^T \frac{d\tau}{T} \left( 1 - \frac{\tau}{T} \right) c_x(\tau).
\]

(A2)

Here, the autocovariance function \( c_x(\tau) = E[(x(t + \tau) - m_x)(x(t) - m_x)] \) appears. Mean values will be presented in the following form:

\[
m_x = m^*_x \pm \frac{s_x^*}{\sqrt{n^*_x}^{1/2}},
\]

(A3)

with the estimated sample variance

\[
s^2[m^*_x] = E[(m^*_x - m_x)^2] = \int_0^T \frac{d\theta}{T - \tau} \left( 1 - \frac{\tau}{T} \right) \left( c_c(\theta) - c_y(\tau) \right) c_{xy}(\theta - \tau).
\]

(A8)

While this equation is exact, the variance will usually be estimated with the approximation for sufficiently large lags (\( \gamma \tau \gg 1 \))

\[
c_c(\tau) = c^*_c(\tau) \pm \frac{s^*_c}{\sqrt{n^*_c}^{1/2}}
\]

(A9)

with the effective degree of freedom

\[
n^*_c = \frac{2}{n^*_c + 1}.
\]

(A10)

using

\[
n^*_c = n^*_c(T - \tau) = \frac{T - \tau}{T - \tau}.
\]

(A11)

2) Scatter of Variances

The scatter of the estimated cross-covariance function

\[
c^*_c(\tau) = \int_0^T \frac{d\tau}{T - \tau} [x(t + \tau) - m_x][y(t) - m_y]
\]

(A7)

is given by (Bartlett 1955, section 9.1 and 9.3; Swerschnikow 1965, section 4.1):

\[
s^2[c^*_c(\tau)] = E[(c^*_c(\tau) - c_c(\tau))^2] = \int_0^T \frac{d\theta}{T - \tau} \left( 1 - \frac{\tau}{T} \right) \left( c_c(\theta) - c_y(\tau) \right) c_{xy}(\theta - \tau).
\]

(A8)

3) Scatter Due to Methods and Physics

The task is to find the total scatter for a mean value:

\[
m^*_t = \frac{1}{P} \sum_{p=1}^P m^*_x(p).
\]

(A12)

This is based on \( P \) estimates, for example, in a Hovmöller diagram. Each of the mean estimates is subject to a certain mean scatter \( \pm s^*_x(p)/n^*_x(p) \), which makes up the methodological sample variance

\[
s^2[m^*_x(p)] = \frac{1}{P} \sum_{p=1}^P s^2_x(p).
\]

(A13)

On the other hand, the variations of the mean values define the physical sample scatter

\[
s^2[m^*_x(p)] = \int_0^T \frac{dt}{T} [x(t) - m^*_x(p)]^2.
\]
\[ s^2_{\text{tot}}(\text{phys}) = \frac{1}{P} \sum_{p=1}^{P} (m^p_\alpha - m^p_\alpha)^2. \]  

This is reflected in the total sample variance

\[ s^2_x[x] = s^2_\text{tot}(\text{meth}) + s^2_\text{tot}(\text{phys}) \]  

and the total mean variance

\[ s^2[m^\alpha_\text{tot}] = \frac{s^2_\text{tot}(\text{meth})}{n^\alpha_\text{tot}(\text{meth})} \cdot \frac{s^2_\text{tot}(\text{phys})}{n^\alpha_\text{tot}(\text{phys})}. \]  

The effective degrees of freedom for the method

\[ n^\alpha_\text{eff}(\text{meth}) = \frac{1}{P} \sum_{p=1}^{P} s^2_\alpha(p). \]  

For a Hovmöller diagram the degrees of freedom from vertical profiles \((n^\alpha_\text{v})\) and the time series \((n^\alpha_\text{m})\) is used for the estimate \(n^\alpha_\text{m}(\text{phys}) = n^\alpha_\text{v} \cdot n^\alpha_\text{m}.\)

4) SCATTER OF DERIVED QUANTITIES

The scatter of derived quantities such as \(f = f(x, y)\) is treated in linear approximation. Hence, the sample variance is

\[ s^2_f = \left( \frac{\partial f}{\partial x} \right)^2 s_x^2 + \left( \frac{\partial f}{\partial y} \right)^2 s_y^2 \]  

and the mean variance is

\[ s^2[m^\alpha_f] = \left( \frac{\partial f}{\partial x} \right)^2 s^2[m^\alpha_x] + \left( \frac{\partial f}{\partial y} \right)^2 s^2[m^\alpha_y] \]  

with the corresponding effective degree of freedom as

\[ n^\alpha_f = \frac{s^2_\alpha_f}{s^2[n^\alpha_f]} \]  

b. IGW parameters from the Vincent method (VIN)

The basic part of the statistical procedure has been downloaded from Robert Vincent’s homepage (available online at http://www.physics.adelaide.edu.au/atmospheric/igw_project.html). Prior to the statistical procedure the background signal is subtracted. The following two methods are used for this:

- VIN2POL, VIN4POL: fit of the background with a polynomial of second or fourth degree,

- VIN5KM: low-pass filtering of the background using a Lanczos filter:

\[ F_L = \frac{\sin(2\pi m/m_\nu)}{\pi m} \frac{\sin(2\pi m/m_\nu)}{\pi m/m_\nu}; \]

\[ m = (-m_\nu, \ldots, +m_\nu), \]

with a cutoff at \(m_\nu = 5 \text{ km}/\Delta z\) and a width of \(m_\nu = m_\nu^*\).

The intrinsic period is found from a Stokes analysis (Vincent and Fritts 1987; Eckermann and Vincent 1989). For a scatter estimate uncertainties in the parameter \(Q\) are taken into account. The frequency estimate is rejected if the range of \(\omega_0 = \omega_0^* \pm s[\omega_0^*]\) includes zero \((\omega_0 < 0)\) or if it exceeds a threshold \((\omega_0 > 10f)\), as it was implemented in the original Vincent program.

The vertical wavenumber \(k_z\) is estimated from the average

\[ k_{z,\text{max}}^* = \int_0 \Delta k_z w(k_z)k_z \]  

with weights according to the sum of the power spectra of zonal and meridional winds \(w = S_u + S_v\). In this sense, the variance is determined with

\[ s^2_{k_z} = \int_0 \Delta k_z w(k_z)(k_z - k_{z,\text{max}}^*)^2. \]

c. IGW parameters from the divergence method (DIV)

Wavelengths are estimated directly from a harmonic analysis. The general approach is described in Taubenheim (1969) and will be briefly detailed here in order to make the paper complete. The zonal wavelength \(\lambda_{z}\) is determined from the zonal profile \(\delta(x)\), where the other variables \((t, v, z)\) have been kept fixed. From this profile a fitted linear trend is subtracted and the autocovariance function \(c_\| (\xi)\) for lags \(\xi = 0 \ldots \xi_{\text{m}} = L/5\) is estimated. The point of first zero crossing \(\xi_{\text{m}}^*\) \([c_\| (\xi_{\text{m}}^*) = 0]\) is closely related to the wavelength

\[ \lambda_{z}^* = \frac{4}{\xi_{\text{m}}^*}. \]

Taubenheim (1969) analytically calculated the variance at this point from Eq. (A8) for the model autocorrelation function:

\[ c_\| (\tau) = s^2_\| \exp(-\lambda_{z}\tau) \cos(\omega_\nu\tau). \]

It reads

\[ s^2[c_\| (\xi = \xi_0)] = \frac{s^2_\|}{2\gamma_{c}(L_z - \xi)} \left\{ 1 - (2\gamma_{c}\xi + 1) \exp(-2\gamma_{c}\xi) + \frac{(\gamma_{c}\xi)^2}{(\gamma_{c}\xi)^2 + (\pi/2)^2}[1 - \exp(-2\gamma_{c}\xi)] \right\}. \]
The points where the empirical covariance function meets the upper and lower levels \([ \xi^* \pm \xi^* \pm ] = \pm \sigma[c^2(\xi^*)] = \pm \sigma_0 \] defines the variance of the zero crossing. Hence, sample scatter and effective degree of freedom are estimated with

\[
s_\delta^2 = 2(\xi^* - \xi^*)
\]

\[
n_\delta^2 = \gamma_\delta^2 L_x \left( \frac{L_x}{2} \right)^2 - 2 \int_0^L \xi^2 d\xi^2 \xi^2 (\xi)
\]

The estimate of the wavelength is accepted, when the range remains bound between zero and the maximum detectable wave length: \(0 < \lambda^*_x = s[\lambda^*_x] < \lambda^*_x < \lambda^*_x + s[\lambda^*_x] < \xi^*_m\). This procedure corresponds to a \(t\) test with \(t(\alpha/2, n) = 1\) and an approximate error level of \(\alpha \approx 34\%\) for a typical degree of freedom \(n = 10\).

For a check of consistency the power spectrum has been estimated from the autocovariance function,

\[
S_\theta^2(k_x) = F_H \left( 2 \int_0^L \xi \cos(k_x \xi) \xi^2 (\xi) \right),
\]

which has been smoothed with a Hamming filter \(F_H\). A peak in the empirical spectrum was accepted if it exceeded the upper level of a fitted red spectrum with the same variance:

\[
S_\text{red}(k_x) = \frac{2\gamma_k^2 \xi_k}{\gamma_k^2 + k_x^2}.
\]

The criterion for the acceptance of a significant peak at \(k_x\)

\[
S_\theta^2(k_x) > S_\text{red}(k_x) + \sigma[S_\text{red}(k_x)]
\]

with the dispersion

\[
\sigma[S_\text{red}] = \frac{S_\text{red}}{n_k^2},
\]

\[
n_k^2 = 2 \left( \frac{L_x}{\xi_m} \right) \left( \frac{1}{3} \right)
\]

corresponds to a chi-squared test with \(\chi^2(\alpha, n) = n + 1\) at an error level of \(\alpha \approx 36\%\) and \(n = 10\).

In the same way the meridional wavelength \(\lambda_y\) is determined from the meridional profile \(\delta(y)\) and the vertical wavelength \(\lambda_z\) from the height profile \(\delta(z)\) (see Table A1). A valid estimate has only been made, when both the horizontal wavelength and the vertical wavelength could be determined with significance. The length of the respective profiles \(L\) allowed for a maximal lag of \(\xi_m = L/5\). If we suppose that a significant zero crossing occurred in the middle of this range \((\xi_m/2) = 0.4L\) we could estimate a maximal wavelength of \(\lambda_m = 4 \xi_m/0.4L\). Hence, the horizontal and vertical profiles allowed for wavelength of maximal \(\lambda_{x,m} \approx 1000\) km and \(\lambda_{z,m} \approx 7\) km.

For a measure of the IGW amplitude, the variance of divergence has been estimated from the zonal and meridional profiles \([c(\xi)\) and \(c(\eta)\)]:

\[
s_\delta^2 = \frac{1}{2} [c_\delta^2(\xi = 0) + c_\delta^2(\eta = 0)].
\]

REFERENCES


Dörnhack, A., M. Leutbecher, R. Kivi, and E. Kyrö, 1999: Moun-


