Investigation of preferred structures of planetary wave dynamics is addressed using multivariate Gaussian mixture models. The number of components in the mixture is obtained using order statistics of the mixing proportions, hence avoiding previous difficulties related to sample sizes and independence issues. The method is first applied to a few low-order stochastic dynamical systems and data from a general circulation model. The method is next applied to winter daily 500-hPa heights from 1949 to 2003 over the Northern Hemisphere. A spatial clustering algorithm is first applied to the leading two principal components (PCs) and shows significant clustering. The clustering is particularly robust for the first half of the record and less for the second half. The mixture model is then used to identify the clusters. Two highly significant extratropical planetary-scale preferred structures are obtained within the first two to four EOF state space. The first pattern shows a Pacific–North American (PNA) pattern and a negative North Atlantic Oscillation (NAO), and the second pattern is nearly opposite to the first one. It is also observed that some subspaces show multivariate Gaussianity, compatible with linearity, whereas others show multivariate non-Gaussianity. The same analysis is also applied to two subperiods, before and after 1978, and shows a similar regime behavior, with a slight stronger support for the first subperiod. In addition a significant regime shift is also observed between the two periods as well as a change in the shape of the distribution. The patterns associated with the regime shifts reflect essentially a PNA pattern and an NAO pattern consistent with the observed global warming effect on climate and the observed shift in sea surface temperature around the mid-1970s.

1. Background and motivation

The dynamics of low-frequency variability have been, and still are, the focus of many research studies. This continuous interest is driven by the need to understand the dynamics of tropospheric planetary waves and other synoptic patterns such as blocking for predictability. The issue of climate change has also brought the question related to the existence of preferred flow regimes up front in science media (Palmer 1999; Corti et al. 1999; Hsu and Zwiers 2001; Christiansen 2003). Origins and mechanisms of large-scale and low-frequency variability have also been discussed extensively in the literature. Linear Rossby waves forced from the Tropics have been proposed to explain mid-latitude large-scale systems (Hoskins and Karoly 1981; Sardeshmukh and Hoskins 1985; Hoskins and Ambrizzi 1993). A significant body of researchers also maintain that these systems are essentially steady wave patterns that can be resonantly excited and stabilized by nonlinear midlatitude processes such as adiabatic heating and eddy vorticity fluxes (Mitchell and Derome 1983; Marshall and So 1990), or resulting from the instability of the climatological winter flows (Simmons et al. 1983). The nonlinear paradigm was also proposed, which led to the concept of multiple equilibria first mentioned by Rossby (1940) and taken steps further by Charney and DeVore (1979), Charney et al. (1981), and others.

Various authors have reported, using empirical studies, robust and statistically significant bimodality in the wave amplitude index (Hansen and Sutera 1986, 1995; Christiansen 2005, 2007; Ruti et al. 2006.) The issue of nonlinearity from observations remains, however, a challenging task given the complexity of the system and the relatively small sample size of reanalyses (see, e.g., Branstator and Berner 2005; Wallace et al. 1991; Nitsche et al. 1994). Recently Sura et al. (2005) pre-

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sented an alternative theory, namely, that low frequency variability is basically linear (Newman et al. 1997, 2003) and that nonlinearity is primarily rapidly decorrelating and can be approximated by a multiplicative noise. Linear dynamics of planetary waves yield, conceptually, systems whose probability distribution function (pdf) is multivariate Gaussian or multinoormal if the forcing is Gaussian (Leith 1971; Delsole and Farrell 1995; Penland 1989; Penland and Sardeshmukh 1995; Toth 1991) but also can result in non-Gaussianity if the external forcing is non-Gaussian. A multivariate non-Gaussian structure of the pdf can reflect nonlinearity (Hannachi et al. 2003; Christiansen 2005, 2007; Berner 2005) but can also provide evidence of state-dependent or multiplicative noise, as shown by Sura et al. (2005).

Tropical forcing is also important in the study of low-frequency variability. An example is the effect of El Niño–Southern Oscillation (ENSO) on the extratropics (Lau and Nath 1996; Horel and Wallace 1981; see also the review of Trenberth et al. 1998). Persistent tropical forcing can cause persistence of extratropical large-scale weather systems, such as the case of ENSO forcing of both phases of the Pacific–North American (PNA) pattern (Livezey et al. 1997; Hoerling et al. 1997, 2001; Hannachi 2001). These extratropical weather patterns, however, are internal to the atmosphere and the (external) tropical forcing can contribute to their excitation and persistence, and can also slightly modify their phase. Therefore nonlinearity and/or nonnormality of tropical forcing could also provide another alternative mechanism. Other alternative views also exist. For example, possible nonlinearity may exist but may result primarily in a roughly piecewise linear response or in a saturation at the extremes.

Investigations using atmospheric models do seem to show some nonlinearity in the space of large-scale flow. Nonlinearity can produce skewness and nondiffusive behavior (Crommelin 2004) and can also yield multiple equilibria as in the theoretical studies using low-order models (Charney and Devore 1979) and intermediate complexity models (Egger 1981; Reinhold and Pierre-humbert 1982; Legras and Ghil 1985; Crommelin et al. 2004; Yang et al. 1997). Similar behavior is also observed using general circulation model (GCM) simulations (Haines and Hannachi 1995; Hannachi 1997a,b; Selten and Branstator 2004; Branstator and Berner 2005) where quasi-stationary (QS) states are identified in the subspace of the large-scale flow. But, it is fair to say that the task is a challenging one; see, for example, the discussion in Lorenz (2006). The difficulty in separating contributions from the linear and the nonlinear dynamics is mainly due to (i) the myriad of spatiotem-
a difficult one since there is no systematic way to identify the independent sample and its size. This difficulty is overcome here by systematically finding the number of regimes in the mixture model using arguments from order statistics. The mixture model and the methodology are described in section 2. In section 3 the method is applied to few simple stochastic dynamical systems and data from a GCM. The application to the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalyses is presented in section 4. A summary and conclusions are presented in the last section.

2. Methodology

a. The Gaussian mixture model

It has been shown (e.g., Christiansen 2003) that winter NH stratospheric planetary waves display a bimodal behavior representing the strong/weak polar vortex or cold/warm stratosphere. By looking at the tropospheric wave amplitude index few authors have reported significant bimodality of daily winter NCEP 500-hPa heights (Hansen and Sutera 1995; Christiansen 2005, 2007). Ruti et al. (2006) have reported a characterization of planetary atmospheric waves in terms of the subtropical jet. They show, using NCEP–NCAR reanalyses and the 40-yr European Centre for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (ERA-40) that for intermediate jet strengths a bimodal behavior of an indicator of planetary waves is obtained. The extratropical troposphere as a whole, however, is characterized by highly chaotic dynamics and low signal-to-noise ratio. The background (or dynamical) noise combined with ergodicity (Branstator and Berner 2005) therefore prevents multimodality from emerging, even if the dynamics reveal intermittent behavior between low-frequency flow regimes resulting in a skewness of the planetary wave pdf. For example, Sura et al. (2005) show that large-scale extratropical flow could well be entirely linear but with multiplicative noise resulting from the rapidly decorrelating nonlinearity. The observed skewness of the pdf of planetary wave dynamics (Sura et al. 2005) is interpreted here in terms of a mixture model to diagnose preferred large-scale structures. But it should be noted that this alternative interpretation does not necessarily rule out any current theories.

A more general framework for the study of the system joint pdf \( f(\mathbf{x}) \) is to use the multivariate Gaussian mixture model. Here the system pdf is expressed as a weighted sum of multivariate Gaussian distributions as

\[
f(\mathbf{x}) = \sum_{k=1}^{c} \alpha_k g_k(\mathbf{x}, \Sigma_k, \mu_k)
\]

where \( \alpha_1, \ldots, \alpha_c \) are the \( c \) mixing proportions of the mixture model and they satisfy

\[
0 < \alpha_k < 1, \quad \text{for} \quad k = 1, \ldots, c, \quad \text{and} \quad \sum_{k=1}^{c} \alpha_k = 1
\]

and \( \mu_k \) and \( \Sigma_k \), \( k = 1, \ldots, c \), are respectively the mean and the covariance matrix of the \( k \)th, \( k = 1, \ldots, c \), multivariate normal density function \( g_k \):

\[
g_k(\mathbf{x}, \Sigma_k, \mu_k) = \frac{1}{(2\pi)^{d/2} |\Sigma_k|^{1/2}} \exp \left[ -\frac{1}{2} (\mathbf{x} - \mu_k)^T \Sigma_k^{-1} (\mathbf{x} - \mu_k) \right].
\]

Note that (2) results from the conservation of the pdf, obtained by integrating (1). Note also from (1) that \( \mu = \Sigma^{-1} \Sigma_k \alpha_k \mu_k \), where \( \mu \) is the pdf mean. In particular, when \( c = 2 \) and \( f(\mathbf{x}) \) is centered, we get \( \alpha_1 \mu_1 + \alpha_2 \mu_2 = 0 \). This mixture framework derives, in fact, from a well-known result, namely that any pdf can be approximated as closely as desired by a finite mixture of Gaussians (Anderson and Moore 1979). This is a very important result, which can also be used to test the system pdf against the null hypothesis of multinormality. Another advantage of this model is the fact that the parameters can be obtained using efficient algorithms based on the expectation–maximization (EM) principle (Everitt and Hand 1981; Titterington et al. 1985; McLachlan and Basford 1988). The only difficulty is the number \( c \) of components, which is addressed next.

b. Model identification

Given a set of \( n \) data points \( \mathbf{x}_1, \ldots, \mathbf{x}_n \), the model parameters in (1), for a given \( c \), are estimated by maximizing the log-likelihood \( L = \sum_{i=1}^{n} \ln(f(\mathbf{x}_i)) \) with respect to the \( [c(d + 1)(d + 2) - 2]/2 \) unknown parameters \( \mu_k \), and \( \Sigma_k \), \( k = 1, \ldots, c \), and \( \alpha_k \), \( k = 1, \ldots, c - 1 \). These parameters are obtained iteratively using the EM algorithm. In HO01, the number \( c \) of multivariate Gaussian components is obtained using a cross-validation procedure (Smyth et al. 1999) based on an “independent sample.” The choice of an independent
sample, however, is not unique and is controversial because of the difficulty in defining an independent sample and its size.\(^1\) In addition, the method suffers a systematic bias as the sample size increases (see, e.g., HO01 or Christiansen 2007 for details). The method proposed here finds the number of components for any given significance level and overcomes the previous drawbacks.

The method is based on finding uncertainties on the estimated parameters. One way to compute these uncertainties is to use information from second derivatives (Hessian) of the likelihood. This, however, defeats the very objective of the EM algorithm, which was introduced, indeed, to avoid direct maximization of the likelihood. Another major problem in finding uncertainties in this model is related to the difficulty in mapping the mixture components of one solution to the mixture components of another solution (of the same data). The simple solution presented here avoids this problem and looks for confidence intervals of the mixing proportions only. This is because the weights \(\alpha_1, \ldots, \alpha_c\) play a central role in the model since they represent explicitly prior probabilities of belonging to the associated Gaussian. If, for example, in a mixture of \(c\) Gaussians one proportion is not significant (at a given significance level), then the model can only support at most \(c - 1\) Gaussian components (at that significance level).

The method developed here is based on arguments from order statistics, as explained below. For a given number \(c\) of Gaussian components the EM algorithm provides, given the initial conditions, the model parameters. For the initial conditions the mixing proportions are randomly selected positive numbers satisfying (2), the centers of the components are randomly chosen from the state space, and the covariance matrices are identical and chosen to be diagonal. To quantify uncertainties on the mixing proportions one hundred such solutions are generated using different initial conditions and the mixing proportions sorted in increasing order. Suppose that the true model has (unknown) proportions\(^2\) that are sorted as \(\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_c\). Suppose now that \(\beta_1, \ldots, \beta_c\) is a realization of these \(c\) proportions obtained using the EM algorithm with a mixture of \(c\) components. Note that at this stage the remaining parameters are not used. Next, these proportions are sorted to yield \(\beta_{1 \leq c} \leq \beta_{2 \leq c} \leq \cdots \leq \beta_{c \leq c}\); that is, \(\beta_{1 \leq c}, \beta_{2 \leq c}, \ldots, \beta_{c \leq c}\) are respectively the smallest, the next smallest \(\ldots\), and the maximum of \(\beta_1, \beta_2, \ldots, \beta_c\). The principle of order statistics implies that the \(k\)th smallest number \(\beta_{k \leq c}\) is precisely a realization of \(\alpha_k\), \(k = 1, \ldots, c\).\(^3\) For details on order statistics see, for example, Wilcox (1997) and also Hannachi (2006) for further references and for an application to climate. Hence, from a generated ensemble of solutions, the mean \(m_k\) and standard deviation \(\sigma_k\) of the estimator of \(\alpha_k\), for each integer \(k\) \((1 \leq k \leq c)\), can be obtained. The interval \([m_k - \sigma_k \Phi^{-1}(1 - \alpha/2), m_k + \sigma_k \Phi^{-1}(1 - \alpha/2)]\) provides approximately the \(100(1 - \alpha)\%\) confidence interval of the mean of the estimator of \(\alpha_k\), where \(\Phi^{-1}(\cdots)\) is the inverse cumulative distribution function of the normal distribution. This is a rough approximation to the confidence intervals and an improvement to this approximation, which could take into account the positivity of the proportions (2), is left for future research. To find the right number of components we then start from \(c = 2\) and successively increase \(c\) by one until the added component becomes insignificant. The method is very successful and provides systematically the correct number of Gaussian components. The method is applied next to few stochastic dynamical systems, and later to NCEP–NCAR reanalyses.

3. Application to few low-order systems and data from a GCM

Two simple low-order systems are considered here, a three-well potential system and the classical Lorenz (1963) model. These models have been used in HO01 and are used here also with the new methodology because they provide a convenient test bed but do not necessarily reflect the dynamics of climate.

a. Three-well potential system

This is a two-dimensional system representing a stochastic particle moving in a three-well potential (Hannachi and Legras 1995). A similar system has been used by Hasselmann (1999) as a very simple paradigm to interpret climate change. The dynamic of the particle is given by

\[
dX(t) = -VW(X)dt + \epsilon dW_t
\]

where \(W_t\) is a two-dimensional random walk and \(\epsilon\) is a small parameter representing the amplitude of the random forcing. The potential well \(V(X)\) is given by

\[
V(X) = \sum_{i=1}^{3} \frac{1}{2} k_i (X_i - x_i)^2
\]

\(1\) The very existence of an independent sample is questionable. The usual way of defining independence based on decorrelation is not conceptually valid since independence and uncorrelation are two different concepts. Independence is stronger and is based on factorization of the joint pdf.

\(2\) In a Bayesian framework these unknown parameters are random variables.

\(3\) As an illustration, suppose that \(c = 2\) and we generate only two solutions. The first one provides, say, \(\beta_1 = 0.3\) and \(\beta_2 = 0.7\), then \(\beta_{1 \leq 2} = 0.3 \leq \beta_{2 \leq 2} = 0.7\). The second solution provides, say, \(\beta_1' = 0.85\) and \(\beta_2' = 0.15\), then \(\beta_{1 \leq 2}' = 0.15 \leq \beta_{2 \leq 2}' = 0.85\). Then \([0.3, 0.15]\) is a sample of \(\alpha_1\), and \([0.7, 0.85]\) is a sample of \(\alpha_2\).
\[ V(x, y) = \nu(x^2 + y^2) + \nu[(x - 2a)^2 + y^2] + a_0 \nu[(x - a)^2 + (y - a\sqrt{3})^2] + b \left[ (x - a)^2 + \left( y - \frac{a}{\sqrt{3}} \right)^2 \right]. \]

where \( \nu(x) = \kappa \exp[1/(x^2 - a^2)] \) if \( |x| < a \) and zero otherwise. The parameters \( \kappa, a, b, \) and \( \nu \) are fixed respectively to 0.05, 0.37, 0.12, and 21 (see HO01 for more details). The parameter \( a_0 \) is fixed to 1 here and will be used later to test the power of the method. A sample size of \( n = 5 \times 10^3 \) is generated using (4). This sample has been chosen so as to have approximately the same number of statistical degrees of freedom (dof) as the reanalyses data, which are analyzed in section 4. For a given number \( c \) of multivariate Gaussian components of the mixture model the EM algorithm has been applied as outlined in the previous section. Hence, in each case one hundred solutions are obtained. These solutions yield, after sorting and using order statistics, one hundred realizations of the (true) proportions \( \alpha_1 \leq \alpha_2 \ldots \leq \alpha_c \), which then provide confidence intervals on the mixing proportions.

When the number of the multivariate Gaussian components in the mixture is less than or equal to three, all components are found to be highly significant (not shown). However, when a four-component model is fitted, the fourth is found to be nonsignificant. The distribution of the mixing proportions for \( c = 4 \) is shown in Fig. 1a, and the corresponding 99% confidence intervals for the means are shown in Fig. 1b. The case of a three-multivariate Gaussian mixture is shown in Fig. 1c. Similar results to those shown in Figs. 1b,c are obtained at a lesser confidence level (e.g., 90% level) and at a higher confidence level (e.g., 99.9%). This is clear evidence that the data contain three components representing the three potential wells. Note from Fig. 1 that, when the number of components increases, the confidence intervals of the mixing proportions can change slightly. However, the strategy adopted here is that we start always with two components, then add an extra component only if the last regime (or component) is significant at a given level, and so on. In this way we avoid the ambiguity that can occur when regime \( K \), in \( K \)
components, is not significant, but it is for \( K + 1 \) components. Figure 1d shows a random subsample of size \( 10^3 \) from the potential system data along with the centers of the bivariate Gaussian components (bold thick dots) and their covariances (continuous circles). The squares show the theoretical centers, that is, the bottom of the wells. Figure 1d shows that the theoretical centers have been well identified. Note that the solution shown in Fig. 1d is chosen among the 100 generated solutions by selecting the one with maximum likelihood. The dashed lines in Fig. 1d show contour plots of the obtained mixture pdf where multimodality is clear in this simple system. The robustness of the method has also been tested by successively splitting the data into smaller trunks of sizes 2500, 1250, 625, 312, 156, and the method applied to these trunks. In all of them, the correct number of components was recovered at the three levels 10\%, 5\%, and 1\%.

The power of the methodology has been addressed by analyzing the rate of type-2 errors, that is, false negatives/misses, which correspond to when the null hypothesis is incorrectly not rejected when, in fact, it is false. The probability \( p_2 \) of making a type-2 error is a function of the significance level and the sample size. The power we proceed by progressively decreasing \( a_0 \) [see Eq. (4)], starting from 0.9, hence reducing the depth of one of the wells (the one centered at \( a \) and \( a\sqrt{3} \)). For each value of \( a_0 \) one hundred samples, each of size \( 10^3 \), are generated. The samples are chosen to have approximately the same decorrelation time as the original sample used above and also as the reanalyses. At each significance level, we compute the number 100\( p_2 \) of cases when the methodology fails to detect three components. Table 1 shows the power \( 1 - p_2 \) of the methodology as a function of \( a_0 \) and the significance level. The table shows that the methodology has, in general, high power. For \( a_0 \approx 0.2 \) the corresponding well is already very shallow and the associated cluster is virtually absent, yielding the slightly low power shown in Table 1.

### b. The Lorenz system

In the previous example the attractive centers of the unperturbed system are stable and the source of variation comes from the random shocks. In addition, the system is multimodal, which makes the search for the centers somehow simple. The next example is slightly more complex and more realistic in which the fixed points of the unperturbed system are unstable, in fact attractive in two directions and repulsive in the third orthogonal direction (metastable or saddle), rather like some situations in atmospheric flows. The example consists of the familiar Lorenz (1963) system with its two unstable (saddle) fixed points. The system is used here by adding a random forcing, as in the previous example, and the objective is to find the fixed points of the system. The equations are given by

\[
\begin{align*}
    d\mathbf{x} &= -10(x - y)dt + ed\mathbf{w}_{1,t} \\
    d\mathbf{y} &= (-xz + 20x - y)dt + ed\mathbf{w}_{2,t} \\
    d\mathbf{z} &= (xy - \frac{8}{3}z)dt + ed\mathbf{w}_{3,t}
\end{align*}
\]

The amplitude of the random shocks is fixed to \( e = 2 \), which is relatively strong, and the fine fractal structure of the (unforced) attractor is suppressed, but the underlying structure of the wings is still manifest. The system has been integrated forward in time to obtain a sample size as in the previous example with approximately the same number of dof and the data scaled to have zero mean and unit variance to avoid bad conditioning. The same procedure of the EM algorithm, as presented above, is applied here with various numbers of bivariate Gaussian components starting from two. Figure 2a shows the 99\% confidence limits on the mixing proportions for a two-component mixture model. The figure indicates that the system supports at least a two-component mixture model. Next, a third component is added and the 99\% confidence limits of the mixing proportions for the obtained three-component mixture model are shown in Fig. 2b. Clearly the number \( c = 2 \) stands out unambiguously. Like Fig. 1d, Fig. 2c shows the centers of the two-component mixture model along with a random subsample of size \( 10^3 \) from the data. Also shown are the covariance matrix isolines of each component (continuous ellipses), the contours of the mixture model pdf (dashed lines), and the theoretical saddle fixed points (squares). Again the method captures the centers well. Note in particular from Fig. 2c that the pdf of the system is unimodal. This is the

<table>
<thead>
<tr>
<th>( a_0 )</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>&gt;99%</td>
<td>&gt;99%</td>
<td>&gt;99%</td>
</tr>
<tr>
<td>0.8</td>
<td>&gt;99%</td>
<td>&gt;99%</td>
<td>&gt;99%</td>
</tr>
<tr>
<td>0.7</td>
<td>&gt;99%</td>
<td>&gt;99%</td>
<td>97%</td>
</tr>
<tr>
<td>0.6</td>
<td>&gt;99%</td>
<td>&gt;99%</td>
<td>98%</td>
</tr>
<tr>
<td>0.5</td>
<td>&gt;99%</td>
<td>99%</td>
<td>93%</td>
</tr>
<tr>
<td>0.4</td>
<td>&gt;99%</td>
<td>99%</td>
<td>87%</td>
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<tr>
<td>0.3</td>
<td>98%</td>
<td>97%</td>
<td>79%</td>
</tr>
<tr>
<td>0.2</td>
<td>96%</td>
<td>85%</td>
<td>67%</td>
</tr>
<tr>
<td>0.1</td>
<td>79%</td>
<td>73%</td>
<td>51%</td>
</tr>
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case for the unperturbed system and even more unimodal for the perturbed system given by (5), yet the system has two unstable foci. This points, in particular, to one of the main difficulties in regime identification through an argument of bimodality (see, e.g., Lorenz 2006). Also, as in Fig. 1d, similar results were obtained at different significance levels. For the three-well system the distance between the theoretical and experimental centers (Fig. 1) is less than 2% whereas for the Lorenz system, scaled to compare with the three-well system, it is around 8%. This difference is due to the fact that the centers of the potential system are stable or attractive, whereas the Lorenz fixed points are unstable (saddle).

The approach has been tested also with a simple bivariate first-order autoregressive (AR1) model. Two AR1 time series, $x_1$ and $x_2$, with respective lag-1 auto-correlations 0.6 and 0.65 have been generated. The red-noise time series have been transformed to yield cross-correlation using $y_1 = x_1/\sqrt{0.025}$ and $y_2 = x_2/\sqrt{0.025}$. Various sample sizes have been used, and the results are reported in Table 2 showing the obtained number of components for various confidence intervals.

c. Application to data from a GCM

Before embarking into the NCEP–NCAR reanalyses, the method is first applied to data from a more complex model with complexity near to that of the real atmosphere. The model is the U.K. Universities Global Atmospheric Modeling Program (UGAMP) GCM, described in Haines and Hannachi (1995). The data are generated from a 10-yr run in perpetual January conditions and consist of a 5-day low-pass Lanczos-filtered streamfunction field at the 500-hPa level. These data were also used in Haines and Hannachi (1995), and the reason they are used here also as a test bed is because low frequency variability in this GCM model was studied in Haines and Hannachi (1995) based on dynamical grounds. The nonlinear flow regimes were identified as quasi-stationary states of the dynamics based on minimizing the streamfunction flow tendency:

$$I = \int S \left( \frac{\partial \psi}{\partial t} \right)^2 dS$$  \hspace{1cm} (6)

over the Pacific sector. The flow tendency in (6) is based on the barotropic vorticity equation:

$$\frac{\partial \nabla^2 \psi}{\partial t} = -J(\psi, \zeta_a)$$  \hspace{1cm} (7)

where $\zeta_a = \nabla^2 \psi + 2\Omega \sin \phi$ is the absolute vorticity, $\Omega$ is the earth rotation rate, $\phi$ is the latitude; and the streamfunction $\psi$ is expressed using the empirical orthogonal functions (EOFs) state space as

$$\psi = \bar{\psi} + \sum_{i=1}^{d} a_i \psi_i,$$  \hspace{1cm} (8)

where $\bar{\psi}$ is the streamfunction climatology and $\psi_k, k = 1, \ldots, d$, are the streamfunction EOFs. The flow ten-

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4 In fact, bimodality of the Lorenz (1963) system shows up after a smoothing has been applied to reduce the effect of transitions near the unstable fixed point (the origin); see Palmer (1993).

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**Table 2.** Obtained number of components of the mixture model using a two-dimensional AR(1) model for various sample sizes and various confidence intervals.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
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<tbody>
<tr>
<td>100</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>250</td>
<td>1</td>
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<td>500</td>
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<td>1</td>
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</tr>
<tr>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1500</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2000</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
The logarithm of the flow tendency $I$, given by Eq. (6), of the streamfunction using (a) two EOFs and (b) three EOFs, and (c), (d) the dynamical quasi-stationary states minimizing the flow tendency using three EOFs and identified as $\pm$PNA; units of $I$ are $3.8 \times 10^{-21}$ m$^2$ s$^{-1}$. Note that (b) is a cross-section through the origin and the two minima, and the contour interval is chosen so that both minima appear simultaneously. Contour interval is $2.37 \times 10^{6}$ in (c) and $3.4 \times 10^{6}$ m$^2$ s$^{-1}$ in (d).

Haines and Hannachi (1995) found that, when the number $d$ of EOFs used in (8) increases, the minima get deeper. Similar behavior is also observed here using the sampled streamfunction trajectory, that is, PCs, and the mixture model. As the state space dimension increases, the uncertainties on the mixing proportions of the two states get smaller, but the third component remains insignificant (not shown). Figure 4 shows the solutions obtained from the two-component mixture model using three EOFs showing $\pm$PNA (Figs. 4c,d) similar to the dynamical QS states (Figs. 3c,d). The QS state 1 (Fig. 3c) is similar to regime 1 (Fig. 4c). The spatial structure of the QS state 2 (Fig. 3d) is, however, a little different from the experimental regime 2 (Fig. 4d). The distance between QS 1 and regime 1 is around 0.1, and that between QS 2 and regime 2 is around 0.3. One candidate responsible for this difference is the effect of high frequency eddies (Haines and Hannachi 1995). In fact, when these latter are taken into account in Eq. (7), the experimental regimes and the QS states become closer (Haines and Hannachi 1995). The other and most important reason is the nature of these QS states. It is shown in Hannachi (1997a), through a stability analysis using both the reduced EOF space and normal modes, that $+$PNA is more unstable than $-$PNA, rather like the situation for the Lorenz system and the three-well...
Note also, as pointed out in the end of section 2a, that the regime centers are colinear when there are two components.

4. Application to NCEP–NCAR geopotential heights

a. Winter 500-hPa height variability

The data used here consist of the NCEP–NCAR geopotential height field at the 500-hPa pressure level ($Z_{500}$). The data have a horizontal resolution of $2.5^\circ \times 2.5^\circ$ and span the period 1 January 1949–31 December 2003. The seasonal cycle is computed using the smoothed daily geopotential heights using a 9-day moving average filter. At each grid point, height anomalies are then computed as the departure of unfiltered daily values from the seasonal cycle. Winter daily anomalies, defined by the months December–January–February (starting 1 January 1949), north of 20°N, are then obtained and are used next for the analysis of planetary wave behavior. The obtained winter gridded data are then weighted by the square root of the cosine of the corresponding latitude and EOFs and PCs obtained. We emphasize here that the NH circulation data used are not low-pass filtered and may include synoptic scale. This has the potential to be compared to most previous studies where low-pass filtered data are used.

Figure 5 shows the spectrum of the covariance matrix, where only the first 40 eigenvalues, given in percentage of explained variances of the corresponding EOFs, are shown. The vertical bars show approximately 95% confidence limits, given by the rule of thumb of North et al. (1982). The two leading eigenvalues are degenerate. Figure 6 shows the leading two EOFs of the geopotential heights, and they look similar to those obtained, for example, by Crommelin (2004). EOF1 (Fig. 6a) has a wave pattern over the North Pacific, the North Atlantic sector, eastern North America, and western Europe. The strongest center of action of

![Figure 4](image_url)
EOF1 is sitting over southern Greenland. The pattern bears some similarities with the PNA pattern (Wallace and Gutzler 1981) except that the center of action over North America is stretched toward southern Greenland. The EOF2 pattern (Fig. 6b) has zonal and meridional structures with its strongest center of action sitting over the Aleutian Islands. The pattern has two positive centers sitting over the Aleutian Islands and southern Greenland, and two opposite centers over North America and northeastern Atlantic Ocean/western Europe. These two leading EOFs are mostly concentrated over the Western Hemisphere.

These two EOFs explain altogether about 18% of the total winter daily 500-hPa geopotential height variance. The next five EOFs are degenerate as are the following four EOFs (Fig. 7). Most of these EOFs have zonal structure. The strongest centers of action of these patterns are located over the North Pacific (EOF3, EOF8), western North Atlantic (EOF9, EOF10, and EOF11), eastern North Atlantic (EOF5, EOF6), eastern North Pacific (EOF7, EOF8), and Scandinavia (EOF5).

b. Planetary wave dynamics and mixture analysis

Figure 8 shows a scatterplot matrix of the daily 500-hPa height PCs within various two-dimensional state spaces using EOFs 1 to 5. Histograms of the individual PCs are also shown. This is a simple and informative way to explore all two-dimensional spaces within this five-dimensional state space. Some PCs show departure from normality (e.g., skewness in PC1 and PC2). There is also an indication of departure from binormality of some of the scatterplots, for example, the PC1/PC2 scatterplot. The skewness in, for example, the leading PCs is consistent with Sura et al. (2005). Here an alternative interpretation is presented. Planetary-scale flows are analyzed using the mixture model applied to various subspaces spanned by the large-scale 500-hPa height EOFs, and preferred structures are interpreted in terms of the mixture components.

A clustering analysis in probability space (Diggle 1983; Martinez and Martinez 2002; Stephenson et al. 2004) has been conducted and applied to the leading two PCs, as in Stephenson et al. (2004). The PC data have been transformed using the marginal cumulative pdfs of the PCs. The procedure then looks for clusters in the probability space based on the (interpoint) distance between pairs of points. The mean number of points that are within a given distance of a target point is then computed. The ratio of this number to that obtained from a uniform (no clustering) distribution...
yields a clustering index $L$. Figure 9 shows an example of a scatterplot of PC1/PC2 in the probability space for the first half of the record (Fig. 9a) and the second half (Fig. 9c) and the associated clustering indexes $L$ (Figs. 9b,d). The dashed curves represent, respectively, the upper and lower envelopes of the clustering indexes obtained using 25 samples simulated from a homogeneous Poisson random process known as complete spatial randomness (CSR; Martinez and Martinez 2002). There is clear indication of significant clustering (i.e., departure from CSR), particularly in the range $[0.025, 0.3]$ of interpoint distances (within the probability space). The clustering is particularly marked for the first half of the record (Fig. 9b) and less so for the second half (Fig. 9d). The test has also been applied to subsamples obtained by considering points that are a number of days apart. Clustering is found to decrease as the sample size decreases but was still observed for data that are 4–5 days apart for the whole record, particularly for the first half of the record but less so for the second half. For example, the slight clustering observed in the second half of the record (Fig. 9d) can still be seen when the data are 2, but not more, days apart.

The mixture model is now used as a way to quantify the departure of the PC1/PC2 scatterplot from bimodality, and also as a clustering algorithm within a probabilistic and inferential framework [see, e.g., Fraley and Raftery 1998, 2003; Everitt et al. 2001; Martinez and Martinez 2002 for further details and references]. The PCs are scaled to have unit variance prior to applying the mixture model. The mixing proportions for a three-Gaussian mixture model are shown in Fig. 10a, and the associated 99% confidence limits on the mixing proportion means in Fig. 10b. The mixing proportions for the two-Gaussian mixture model (not shown) produce two significant components at the 99% confidence level. Figure 10b shows clear evidence for the presence of two components. Robustness tests against changes in sample size have also been carried out as for the three-well potential system. The data have been resampled by

![Fig. 7.](image-url)
scrambling years, that is, 90-day blocks and also 45-day blocks. The approach is then applied to the first trunks of respective sizes 2475, 1237, 618, and 308, and they all produce two regimes at 10%, 5%, and 1% significance levels except for the last trunk (of size 308) where the two components are only significant at the 10% level. So at least 4 years of daily data are required to reproduce the results. Other two-dimensional scatterplots have also been investigated for nonbinormality, and the results show some nonuniformity. This nonuniformity can also be obtained by visual inspection of Fig. 8. At 99% confidence limits only (PC1, PC2) and (PC2, PC3) scatterplots each fit a two-bivariate Gaussian mixture model. In the (PC2, PC3) state space the two-component mixture model is significant also at the 99.9% confidence level. It can be seen in fact, from Fig. 8, that the highest density of the scatter is shifted toward the lower right corner in the (PC1, PC2) plane and toward the upper left corner in the (PC2, PC3) plane, that is, evidence for skewness or departure from bivariate normality in agreement with Sura et al. (2005). In all other scatterplots shown in Fig. 8 the mixture model supports only one single two-dimensional Gaussian component model. This also can be noted from these scatterplots where the highest density (dark) region is located in the middle of the scatter. The scatterplot of the (PC1, PC3) from the GCM of the previous section (not shown), where non-Gaussianity is manifest, is similar to the (PC1, PC2) scatterplot of the reanalyses (Fig. 8), showing similar skewness in their respective pdfs. Branstator and Berner (2005) also reported that the linear/nonlinear signature of their two-dimensional phase space tendencies from a long run of a GCM varies a great deal from plane to plane. They found that the projected dynamics in some planes is exclusively nonlinear, while in others it is nearly linear.

The centers of the mixture model in the (PC1, PC2) plane (Table 3) are shown in Fig. 10c along with a random subsample of size $10^3$ from the data, the covariance matrix of each component (continuous line), and contours of the obtained mixture model pdf (dashed lines). The associated preferred patterns are discussed later. Analyses performed on five consecutive 11-yr periods also indicate significant low frequency variability between these periods, as is observed in Christiansen (2005).

Extension of the mixture model to higher state space dimensions is straightforward conceptually. Technically, however, two impediments prevent this extension. The empty space phenomenon in high dimensional spaces requires a power increase of sample size with dimension. For instance, with the available sample size of daily geopotential heights, one can only use a maximum of three or four state space dimensions. The other difficulty is related to the degeneracy of the spectrum. The winter 500-hPa height spectrum (Fig. 5) shows, for example, that EOFs 3 to 7 (Fig. 7) are degenerate and, hence, if one decides to use three-dimensional state space, there is no reason why one should use the leading three EOFs. One could, for example, equally choose EOFs 1, 2, and 6. In the literature, either or both of these difficulties are not taken care of in general (e.g., Kimoto and Ghil 1993; Crommelin 2004).

To get away with these difficulties, rotated EOFs (REOFs) are used. Rotation of a set of $m$ EOFs $\mathbf{A}_m = (a_1, \ldots, a_m)^T$ (see, e.g., Richman 1986; Jolliffe 2002) is achieved by finding a rotation matrix $\mathbf{R}$ such that the rotated loadings:

$$\mathbf{B} = \mathbf{A}_m \mathbf{R}$$  \hspace{1cm} (9)

optimize a simplicity criterion. The rotation is orthogonal whenever $\mathbf{R}$ is orthogonal and oblique otherwise; see, for example, Hannachi et al. (2006) for properties of and alternatives to rotation. A VARIMAX orthogonal rotation (Kaiser 1958), which maximizes the functional

$$g(\mathbf{B}) = \sum_{k=1}^{m} \left[ \frac{1}{p} \sum_{j=1}^{p} b_{kj}^2 - \left( \frac{1}{p} \sum_{j=1}^{p} b_{kj} \right)^2 \right],$$  \hspace{1cm} (10)

where $b_{jk}, j = 1, \ldots, p$, and $k = 1, \ldots, m$ are the elements of the matrix $\mathbf{B}$ given in (9), has been applied here. When the REOF procedure is applied, the familiar and
useful geometric properties of EOFs, namely spatial orthogonality and temporal uncorrelation, are lost. To keep these properties, rotation has been applied only to the degenerate eigenvectors. In this case, for example, EOFs 3 to 7 are rotated using VARIMAX, then the variances of the associated time series, obtained by projecting the winter 500-hPa height anomalies onto these REOFs, are computed. Only the rotated EOF with the

![Fig. 9. Scatterplot of PC1/PC2 data within the probability space for (a) the first half of the record and (c) the second half and (b), (d) the associated clustering indexes L. The dashed curves represent respectively the upper and lower envelopes of 25 clustering index curves obtained using simulations from a homogeneous Poisson random process.](image1)

![Fig. 10. Distributions (a) of the mixture proportions for a three-component bivariate Gaussian mixture model, (b) the 99% confidence limits on the means of the previous three-component mixture proportions, and (c) the centers along with the covariances of the bivariate normals of a two-component mixture model and a random subsample of the data of size 10^3 using the leading two 500-hPa height PCs. The mixture pdf is also shown in (c) by the dashed line.](image2)
The largest variance is retained and labeled REOF3. The same procedure is applied to EOFs 8 to 11 and the retained pattern labeled REOF4. Now the set of patterns EOF1, EOF2, REOF3, and REOF4 are orthogonal and have their associated time series uncorrelated; that is, together they are similar to a subset of EOFs. The patterns of REOF3 and REOF4 are shown in Fig. 11. The structure of REOF3 is nearly zonal, with its main centers of action located over the North Pacific and western North American coast, and explains around 8% of the total winter variance. REOF4 has a strong center located over the Canadian west coast extending into the east Pacific with two smaller centers over northeastern Siberia and southern Greenland, and two opposite centers over North America/Aleutian and Canary Islands. This pattern explains around 7% of the total variance. Both rotated patterns are large scale but have a tendency to be localized over the Western Hemisphere. Note that not much variance is lost by using EOF1, EOF2, REOF3, and REOF4 instead of, for example, EOF1, EOF2, EOF3, and EOF4; the loss of variance is on the order of 1.25%.

The data anomalies are next projected onto the obtained rotated EOFs, that is, REOF3 and REOF4, to yield, respectively, RPC3 and RPC4 time series. The mixture model has been extended to three- and four-dimensional state spaces spanned, respectively, by (EOF1, EOF2, REOF3) and (EOF1, EOF2, REOF3, REOF4). In both cases a strong support for a two-component mixture model is obtained. For example, Fig. 12 shows the 99% confidence limits of the mixing proportions for a two- (Fig. 12a) and a three-component (Fig. 12b) model using the winter daily 500-hPa

Fig. 11. VARIMAX (a) REOF3 obtained by rotating the five EOFs, EOF3 to EOF7, and (b) REOF4 obtained by rotating the next four EOFs, EOF8 to EOF11.

<table>
<thead>
<tr>
<th>Proportion and centers of a mixture of two Gaussians</th>
<th>Leading two PCs</th>
<th>Leading (new) three PCs</th>
<th>Leading (new) four PCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949–2003</td>
<td>54%; 46%</td>
<td>53%; 46%</td>
<td>49%; 51%</td>
</tr>
<tr>
<td></td>
<td>(0.04, −0.59)</td>
<td>(−0.08, 0.50, −0.34)</td>
<td>(0.07, −0.58, 0.35, −0.06)</td>
</tr>
<tr>
<td></td>
<td>(−0.05, 0.69)</td>
<td>(0.09, −0.58, 0.39)</td>
<td>(−0.07, 0.55, −0.34, 0.06)</td>
</tr>
<tr>
<td>1949–78</td>
<td>45%; 55%</td>
<td>51%; 49%</td>
<td>48%; 52%</td>
</tr>
<tr>
<td></td>
<td>(−0.17, 0.80)</td>
<td>(0.13, −0.5, 0.16)</td>
<td>(0.12, −0.52, 0.2, 0.04)</td>
</tr>
<tr>
<td></td>
<td>(0.15, −0.49)</td>
<td>(−0.12, 0.71, −0.5)</td>
<td>(−0.11, 0.68, −0.50, 0.13)</td>
</tr>
<tr>
<td>1979–2003</td>
<td>48%; 52%</td>
<td>63%; 37%</td>
<td>34%; 67%</td>
</tr>
<tr>
<td></td>
<td>(0.27, 0.51)</td>
<td>(0.26, −0.39, 0.57)</td>
<td>(−0.59, 0.36, −0.45, 0.07)</td>
</tr>
<tr>
<td></td>
<td>(−0.26, −0.69)</td>
<td>(−0.44, 0.34, −0.44)</td>
<td>(0.29, −0.36, 0.51, −0.18)</td>
</tr>
</tbody>
</table>

Note, however, that these patterns (or their time series) are not orthogonal to (or uncorrelated with) the residual; there is no clear partition between this four-dimensional space and its complement.
height trajectory within the newly formed four-dimensional state space; see also Table 3. Figure 13 shows the anomaly patterns, that is, flow regimes, associated with the centers of a two-component mixture when PC1 and PC2 are used. These nearly opposite patterns are aligned mainly along PC2 (Fig. 10c) in the two-dimensional case. The solutions are quite robust to changes in the basis, and in three or four dimensions the patterns still look like those of Fig. 13 when either PC1, PC2, PC3, and PC4 or PC1, PC2, RPC3, and RPC4 are used.

The solutions have a strong projection onto ±PNA patterns on the Pacific side and a slightly weaker projection onto the negative/positive North Atlantic Oscillation (NAO) patterns on the Atlantic side, respectively. The first solution (Fig. 13a) projects particularly well onto the so-called cold-ocean/warm-land (COWL) pattern (Wallace et al. 1996; Corti et al. 1999) but with a slightly weaker contribution over the North Atlantic compared to the North Pacific. The COWL pattern describes much of the recent climate change of NH surface air temperature (Houghton et al. 1996; Wallace et al. 1996).

c. Climate change and regime shift

Christiansen (2003) analyzed stratospheric winter-averaged geopotential heights and found a regime shift in the last half of the 1970s from the weak vortex regime to the more zonal strong vortex regime resulting in a change of frequency of occupation of the regimes. We have attempted here to see whether one can observe such a shift in the high frequency part of planetary wave variability. The newly formed four winter PCs, that is, PC1, PC2, RPC3, and RPC4, have been split into two subsamples corresponding respectively to two periods: January 1949–February 1978 and December 1978–December 2003. Both subsets have been sub-

Fig. 13. Patterns of the regime centers obtained from a two-component mixture model using the leading two height PCs. Contour interval is 10 m.
mitted, separately, to the same analysis of the mixture model as in the previous section.

For both of the periods highly significant support for a two-multivariate Gaussian mixture is obtained with slightly stronger support for the first period than for the second period, in agreement with the results of the clustering test discussed in section 4b. For example, using the leading two PCs, the preferred structures for the first period are significant at the 0.1% level, whereas they are significant at only the 1% level for the second period. The mixing proportions and the centers of the corresponding mixtures are shown in Table 3. Figure 14 shows the scatterplot, the covariances of each component, and the mixture pdf for the two periods: January 1949–February 1978 (Fig. 14a) and December 1978–December 2003 (Fig. 14b). The filled small circles refer to the regime centers for the first period, whereas the squares refer to the last period. The centers for the whole period, that is, January 1949–December 2003, are indicated by small circles (see also Fig. 10c). The arrows shown in Fig. 14 indicate the shift from the regime centers of the first period to those of the second period.

Figure 14 shows a clear shift in preferred structures between the two periods. The axis of the regime centers is rotated by about 45° clockwise between the two periods. This has resulted in the axis of the regime centers corresponding to the whole period being nearly aligned with the EOF2 direction (Fig. 14; see also Corti et al. 1999). The observed skewness in PC2 (Stephenson et al. 2004) could be a result of this regime shift. Figure 15 shows the regime centers using the first three PCs, that is, (PC1, PC2, RPC3) for the first period (Figs. 15a,b) and the second period (Figs. 15c,d). The Aleutian low center in Fig. 15c is deeper than its counterpart in Fig. 15a, and similarly for the positive centers over North America. Both centers have shifted southwestward. For the second regime, the high center over southern Greenland disappeared during the second period, accompanied by a weakening of the low over the North Atlantic and a strengthening of the high over the southeastern United States.

To investigate the significance of the observed regime shift a Monte Carlo test is conducted to check whether the difference between the pre- and post-1978 regimes could be simply an artifact of sampling. One hundred random subsamples, each with approximately half the size of the entire record, are drawn from the winter 1949–2003 height PCs. To take seriality, and also possible autocorrelation of seasonal forcing into account, each subsample is composed of randomly drawn 90-day blocks. Solution from a two-component mixture model is then obtained for each subsample. The pre- and post-1978 regime centers are then compared to the distribution obtained from the clusters of the Monte Carlo centers. It is found that the regimes from the two periods, particularly those of the second period, are well outside the bulk of the Monte Carlo clusters (not shown). The centers of the first period are on the 85% confidence ellipse of the clusters. For the second period, however, the first regime, associated with a deepening of the Aleutian low (Fig. 15c), is nearly on the 95% confidence ellipse and the second regime, associated with a deepening of the Icelandic low (Fig. 15d), is
nearly on the 99% confidence ellipse. So the regime shift is highly significant and points mainly to changes occurring in the second half of the record. To test whether there is a change in the shape of the pdf, both subsamples have been rescaled to have zero mean and unit variance and changes in the pdf shape along various directions, in the (PC1, PC2) plane, are analyzed using a two-sample Kolmogorov–Smirnov test. A significant change in the shape is found along all directions, with EOF1 angles between 138° and 161° at the 5% level and between 141° and 154° (around the second diagonal in, e.g., Fig. 14a) at the 1% level. For example, along the direction 148°, the sample is significantly skewed in the first period (Fig. 16a) and with significant kurtosis in the second period (Fig. 16b). Although the changes in regimes are significant, the spatial correlations between the associated regimes are very high. For example, the correlations between the patterns of Figs. 15a,b and those of Figs. 15c,d are larger than 0.8. The same high correlations are also observed for different state space dimensions. There is also a change in the frequency of regime occupation with an increase (decrease) for the first (Fig. 15c) (second) (Fig. 15d) regime. Note that changes in regime occupation frequency are directly related to changes in the mixing proportions (Table 3). The regimes have therefore undergone some change in frequency of occurrence (Palmer 1999; Corti et al. 1999), also in amplitude but not much in regime phase.

Figure 17 shows the difference between the associated regimes corresponding to the first and second periods. The patterns of change (Fig. 17) reflect the effect...
of forcing involved in the observed regime shift. The first pattern (Fig. 17a) projects onto +PNA and shows a deepening in the Aleutian low system by about 40 m. The second pattern (Fig. 17b), with a low over southern Greenland and a high stretching from the eastern U.S. coast through to Europe, projects onto the positive phase of the NAO. It is known that the NAO has gone through a positive phase since around the mid-1970s, characterized by a deepening of the Icelandic low and an increase of the subtropical North Atlantic high (see, e.g., Hurrell et al. 2003). This trend toward the positive phase is characterized by a strengthening of westerly winds bringing stormy and rainy weather over Europe.

Changes in external forcing can also have an influence on regime shift. On the Pacific side, for example, an increase in ENSO activity has also been observed. El Niño events observed in the last quarter century, for example, the events of 1982/83 or 2001, are undoubtedly stronger than all previously observed events since records began. Since El Niño forces the positive phase of the PNA, the increasing warm ENSO events in the second period have contributed therefore to the observed increase in the PNA (Fig. 17a) (see, e.g., Hoerling et al. 2001; Livezey et al. 1997; Hannachi 2001).

Another possibility of external influence could be the observed changes in SST that occurred in the mid-

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**Fig. 16.** Histograms of PC1/PC2 along a direction making 148° with PC1 axis for (a) the first (scaled) subsample and (b) the second (scaled) subsample. The shapes of the distributions are significantly different at less than 1% level. The standard normal density function is also shown.

**Fig. 17.** Difference between the corresponding regime centers of Fig. 15, showing (a) the difference between Figs. 15c and 15a and (b) the difference between Figs. 15d and 15b.
1970s, when a shift was identified in fall and winter 1976–77. This change was caused by a marked cooling of SSTs in the central North Pacific and a warming of SSTs along the west coast of North America (Venrick et al. 1987; Ebbesmeyer et al. 1990; Trenberth and Hurrell 1994; Miller et al. 1994). This SST shift along with some instigation by forcing from the tropical Pacific could have been partially involved in the basin-scale deepening of the wintertime Aleutian low system (Graham 1994). The deepening of the Aleutian low, by about 4 hPa as a result of tropical SST forcing of the 1976/77 climate transition over the North Pacific, has been simulated recently by Deser and Phillips (2006) using a combined observational and modeling approach.

5. Summary and conclusions

Preferred flow structures in the high-frequency and large-scale flows are investigated using winter Northern Hemispheric 500-hPa NCEP–NCAR heights for the period January 1949–December 2003. The approach followed here is based on the mixture model (Haines and Hannachi 1995; Smyth et al. 1999; Hannachi and O’Neill 2001) whereby the probability distribution function of the atmospheric state is expressed as a mixture of a finite number of multivariate Gaussian distributions. The centers of the individual multivariate Gaussians are interpreted as the regime centers. The components of the mixture are obtained using the efficient expectation–maximization algorithm. Because all of the information regarding the significant components is contained in the mixing proportions, the number of components is obtained using order statistics applied to the mixing proportions of the mixture model. A large number of solutions of the mixture model is obtained, then the mixing proportions sorted and confidence intervals obtained. This method is efficient in finding the number of components and avoids all known problems encountered, for example in cross-validation such as the issues of sample size and independence (Hannachi and O’Neill 2001; Christiansen 2005).

The method has been successfully applied to a simple three-well potential system, the perturbed Lorenz system, and to the output of a GCM. Next, daily winter (December–February) 500-hPa geopotential heights from NCEP–NCAR (January 1949–December 2003) are used and EOFs/PCs computed. A clustering algorithm is applied to the leading two PCs. Significant clustering is found in the data. Clustering is found to decrease when seriality is taken into account, but the data remain clustered, particularly for the first half of the record and less so for the second half. The mixture model is then used to identify the clusters. The model has been applied to the leading two, three, and four PCs where the third and fourth PCs are obtained using rotation of EOFs 3 to 7 and EOFs 8 to 11, respectively. This rotation procedure has been chosen because of the degeneracy of the eigenvalue spectra. The method shows a strong support for a mixture of two multivariate Gaussians. The associated preferred regimes have structures over the North Pacific and the North Atlantic sectors. The preferred large-scale structures show a +PNA/−NAO pattern and its opposite. This behavior is not observed in all EOF subspaces, and there are subspaces in which the distribution is multinormal.

To investigate possible changes in preferred large-scale structures the PCs have been divided into two subperiods: pre- and post-1978. For both periods two highly significant regimes are again observed with a slight support for the first period. In addition, a significant regime shift between the two periods is obtained along with a significant change in the shape of the distribution. For the regime shift, the main changes are in amplitude, but not much in the structure (phase) of the regimes. The pattern of regime difference between the associated regimes of the two periods are the +PNA, associated with a deepening of the Aleutian low by about 40 m, and the positive phase of the NAO, associated with a deepening of the Icelandic low by about the same amount. The patterns associated with the regime shift are consistent with 1) the positive phase shift of the NAO, and the increase in positive ENSO events during the last quarter of the century, and 2) the SST shift observed in the North Pacific around the mid-1970s.

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