A Lagrangian Spectral Parameterization of Gravity Wave Drag Induced by Cumulus Convection

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ABSTRACT

A Lagrangian spectral parameterization of gravity wave drag (GWD) induced by cumulus convection (GWDC) is developed based on ray theory and several assumptions and implemented into the NCAR Whole Atmosphere Community Climate Model. The Lagrangian parameterization calculates explicitly gravity wave (GW) propagation that has been treated too simply in existing column-based parameterizations. For comparison with column-based parameterization, a hydrostatic and Boussinesq version of the Lagrangian parameterization is used in the present study. One-day convective GW-packet trajectories demonstrate that the Lagrangian parameterization calculates reasonably the GW-packet propagation, and GW packets propagate upward along curved paths determined by Doppler shifting and the variation of stability. The GW trajectories show that the horizontal extent of GW propagation can be as large as 20° as GWs approach critical levels. Comparison with column-based parameterization through one-month simulations indicates that the magnitude of GWDC is much increased due mainly to the vertical convergence of GW packets in the lower stratosphere and equatorial troposphere with the Lagrangian parameterization. However, this increase in GWDC is found to be essentially dependent on the horizontal propagation characteristics of GWs. In climate simulations, it is found that the easterly flow in the equatorial stratosphere and mesospheric subtropical jet are improved through the Lagrangian parameterization. With the Lagrangian parameterization, interannual variability is significantly enhanced in the equatorial lower stratosphere and exhibits a structure related to the onset of the westerly phase of the stratospheric quasi-biennial oscillations. Finally, limitations of the current Lagrangian parameterization and required improvements are noted.

1. Introduction

Momentum forcing due to gravity waves (GWs) is believed to be crucial in driving the large-scale circulations in the middle atmosphere and in accounting for the global angular momentum budget. In general circulation models (GCMs), such wave-induced force has been represented through gravity wave drag (GWD) parameterizations since neither GWs themselves nor GW momentum deposition processes are fully resolved even in up-to-date GCMs. For these reasons, GWD parameterization has been accepted as a standard component of GCMs (Fritts and Alexander 2003; Kim et al. 2003).

Through several GWD parameterizations, GCMs have effectively simulated observed climatology in the middle atmosphere (Norton and Thuburn 1999; Fomichev et al. 2002) and variabilities such as the semiannual oscillation (SAO; Manzini et al. 1997; Medvedev and Klaassen 2001) and the quasi-biennial oscillation (QBO; Scaife et al. 2002; Giorgetta et al. 2002; Shibata and Deushi 2005) in the equatorial stratosphere. However, in spite of these successes, it has been pointed out that there are still large uncertainties in the reference-level GW momentum in the parameterizations. To remove the uncertainties, several efforts have been made to explicitly include the properties of GWs induced by various sources in parameterizations.

Among various sources for GWs, convection has long been a target of significant interest. There have been several studies to represent reasonably well convectively forced GWs in the reference-level GW momentum of GWD parameterizations (e.g., Chun and Baik 1998, 2002; Beres 2004; Song and Chun 2005). GW momentum with physical information about convec-
tively forced GWs has contributed to more realistic GCM simulations of wind and temperature climatologies and variabilities in the equatorial middle atmosphere (Chun et al. 2001, 2004; Beres et al. 2005; Song et al. 2007).

However, better representations of GW momentum do not necessarily ensure physically consistent wave-induced force since there are several other problems inherent in the current GWD parameterizations. One of the problems is that the propagation of GWs is treated too simply in parameterizations. In general, GWs propagate horizontally as well as vertically, and their properties are affected by the variation in large-scale flow (Lighthill 1978). Nonetheless, in the current GWD parameterizations, GWs are assumed to propagate only in the vertical direction and to reach the top of the model within a model time step. These assumptions have been commonly made since most of the GWD parameterizations are based on column dynamics as in other physics parameterizations in GCMs, but those assumptions are not consistent with general propagation properties of GWs.

GW propagation properties have been studied using models developed based on the ray theory (e.g., Dunkerton 1984; Schoeberl 1985; Marks and Eckermann 1995; Eckermann and Marks 1997). However, such models have been used in the interpretation of GW observations or identification of GW sources (e.g., Eckermann and Preusse 1999; Guest et al. 2000; Jiang et al. 2004) rather than in the GWD parameterizations used in GCMs. Therefore, for more reliable GWD parameterizations, physically consistent representations of GW propagation properties need to be included in the GWD parameterizations.

In this study, using the ray theory and several assumptions on the GW propagation and convectively forced GW spectrum developed by Song and Chun (2005), we propose a Lagrangian spectral parameterization of GWD induced by cumulus convection (GWDC) in which GW propagation properties are explicitly considered and implement this parameterization in the National Center for Atmospheric Research (NCAR) Whole Atmosphere Community Climate Model, version 1b (WACCM1b; Sassi et al. 2002). Using WACCM1b with the Lagrangian parameterization, we investigate effects of three-dimensional GW propagation on the interaction between convective GWs and large-scale flow. Also, we examine impacts of the new parameterization on WACCM1b, and compare them with impacts of the column-based spectral GWDC parameterization examined by Song et al. (2007, hereafter SCGB07).

This paper is organized as follows. In section 2, the Lagrangian spectral GWDC parameterization is described. In section 3, results obtained from the Lagrangian parameterization are presented and compared with results obtained from column-based parameterization. In section 4, results of additional tests are presented, and issues regarding the limitations of the current Lagrangian parameterization are discussed. A summary and conclusions are given in the last section.

2. Lagrangian spectral GWDC parameterization

a. Linearized system of equations

The Lagrangian spectral (LS) GWDC parameterization is based on the following equation set in a Cartesian geometry for perturbations linearized about time-independent and spatially slowly varying background flow in a nonhydrostatic, rotating, and anelastic system:

\[
\overrightarrow{D}_{t} u' + v' \cdot \nabla u - f v' + \partial_{y}(p'/\overline{\rho}) = 0,
\]

\[
\overrightarrow{D}_{t} v' + w' \cdot \nabla v + f u' + \partial_{y}(p'/\overline{\rho}) = 0,
\]

\[
\overrightarrow{D}_{t} w' + \partial_{x}(p'/\overline{\rho}) - g(\theta'/\overline{\theta}) = 0,
\]

\[
(\partial_{x} u' + \partial_{y} v' + \partial_{z} w') - w'/H = 0,
\]

and

\[
\overrightarrow{D}_{i}[g(\theta'/\overline{\theta})] + N^2 w' = 0,
\]

where \(v' = (u', v', w')\), \(p'\), and \(\theta'\) are the perturbations for wind vector, pressure, and potential temperature, respectively; \(\overline{\rho}\) and \(\overline{\theta}\) are the background density and potential temperature, respectively; \(f\), \(N\), and \(H\) are the Coriolis parameter, buoyancy frequency, and density scale height, respectively; \(\overrightarrow{V}\) is the three-dimensional gradient operator, \(\partial_{x}\), \(\partial_{y}\), and \(\partial_{z}\) are the partial derivatives with respect to \(x\), \(y\), and \(z\), respectively; and \(\overrightarrow{D}_{i}\) (= \(\partial_{x} + i \pi \partial_{x} + v \partial_{z}\)) is the rate of change following the background wind (\(\overline{n}, \pi, 0\)).

In (4) and (5), horizontal advections of \(\overline{\rho}\) and \(\overline{\theta}\) by the perturbation wind are ignored, given that the ratio of the horizontal variations of \(\overline{\rho}\) and \(\overline{\theta}\) to their vertical variations is less than \(10^{-1}\) for the large-scale flow in the tropics (Browning et al. 2000) as well as in the midlatitudes (Pedlosky 1987).

The Wentzel–Kramers–Brillouin (WKB) solutions of (1)–(5) can be locally expressed as monochromatic GWs:

\[
\psi'(x, y, z, t) = \text{Re}[e^{i(kx + ly + mz - \omega t)}].
\]

where \(\psi'\) stands for \(u', v', w', p'/\overline{\rho}\), or \(\theta'/\overline{\theta}\); \(\overline{p}_{\omega}\) is a constant reference air density; \((k, l, m)\) is the wavenumber vector; and \(\omega (> 0)\) is the ground-relative frequency. Substituting (6) into (1)–(5) (without considering
\( \mathbf{v} \cdot \nabla \mathbf{u} \) and \( \mathbf{v} \cdot \nabla \mathbf{v} \) gives the following the dispersion relationship found in the leading-order terms of the WKB expansion:

\[
\omega^2 = [N^2(k^2 + \ell^2) + f^2(m^2 + \alpha^2)]/\alpha^2,
\]

where \( \omega = -\omega - \kappa \vec{u} - l \hat{\mathbf{n}} \) is the intrinsic frequency, \( \alpha = 1/(2\hat{\mathbf{n}}) \), and \( \alpha^2 = k^2 + \ell^2 + m^2 + \alpha^2 \). By setting \( m = 0 \), the cutoff frequency \( \omega_c \) for large-\( \omega \) GWs is obtained. In terms of \( \omega_c \), the condition for the vertical propagation of GWs is given as follows: \( f^2 < \omega^2 < \omega_c^2 < N^2 \).

### b. Reference-level gravity wave properties

In the LS GWDC parameterization, reference-level pseudomomentum flux is calculated at the top of individual deep cumulus clouds using the following phase speed spectra of GW momentum flux \( (M_{ct}) \) and horizontal wavenumber \( (k_{ct}) \):

\[
M_{ct}(c, \varphi) = \text{sgn}(c - U_{ct}(\varphi))|\vec{p}_{ct}|^2 \frac{2(2\pi)^3}{A_\theta L_\varphi} \left( \frac{g}{c_p T_{ct} N_g^2} \right)^2 \frac{N_{ct} |\mathbf{X}|^2}{|U_{ct}(\varphi) - c|} \Phi(c, \varphi)
\]

and

\[
k_{ct}(c, \varphi) = \frac{2\pi/\delta_n \sqrt{\pi}}{1 + (c - c_{gh})^2/c_0^2},
\]

where \( c \) and \( \varphi \) are the phase speed and propagation direction of GWs, respectively; \( \vec{p}_{ct}, T_{ct}, \) and \( N_{ct} \) are the cloud-top air density, temperature, and buoyancy frequency, respectively; \( N_g \) is the mean buoyancy frequency below cloud top; \( U_{ct}(\varphi) \) is the cloud-top wind projected onto the direction of \( \varphi \); \( A_\theta \) and \( L_\varphi \) are horizontal and time scales, respectively, used for averaging \( M_{ct}; c_{gh} \) is convective cell’s moving speed in the direction of \( \varphi; c_0 = \delta_n/\delta_n; \) and \( \delta_n \) and \( \delta_n \) are the horizontal and time scales of a convective cell, respectively. The spectra in (8) and (9) are identical to those used in the column-based spectral (CS) GWDC parameterization (SCGB07), and details of their calculations are described therein.

In the LS parameterization, GWs are assumed to consist of wave packets. The properties of each packet at cloud top are determined as follows. First, (8) and (9) are discretized in a phase speed \( (c) \) grid ranging from \( -100 \) to \( 100 \) m s\(^{-1} \) at an interval of \( \delta c = (2 \) m s\(^{-1} \) for two \( \varphi \)‘s of 0° and 90°. Second, from the discretized (9), the \( k \) and \( l \) of packets are calculated as \( \text{sgn}(c) k_{00} \cos \varphi \) and \( \text{sgn}(c) k_{00} \sin \varphi \), respectively. When \( c = 0 \) for each \( \varphi \), two packets are assumed to exist and to have \( k \) and \( l \) with opposite signs. Third, \( \omega \) is set equal to \( |\lambda k_{00}| \), and \( \omega \) is given by \( \omega - \kappa \vec{u}_{ct} - l \vec{n}_{ct} \), where \( \vec{n}_{ct} \) and \( \vec{u}_{ct} \) are cloud-top zonal and meridional wind, respectively. Fourth, \( m \) is calculated from (7) as follows: \( \text{sgn}(\omega) [(k^2 + \ell^2)(N_{ct}^2 - \omega_c^2)(\omega^2 - f^2 - \alpha^2)^{1/2}], \) where \( \text{sgn}(\omega) \) is a factor to ensure upward GW energy propagation.

Finally, the vertical fluxes of the pseudomomentum of GW packets at cloud top are calculated from the discretized (8). In terms of the wave action density \( A \) \( ( = E/\omega \), where \( E \) is the total GW energy density; Bretherton and Garrett (1969)\), the horizontal pseudomomentum is represented as \( (kA, IA) \) (McIntyre 1980), and its vertical flux \( (F_{px}, F_{py}) \) is given as \( (k_{gA}, l_{gA}) \). Here, \( c_{gA} \) is the vertical group velocity. The \( F_{px} \) and \( F_{py} \) are related to GW momentum flux as follows:

\[
F_{px} = \text{sgn}(k\omega)M_{ct}(c, \varphi)\delta c(1 - f^2/\omega^2)\cos \varphi
\]

and

\[
F_{py} = \text{sgn}(l\omega)M_{ct}(c, \varphi)\delta c(1 - f^2/\omega^2)\sin \varphi,
\]

where \( \text{sgn}(k\omega) \) and \( \text{sgn}(l\omega) \) are factors for consistency with the sign of (8). When \( c = 0 \), two GW packets are assumed to have wavenumber vectors of the opposite direction, and for each one, \( F_{px} \) and \( F_{py} \) are reduced by half.

### c. Three-dimensional propagation of GWs

After the properties of GW packets at cloud top \( (k, l, m, \omega, \omega_c, k_{gA}, A, l_{gA}, A) \) are determined, the packets are launched if \( |f| < |\omega_c| < |\omega| \). The three-dimensional propagation of the launched GW packets and the variation of the GW properties are explicitly calculated using the following ray-tracing equations for the steady background flow (Lighthill 1978):

\[
dx = c_{gx} = \vec{u} + k(N^2 - \omega^2)/(\omega \sigma^2),
\]

\[
dy = c_{gy} = \vec{v} + l(N^2 - \omega^2)/(\omega \sigma^2),
\]

\[
dz = c_{gz} = -m(\omega^2 - f^2)/(\omega \sigma^2),
\]

\[
dk = -k\partial_x \vec{u} - l\partial_x \vec{v} - (A\partial_x N^2 - B\partial_x \sigma^2)/(2\omega \sigma^2),
\]

\[
dl = -k\partial_y \vec{u} - l\partial_y \vec{v} - (A\partial_y N^2 - B\partial_y \sigma^2 + C\partial_y \sigma^2)/(2\omega \sigma^2),
\]

and

\[
dm = -k\partial_z \vec{u} - l\partial_z \vec{v} - (A\partial_z N^2 - B\partial_z \sigma^2)/(2\omega \sigma^2),
\]
where \((c_{px}, c_{py}, c_{pz})\) is the group velocity vector, \(d_t\) is the rate of change following GW packets moving at their group velocities, \(A = k^2 + f^2\), \(B = \omega^2 - f^2\), and \(C = m^2 + \alpha^2\).

These ray-tracing equations are numerically integrated in time within a model time step \((\Delta t)\) (see the appendix). Since GW packets propagate at finite velocities determined from (12)–(14), the GW packets can remain in the model for a much longer time period than \(\Delta t\). To consider this situation, the LS parameterization is designed to be able to treat GW packets launched at previous, as well as current, instants of time. However, the lifetime of GW packets is limited to 1 day in preparation for limited computing resources. For implementation into the global model, the ray-tracing Eqs. (12)–(17) expressed in Cartesian coordinate system are transformed using the simple relationships \(\Delta \text{x} = (a \cos \phi) \Delta \lambda\) and \(\Delta \text{y} = a \Delta \phi\), as in Marks and Eckermann (1995). Here, \(a\) is Earth’s radius, \(\lambda\) is the longitude, and \(\phi\) is the latitude.

As noted earlier, the LS parameterization is based on ray formulations derived in Cartesian coordinates. However, ray-tracing formulations based on such a simple geometry are essentially approximate and can yield inaccurate results when they are applied to a spherical geometry. Especially, the ray formulations derived in a simple geometry may lead to serious problems in the calculation of the trajectories of GW packets traveling long horizontal distance since the trajectories of such GW packets should be greatly affected by Earth’s curvature. This issue is discussed in section 4.

In the parameterization, GW propagation is not calculated further in the following five cases: (i) when GWs encounter critical levels, (ii) when the lifetime limit is exceeded, (iii) when the WKB assumption is not valid, (iv) when wave reflection occurs, or (v) when GWs pass through the top boundaries of the model, or either the North or South Pole. For the first case, GWs are assumed to encounter critical levels when the vertical wavelength \((\lambda_z = 2\pi/ml)\) is less than 100 m. In this case, the pseudomomentum contained in the GW packet is completely deposited to the large-scale flow, and then the GW packet is removed. For the other cases, the GW packets are removed without momentum deposition. The WKB assumption is examined using the parameter \(R = [1/(2m^3)] \frac{\rho c^2}{\rho_z} \text{m} - 3/(4m^4) (\hat{\omega} \cdot \text{m})^2\); Einaudi and Hines (1970), and it is assumed to be invalid when \(R > 1\). In the calculation of \(R\), \(\hat{\omega}\) is approximated by \((1/c_{gz}) \cdot d_t\), as in Marks and Eckermann (1995).

This approximation is required since any local change in wave properties cannot be directly evaluated when using ray-tracing equations. In the LS parameterization, the reflection is checked using \(\hat{\omega}\) (\(\hat{\omega}\) when \(\lambda_z\) approaches \(\infty\)), and GWs are regarded as being reflected when \(|\hat{\omega}| > |\hat{\omega}|\). In addition, the upper limit of \(\lambda_z\) is set to 100 km, and GWs with \(\lambda_z > 100\) km are also considered being reflected. This upper limit is essentially arbitrary, but is motivated by the idea that waves with \(\lambda_z\) much greater than an atmospheric scale height can be more subject to partial internal reflection (Hines 1997). Issues with regard to this upper limit are again discussed later.

The above-mentioned termination conditions of ray integration may lead to important issues. One of the issues is that the conditions can force GWs to simply disappear without momentum deposition and thus can lead to problems in a view of the angular momentum conservation (Shepherd and Shaw 2004). This issue is further discussed in section 4.

d. Saturation and GW momentum forcing

Wave-induced force is associated with the vertical divergence of the pseudomomentum flux \((F_{_{px}}\) and \(F_{_{py}}\)) of GWs. To calculate the variation of \(F_{_{px}}\) and \(F_{_{py}}\), the LS parameterization solves the following approximate equation (see discussion section for details) for the vertical flux of wave action density \((c_{gz} A)\) along rays as in Marks and Eckermann (1995):

\[d_t(c_{gz} A) \approx -2c_{gz}A/\tau,\]  

(18)

where \(\tau\) is a time scale related to the radiative damping and turbulent diffusion.

After (18) is calculated, the saturation of GWs is checked using Lindzen-type method so that the magnitude of GW pseudomomentum fluxes \([F_p] = (k^2 + f^2)^{1/2}|c_{gz} A|\) cannot exceed \([F_p]\) for saturated GWs. The Lindzen-type method is identical to that in the CS parameterization (see SCGB07). While GWs are saturated, \(k\) and \(l\) are assumed to remain unchanged. As a result, saturation process is accomplished by reducing \(c_{gz} A\). These change in \(c_{gz} A\) and change in \(k\) and \(l\) obtained from (15) and (16) produce the variations of \(F_{_{px}}\) and \(F_{_{py}}\) along GW packets. From these variations, GW momentum forcing at \((x, y, z)\) is calculated as follows:

1 When waves are reflected, they can intersect one another below reflection points. Near such intersection points (caustics), ray theory breaks down since wave properties do not vary slowly in space (Broutman et al. 2004).
\[
\Delta t \Delta \bar{u}(x, y, z) = -\frac{1}{\bar{p}(x, y, z)} \sum_{i=1}^{n_w} \left( \delta F_{px,i} \right) J_i(x, y, z), \tag{19}
\]
\[
\Delta t \Delta \bar{v}(x, y, z) = -\frac{1}{\bar{p}(x, y, z)} \sum_{i=1}^{n_w} \left( \delta F_{py,i} \right) J_i(x, y, z). \tag{20}
\]

Here, \(n_w\) is the total number of GW packets passing through \((x, y, z)\) during \(\Delta t\) and is calculated using the gridding process in section c of the appendix; \(\delta F_{px,i}\) and \(\delta F_{py,i}\) are change in the \(F_{px}\) and \(F_{py}\) along the \(i\)th packet, respectively; \(\delta z_i\) is the vertical displacement of the \(i\)th packet; and \(J_i = \min(\delta z/\Delta z, 1)\) is a factor to obtain divergence over \(\Delta z\) when \(\delta z_i < \Delta z\).

**e. Simplification of the parameterization**

The Lindzen-type saturation scheme used in the LS GWDC parameterization is based on the hydrostatic, nonrotating, and Boussinesq flow system. For theoretical consistency, in (10)–(17), \(f\) and \(\alpha\) are set to zero, respectively, and the hydrostatic and Boussinesq approximations are applied to (10)–(17) by letting \(1 - \hat{\omega}^2/N^2 \approx 1\) and \(\sigma^2 \approx m^2\), respectively. Also, the damping term in (18) is ignored because the Lindzen-type method has its own dissipation process. In addition, the horizontal derivatives of the background variables are ignored, and thus \(k\) and \(l\) do not vary in time along ray trajectories.\(^2\) By doing this, GWs deposit momentum only when they are dissipated or saturated. Through these simplifications, ray tracing and action flux equations that are actually used in the parameterization are given as follows:

\[
d_t(x, y, z) = (c_{gx}c_{gy}c_{gz})
\]
\[
= [\bar{u} + kN^2/(\hat{\omega}m^2), \bar{v} + LN^2/(\hat{\omega}m^2), -\hat{\omega}m],
\]
\[
d_t(k, l, m) = [(0, 0) - k\partial_z\bar{u} - l\partial_z\bar{v} - (k^2 + l^2)\partial_zN^2/(2\hat{\omega}m^2)], \quad \text{and} \tag{22}
\]
\[
d_t(c_{gz}A) = 0. \tag{23}
\]

This section mainly describes the theoretical parts of the LS GWDC parameterization, but there are many other considerations for implementation. Details are in the appendix.

**3. Results**

**a. Gravity wave packet trajectory**

In this section, we examine whether the LS parameterization reasonably calculates GW-packet propagation and GW properties along packet trajectories. For this, a one-day simulation for 1 January 1979 using WACCM1b with the LS GWDC parameterization is conducted. In this simulation, feedback between GWDC and flow in the model is ignored to examine the behavior of GW packets for the given background flow conditions.

Figure 1 demonstrates one-day trajectories of GW packets generated by deep convection occurring at 18.1°S and 129.3°E at 0000 UTC 1 January 1979. Initially, GW packets launched at cloud top \((z = 10\text{ km})\) propagate in the four cardinal directions, parallel to...
horizontal wavenumber vectors \([k_h = (k, l)]\). The \(k_h\) is constant along GW packets since the horizontal derivative of the background flow is ignored in the parameterization. Each trajectory is plotted in different colors at every 10-km height range, and plus symbols are plotted in case of GW packets with zonal \(k_h\). For convenience, GW packets with eastward, westward, northward, and southward \(k_h\) will be referred to as GWE, GWW, GWN, and GWS, respectively.

First of all, Fig. 1 shows that GW packets propagate horizontally as well as vertically along curved paths, and the horizontal extent of propagation can be 10°–20° in some height ranges. This large horizontal extent of GW propagation demonstrates the necessity of including a realistic representation of GW propagation in GWD parameterizations. The GW propagation shown in Fig. 1 exhibits characteristic features depending on the initial propagation directions. In case of GWEs, GWs propagate mainly in the vertical direction at initial time, but they propagate northeastward above \(z = 80\) km. In contrast, GWWs propagate mainly in the horizontal direction in the troposphere and lower stratosphere, and as a result, they move as far as about 20° from the source region. This asymmetric zonal propagation resembles the zonal propagation of GWs affected by Doppler shifting by the easterly flow. On the other hand, GWNs and GWSs propagate symmetrically with respect to the vertical direction compared with GWEs and GWWs. Overall, GWNs and GWSs propagate gradually in the vertical direction, but some of the GWSs propagate suddenly in the vertical direction near 25°S. In Fig. 1, it is also found that the GWNs and GWSs propagate a great distance westward, but the GWEs and GWWs propagate mainly in the zonal direction without latitudinal spread.

To understand the GW propagation shown in Fig. 1, we examine the background wind and stability and the vertical variation of the vertical wavelength \((\Lambda_z)\) following GW packets (Fig. 2). For better illustration, the \(\lambda_z\)'s are plotted only when GWs propagate above \(z = 15\) km and are categorized according to their phase speeds in the direction of \(k_h\). The background wind and stability

![Fig. 2. Vertical profiles of zonal or meridional wind (solid), static stability (dotted), and vertical wavelengths of convectively forced GW packets with (a) eastward, (b) westward, (c) northward, and (d) southward horizontal wavenumber vectors. Wind and stability profiles are averaged over the horizontal area where GW packets propagate. Vertical wavelength profiles are plotted for GW packets propagating above \(z = 15\) km and are categorized depending on the ranges of phase speeds in the direction of wavenumber.](image)
are horizontally averaged over four regions [(20°–10°S, 130°–145°E), (20°–15°S, 110°–130°E), (18°–10°S, 120°–130°E), and (30°–18°S, 110°–120°E)] considering the propagation paths of GWEs, GWWs, GWNs, and GWSs. For the background wind, the $\pi$ profile is plotted in the case of GWEs and GWWs since their $\lambda_z$'s are not affected by $\bar{v}$. For a similar reason, the $\bar{v}$ profile is plotted in the case of GWNs and GWSs. The zonal wind is characterized by strong westward flow with its maximum near $z = 50$ km. The meridional wind is weak compared with the zonal wind, but moderate northward flow exists at around $z = 80$–90 km. The stability has a large vertical gradient near the tropopause level ($z = 15$ km), but it is nearly constant with height in the middle atmosphere.

One interesting point found in Fig. 2 is that the $\lambda_z$ is reduced near the tropopause regardless of the direction of $k_z$. This reduction of $\lambda_z$ is due mainly to the vertical increase of the stability near the tropopause. Although $\lambda_z$ can be affected by wind, the effects of wind are likely to be small because wind does not vary rapidly with height near the tropopause. In the case of GWEs (Fig. 2a), the $\lambda_z$'s increase with height and reach maximum values near $z = 50$ km, due to Doppler shifting by the easterly flow. As a result, GWEs propagate fast in the vertical direction around $z = 50$ km, as shown in Fig. 1. Above $z = 70$ km, the $\lambda_z$'s of GWEs are again reduced owing to Doppler shifting by the zonal wind with eastward vertical shear, and some GWs ($c_{px} = 0$–25 m s$^{-1}$) approach their critical levels around $z = 90$ km. Around $z = 80$–90 km, moderate northward flow exists, and this flow advects the GWEs northward, as shown in Fig. 1. In the case of GWWs, all waves approach critical levels around $z = 15$ or $35$ km. Near the critical levels, they have quite small $\lambda_z$'s, and thus their $c_{px}$'s are also small. This accounts for the dominance of the horizontal propagation of the GWWs shown in Fig. 1.

In case of GWNs and GWSs, the vertical variation of $\lambda_z$ increases upward from $z = 40$ km. At $z = 40$–55 km, GWs are Doppler shifted by the meridional wind with weak southward vertical shear, and thus the $\lambda_z$ of the GWNs (GWSs) increases (decreases) with height. Above $z = 60$ km, some of the GWNs are filtered by the wind (e.g., GWs with $c_{py} = 0$–35 m s$^{-1}$). On the other hand, the $\lambda_z$ of GWSs increases suddenly with height above $z = 60$ km owing to Doppler shifting, and as a result, the GWSs propagate fast in the vertical direction near 25°S (see Fig. 1). In the case of GWNs and GWSs, $c_{gx}$ is simply equal to $\bar{v}$ since $k = 0$ [see (12)], but in the case of GWEs and GWWs, $c_{gy} = \bar{v}$ since $l = 0$. Therefore, since $\bar{v}$ is generally larger than $\bar{v}$, the zonal propagation of GWNs or GWSs can be significant compared with the meridional propagation of GWEs or GWWs. This accounts for the zonally confined propagation of the GWEs and GWWs.

b. Comparison with column-based parameterization

In this section, the results of the LS parameterization are compared with those of the CS parameterization. For this, three one-month simulations (LS, CS, and CSRW) are carried out for January and July 1979 using WACCM1b with the two parameterizations. The LS and CS simulations are conducted using the model with the LS and CS parameterizations, respectively. In the CSRW simulation, the CS parameterization is used together with the criteria of the reflection and the WKB violation used in the LS parameterization (see section 2c); thus, the processes involved in representing the effects of GWs in the CSRW simulation become identical to those in the LS simulation. In these simulations, feedback between the GWDC and flow in the model is also ignored, as in the one-day simulation.

Figure 3 demonstrates latitude–height cross sections of zonal-mean zonal wind and GWDC in the three one-month simulations for January and July. The zonal-mean zonal wind shown in Fig. 3 exhibits the characteristic features of the zonal wind observed in solstice seasons such as the hemispheric difference in the strength of the polar night jets, zonal wind reversal near the mesopause level, and the cross-equatorial westerly flow from the winter mesosphere to the summer thermosphere (see Swinbank and Ortland 2003). The overall structures of the zonal-mean GWDC obtained from these three simulations are similar to one another, but the magnitude of the GWDC in the LS simulation exhibits the following clear differences from the magnitude in the CS simulation: (i) in the troposphere, the eastward GWDC in the equatorial region is much larger, (ii) in the lower stratosphere, the GWDC is at least 3–4 times stronger in the midlatitude regions as well as in the equatorial regions, (iii) near $z = 75$ km, the westward GWDC in the midlatitude winter hemisphere is reduced, and (iv) in the equatorial lower thermosphere ($z > 100$ km), the eastward GWDC is significantly reduced, and the reduced eastward GWDC is extended to the winter hemisphere.

Among these differences, reasons for the reduction of the GWDC in the LS simulation can be revealed from the CSRW simulation. A comparison of the two simulations shows that the reduction of the GWDC in the LS is similarly found in the equatorial lower thermosphere and near $z = 75$ km in the midlatitude winter hemisphere in the CSRW. This result indicates that the reduction of the GWDC in the LS is related to the elimination of packets through the criteria of the reflection and the WKB violation. However, in contrast
to the LS, an increase in the GWDC does not appear in the CSRW. Given that the processes used to parameterize the effects of GWs are identical in the two simulations, the increase in the GWDC should be attributed to the GW propagation properties considered in the LS simulation. Here, it should be noted that the reduction of the thermospheric GWDC in the LS and CSRW may not be physically very reasonable, since it is the consequence of too strict application of the WKB violation to avoid singularities and the use of the empirical (but arbitrary) reflection condition (see section 2c). In practice, the ray theory might possibly perform well except for some cases even if there are theoretical singularities due to the WKB violation and reflection (e.g., Sobel and Bretherton 1999; Harnik 2002; Sartelet 2003).

Figure 4 demonstrates latitude–height cross sections of the monthly and zonally averaged energy-weighted mean age of GW packets with the eastward or westward $F_{p_n}$ in January and July. The age of the GW packet is defined as the time since launch and is saved at model grid points using the gridding process presented in section c of the appendix. Energy weighting is used to consider that properties of GW packets with larger amplitudes contribute more to the averaged properties of GWs. Time and zonal averaging is accomplished using mean age at grid points (square-marked points in Fig. A2). Stippled areas denote regions in which no GW packets are found in the zonal direction for the period of 1 month.

Overall, the ages of GWs increase with height since GW packets propagate upward from tropospheric convection. The values of the ages indicate that it can take 2–13 h (7–15 h) for GW packets to reach $z = 50$ km (100 km), and they are much larger than the time step size of the current model (30 min). This result is another evidence of the need for the Lagrangian approach in the GWD parameterization. Comparison of the ages of GWs with the GWDC shown in Fig. 3 indicates that...

**Fig. 3.** Latitude–height cross sections of monthly and zonally averaged zonal wind and zonal GWDC in three 1-month simulations (LS, CS, and CSRW) without feedback between GWDC and flow in the model for (a) January and (b) July. Contour intervals for zonal wind and GWDC are 10 m s$^{-1}$ and 10 m s$^{-1}$ day$^{-1}$, respectively, but in case of GWDC, the contours of $0.3$, $1$, $0.3$, $0.1$, $0.03$, and $0.01$ m s$^{-1}$ day$^{-1}$ are also plotted. Negative values are shaded, and zero lines are omitted.
the ages of GWs with eastward (westward) $F_{px}$ increase rapidly in the vertical direction near the regions of strong eastward (westward) GWDC. In January, for example, rapid vertical increase of the ages of GWs with westward $F_{px}$ is found near the regions (30°S and 30°N) of the westward GWDC around $z = 25$ km. Such rapid vertical increase of the ages of GWs indicates that the vertical propagation of GWs suddenly slows down. Given that $c_{gc} \approx \lambda^2_z$ in the hydrostatic and Boussinesq flow system, most of the strong GWDC is expected to be induced mainly by GW packets with small $\lambda_z$'s.

Figure 5 shows latitude–height cross sections of monthly and zonally averaged energy-weighted mean vertical wavelengths of GW packets in January. GW packets with westward $F_{px}$ have quite small $\lambda_z$'s (<700 m) in the lower stratosphere near 50°–25°S and 30°N, in the equatorial stratosphere near 5°S and 15°N, and in the high-latitude regions (65°N). The $\lambda_z$'s of GW packets with eastward $F_{px}$ are quite reduced below $z = 10$ km near 30°S and 30°N, in the lower stratosphere near the equator, and near the equatorial stratopause level. A comparison with the GWDC shown in Fig. 3a indicates that regions of small $\lambda_z$'s of GW packets with westward (eastward) momentum flux correlate well with regions of locally strong westward (eastward) GWDC. This result is also consistent with the expectation from the vertical variation of ages of GW packets shown in Fig. 4.

However, the small-$\lambda_z$ regions (Fig. 5) also agree roughly with the regions of the local maxima of GWDC in the CS and CSRW simulations. This indicates that the small-$\lambda_z$ regions do not explain the increase of the GWDC in the LS simulation. Regarding the increase in the GWDC, it should be noted that GW packets in the LS simulation can be concentrated in some regions since GW propagation is explicitly calculated. If a large number of GW packets are concentrated in some regions, some filtered packets among the packets accu-
mulated in those regions will enhance wind shear, and then the enhanced wind shear will increase the vertical wavenumbers of many remaining packets. As a result, the many remaining packets can be more likely to become unstable and can produce stronger wave-induced force in those regions. To confirm such GW-packet accumulations, the number of packets \( n_w \) with westward and eastward \( F_{px} \) \( n_{ww} \) and \( n_{ee} \) is examined and shown in Fig. 6. The values of \( n_w \) are obtained by counting GW packets at grid points lying along the paths of packets during every model time step. Such grid points are found using the gridding process described in section c of the appendix. The \( n_w \) in the CSRW is also plotted in Fig. 6 for comparison.

In contrast to the vertical wavelength (Fig. 5), the \( n_{ww} \) exhibits a structure that can be related to the increase in GWDC. In the stratosphere, a large value of \( n_{ww} \) is present near 50° and 25°S–0° at \( z = 20–30 \) km and near 15°N at \( z = 45 \) km, and these regions roughly correspond to most of the regions of the increased westward GWDC (Fig. 3a), except for the westward GWDC near 35°N, 22 km. Around 35°N, 22 km, the value of \( n_{ww} \) is not large, but it has a local maximum that can be related to the local increase of the westward GWDC. Above the stratopause (\( z > 50 \) km), the \( n_{ww} \) has two local maxima near 40°N, 75 km and 55°N, 65 km, and these two regions coincide roughly with the regions of the maxima of the westward GWDC (Fig. 3a). The values of \( n_w \) in these regions are larger than those in the CSRW, which results in the larger GWDC near 40°N, 75 km and 55°N, 65 km in the LS than in the CSRW.

In case of GW packets with the eastward \( F_{px} \), they are concentrated in two regions of 20°–10°S, 75–85 km and 5°S–10°N, 60–70 km. These regions roughly agree with two regions of the strong eastward GWDC in the mesosphere (Fig. 3a). As in case of \( n_{ww} \), the values of \( n_{ee} \) are somewhat larger in such regions in the LS simulation, and thus the eastward GWDC is stronger than in the CS. In the troposphere, large values of \( n_{ww} \) and \( n_{ee} \) are present over a wide range of latitudes (60°S–60°N). However, the values of \( n_{ee} \) are somewhat larger than those of \( n_{ww} \) near 30°S, 10 km and 5°N, 10 km, and these two regions coincide with the regions of increased eastward GWDC in the troposphere (not shown).

In the LS and CSRW simulations, the number of launched GW packets is almost the same, and GW packets are removed from the model atmosphere in the same way. Nonetheless, the values of \( n_w \) are locally larger in the LS simulation. As shown in Figs. 4–6, this difference should be attributed to the convergence of packets with small \( c_{gz} \) in the LS simulation.

Figure 7 demonstrates latitude–height cross sections of monthly and zonally averaged three-dimensional divergence and vertical group velocities of GW packets with \( c_{gz} < 5 \) m s\(^{-1} \) and westward and eastward \( F_{px} \) in January. The divergence is written as follows:

\[
(a \cos \phi)^{-1} \left[ \partial_x(n_w c_{gx}) + \partial_y(n_w c_{gy} \cos \phi) \right] + \partial_z(n_w c_{gz}).
\]  

Note that this divergence is a derived quantity in that its calculation involves the horizontal and vertical derivatives of gridded output variables \( c_{gx}, c_{gy}, c_{gz}, \) and \( n_w \). Therefore, the divergence at a grid point can be affected by missing values at neighboring grid points, and thus stippled regions for the divergence are more widespread than stippled regions for \( c_{gz} \).

As shown in Fig. 7, GWs with westward \( F_{px} \) converge throughout broad regions around \( z = 15–30 \) km from
80°S to 60°N, and such convergence also occurs near \( z = 45 \) km in the tropics and \( z = 60-80 \) km in the Northern Hemisphere midlatitudes. Overall, regions of the local convergence correspond to those of strong westward GWDC (Fig. 3a) and a large number of GW packets with westward \( F_{px} \) (Fig. 6a) except near 50°S, 25 km. Around 50°S, 25 km, a localized peak of the convergence does not exist. However, in this region, convergence still appears even if the criterion of \( c_{gz} \) is reduced to a few tens of centimeters per second (not shown). This implies that the westward GWDC and large number of GW packets near 50°S, 25 km are due to GWs with very small \( c_{gz} \). In this case, GWs can remain within a model vertical layer for a long time, and the convergence of such GWs might not be correctly calculated using (20).

Strong convergence of GW packets with eastward \( F_{px} \) occurs in the troposphere, near the equatorial stratopause level and in the lower thermosphere. These convergence regions also correlate with regions of strong eastward GWDC (Fig. 3a). Correlations of this convergence with the number of GW packets are not as clear in the lower thermosphere as in the equatorial stratopause level. This indicates that the extension of the eastward GWDC to the winter mesosphere in the lower thermosphere in the LS parameterization is the result of the convergence of sparsely distributed GW packets with eastward \( F_{px} \).

A close inspection of the vertical gradient of \( c_{gz} \) demonstrates that the positive (negative) vertical gradient of \( c_{gz} \) correlates roughly with the wave divergence (convergence) region shown in Fig. 7. The correlation indicates that wave convergence regions appear as the upward movement of GWs slows down. Given that wave convergence is related to the structure of the increase of the GWDC, the local vertical accumulation of the GW packets can explain the difference between LS and column-based parameterizations. However, these re-
results do not imply the relative importance of the vertical propagation of GWs to their horizontal propagation since the divergence calculated using (24) is not capable of explaining the origin of the vertically accumulated GW packets. In fact, the horizontal propagation characteristics of GWs have important effects on the structure and magnitude of GWDC. One example is presented in the discussion section (see Fig. 12).

c. Impacts in climate simulations

In this section, we examine impacts of the explicit consideration of GW propagation in the climatology and variability of WACCM1b through 7.5-yr LS and CSRW simulations. The two simulations start from an identical initial condition (1 July 1978) and are driven by climatological ozone and sea surface temperature. In contrast to previous simulations, feedback between the model and parameterizations is considered in these two simulations. For validation of model climatology and variabilities, we use Upper Atmosphere Research Satellite (UARS) Reference Atmosphere Project (URAP) zonal-mean zonal wind from 1992 to 1998 (Swinbank and Ortland 2003) and 40-yr European Centre for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (Uppala et al. 2005) data from 1978 to 2001. The URAP and ECMWF data are used above and below the stratopause level, respectively.
Figure 8 shows latitude–height cross sections of zonal-mean zonal wind in the observations (URAP and ECMWF) and the LS simulation with feedback, and zonal wind difference between the 6-yr CSRW simulations with feedback and observations (CSRW–URAP and CSRW–ECMWF) and between the LS and CSRW simulations (LS–CSRW). For observations, the URAP and ECMWF data are used above and below \( z = 45 \text{ km} \), respectively. Contour intervals for zonal wind, wind bias (CSRW–URAP and CSRW–ECMWF), and wind difference (LS–CSRW) are 10, 5, and 2 m s\(^{-1}\), respectively, and negative values are plotted with dotted lines. Shading denotes regions where biases in the CSRW simulation are reduced in the LS simulation.

Figure 8 shows latitude–height cross sections of zonal-mean zonal wind in the observations (URAP and ECMWF) and the differences between CSRW and observations and between LS and CSRW simulations in January and July. The model climatology is made by averaging results over 6 yr after a spinup period of 1.5 yr. The zonal-mean zonal wind in the LS simulation is similar in magnitude and structure to the URAP and ECMWF wind, although the zonal-mean zonal wind in the model exhibits some unrealistic features such as an upright polar night jet, a too-strong mesospheric subtropical westerly jet, rapid reversal of zonal wind in the winter mesosphere, and the downward shift of the equatorward intrusion of the westerly flow from the polar night jet.

Shading in Fig. 8 denotes regions where wind biases of the CSRW simulation with respect to the URAP and ECMWF are reduced in the LS simulation. Shaded regions are located near the equatorial stratosphere, some regions in the equatorial mesosphere, mesospheric subtropical westerly jet regions, and near the axis of the polar night jets. Large reductions of biases are found in the equatorial stratosphere. The largest bias in the equatorial stratosphere is about \(-15 \text{ m s}^{-1}\) \((-40 \text{ m s}^{-1}\)) and appears at \( z = 35 \text{ km} \) (45 km) in January (July). These biases are alleviated by the positive differences between the LS and CSRW simulations. However, the largest positive differences (14 m s\(^{-1}\) in January and July) are located at somewhat lower altitudes compared with the negative bias re-
regions, resulting in somewhat weak easterly flow near \( z = 30 \) km in the equatorial regions. In the mesospheric subtropical jet regions, biases reach about 20 and 15 m s\(^{-1}\) in January and July, respectively, and are reduced by a negative difference of about \(-6\) and \(-4\) m s\(^{-1}\), respectively.

Figure 9 demonstrates latitude–height cross sections of the zonal-mean GWDC in the LS and CSRW simulations and their differences in January and July. The overall structures of the GWDC in the two simulations are similar to each other, but clear differences in the magnitude of the GWDC are found, as in the one-month simulations. In the LS simulation, the eastward GWDC is enhanced in the equatorial troposphere and lower stratosphere, the westward GWDC is increased in the subtropical and midlatitude lower stratosphere. However, in contrast to the one-month simulations, the westward GWDC near \( z = 60\)–70 km in the winter mid- to high-latitude regions is somewhat increased. This difference is likely related to the fact that such westward GWDC maximum in climate LS simulation is located \(~3\)–5 km higher than in the CSRW simulation. The GWDC at higher altitudes can be larger since GWDC is inversely proportional to the air density decreasing exponentially with height.

Figure 10 demonstrates time–height cross sections of monthly and zonal-mean stratospheric zonal wind around the equator in the ECMWF data from January.
1978, and the LS and CSRW simulations for a period of 6 yr. The ECMWF data clearly exhibit the SAO in the upper stratosphere and the QBO in the lower stratosphere. In the model results, the SAO is found, but no QBO signal exists, irrespective of the parameterizations used. Although the SAO is simulated in the model, the SAO's temporal variation is quite regular in contrast to that in the ECMWF. This regular SAO is attributable to the absence of the modulation due to the QBO (see Garcia et al. 1997). In addition, the SAO in the LS simulation descends down to $z = 35$ km, but this feature is somewhat unrealistic compared with observations. This strong descent of the SAO can also be related to the absence of the QBO (Giorgetta et al. 2006).

As is also demonstrated by SCGB07, interannual variabilities in the equatorial lower stratosphere are increased in the model with convective GWD parameterizations. In the LS simulation, such interannual variabilities are far more enhanced, and in addition, they exhibit a structure that can be related to the QBO westerly phase. In the ECMWF data, the easterly phase of the SAO tends to be weak (its magnitude is less than $5 \text{ m s}^{-1}$) between two westerly phases of the SAO before the onset of QBO westerly phases near $z = 35$ km at 18–24 and 48–54 months. A similar tendency also appears near $z = 33$ km at 18–24, 54–60, and 66–72 months in the LS simulation. During these three time periods, two eastward GWDC phases are merged, and the merged eastward GWDC descends down to the troposphere (Fig. 11).

4. Discussions

In the LS parameterization, the horizontal propagation depends on the initial propagation direction of GWs at launch level. As an example, the meridional propagation can be more enhanced by rotating the initial propagation direction ($\varphi$) $45^\circ$ counterclockwise (see LS45 in Fig. 12) because in this case GW packets with zonal $F_{pr}$ have nonzero $l$ and can have a meridional group velocity larger than $\vec{v}$. The zonal-mean GWDC shown in Fig. 12 is obtained from the one-month simulation without feedback for January. The GWDC for the LS in Fig. 12 is identical to that in Fig. 3. In the LS45, GWDC is more spread out in the latitudinal and vertical directions, and in addition, its magnitude is increased. Especially, the GWDC is significantly enhanced in the stratosphere. This result indicates that GWDC obtained from the LS parameter-
zation is essentially dependent on horizontal propagation characteristics of GW packets. In a practical sense, this result indicates that appropriate choice of propagation directions can be important in the implementation.

In the LS parameterization, GWs are assumed to approach critical levels when their $z'$ are less than 100 m. This criterion is somewhat arbitrary, but is in practice necessary for numerically calculating momentum deposition near critical levels using (20) and (21) because the vertical displacement of GW packets can become zero at critical levels. To examine sensitivity to $z'$ criteria for the critical-level filtering, the $z'$ criterion is set to 1 km, and an additional one-month simulation without feedback (LS1KM) is carried out. The magnitude and structure of zonal-mean GWDC in the LS1KM shown in Fig. 12 are quite similar to those in the LS simulation. This implies that the GWDC is not sensitive to the criteria for the critical-level filtering as long as $z'$ criteria between 100 m and 1 km are used.

The current LS parameterization launches about 200 convective GW packets upward from deep convection at every model time step and calculates the propagation and dissipation of the GW packets generated even at previous, as well as current, instants of time. As a result, this parameterization requires a significant amount of computation (about 2–3 million GW packets) at every model time step, and in fact makes the model about 5–6 times slower than when column-based parameterization is used. Practically, it would be useful to decrease the number of GW packets by reducing the launching frequency and then to appropriately scale up the magnitude of reference-level GW momentum. A series of one-month simulations with feedback can provide information on how much the momentum can be increased (see Fig. 13). Figure 13 indicates that when the launching frequency is reduced to 12 per day from 48 per day, the magnitude of maximum or minimum zonal-mean GWD is reduced by about half. Therefore, although the frequency is 4 times reduced, it would be possible to obtain similar results to when the frequency is 48 per day if the reference-level momentum is doubled. Although reducing launching frequency looks somewhat arbitrary, it is necessary to note that launching GW packets at every model time step can also be unrealistic because convective GWs are not expected to be always generated whenever convection exists.

The LS parameterization presents an advanced approach in GWD parameterizations in that it can give substantial realism in the propagation of the parameterized GWs. However, this parameterization is formulated based on several assumptions and thus may include several physical inconsistencies associated with the assumptions.

First, the ray-tracing Eqs. (12)–(17) are formulated without considering spherical geometry, and thus they may not be valid in a case in which GWs propagate a great horizontal distance. In fact, given that GWs can propagate in the horizontal direction as far as 20° as shown in Fig. 1, effects of Earth’s curvature can be important, and results obtained from the ray-tracing Eqs. (12)–(17) may include errors. To consider curvature effects of Earth, the ray-tracing equations should be reformulated in spherical coordinates. However, given that most of current ray theories highly depend on a simple geometry such as Cartesian coordinates,
this reformulation would require a more generalized ray theory. This formulation and development of a generalized theory should be important parts of continuing researches.

Second, the current LS parameterization may not respect the angular momentum constraint (Shepherd and Shaw 2004; Shaw and Shepherd 2007) since GW packets can disappear without momentum deposition as described in section 2. According to an additional 1-month simulation (not shown) where GWs are made to deposit momentum before they disappear, GW packets disappear mainly in association with escape through the top boundary ($z = 150 \text{ km}$) and vertical reflection over the wide regions of the tropical stratosphere (see section 2c for the conditions for the termination of ray integration). It is found that GWs escaping the model top can produce zonal-mean GWDC of 50–60 m s$^{-1}$ day$^{-1}$ near the top boundary. Although this GWDC appears in localized regions in the tropics, its magnitude is comparable to that of GWDC in the mesosphere in the LS simulation. Besides, if GWs disappearing due to the reflection condition can keep propagating upward, they would either enhance the GWDC in the middle atmosphere or produce stronger GWDC in wider latitudinal range near the top boundary. In this regard, effects of GWs disappearing without momentum transport cannot be ignored. Therefore, due care should be taken in judging the reality of the impacts of climate LS simulation. To obtain more realistic impacts of the LS parameterization, physically more reasonable treatments of the GW propagation are required and should be taken into account in the future version of the LS parameterization.

Third, $c_{gz} A$ remains constant following GW packets in the absence of nonconservative forcing, but this constancy may not be in general valid since action flux equation [(18) or (23)] does not include two forcing terms $A \partial c_{gz}/\partial t$ and $-c_{gz} A c_{gz} \mathbf{v} \cdot \mathbf{l}$, where $\mathbf{l} = c_{gx} c_{gz} \mathbf{i} + c_{gy} c_{gz} \mathbf{j} + \mathbf{k}$, and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors in the zonal, meridional, and vertical directions, respectively. The term $A \partial c_{gz}/\partial t$ can be ignored for the steady background flow, but $-c_{gz} A c_{gz} \mathbf{v} \cdot \mathbf{l}$, related to the geometry of ray paths, may become important. In fact, for ray paths shown in Fig. 1, it is found that the relaxation time scale $(c_{gz} \mathbf{v} \cdot \mathbf{l})^{-1}$ for the forcing term can be quite short (3–6 h). The time scale is calculated by gridding the irregularly distributed group velocities. However, as long as ray paths are individually calculated for GW packets as in the LS parameterization, it is not possible to directly evaluate the term $-c_{gz} A c_{gz} \mathbf{v} \cdot \mathbf{l}$, which requires the information of neighboring rays. To explicitly consider the forcing term, the initial horizontal distribution of parameterized GW packets is required, and the LS parameterization will need to be reformulated so that it can use the ray-tubing technique as in Broad (1999).

Fourth, in the LS parameterization, the horizontal derivative of the background flow is ignored in the ray-tracing equations, even though GWs can propagate horizontally through several model grid boxes. If the horizontal derivative of background flow is considered for a complete calculation, the GW pseudomomentum can be changed by the horizontal refraction, and the change in momentum will appear as a force in remote regions (Bühl er and McIntyre 2003). This remote momentum deposition in GWD parameterizations has been investigated (e.g., Hassler and Warner 2006), but reliable methodologies are not yet proposed.

Finally, in the LS parameterization, several important considerations are ignored for simplicity, but they should be included in the future version of the LS parameterization: 1) The hydrostatic and Boussinesq dy-
dynamic framework can be replaced so that the effects of nonhydrostaticity or compressibility of the atmosphere can be considered. 2) If the dynamic framework is changed, other GW saturation schemes (e.g., Fritts and Rastogi 1985; Hines 1988) should be used for consistency. 3) Physical dissipative processes (e.g., radiative damping or turbulent diffusion) need to be explicitly considered, as in Marks and Eckermann (1995). 4) The size of convective GW packets (see Alexander 1996) needs to be appropriately determined to exactly calculate grid-volume averaged GW-induced force.

5. Summary and conclusions

A Lagrangian spectral parameterization of gravity wave drag (GWD) induced by cumulus convection (GWDC) is developed using the phase speed spectra of the momentum flux and horizontal wavenumber of convectively forced gravity waves (GWs; Song and Chun 2005), and implemented in the NCAR Whole Atmosphere Community Climate Model version 1b (WACCM1b). This Lagrangian parameterization presents a new way to give substantial realism to GW propagation treated inappropriately in the conventional column-based GWD parameterization by explicitly calculating the three-dimensional propagation of GW packets using the ray-tracing equations.

In the present study, we compare the results of Lagrangian spectral (LS) GWDC parameterization with those of column-based spectral GWDC parameterization presented by SCGB06, and examine the impacts of the LS parameterization on the climatology and variabilities of WACCM1b. One-day GW-packet trajectories demonstrate that the Lagrangian parameterization calculates reasonably well the propagation of GW packets, and GW packets propagate upward along curved paths affected by Doppler shifting and the variation of stability. The GW trajectories also show that the horizontal extent of GW propagation can be as large as 20° when GWs approach their critical levels. For comparison with column-based parameterizations, three one-month simulations without feedback are carried out. Results of the one-month simulations indicate that the overall structure of the GWDC is similar in the three simulations, but the magnitude is quite increased in the lower stratosphere and troposphere in the LS simulation. This difference in magnitude is found to be due mainly to the vertical convergence of GW packets through the analysis of GW properties (wave age and vertical wavelength, the number of GW packets, and the convergence of GW group velocity).

In the climate simulations, it is found that the zonal-mean zonal wind in the equatorial lower stratosphere and near the axis of the polar night jet is improved through the new parameterization. Even in the model with the LS parameterization, the QBO in the equatorial lower stratosphere is not simulated. However, interannual variability in the lower stratosphere is significantly enhanced and exhibits a feature associated with the onset of the QBO westerly.

Finally, sensitivity to the initial GW propagation direction and criteria for the critical-level filtering are examined, and the degradation in the speed of calculation that can be caused by implementing the LS parameterization into the model is mentioned and one solution is suggested. Several issues for important limitations and future improvements of the current LS parameterization are discussed.

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APPENDIX

Considerations for Implementation to GCMs

a. Integration of ray-tracing equations

The ray-tracing equations [(12)–(17)] are numerically integrated using the Runge–Kutta fourth-order scheme with a variable ray time step (Δt) (Press et al. 1992). In this scheme, a significant amount of interpolation is required to obtain model variables at wave-packet positions. In the LS parameterization, linear interpolation is used successively in the vertical, zonal, and meridional directions using six indices of $i_0$, $j_0$, $k_0$, $k_1$, $k_2$, and $k_3$ (see Fig. A1), and the indices are updated using the bisection method (Press et al. 1992) every $Δt$. Since model grid boxes are in general non-Cartesian and non-orthogonal, this successive linear interpolation is not exactly same as the well-known trilinear interpolation. In the parameterization, the ray-tracing equations should be integrated during the period of a model time step (Δt), and thus time-varying $Δt$ is adjusted so that
total integration period does not exceed $\Delta t$. In addition, $\delta t$ is constrained by checking the horizontal displacements of the wave packets in preparation for implementation to parallelized GCMs. This is achieved by reducing $\delta t$ until changes in $i_0$ and $j_0$ do not exceed 2 during $\Delta t$.

b. Parallelization and dynamic data structure

The LS parameterization, in which the horizontal propagation of GWs is explicitly considered, is parallelized in the latitudinal direction using message passing interface (MPI) routines (Gropp et al. 1999). For the parallelization, it is necessary to check whether wave packets can be calculated in a specific latitudinal domain or should be moved to neighboring domains. For this check, the index $j_0$ is used. In the current parameterization, it is not possible to know how many wave packets will be generated and moved to neighboring domains in advance, and therefore it is not convenient to save the properties of the wave packets in arrays with fixed boundaries. To avoid this inconvenience, the parameterization uses a dynamic data structure (Meissner 1998) that can be expanded or shrunk during the run time.

c. Gridding of wave packet information

In the LS parameterization, GW momentum forcing is calculated following wave packets [see (19) and (20)]. However, for interaction with gridded model variables, the forcing obtained from such along-ray calculations should be assigned to model grid points considering the trace of wave packets. Figure A2 accounts for this gridding process for two upward propagating wave packets, initially located at $z = ZB1$ and $ZB2$, respectively. In the case of the westward propagating wave packet, for example, the parameterization first searches for a triangle-marked grid point nearest to the position of the packet that reaches $z = ZT1$, and then finds square-marked grid points by searching downward for grid points between $z = ZB1$ and $ZT1$ from the triangle-marked grid point. If the vertical displacement of a GW packet is larger than a model vertical grid spacing, as illustrated in Fig. A2, forcing is interpolated in the vertical direction to square-marked grid points. Otherwise, forcing at $z = ZT1$ is multiplied by the factor $J_i$ in (19)–(20) and is assigned to a triangle-marked grid point. This gridding process is also used to make gridded outputs of the GW properties (e.g., pseudomomentum flux, action density, wavenumbers, and group velocities), but in this case the factor $J_i$ is not considered.

REFERENCES


——, R. R. Garcia, B. A. Boville, and F. Sassi, 2005: Implementation of a gravity wave source spectrum parameterization dependent on the properties of convection in the Whole At-


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