Effects of Nonlinearity on Convectively Forced Internal Gravity Waves: Application to a Gravity Wave Drag Parameterization

HYE-YEONG CHUN, HYUN-JOO CHOI, AND IN-SUN SONG*

Department of Atmospheric Sciences, Yonsei University, Seoul, South Korea

(Manuscript received 21 July 2006, in final form 14 May 2007)

ABSTRACT

In the present study, the authors propose a way to include a nonlinear forcing effect on the momentum flux spectrum of convectively forced internal gravity waves using a nondimensional numerical model (NDM) in a two-dimensional framework. In NDM, the nonlinear forcing is represented by nonlinear advection terms multiplied by the nonlinearity factor (NF) of the thermally induced internal gravity waves for a given specified diabatic forcing. It was found that the magnitudes of the waves and resultant momentum flux above the specified forcing decrease with increasing NF due to cancellation between the two forcing mechanisms. Using the momentum flux spectrum obtained by the NDM simulations with various NFs, a scale factor for the momentum flux, normalized by the momentum flux induced by diabatic forcing alone, is formulated as a function of NF. Inclusion of the nonlinear forcing effect into current convective gravity wave drag (GWD) parameterizations, which consider diabatic forcing alone by multiplying the cloud-top momentum flux spectrum by the scale factor, is proposed. An updated convective GWD parameterization using the scale factor is implemented into the NCAR Whole Atmosphere Community Climate Model (WACCM). The 10-yr simulation results, compared with those by the original convective GWD parameterization considering diabatic forcing alone, showed that the magnitude of the zonal-mean cloud-top momentum flux is reduced for wide range of phase speed spectrum by about 10%, except in the middle latitude storm-track regions where the cloud-top momentum flux is amplified. The zonal drag forcing is determined largely by the wave propagation condition under the reduced magnitude of the cloud-top momentum flux, and its magnitude decreases in many regions, but there are several areas of increasing drag forcing, especially in the tropical upper mesosphere and lower thermosphere.

1. Introduction

Vertically propagating gravity waves generated by various sources in the troposphere have profound effects on the large-scale circulation in the middle atmosphere where waves are broken, filtered at their critical level, or dissipated by eddy viscosity during their propagation (Lindzen 1981; Matsuno 1982). Among the various sources, cumulus convection is one of the major sources of nonstationary gravity waves with a wide phase speed spectrum. In the tropics, convectively forced gravity waves can contribute to momentum forcing required to drive the quasi-biennial oscillation and semiannual oscillation (Alexander and Holton 1997; Sassi and Garcia 1997).

There are several numerical modeling studies of convectively forced gravity waves and their generation mechanisms. Pandya and Alexander (1999) showed that the spectral characteristics of convective gravity waves in a quasi-linear simulation forced by diabatic forcing alone is similar to that in a fully nonlinear simulation, although the amplitude of waves is much larger than that in the fully nonlinear simulation. Lane et al. (2001) showed that convective gravity waves are generated by two forcings, diabatic and nonlinear forcing, and demonstrated the importance of the nonlinear forcing mechanism in the generation of convective gravity waves by comparing the magnitude of the nonlinear and diabatic forcing of a simulated storm. Song et al. (2003, hereafter SCL) performed quasi-linear simulations of gravity waves generated by the diabatic forcing and nonlinear forcing separately and found that the
characteristics of the gravity waves induced by either of the forcings are determined by the effective forcing that is filtered by the vertical propagation condition of gravity waves in the spectral domain. The effective diabatic forcing and effective nonlinear forcing are comparable and largely out of phase with each other, suggesting the importance of both forcing mechanisms in generation of convective gravity waves.

To understand the implications of SCL on convective gravity parameterization, Chun et al. (2005) calculated the momentum flux spectrum of convective gravity waves in the stratosphere using SCL's numerical simulations. They showed that the momentum flux induced by either forcing differs significantly from each other as well as from the momentum flux obtained from the control simulation including both forcings. This is because cancellation of momentum flux by cross-correlation terms between the two forcings cannot be represented in the momentum flux by a single forcing. Based on this result, Chun et al. suggested the inclusion of nonlinear forcing in recently developing convective gravity wave drag (GWD) parameterizations with a diabatic forcing alone, such as Beres (2004) and Song and Chun (2005, hereafter SC05), although those parameterizations are still much more realistic than the spectral gravity wave drag parameterization used in current large-scale models that have a prescribed source spectrum.

A most straightforward method that includes the nonlinear forcing effect on the momentum flux spectrum is likely to obtain an analytical solution of gravity waves induced by both forcings. However, one of the problems, aside from the additional effort required for its mathematical treatment, is that compared with the diabatic forcing we do not know much about the structure of nonlinear forcing and its relationship with the diabatic forcing, which might be strongly coupled with each other. Given this situation, one feasible way to include the nonlinear forcing effect on the momentum flux of convective gravity waves is to solve weakly nonlinear perturbations forced by diabatic forcing, as was done by Chun and Baik (1994) in a uniform background wind and stability condition and by Chun (1997) that included background wind shear. In weakly nonlinear systems the zeroth-order solution represents waves induced by diabatic forcing alone, while the first-order solution represents waves induced by nonlinear forcing (nonlinear advections of the zeroth-order momentum and heat fluxes).

There are, however, several difficulties in this approach. First, obtaining an analytical formulation of momentum flux [the sum of at least three terms, \( u_0w_0 \), \( \mu u_1w_0 \), and \( \mu u_1^2w_0 \), where \( u_0, w_0 (u_1, w_1) \) are the zeroth (first)-order perturbation zonal and vertical velocities and \( \mu \) is the nonlinearity factor (NF) of the thermally induced gravity waves, respectively] is very tedious mathematically, especially under a relatively realistic background condition such as the three-layer structure considered in SC05. For some terms in the first-order solution, numerical calculation might be involved (Chun 1997). Second, and perhaps more importantly, the weakly nonlinear solution may not apply directly to the highly nonlinear flow regime such as mesoscale convective storms considered in the present study.

To overcome these analytical difficulties, while still obtaining some insight into the influence of nonlinear forcing on the momentum flux of convective gravity waves in a simple dynamic system, in the present study we use a nondimensional numerical model (NDM) in a two-dimensional framework, which is similar to that used in Baik and Chun (1996, hereafter BC96). The NDM simulates gravity waves forced by a specified diabatic forcing according to a nonlinearity factor of the thermally induced gravity waves. In NDM, nonlinear advection can be considered as the nonlinear forcing, of which its magnitude depends largely on the nonlinearity factor.

This paper is organized as follows: section 2 describes the model hierarchy. In section 3, perturbations induced by a specified diabatic forcing under various nonlinearity factors are presented. In section 4, the cloud-top momentum flux spectrum is calculated and compared with that derived from diabatic forcing alone. Based on this result, we determine a scale factor for the momentum flux normalized by the momentum flux induced by the diabatic forcing alone. The scale factor is then included in the cloud-top momentum flux formulation by SC05, and impact of the new convective GWD parameterization in a GCM is examined in section 5. Summary and discussion are given in the final section.

2. Nondimensional numerical model

The equations governing perturbations in a two-dimensional, hydrostatic, nonrotating, Boussinesq airflow system forced by diabatic forcing can be written as

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = - \frac{\partial \pi}{\partial x},
\]

(1)

\[
\frac{\partial b}{\partial t} + U \frac{\partial b}{\partial x} + N^2w + u \frac{\partial b}{\partial x} + w \frac{\partial b}{\partial z} = \frac{gQ}{\varepsilon_p T_0},
\]

(2)

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,
\]

(3)

\[
\frac{\partial \pi}{\partial z} = b.
\]

(4)
Here \( u \) and \( w \) are the perturbation zonal and vertical wind velocities, \( \pi \) is the perturbation kinematic pressure, \( b \) is the perturbation buoyancy, \( U \) is the basic-state wind, \( N \) is the Brunt–Väisälä frequency, \( c_p \) is the specific heat of air at constant pressure, \( T_0 \) is a constant reference temperature, \( g \) is the gravitational acceleration, and \( Q \) is the diabatic forcing. For direct comparison with SC05’s linear analytical calculation of the momentum flux spectrum, we consider the basic-state motion, and \( Q \) for \( \text{diabatic forcing as in SC05 (see Fig. 1 of SC05):} \)

\[
U(z) = \begin{cases} 
U_0 + \alpha z & \text{for } 0 \leq z \leq z_s, \\
U_t & \text{for } z > z_s,
\end{cases}
\]

(5)

\[
N(z) = \begin{cases} 
N_1 & \text{for } 0 \leq z \leq z_t, \\
N_2 & \text{for } z > z_t.
\end{cases}
\]

(6)

\[
Q(x, z, t) = q(x, t)s(z),
\]

(7)

\[
q(x, z) = Q_0 \exp \left\{ -\frac{(x-x_0-c_p(t-t_0))^2}{(\delta_x/2)^2} \right\}
\]

\[
\times \exp \left\{ -\frac{(t-t_0)^2}{(\delta_t/2)^2} \right\},
\]

(8)

\[
s(z) = \begin{cases} 
1 - \frac{(z-z_m)/z_d}{2} & \text{for } z_b \leq z \leq z_t, \\
0 & \text{elsewhere}.
\end{cases}
\]

(9)

Here \( U_0 \) is the basic-state wind at the surface, \( z_b \) and \( z_t \) are the bottom and top heights of the specified diabatic forcing respectively, \( z_s \) is the top of the shear layer, \( U_t \) is the background wind above \( z_s \), \( N_1 \) and \( N_2 \) are the buoyancy frequencies below and above \( z_s \) respectively, \( \alpha \) is the wind shear \([=(U_t - U_0)/z_s]\), \( Q_0 \) is the magnitude of the diabatic heating, \( x_0 \) and \( t_0 \) are the center of the diabatic forcing in the zonal direction and time for the maximum forcing respectively, \( c_p \) is the moving speed, \( \delta_x \) and \( \delta_t \) are the horizontal and temporal scales of the diabatic forcing, respectively, \( z_m = (z_s + z_b)/2 \), and \( z_d = (z_s - z_b)/2 \).

As suggested by BC96, the following nondimensional variables are introduced:

\[
t = \frac{L}{U_c}, \quad x = L \hat{x}, \quad z = \frac{U_c}{N_c} \hat{z}, \quad U = U_c \hat{U},
\]

\[
N = N_c \hat{N}, \quad \alpha = N_c \hat{\alpha},
\]

\[
u = \frac{gQ_0L}{c_p T_0 N_c^2}, \quad w = \frac{gQ_0}{c_p T_0 N_c^2}, \quad \pi = \frac{gQ_0}{c_p T_0 N_c},
\]

\[
b = \frac{gQ_0}{c_p T_0 N_c}, \quad Q = \frac{Q_0}{\hat{Q}}.
\]

(10)

Here \( L, U_c \), and \( N_c \) are the characteristic scales of the horizontal length, basic-state wind, and stability, respectively, and the caret quantities are dimensionless. The other variables in (1)–(9) related to height \((z_p, z_r, z_s, z_m, \text{and } z_d)\), horizontal \((\delta x \text{ and } x_0)\) and time \((\delta t \text{ and } t_0)\) scales, wind \((U_0, U_t, \text{and } c_p)\), and stability \((N_1 \text{ and } N_2)\) are nondimensionalized by using the corresponding characteristic scales in (10). Substituting (10) into (1)–(9) yields (with all the caret quantities dropped hereafter) the following nondimensional governing equations:

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} + \mu \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \right) = \frac{\partial \pi}{\partial x},
\]

(11)

\[
\frac{\partial b}{\partial t} + U \frac{\partial b}{\partial x} + N^2 w + \mu \left( \frac{\partial b}{\partial x} + \frac{\partial b}{\partial z} \right) = Q,
\]

(12)

\[
\frac{\partial u}{\partial x} + \frac{\partial b}{\partial z} = 0,
\]

(13)

\[
\frac{\partial \pi}{\partial z} = b.
\]

(14)

Here \( \mu \) is the nonlinearity factor of the thermally induced gravity waves defined by Lin and Chun (1991) as

\[
\mu = \frac{gQ_0 L}{c_p T_0 N_c U_c^2}.
\]

(15)

The nonlinearity factor can be interpreted as a scale ratio of the perturbation horizontal wind to the basic-state wind. The nonlinearity factor is proportional to the heating rate and the horizontal scale of heating, while it is inversely proportional to the square of the basic-state wind speed and the Brunt–Väisälä frequency. For cases with stronger inflow in the convective forcing region, convective cells may blow downstream rather quickly before they generate gravity waves, and the amplitude of perturbations will be smaller. When the stability is smaller in the region where diabatic forcing exists, stronger perturbations are expected, as usually seen from the numerical simulations of mesoscale circulations associated with convection. The dependence of the basic-state wind and stability as well as diabatic heating on thermally induced internal gravity waves has been demonstrated clearly in previous analytical studies (e.g., Smith and Lin 1982; Lin and Smith 1986; Lin and Chun 1991). The nonlinearity factor is also a key parameter in the convective GWD parameterization scheme proposed by Chun and Baik (1998).

Note that calculating the nonlinearity factor for real or numerically simulated storms is not straightforward, especially under vertically various basic-state wind and stability conditions, since there are multiple convective cells with different horizontal scales. For the SCL case, one can estimate \( \mu \sim 2 \) with \( L = 50 \text{ km} \) (the dominant
horizontal wavelength of diabatic forcing as shown in Fig. 5b of SCL), \( Q_0 = 3.63 \text{ J kg}^{-1} \text{s}^{-1} \) (diabatic heating averaged over 50 km in the major convection region), \( U_c = 17.8 \text{ m s}^{-1}, \ N_c = 0.01 \text{ s}^{-1}, \) and \( T_0 = 273 \text{ K}. \) Here \( U_c \) is estimated by \( U_{m} - c_m, \) where \( U_{m} (2 \text{ m s}^{-1}) \) is the basic-state wind in the middle region of convection \((z = 6 \text{ km})\) and \( c_m (-15.8 \text{ m s}^{-1}) \) is the mean speed of convective cells. Even if we consider a single convective cell with a smaller horizontal scale (say \( L = 5 \text{ km} \)), the nonlinearity factor will not be changed significantly, given that the heating rate averaged over a smaller horizontal domain will be larger. The nonlinearity factor is sensitive to the magnitude of the basic-state wind and stability for a given diabatic heating structure. One can calculate the nonlinearity factor using the basic-state variables at the cloud top, as in the convective GWD parameterization by Chun and Baik (1998).

However, for the SCL case, the basic-state wind is constant with height above \( z = 6 \text{ km} \) and the basic-state stability is almost constant in the troposphere. Consequently, NF evaluated at the cloud top is the same as that in the middle region of convection.

The basic-state wind, stability, and diabatic forcing in \((5)-(9)\) are nondimensionalized with exactly the same forms. Note that \((11)-(14)\) are identical to those of the NDM used in BC96 except that the Rayleigh friction and Newtonian cooling terms are excluded in \((11)\) and \((12)\), respectively. Because an inviscid flow is considered in the current NDM, the linear part of the governing equations \((11)-(14)\) is identical to the governing equation set of SC05. Therefore, the current NDM results, compared with SC05’s linear solution, can isolate nonlinearity effects of the thermally induced internal gravity waves. Equations \((11)-(14)\) are solved numerically using a finite-difference method, where the time derivative is calculated using a leapfrog scheme, and a fourth-order compact implicit scheme (Navon and Riphagen 1979) and a centered difference scheme are used for the first derivative terms with respect to \( x \) and \( z \), respectively. A flat bottom boundary condition \((w = 0 \text{ at } z = 0)\) and the upper radiation condition as proposed by Klemp and Durran (1983) are imposed. At the lateral boundary, the radiation condition proposed by Betz and Mittra (1992) is imposed, which allows waves to propagate at the lateral boundary more efficiently, at least in the present simulations, compared with the radiation condition of Orlanski (1976) used in BC96. The Asselin (1972) time filter and the space smoothing of the fourth-order type (Perkey 1976) also are applied.

The model domain is 200 wide and 20 deep with a horizontal and vertical grid size of 0.1. The model is integrated up to 25.92 with a time interval of 0.0018. For direct comparison with SC05’s analytically calculated momentum flux spectrum we choose \( L = 5 \text{ km}, \ U_c = 18 \text{ m s}^{-1}, \) and \( N_c = 0.01 \text{ s}^{-1}. \) For the given characteristic scales, the dimensional model domain is \( 1000 \text{ km} \times 36 \text{ km} \) with horizontal and vertical grid sizes of 500 and 180 m, respectively. In the present simulations, we set \( z_b = 1.53, \ z_p = 5.83, \ z_f = 3.33, \ U_b = -1, \ U_t = 0.11, \ \delta x = 1, \ \delta t = 4.32, \ c_q = -1.1, \ x_0 = 100, \ g_0 = 12.96, \) and \( N_1 = N_2 = 1. \) Note that a vertically uniform Brunt–Väisälä frequency was assumed as in SC05 since the top of clouds is far below the tropopause in SCL. For the given diabatic forcing structure, the nonlinearity factor is varied through the range \( NF = 0-5 \) between the different experiments, and the influence of the nonlinearity on the momentum flux spectrum above the diabatic forcing is investigated.

In the present NDM simulations, nonlinear advection terms are retained from the surface to model top. Therefore, two factors associated with the nonlinearity affect the momentum flux above the forcing region: nonlinear forcing in the troposphere and nonlinearity above the forcing region. Note that in the present study we call the nonlinear advection below and above the top of the specified diabatic forcing as “nonlinear forcing” and “nonlinearity,” respectively, because the nonlinear advection in the region of the specified diabatic forcing is largely determined by the forcing itself, while nonlinear advection above the forcing represents purely a nonlinearity of wave disturbance. The latter factor is not likely to be significant in the momentum flux spectrum above the forcing region, as shown in SCL (their Fig. 8c), except for extremely high values of NF. (This is examined through additional simulations, discussed in detail in section 4, in which nonlinear forcing is limited to the height of the specified diabatic heating.) Therefore, nonlinear forcing and nonlinearity are used interchangeably in the present paper.

3. Nondimensional model results

Figure 1 shows the vertical velocity field from the original NDM simulation (CTL hereafter) with respect to NF at \( t = 12.96, \) the time for the maximum forcing. The region for the specified diabatic forcing defined in \((7)-(9)\) is overlaid with thick gray line. It is clear that as NF increases, the magnitude of the vertical velocity above the forcing region decreases, while the maximum updraft in the forcing region increases slightly. At this time, the maximum updraft exists near the center \((x_0 = 100)\) of the maximum forcing but at different heights with respect to NF, slightly below the maximum heating height \((z_m = 3.68)\) for \( NF = 0 \) \((z = 3.4)\) and above it for other NFs ranging from \( z = 3.7 \) to 4.5. As NF increases, an eastward-tilted updraft below the forcing top ex-
tends upward with a relatively smaller horizontal size and larger magnitude. On the other hand, the wave amplitude above the forcing deceases as NF increases.

The dependency of the magnitude of waves on the nonlinearity factor can be explained by Fig. 2, which shows the difference of the vertical velocity between each nonlinear case and the linear (NF = 0) case. The difference field represents the impact of nonlinearity on the vertical velocity for a given diabatic heating. Shading in Fig. 2 represents the area where the vertical velocity induced by diabatic forcing alone \( (w_{NF=0}) \) is out of phase with that induced by nonlinear forcing alone \( (w_{NF} - w_{NF=0}) \). When the nonlinearity is included, downdraft on the upstream and updraft on the down-stream of the specified heating are induced in the forcing region, and their magnitudes increase as NF increases (Figs. 2b–f). Note that the upstream is east (west) of the heating center below (above) \( z = 3 \), given that the basic-state wind blows from east to west below \( z = 3 \) and changes its direction above. In most regions, in particular where nonlinearity-induced perturbations are strong, vertical velocity perturbations induced by the nonlinear forcing are out of phase with those induced by diabatic forcing alone. The only exception occurs in the region where the two centers of the maximum positive difference connect (from the surface to about \( z = 6 \), tilted eastward). As NF increases, the area of this positive phase shrinks, while the magnitude of

---

**Fig. 1.** The perturbation vertical velocity fields for the nonlinearity factors of (a) 0, (b) 1, (c) 2, (d) 3, (e) 4, and (f) 5 at a time step \( t = 12.96 \). The contour interval is 0.05. The thick solid box denotes the area of diabatic forcing specified by (7)–(9).
the positive difference between the two fields increases. Because of this nonlinearity-induced pattern of the vertical velocity in the specified heating region, the location of the maximum updraft shifts upward and eastward as NF increases (Fig. 1). Above the forcing region, the magnitude of the nonlinearity-induced upward propagating waves also increases as NF increases. However, since those nonlinearity-induced waves (Figs. 2b–f) are largely out of phase with the waves induced by diabatic forcing alone (Fig. 2a), the magnitude of waves above the forcing top is smaller than that induced by diabatic forcing alone, as shown in Fig. 1.

Note that the present result for vertical velocity with respect to NF is significantly different from that reported in BC96 based on a similar NDM. In BC96, the magnitude of the vertical velocity is about 5 times larger than in the current simulation, and the domain-maximum vertical velocity shows a periodic oscillation for NF > 3, with multiple moving updraft and downdraft cells that propagate downstream of the specified heating. The differences between the present results and BC96 likely are due to the different background condition and forcing structure imposed, along with a minor contribution from use of different numerical schemes. BC96 imposed a uniform basic-state wind and stability condition along with vertically uniform (from the surface to \(z = 2\)) and stationary heating. It remains for future research to determine the response of a sta-
bly stratified nonlinear flow to specified heating under different basic-state conditions and imposed forcing.

To understand the structure of the nonlinear forcing with respect to the diabatic forcing, we calculate the nonlinear and diabatic sources of internal gravity waves, defined per SCL [see Eq. (16) of SCL], in nondimensional form:

\[
- \frac{\partial^2}{\partial x \partial z} \left( \frac{\partial F}{\partial t} \right) + \frac{\partial^2 F_u}{\partial x^2} + \frac{\partial^2 F_b}{\partial z^2} + \frac{\partial^2 Q}{\partial x^2},
\]

(16)

where,

\[
F_u = -\mu \left[ \frac{\partial}{\partial x} (u^2 u) + \frac{\partial}{\partial z} (uw) \right],
\]

(18)

\[
F_b = -\mu \left[ \frac{\partial}{\partial x} (bu) + \frac{\partial}{\partial z} (b^2 u) \right].
\]

(19)

Here \( \frac{\partial}{\partial t} \) is the simplified form under hydrostatic approximation, as considered in the present NDM.

Figure 3 shows the nonlinear source of (16) superimposed on the diabatic source of (17) for different NFs at \( t = 12.96 \). Note that in the present NDM system, the diabatic source is fixed while the nonlinear source can be changed according to NF, directly by the NF value itself and indirectly through changed magnitude of perturbations resulting from different NF values. The diabatic source (Fig. 3a) has its minimum value near the center of the specified forcing and is positive on the upstream and downstream edges of the forcing, due to the second-order derivative of the Gaussian-type heating structure given by (8). As NF increases, a positive nonlinear source is induced at the center of the negative diabatic source, while a negative nonlinear source is induced near the boundaries of positive and negative diabatic sources. As a result, cancellation between the diabatic and nonlinear sources occurs in the center of the specified heating, while the diabatic source located at the bottom right and top left of the negative region is enhanced by the negative nonlinear source there. It is noteworthy that not all sources shown in Fig. 3 can be used to generate gravity waves that propagate above forcing region, but the effective forcing that satisfies the vertical propagation condition of internal gravity waves in the spectral domain \( k - \omega \) for a given basic-state wind and stability condition (Song et al. 2003) can. Further discussions about this issue will be given in the next section.

The nonlinear source has a wave-like structure, as expected, since it originates from the nonlinear advection of the waves induced by diabatic forcing, and it has smaller horizontal and vertical scales than those of the diabatic source. The structure of the nonlinear source shown in Fig. 3 is very complicated, even for a single convective cell with a Gaussian-type spatial and temporal structure, as considered in the present study. For more realistic situations with multiple convective cells with different temporal and spatial scales, nonlinear forcing is likely to be much more complicated than the present result. This is one of the factors making any analytical approach to formulate a momentum flux spectrum by including nonlinear forcing technically difficult.

4. Momentum flux spectrum

Figure 4 shows the gravity wave momentum flux above forcing (at \( z = 10 \)) with respect to zonal phase speed \( (c_p) \) for different NFs. It is clear that the magnitude of the momentum flux above the convective forcing decreases gradually as NF increases. This is particularly pronounced at phase speeds from \(-0.4 \) to \(-1.6 \), where the maximum magnitude of momentum flux exists for each NF case. The maximum magnitude of momentum flux for NF = 5 is one order of magnitude smaller than that induced by diabatic forcing alone (NF = 0). Besides the magnitude, the phase speed at which the maximum magnitude of momentum flux occurs shifts slightly toward the right (a slightly less negative value) with an increase in NF. For westward propagating components with larger phase speeds \( (c_p < -1.6) \) and for eastward propagating components, the magnitude of the momentum flux decreases with NF but without significant difference. This implies that the magnitude change of the momentum flux by including the nonlinear forcing is phase speed dependent.

To get some insight into how each forcing mechanism contributes to the momentum flux spectrum above the convective forcing, as shown in Fig. 4, additional numerical simulation is conducted with nonlinear forcing alone, which is similar to the dry simulation forced by nonlinear forcing (DRYMH) in SCL. The nonlinear forcing by (18) and (19), obtained in the CTL simulation from the surface to the top of diabatic forcing, is saved at intervals of 0.036 (10 s in the time dimension) and is then included on the right side of the linearized version of the NDM equations without specified diabatic forcing \( Q \). Note that the CTL simulation with NF = 0 is equivalent to the linear simulation with diabatic forcing alone, similar to the dry simulation forced by diabatic forcing (DRYQ) in SCL. Figure 5 shows the momentum fluxes induced by the CTL simulation and linear simulations by diabatic forcing alone \((u, w, f)\) and
nonlinear forcing \((u_1 w_1)\) alone, cross-correlation momentum fluxes between the two forcings \((u_0 w_1\) and \(u_1 w_0)\), and momentum flux induced by the sum of the diabatic and nonlinear forcings \((u_0 + u_1)(w_0 + w_1)\) for different NFs. As NF increases, the momentum flux by nonlinear forcing increases with the same sign as the momentum flux by diabatic forcing alone. This is somewhat expected, given that the magnitude of momentum flux by any single source is proportional to the square of vertical velocity that must be proportional to forcing magnitude, in general, as shown analytically by Chun et al. (2005). Nonetheless, the momentum fluxes of CTL

Fig. 3. The diabatic (solid and dashed lines) and nonlinear (shading) sources of internal gravity waves for different NFs of (a) 0, (b) 1, (c) 2, (d) 3, (e) 4, and (f) 5 at a time step \(t = 12.96\), calculated by (17) and (16), respectively. Solid (dashed) lines denote positive (negative) values in the diabatic forcing. Contour intervals of diabatic and nonlinear forcing are 1 and 2, respectively.
and \((u_0 + u_1)(w_0 + w_1)\) decrease as NF increases. This is because cross-correlation momentum fluxes with a sign opposite to the momentum fluxes by the single sources increase more rapidly with NF and, consequently, the resultant magnitude of total momentum flux becomes smaller as NF increases. The present NDM simulation, which isolates specified diabatic forcing and nonlinear forcing associated in a simple dynamical framework, clearly demonstrates that the cancellation of the momentum flux by the cross-correlation terms is due to the fundamental characteristics of the diabatic forcing and induced nonlinear forcing. This implies that the magnitude of the momentum flux induced by both the diabatic forcing and nonlinear forcing should be smaller in any case than that induced by diabatic forcing alone, and this effect should be included in convective GWD parameterizations.

Figure 4 shows the maximum magnitude of momentum flux as a function of NF normalized by the maximum magnitude of momentum flux by diabatic forcing obtained in the present NDM result.
alone, obtained using Fig. 4 (dots), and its best fitting curve (solid line) that represents the scale factor for the momentum flux by including nonlinearity effect. The scale factor is obtained as

$$F(\mu) = \frac{1}{1 + a \mu^b},$$

where $a = 0.487$ and $b = 1.648$ 96. According to (20), the magnitude of cloud-top momentum flux reduces to be less than 0.4 when the nonlinearity becomes larger than 2. It is rather straightforward to include the nonlinear forcing effect in current convective GWD parameterizations that consider diabatic forcing alone by simply multiplying the cloud-top momentum flux spectrum by the scale factor. For this, the nonlinearity factor (15) can be obtained at each grid using the model-produced diabatic forcing rate (from cumulus parameterization) and wind and stability with a specified horizontal size of cloud (5 km is used in the present study, as in SC05).

![Fig. 5. The momentum flux spectrum at $z=10$ induced by CTL simulation, diabatic forcing alone ($u'w'_0$), nonlinear forcing alone ($u'w'_1$), cross-correlation terms between the two forcings ($u'w'_1 + u'_0w'_0$), and sum of diabatic and nonlinear forcings ($[u'_0 + u'_1](w'_0 + w'_1)$) for different NFs.](image-url)
to NF normalized by the maximum magnitude of the momentum flux in a quasi-linear simulation forced by diabatic forcing alone (CTL with NF = 0) at z = 10 calculated using the result in Fig. 4 (solid dots) and its best fitting curve (solid line) as a function of nonlinearity factor.

![Graph](image)

Fig. 6. Maximum magnitude of the momentum flux with respect to NF normalized by the maximum magnitude of the momentum flux induced by diabatic forcing alone (NF = 0) at z = 10 calculated using the result in Fig. 4 (solid dots) and its best fitting curve (solid line) as a function of nonlinearity factor.

Figure 7 compares the momentum flux spectrum obtained from diabatic forcing alone (CTL with NF = 0), the CTL simulation with nonzero NF, and diabatic forcing alone multiplied by the scale factor for NFs of 1, 2, 3, 4, and 5. The updated momentum flux spectrum is reasonably good and indeed much better than the momentum flux induced by diabatic forcing alone as compared with that of CTL simulation. Discrepancies in the momentum flux between the updated one and the CTL simulation exist, as expected, mainly due to using a scale factor that is phase speed independent: (i) the phase speed at which the maximum magnitude of momentum occurs shifts to the west, (ii) the magnitudes of momentum flux with relatively high negative and positive phase speeds are smaller than those by CTL simulation, and (iii) the momentum flux at c = 0 for the updated one has a sign opposite to that for the CTL simulation with NFs larger than 3. However, these discrepancies occur mostly at phase speeds where the magnitude of the momentum flux is insignificant. Overall, the updated momentum flux is much better than that induced by diabatic forcing alone, and the scale factor obtained in the present study is considered to be reasonably good to use in convective GWD parameterizations.

To examine the generality of the fitting-curve formulation, NFs calculated by (15) and that estimated by (20)–(21) using the maximum magnitude of the momentum flux in a nonlinear simulation (M_{CTL}) and that in a quasi-linear simulation forced by diabatic forcing alone (M_{DRYQ}) are compared for all cases simulated in Choi et al. (2007) and shown in Fig. 8. Choi et al. performed ensemble numerical simulations of convective gravity waves under various ideal and real convection cases. For ideal cases, the basic-state wind profiles are assumed to linearly increase with height from the ground to z = 6 km with different surface winds (U_s = -27, -24, -21, -18, -15, -12, and -9 m s\(^{-1}\)) and to be uniform (2 m s\(^{-1}\)) above z = 6 km. The U_s = -18 m s\(^{-1}\) case is identical to the SCL case. In addition, a nonshear case with zero basic-state wind (U_s = 0 m s\(^{-1}\)) is conducted. For real storm cases, a case observed during the Tropical Ocean and Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE) on 22 February 1993 (Trier et al. 1996) and a case observed in Koto Tabang, Indonesia, on 11 April 2004 (Dhaka et al. 2005) have been considered.

The variables for calculating NFs for each storm case are shown in Table 1. For calculating (15), the horizontal scale (L) of convection is assumed to be 50 km (the dominant horizontal wavelength of diabatic forcing for most cases) and Q_D is the heating rate averaged over 50 km in the major convective region for 1 h from t = 5 to 6 h for ideal cases and from t = 2 to 3 h for real cases. The U_s is estimated by U_m - c_m, where U_m is the basic-state wind in the middle region of convection (z = 6 km) and c_m is the mean speed of convective cells in the direction of the maximum momentum flux (eastward direction for positive momentum flux and westward direction for negative momentum flux). The reference temperature T_0 is fixed to be 273 K and the momentum flux is evaluated at z = 18 km. Figure 8 demonstrates that NFs calculated by (15) and (20)–(21) are reasonably well matched with each other for most cases, except for the TOGA COARE case. Also, a scatterplot of NF versus M_{CTL}/M_{DRYQ} (not shown) for all the cases considered in Choi et al. (2007) shows that the result locates mostly near the fitting curve, except for the TOGA COARE case, where NFs calculated by (15) and (20)–(21) are not matched well, as shown in Fig. 8. Note that NF by (15) is very sensitive to the magnitude of U_s, and that U_s is sensitive to the moving speed of convective cells (c_m), which is not straightforward to determine when there are multiple convective cells with different speeds. Also, the height at which the basic-state wind and stability are evaluated is important to determine NF by (15). The same calculation can be conducted using the variables at the cloud top. For ideal simulations, NFs evaluated at the cloud top are the same as those in the middle region of convection since the basic-state wind is constant with height above z = 6 km. For real cases, NF evaluated at the cloud top is larger (smaller) for the TOGA COARE case (Indo-
nesia case). Overall, NFs calculated by (15) and (20)–(21) are reasonably well matched with each other for most simulated convection cases. This implies that the scale factor obtained in the present study can be applied to various types of convective storms.

5. Impact of the updated convective GWD parameterization on a GCM

The cloud-top momentum flux formulation by SC05 is updated by including the nonlinear forcing effect as

\[ M_{\text{ct}}(\psi, c, \mu) = M_{\text{ct}}^0(\psi, c) F(\mu), \]

where \( M_{\text{ct}}^0(\psi, c) \) is the cloud-top momentum flux originally proposed by SC05, \( \psi \) is the wave propagation direction, and \( c \) is the phase speed in the wave propagation direction. The detailed formulation of \( M_{\text{ct}}^0(\psi, c) \) can be found in SC05. Song and Chun (2006) developed convective GWD parameterizations based on the cloud-top momentum flux spectrum by SC05 with two different wave saturation methods and performed off-
line tests of the schemes using global reanalysis data. Then, Song et al. (2007) implemented the scheme, using the Lindzen-type wave saturation method, into the Whole Atmosphere Community Climate Model version 1b (WACCM1b) developed at the National Center for Atmospheric Research (Sassi et al. 2002) and investigated the effects of convective GWD parameterization on large-scale circulations. In this study, we perform the numerical simulation identical to Song et al. (2007) [spectral GWDC (SGWDC) simulation hereafter] but with an updated cloud-top momentum flux spectrum by (21).

WACCM1b is a global spectral model with T63 horizontal resolution at 66 vertical levels from the surface to about 140 km. The model description and physical processes included can be found in Song et al. (2007) and references therein. As in Song et al., in the current simulation with the updated parameterization (SGWDC_NL simulation hereafter), WACCM1b runs for 12 years starting from an initial condition (1 July 1978) using the climatological ozone and sea surface temperature. The result shown is the last 10-yr average. In this study, we only present a cloud-top momentum flux and zonal drag forcing to show how the results from the updated parameterization, including a nonlinearity effect on the cloud-top momentum flux spectrum, differs from the results by the SGWDC simulation that was analyzed in detail in Song et al. (2007).

Figure 9 shows the 10-yr averaged nonlinearity factor by (15) at cloud top, zonal-mean zonal momentum flux spectrum at the cloud top by the SGWDC simulation and its difference between SGWDC_NL and SGWDC simulations in January and July. Shading in Fig. 9c denotes area where the magnitude of the cloud-top momentum flux in the SGWDC simulation is reduced in the SGWDC_NL simulations. NF is calculated using

Table 1. The variables used to calculate NFs by (15) and (20)–(21) and resultant NFs. The variables are obtained from the numerical simulations for eight ideal and two real storm cases considered in Choi et al. (2007). Details are given in the text.

<table>
<thead>
<tr>
<th>CASE</th>
<th>$Q_0$ (J kg$^{-1}$ s$^{-1}$)</th>
<th>$U_m$ (m s$^{-1}$)</th>
<th>$c_m$ (m s$^{-1}$)</th>
<th>$U_c$ = $U_m$ - $c_m$ (m s$^{-1}$)</th>
<th>$N$ (s$^{-1}$)</th>
<th>$M_{CTL}$ (10$^{-2}$ N m$^{-2}$)</th>
<th>$M_{DRYQ}$ (10$^{-2}$ N m$^{-2}$)</th>
<th>$F(\mu) = M_{CTL}/M_{DRYQ}$</th>
<th>NF by (15)</th>
<th>NF by (20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_s = -27$</td>
<td>2.84</td>
<td>2.0</td>
<td>18.6</td>
<td>-16.6</td>
<td>0.01</td>
<td>0.277</td>
<td>0.734</td>
<td>0.378</td>
<td>1.85</td>
<td>2.09</td>
</tr>
<tr>
<td>$U_s = -24$</td>
<td>2.68</td>
<td>2.0</td>
<td>14.2</td>
<td>-12.2</td>
<td>0.01</td>
<td>0.188</td>
<td>0.674</td>
<td>0.279</td>
<td>3.23</td>
<td>2.75</td>
</tr>
<tr>
<td>$U_s = -21$</td>
<td>4.14</td>
<td>2.0</td>
<td>-10.9</td>
<td>12.9</td>
<td>0.01</td>
<td>-0.132</td>
<td>-0.936</td>
<td>0.141</td>
<td>4.47</td>
<td>4.62</td>
</tr>
<tr>
<td>$U_s = -18$</td>
<td>3.62</td>
<td>2.0</td>
<td>-15.8</td>
<td>17.8</td>
<td>0.01</td>
<td>-0.232</td>
<td>-0.651</td>
<td>0.356</td>
<td>2.05</td>
<td>2.21</td>
</tr>
<tr>
<td>$U_s = -15$</td>
<td>3.56</td>
<td>2.0</td>
<td>-14.1</td>
<td>16.1</td>
<td>0.01</td>
<td>-0.338</td>
<td>-0.993</td>
<td>0.340</td>
<td>2.47</td>
<td>2.31</td>
</tr>
<tr>
<td>$U_s = -12$</td>
<td>3.41</td>
<td>2.0</td>
<td>-10.8</td>
<td>12.8</td>
<td>0.01</td>
<td>-0.318</td>
<td>-1.32</td>
<td>0.241</td>
<td>3.74</td>
<td>3.10</td>
</tr>
<tr>
<td>$U_s = -9$</td>
<td>3.22</td>
<td>2.0</td>
<td>-9.3</td>
<td>11.3</td>
<td>0.01</td>
<td>-0.337</td>
<td>-2.37</td>
<td>0.142</td>
<td>4.53</td>
<td>4.60</td>
</tr>
<tr>
<td>$U_s = 0$</td>
<td>1.73</td>
<td>0.0</td>
<td>±6.7</td>
<td>±6.7</td>
<td>0.01</td>
<td>±0.106</td>
<td>±1.23</td>
<td>0.086</td>
<td>6.92</td>
<td>6.47</td>
</tr>
<tr>
<td>TOGA</td>
<td>1.57</td>
<td>-6.3</td>
<td>-26.2</td>
<td>19.9</td>
<td>0.012</td>
<td>-0.110</td>
<td>-0.43</td>
<td>0.256</td>
<td>0.59</td>
<td>2.95</td>
</tr>
<tr>
<td>Indo</td>
<td>0.65</td>
<td>-4.0</td>
<td>3.0</td>
<td>-7.0</td>
<td>0.013</td>
<td>0.160</td>
<td>0.41</td>
<td>0.390</td>
<td>1.83</td>
<td>2.03</td>
</tr>
</tbody>
</table>
the basic-state wind and stability at the cloud top with a maximum heating rate from convective heating profile at each model grid. Relatively large values of NF exist in the tropical region, where convective clouds frequently occur, and in the Southern Hemisphere (SH) at high latitudes and the Northern Hemisphere (NH) middle latitudes in July. However, even in the tropical region, the 10-yr mean of NF is mostly between 0.2 and 1.0 and is larger than 2.0 at only a few local points where horizontal wind speed at the cloud top is very small. Given that the scale factor $F(\mu)$ is 0.67 for NF = 1 and 0.97 for NF = 0.2, the magnitude change of the momentum flux due to the nonlinearity factor cannot be significant. Therefore, the difference in the zonal momentum flux spectrum at cloud top between the SGWDC_NL and SGWDC simulations (Fig. 9c) is one order of magnitude smaller than the cloud-top momentum flux obtained from the SGWDC simulation. The cloud-top momentum flux spectrum from the SGWDC_NL simulation (not shown) is basically simi-
lar to that of the SGWDC simulation (Fig. 9b), but its magnitude is reduced in most latitudes and phase speed ranges, except in the NH (SH) middle latitudes of storm track regions over 30°–60° in January (July), where the positive momentum flux with phase speed ranging from about 20 to 50 m s⁻¹ and the negative momentum flux with phase speed ranging from about −10 to +10 m s⁻¹ are amplified.

Figure 10 shows the zonally averaged zonal drag forcing due only to the convective GWD process in the SGWDC simulation and its difference between the SGWDC_NL and SGWDC simulations for January and July. The contour intervals of (a) and (b) are 3 and 0.3 m s⁻¹ day⁻¹, respectively; negative values are dashed. Areas where the magnitude of drag forcing in the SGWDC simulation is reduced in the SGWDC_NL simulation are shaded.

![Fig. 10. The WACCM simulation results in 10-yr average of zonal-mean zonal drag forcing induced exclusively by convective GWD parameterization (a) in the SGWDC simulation and (b) its difference from the SGWDC_NL simulation in (top) January and (bottom) July. The contour intervals of (a) and (b) are 3 and 0.3 m s⁻¹ day⁻¹, respectively; negative values are dashed. Areas where the magnitude of drag forcing in the SGWDC simulation is reduced in the SGWDC_NL simulation are shaded.](image-url)

The difference is concentrated where the drag forcing was large, with maximum (minimum) values of 1.5 m s⁻¹ day⁻¹ (−1.8 m s⁻¹ day⁻¹) in January and of 3.7 m s⁻¹ day⁻¹ (−5.4 m s⁻¹ day⁻¹) in July, which is one order of magnitude smaller than the drag value in the SGWDC simulation. This is somewhat to be expected since the shape of the cloud-top momentum flux in the SGWDC_NL simulation is not significantly different from that in the SGWDC simulation. The nonlinear forcing effect reduces the magnitude of drag forcing in many regions, but there are several areas (the tropical upper mesosphere and lower thermosphere between about 20°S–0° in January and 10°S–10°N in July, and the lower mesosphere in the SH middle latitudes) of increasing drag forcing.
In the stratosphere, the magnitude of drag forcing is much smaller than that in the mesosphere, thus it is not well represented using the same contour intervals as Fig. 10. Figure 11 shows the same fields as Fig. 10 but with \( z < 45 \) km. The difference occurs mainly in the NH (SH) upper stratosphere in January (July) with maximum (minimum) values of 0.081 m s\(^{-1}\) day\(^{-1}\) (−0.023 m s\(^{-1}\) day\(^{-1}\)) in January and of 0.076 m s\(^{-1}\) day\(^{-1}\) (−0.066 m s\(^{-1}\) day\(^{-1}\)) in July. Although the difference is much less than that in the mesosphere, it is greater than 20% of the maximum magnitude of drag forcing in the SGWDC simulation. This implies that the nonlinear forcing effect on the zonal drag forcing is even more significant in the upper stratosphere than in the upper mesosphere. In addition, the difference in the drag forcing occurs mostly in the shaded area, indicating that including the nonlinear forcing effect generally reduces the magnitude of drag forcing in the stratosphere. The only exception exists in the winter hemisphere middle latitudes of storm track regions, where the magnitude of the cloud-top momentum flux increases by including the nonlinear forcing effect, as shown in Fig. 9c.

It is noteworthy that the reduced magnitude of the cloud-top momentum flux resulting from the nonlinear forcing effect can either increase or decrease the magnitude of the drag forcing in convective GWD parameterizations, depending on wave dissipation processes. When the waves are dissipated through the critical-level filtering process, drag forcing can be decreased by reduced magnitude of the cloud-top momentum flux because the vertical gradient of the momentum flux decreases by reduced magnitude of cloud-top momentum flux. On the other hand, for vertically propagating waves, reduced magnitude of the momentum flux can allow the waves to reach relatively higher altitudes before they are saturated and break, and consequently can produce a larger magnitude of drag forcing due to lower density, if the vertical gradient of the momentum flux is assumed to be the same. In the present WACCM
simulation with Lindzen-type linear wave saturation process (Song et al. 2007), wave breaking occurs mostly in the upper mesosphere, so the reduced magnitude of cloud-top momentum flux decreases drag forcing in the stratosphere but increases in the upper mesosphere.

Figure 12 shows time series of the zonal-mean zonal momentum forcing by convective GWD parameterization in the tropical region averaged over 15°S–15°N at 10 and 30 hPa. It shows that zonal momentum forcing in the SGWDC_NL simulation either increases or decreases compared with that in the SGWDC simulation, although the overall magnitude of momentum forcing in SGWDC_NL decreases somewhat, especially at 30 hPa. According to the relationship between drag forcing and wave dissipation processes, the momentum forcing by convective GWD, shown in Fig. 12, should decrease under the reduced magnitude of cloud-top momentum flux, given that the critical-level filtering process is the major wave dissipation process in the stratosphere. The argument of wave dissipation, however, is valid only for a single wave at an instant time when the other conditions (source spectrum, wind, stability, etc.) are exactly the same. What is likely to happen in GCMs is that, once reduced magnitude of the cloud-top momentum flux changes drag forcing, the wave source spectrum as well as wave propagation condition can be changed through the change of dynamic and thermodynamic variables at each model grid. Therefore, the difference in drag forcing between the two simulations results from multiple positive and negative feedback processes.

6. Summary and discussion

The nonlinear-forcing effect on the momentum flux spectrum of convectively forced internal gravity wave was investigated using a nondimensional numerical model (NDM) in a two-dimensional framework. To isolate nonlinearity effects, the governing equations of the

---

**Fig. 12.** Time series of zonal-mean zonal drag forcing in the tropical stratosphere averaged over 15°S–15°N at (upper) 10 and (lower) 30 hPa by the SGWDC (gray curves) and SGWDC_NL (black curves) simulations.
NDM, basic-state wind and stability conditions, and the diabatic forcing structure used for all simulations are made identical to those used in the linear analytical study of Song and Chun (2005), which considered diabatic forcing alone. The NDM simulations with various NFs revealed that the magnitude of wave perturbation above the specified diabatic forcing decreases as NF increases because the waves induced by nonlinear forcing are largely out of phase with those induced by diabatic forcing.

The magnitude of the momentum flux above the forcing decreases as NF increases. This is because the momentum flux induced by nonlinear forcing alone increases with NF with the same sign of the momentum flux induced by diabatic forcing alone, but the cross-correlation momentum fluxes with an opposite sign increase more rapidly with NF. This represents a reduced magnitude of total effective forcing as NF increases, due to the cancellation of the effective diabatic forcing by the effective nonlinear forcing that increases as NF increases. Using the maximum magnitude of the momentum flux obtained by the NDM simulations for each NF case, we obtained an analytical formulation of a scale factor for the momentum flux, normalized by the momentum flux induced by diabatic forcing alone, as a function of NF. The updated momentum flux induced by diabatic forcing alone and using the scale factor is reasonably good as compared with the CTL simulation including both forcings.

The updated convective GWD parameterization of SC05 using the updated cloud-top momentum flux spectrum was implemented into a GCM (WACCM1b), and 10-yr simulation (SGWDC_NL) results were compared with those (SGWDC) by the original SC05 parameterization. It was shown that the magnitude of the cloud-top momentum flux by the SGWDC_NL simulation is generally smaller than that by the SGWDC simulation in most latitudes and phase speed ranges, except in the middle latitudes (30°–60°) of storm track regions in both January and July, where the positive (negative) momentum flux with phase speed ranging from about 20 to 50 m s⁻¹ (–10 to +10 m s⁻¹) is amplified. The difference in the cloud-top momentum flux between the SGWDC_NL and SGWDC simulations was found to be one order of magnitude smaller than the cloud-top momentum flux by the SGWDC simulation. This rather small difference results mainly because the nonlinearity factor of the gravity waves, calculated using the GCM-produced variables (wind, temperature, diabatic heating rate), is mostly less than 1.0 even in the tropical region and, consequently, the scale factor is close to 1 in most regions.

The impact of the new parameterization on zonal-mean zonal drag forcing is to reduce its magnitude in many regions, for both positive and negative drags, but there are several regions with increased magnitude of drag forcing, especially in the tropical upper mesosphere and the SH middle latitude lower mesosphere. Under the reduced magnitude of cloud-top momentum flux in the updated parameterization, drag forcing can be either increased or decreased depending on wave dissipation processes: It generally increases in the upper mesosphere where wave saturation is the major dissipation process, while it decreases in the stratosphere where critical-level filtering is the major wave dissipation process. However, in GCM, drag forcing is determined not only by wave dissipation process but also by the complicated relationship between the wave sources and wave propagation condition through the multiple positive and negative feedback processes.

In this study, we presented a way to include a nonlinear forcing effect on the cloud-top momentum flux of convective gravity waves by using a scale factor derived from NDM results with respect to NF. Although this approach is rather straightforward, there are several factors that could influence the results, mainly the way to derive the scale factor. First, the scale factor is obtained using the NDM for a single convection case with a simplified heating structure and basic-state wind and stability condition. Although we considered such an idealized situation to directly apply the nonlinear forcing effect to SC05’s convective GWD parameterization, one may ask whether the current result can be applied to various convective storms. For the SCL case, the maximum value of the cloud-top momentum flux in the CTL simulation (M_{CTL}) is −0.232 × 10⁻² N m⁻², while that in the DRYQ simulation (M_{DRYQ}) is −0.651 × 10⁻² N m⁻². Therefore, M_{CTL}/M_{DRYQ} = 0.357. Given that M_{CTL}/M_{DRYQ} = F(μ) in (21), 0.357 corresponds to μ = 2.2 according to the curve fitting formulation of (20). This value of NF matches well with that previously estimated (μ ∼2) using (15). To examine the generality of the fitting-curve formulation, we used the result of ensemble numerical simulations of convective gravity waves by Choi et al. (2007), who considered eight ideal cases with different low-level wind shears and two real cases observed in the tropics. We found that the fitting-curve formulation by (20) represents reasonably well the result of most cases considered in Choi et al. (2007).

Sensitivity of the fitting-curve formulation to factors other than the low-level shear, such as different convective available potential energy (CAPE), three-dimensionality, directional shear, and upper-level shear, remains to be done in a future study.

Second, GCM-produced diabatic heating rate with a horizontal grid area of ∼40 000 km² can be smaller than
the diabatic heating by mesoscale convective storms considered in the present study and, consequently, the nonlinearity factor obtained at the GCM grid can be underestimated somewhat. Although this might be an inevitable limitation to parameterization of any mesoscale processes to GCM, given that the nonlinearity of thermally induced internal gravity waves cannot be directly calculated in the current GCM, some correction can be included to represent the mesoscale impact properly. A reasonable way to include mesoscale impact on the nonlinearity factor needs to be found.

Acknowledgments. The authors thank two anonymous reviewers for their helpful comments. This work was supported by the Korea Science and Engineering Foundation (KOSEF) through the National Research Lab. The program was funded by the Ministry of Science and Technology (M10500000114-060000-11410) and by the Korea Research Foundation Grant funded by the Korean government (MOEHRD) (R02-2004-000-10027-02).

REFERENCES


