The Dimensional Characteristics of Ice Crystal Aggregates from Fractal Geometry

C. G. SCHMITT AND A. J. HEYMSFIELD
National Center for Atmospheric Research,* Boulder, Colorado

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ABSTRACT

Ice crystal aggregates imaged by aircraft particle imaging probes often appear to be fractal in nature. As such, their dimensional properties, mass, and projected area can be related using fractal geometry. In cloud microphysics, power-law mass \( m \)– and area \( A \)–dimensional \( D \) relationships (e.g., \( m \propto aD^b \)) incorporate different manifestations of the fractal dimension as the exponent \( b \). In this study a self-consistent technique is derived for determining the mass and projected area properties of ice particles from fractal geometry. A computer program was developed to simulate the crystal aggregation process. The fractal dimension of the simulated aggregates was estimated using the box counting method in three dimensions as well as for two-dimensional projected images of the aggregates. The two- and three-dimensional fractal dimension values were found to be simply related. This relationship enabled the development of mass–dimensional relationships analytically from cloud particle images. This technique was applied to data collected during two field projects. The exponent in the mass–dimensional relationship, the fractal dimension, was found to be between 2.0 and 2.3 with a dependence on temperature noted for both datasets. The coefficient \( a \) in the mass–dimensional relationships was derived in a self-consistent manner. Temperature-dependent mass–dimensional relationships have been developed. Cloud ice water content estimated using the temperature-dependent relationship and particle size distributions agreed well with directly measured ice water content values. The results are appropriate for characterizing cloud particle properties in clouds with high concentrations of ice crystal aggregates.

1. Introduction

The fractal dimension \( D_f \) is an indicator of how much space a mathematically described set occupies near each of its points (Falconer 2003). The fractal dimension has been used in numerous areas of atmospheric science research. The structures of smoke plumes have been studied using fractal geometry (Praskovsky et al. 1996). Fassnacht et al. (1999) used fractal geometry to estimate the specific surface area of pristine snow crystals. The fractal dimension of aggregating aerosols has been used to estimate aerosol optical properties (Xiong and Friedlander 2001).

Fractal particles generally have a random appearance, although the average structural properties can be estimated using \( D_f \). Figure 1 shows ice crystal aggregates imaged by the Cloud Particle Imager (CPI) probe in convective clouds during the Cirrus Regional Study of Tropical Anvils and Cirrus Layers–Florida Area Cirrus Experiment (CRYSTAL-FACE) in July 2002. The crystals have a highly irregular structure, although it can be seen that the aggregates are composed mostly of individual hexagonal crystals. Their irregular appearance is due to the randomness inherent in their growth via the aggregation process and suggests that fractal geometry may be useful for understanding their mass and projected area properties.

Numerous projects have been conducted to investigate the relationship between particle maximum dimension \( D \) and particle mass. A common form for mass–dimensional \((m–D)\) relationships is \( m = aD^b \), where \( m \) is the particle mass and \( a \) and \( b \) are constants. In this form of \( m–D \) relationship, the constant \( b \) is \( D_f \). The fact that \( m–D \) relationships of this form often fit data well suggests that atmospheric ice crystal properties can be represented using fractal techniques. Using aircraft particle observations, Heymsfield et al. (2004) calculated a \( D_f \) of 2.05 for ice crystal populations observed during CRYSTAL-FACE and 2.4 for populations observed during the Atmospheric Radiation Measurement (ARM) project in March 2000. Their technique involved calculating the cloud ice water content (IWC) using the power-law...
format $m-D$ relationship and comparing values to directly measured IWC values. There $D_f$ and the pre-fractal value $a$ were iteratively determined by reducing the error between the calculated IWC and the measured IWC. Brown and Francis (1995) used a similar technique that compared calculated IWC using previously developed $m-D$ relationships to the measured IWC. They concluded that the Locatelli and Hobbs (1974) $m-D$ relationship with a $D_f$ value of 1.9 was useful in cirrus cloud IWC estimates. Locatelli and Hobbs (1974) and Mitchell et al. (1990) photographed ice crystals collected at the surface and then melted the crystals to determine the particle mass. Relationships for different particle habits were determined by calculating the best fit to the observations of several hundred particles collected on different days. The $D_f$ values from these studies ranged from 1.4 for aggregates of unrimed side planes to 3.0 for lump graupel, although values between 1.8 and 2.3 were common.

Relationships between $D$ and particle projected area may also be expressed in terms of $D_f$, although $D_f$ in two dimensions ($D_{2D}$) is different from the $D_f$ value for the three-dimensional extent of an object ($D_{3D}$), which is used in $m-D$ relationships. Mitchell (1996) compiled a list of $D_{2D}$ values ranging from 1.4 to 2.0. Heymsfield and Miloshevich (2003) fit curves to CPI probe measurements of area and $D$ and calculated $D_{2D}$ values between 1.65 and 1.75 for bullet rosette-shaped ice crystals. The Heymsfield and Miloshevich (2003) relationships are in terms of area ratio $A_r$, which is the ratio of the projected area of the particle to the area of the smallest circle that would completely cover the 2D image of the particle. Developing relationships to particle Ar leads to a factor of length squared already being included in their calculation, meaning that the $D_{2D}$ is two plus the power given in their equations (which are negative).

Note that $D_{2D}$ and the $D_{3D}$ are related but not equal: $D_{2D}$ gives a measure of how a structure fills 2D space whereas $D_{3D}$ gives a measure of how a structure fills 3D space. As an example, a circle on a plane fills 2D space uniformly and the relationship between a one-dimensional measure of a circle (radius) and the area is well known. The circle covers area on the plane according to the equation for area ($A = \pi r^2$), which includes radius to the second power, meaning that area is proportional to a one-dimensional measure squared. While not technically a “fractal” dimension, 2.0 is the power that relates the measure of length to the area. A 2D structure that covers less area than the circle with an equal increase in the length measure will have a fractal dimension of less than 2.0. The 3D analog of a circle is a sphere, which increases in volume, the 3D measure of extent, with a function of radius to the third power ($v = 4/3 \pi r^3$). A 3D structure that increases its volume (or mass) with increased size more slowly than a sphere would have $D_{3D}$ of less than 3.0. For 3D fractal particles, the fractal structure is uniform in the three dimensions. With these facts in mind, it should be apparent that $D_{3D}/3 = D_{2D}/2$ if the particle is uniform in all three dimensions. The common practice of equating $D_{3D}$ with $D_{2D} + 1$ is not appropriate with
ice crystal aggregates. Tang and Marangoni (2006) showed mathematically that if \( D_{3D} \) were calculated on a 2D slice through a 3D object, \( D_{3D} = D_{2D} + 1 \) would only be true if every other 2D slice of the structure parallel to the original were identical. For structures such as atmospheric ice crystal aggregates, the mass distribution inhomogeneity is likely to be similar in all three dimensions.

In practice, 2D structures are often represented by 2D images of 3D particles, which hide some of the internal 3D structure. To use \( D_{3D} = 1.5 \times D_{2D} \) as suggested by Tang and Marangoni (2006), it is important to calculate \( D_{2D} \) from slices through the particle rather than from two-dimensional projected images. This is not possible with data from aircraft imaging probes that measure projected images. Uncertainties introduced by calculating \( D_{2D} \) from projected images of 3D structures can be circumvented if the relationship between \( D_{2D} \) and \( D_{3D} \) is consistent across scales.

Mass–dimensional relationships in cloud microphysics have typically been empirically calculated. Ice particle \( m–D \) relationships have been calculated from individual particle data or by using bulk statistical methods. The technique of calculating \( m–D \) properties by collecting, photographing, and weighing ice crystals at the surface has limited utility because measured particles tend to be larger than cloud particles, and falling snow particles at the surface do not necessarily have the same properties as cloud particles higher in the cloud. While the technique employed by Heymsfield et al. (2004) and Brown and Francis (1995) works well for large datasets, this technique has fundamental disadvantages. The \( m–D \) relationship is derived empirically by minimizing uncertainty between calculated IWC values and measured values over a large dataset without taking individual particle properties into account. This process necessarily ignores the fact that there can be differences in the \( m–D \) properties at different locations within the clouds. Additionally, there is information available in the form of particle projected area measurements that can be used to understand the 3D structure of the particles. In this work, a technique will be described that utilizes fractal geometry to calculate \( m–D \) relationships for small cloud particle datasets. This will facilitate the investigation of changes in the \( m–D \) properties in different cloud types and in different parts of clouds, which has not previously been possible.

An additional fundamental weakness in cloud microphysical research is that \( m–D \) and area–dimensional relationships have not been synergistically linked. Area–dimensional and mass–dimensional relationships are rarely developed from the same dataset. There are numerous situations in atmospheric science that require both particle projected area and particle mass. Quantities involving particle terminal velocity require both particle area and mass for accurate estimation. For fractal particles, the 2D and 3D measures of particles are related by the fractal dimension. In this work it will be shown that there is a direct link between particle projected area and particle mass for ice crystal aggregates that will facilitate the development of more realistic relationships for cloud particle properties. Particle projected area–dimensional measurements are used directly to derive particle \( m–D \) relationships through the use of fractal geometry. The derived \( m–D \) relationships will be shown to independently predict the total ice water content when combined with particle size distribution measurements.

The results of this study are applicable to clouds composed mainly of ice crystal aggregates. Aggregates are common in clouds with IWC values higher than 0.1 g m\(^{-3}\). The results of this study should not be used for the top 500 m of a cloud where the aggregation processes are not likely to have created a significant concentration of aggregates.

In this study, \( D_f \) of ice crystal aggregates is investigated. A computer model has been developed to simulate the aggregation process in clouds. Section 2 describes the aggregate simulation program. The results of the simulations are used to better understand the fractal properties of aggregate crystals. In section 3, these findings are used to interpret results from aircraft field projects by using particle size distribution measurements to calculate bulk properties. The results represent a purely mathematical approach to calculate particle dimensional properties. Conclusions are drawn in section 4.

2. Aggregation simulations

Common aircraft cloud particle measurement probes provide 2D projected images of cloud particles. To better understand the relationship between the 2D projections and the 3D structure of ice crystal aggregates, a simulation program was developed to simulate 3D ice crystals. The program was developed to create theoretical ice crystal aggregates that were similar in appearance to atmospheric aggregates and that were created by simulating natural aggregation processes in the atmosphere. In the atmosphere, ice crystals aggregate by differential sedimentation. Larger ice crystals have higher terminal velocities, which allows them to sweep out and capture smaller ice crystals as they fall through a cloud. Computer-simulated aggregates have been used frequently to simulate aggregation of aerosols by Brownian motion (Brasil et al. 1999). The simulation program developed for this study is similar to the aerosol simulation algorithms except that the primary particles are
hexagonal and the aggregation process was designed to simulate aggregation by differential sedimentation.

The simulation program uses individual hexagonal crystals to build aggregates. The aspect ratio of the crystals comprising the aggregate was generally held constant through the simulation of each aggregate crystal, although it could be varied. Tests showed that there was insignificant change in the fractal properties of the simulated aggregates when component crystal size and aspect ratio values were varied during the creation process. Simulated aggregates were created from hexagonal crystals with aspect ratios ranging from length/width of 5.0 (columns) to 0.2 (plates). Figure 2 shows images of simulated aggregates composed of the five different aspect ratio particles. The aggregates are shown with varying numbers of component crystals. In the program, each crystal is represented by the 3D coordinate location of the 12 points defining the corners of the end hexagons.

FIG. 2. Images of simulated ice crystal aggregates. Aggregates are shown at four different stages of growth with 8, 20, 38, or 74 crystals. Aggregates composed of hexagonal crystals with five different aspect ratios are shown.
The simulation program begins with a randomly oriented individual crystal. The program then loops a specified number of times, adding an additional crystal each time to the aggregate. In subsequent loops, the aggregate is centered in the middle of the 3D grid. Cloud particles larger than 30 μm are thought to be oriented by airflow (Cho et al. 1981) with their maximum projected area perpendicular to the airflow. Orientation of the simulated aggregates was accomplished by measuring the projected area for 50 random orientations. The random orientation that resulted in the largest projected area was the orientation that was used to represent the crystal orientation while falling. While it is possible to mathematically calculate this orientation for single crystals, it is much more challenging for large complex aggregates. Therefore, the random orientation method was used at all stages.

The aggregation process was simulated by introducing another crystal at a random location onto the x–y coordinate plane, as particles were assumed to be falling in the z direction. This particle will be referred to as the “collected” crystal. This crystal represents a smaller, more slowly falling ice crystal within the cloud through which the larger aggregate was falling. The projected image of the collected crystal combined with the projected image of the aggregate was used to determine if the particles would contact. The collected crystal was also oriented for airflow, although some randomness is introduced depending on where it was expected to contact the aggregate. It was found, using Fluent computational fluid dynamics (CFD) software (Fluent, Inc., Lebanon, New Hampshire), that the airflow near the edge of a falling particle was highly variable, suggesting that a particle passing nearby would encounter varying forces across the crystal that would cause rotation to occur. The randomness was simulated based on the distance between the center of the aggregate and the collected crystal in the x–y space. Collected crystals expected to contact the aggregate near the center were given a higher degree of orientation than those collected crystals expected to contact the aggregate near the edge. The flow modeling did not show a significant regional preference regarding the location at which the collected crystals would impact the aggregate. The airflow diverted particles radially away from the center of the aggregate, but contacts were still expected to be relatively uniform over the surface.

Once the orientation of the collected crystal was found, the exact location of the new crystal in the aggregate structure was determined by moving the collected crystal toward the aggregate until the first contact. To do this, the collected particle was taken to be distant from the aggregate in the z direction. The distance between each potential contact point between the aggregate and the collected crystal was determined. The collected crystal could contact the aggregate by edge contact: a line segment drawn between two points on the collected crystal and a line segment between two points of any crystal in the aggregate structure could come into contact. Alternatively, contact could also occur by point–face contact: a corner point from either the collected crystal or a crystal component of the aggregate contact with a planar surface of another crystal. Once all potential contact distances were calculated, the crystal was moved the minimum distance necessary to come into contact with the aggregate. At this point, the collected particle becomes part of the aggregate and its location and orientation with respect to the other crystals making up the aggregate are held constant. Particle restructuring, the act of rotating the newly attached crystal around the contact point until a second contact occurred, was not done, as the simulated aggregates were similar in appearance to atmospheric ice crystal aggregates. Restructuring would have led to higher area ratio and higher density aggregates similar to rimed aggregates or graupel. Irregularities in individual crystals were not considered because the irregular structure caused by the aggregation process appeared to dominate over shape effects for the S value calculation.

In general, ice crystal aggregates were simulated until they were composed of about 100 crystals. Information on the properties of the simulated aggregates was collected after each new crystal was added to the aggregate. The data collection function calculated numerous properties of the aggregate from several different orientations to simulate cloud probe measurements. Because the fractal calculations were computationally time consuming, the fractal properties were only calculated when the aggregates contained an even multiple of three crystals. Information was written to an ascii file for later analysis. Additionally, 2D projected images were saved and the array containing the points of crystals that make up the aggregate was also saved.

Here $D_{2D}$ and $D_{3D}$ are estimated using the box counting method. To estimate the fractal dimension of a 2D image, a square grid is superimposed over the image. In this project, the initial size of the squares that make up the grid is two pixels on a side. Next, the number of grid squares that include part of the particle is counted. The number of occupied grid squares and the size of the grid are saved. The grid size is then increased, and again the number of grid squares that include part of the particle is counted. This process is repeated by increasing the grid size slowly until the grid squares reach 15% of the maximum size of the particle. A graph is then created showing the number of grid squares occupied (N) plotted versus the side length G of the grid squares.
A power-law equation of the form $N = aG^{-b}$ is then fit to the data, where $a$ is a constant and $b$ is $D_f$. The $D_f$ results were compared directly to the FracLac plug-in (available online at http://rsb.info.nih.gov/ij/plugins/fraclac/FLHelp/Introduction.htm) for ImageJ software (see http://rsb.info.nih.gov/ij for more information) and found to agree well. The $D_{3D}$ was calculated by extending the above algorithm to 3D by superimposing a three-dimensional grid through the space containing a 3D representation of the object and counting the number of cubes that are occupied for each grid size.

The simulated aggregates tended to have higher area ratio values than those observed in atmospheric data. It was determined that this was the result of the simulated aggregates being formed by adding one crystal at a time while large atmospheric particles are likely created when multiple aggregates come into contact and stick together. In an effort to understand the fractal properties of low-area ratio, large atmospheric particles, it was necessary to simulate the aggregation of aggregates as well. A second program randomly chose between two and five previously saved aggregates and combined them into one large crystal aggregate. The resulting aggregates typically had lower area ratio values more in line with observations. The same data collection function was then used to calculate the properties of the combined aggregate. Figure 3 shows some examples of aggregates of aggregates.

A total of 50 aggregates of about 100 particles each were created for each aspect ratio. As data were collected after the addition of each crystal, a total of 25 000 simulated crystal aggregates were studied; $D_{3D}$ was calculated more than 8000 times along the way and $D_{2D}$ was calculated from projected images taken from three mutually perpendicular views each time $D_{3D}$ was calculated. Figure 4 shows the relationship between the number of crystals and $D_{2D}$ and $D_{3D}$ for aggregates composed of column-shaped (aspect ratio $= 2.0$) crystals. The relationship between $D_{2D}$ and $D_{3D}$ can be expressed as

$$D_{2D} \times S = D_{3D},$$

where $S$ is the scaling factor between $D_{2D}$ of the projected images and $D_{3D}$. Analysis of the data shown in Fig. 4 gave an average $S$ value of 1.30 for aggregates containing more than 25 crystals. Because Tang and Marangoni (2006) suggested that $D_{2D}$ needed to be calculated from slices through the 3D object, $D_{2D}$ was also calculated an additional three times for 2D images created using only the particles that passed through the $x = 0, y = 0$, and $z = 0$ planes. While not true slices, these representations allow the 2D box counting algorithm to see more of the internal structure of the aggregate. Figure 5 shows examples of the 2D projected image and the corresponding 2D slices for several particles. The $D_f$ values calculated for each of the images as well as for the three-dimensional aggregates are shown. The $D_f$ calculated for the 2D slices was significantly lower than that calculated for the projected images. Projected area images hide internal details, making the aggregate appear to be solid, leading to the higher $D_f$ value. The slices show that the internal structure of the particle is more open, which leads to a lower $D_f$ value. Table 1 shows the mean values and the standard deviations of $D_f$ calculated for the various 2D and 3D methods for the five different aspect ratio component particles. Only aggregates containing more than 25 crystals were used in calculating the averages, as $D_f$ varied significantly for simulated aggregates containing fewer particles (see Fig. 4). Note that the ratio of $D_{2D}$ of slices and $D_{3D}$ is closest to 1.5 for unit aspect ratio particles. This suggests that the remaining difference is due to the shape of the crystals that make up the aggregate.

The simulated aggregates were used to estimate the $S$ factor for use with aircraft data. The results of the $S$ value estimates from the program that combined multiple aggregates were found not to differ significantly from
initial simulation results. Table 2 shows the $S$ factor results for aggregates formed by combining aggregates. Data are shown by aspect ratio of component crystal and are grouped by area ratio.

### 3. Application to aircraft data

The information gleaned from the simulated aggregates has been used to directly interpret measurements of particle properties from aircraft data. During the CRYSTAL-FACE and ARM research projects, instruments on the University of North Dakota (UND) Citation aircraft took measurements of particle properties in cloud regions where aggregate particles were common. The clouds encountered by the UND Citation during the CRYSTAL-FACE field project were convective in nature and the particle probe data showed that there were high concentrations of aggregate particles, providing an ideal dataset for this study. The dates of the CRYSTAL-FACE flights in this study were 9, 11, 16, 18, 19, 21, 23, 25, 26, and 28 July 2002. The ARM dataset included midlatitude cirrus clouds generated from large-scale uplift including flights where bullet rosette-shaped crystals were common. The dates of the ARM flights used in this study were 9, 10, 12, and 13 March 2000. A full description of the particle probe dataset is omitted as the dataset has been used frequently and a full description can be found elsewhere (Heymsfield et al. 2004; Schmitt and Heymsfield 2005). The particle measurement probes were mounted to measure a top view of the particles. Field et al. (2006) give a description of a data processing method used to remove artifacts caused by particles shattering on probe inlets. The method removes particles based on particle interarrival times. For both CRYSTAL-FACE and ARM, the Particle Measuring Systems (PMS) 2DC probe was used to measure sub-1000-μm particles. For larger particles in the ARM project, the PMS 2D precipitation (2D-P) probe was used, while during CRYSTAL-FACE the Stratton Park Engineering Company (SPEC) High Volume Particle Spectrometer (HVPS) was used to image large particles. In both projects the PMS forward scattering spectrometer probe (FSSP) was used to measure particles smaller than 50 μm and the counterflow virtual impactor (CVI) was used for measurements of IWC. The CVI operates by slowly emitting dry air ahead of its inlet. Cloud particles larger than 7 μm cross the streamlines and enter the inlet where they are sublimated or evaporated and the residual vapor is measured with a Lyman-α hygrometer (Twohy et al. 1997).

<table>
<thead>
<tr>
<th>Aspect ratio</th>
<th>$D_{3D}$</th>
<th>$D_{2D}$</th>
<th>2D slice $D_f$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>1.99 ± 0.031</td>
<td>1.62 ± 0.036</td>
<td>1.48 ± 0.045</td>
<td>1.23</td>
</tr>
<tr>
<td>2.0</td>
<td>2.17 ± 0.032</td>
<td>1.67 ± 0.025</td>
<td>1.53 ± 0.033</td>
<td>1.30</td>
</tr>
<tr>
<td>1.0</td>
<td>2.23 ± 0.034</td>
<td>1.68 ± 0.026</td>
<td>1.55 ± 0.037</td>
<td>1.33</td>
</tr>
<tr>
<td>0.5</td>
<td>2.22 ± 0.031</td>
<td>1.68 ± 0.025</td>
<td>1.55 ± 0.032</td>
<td>1.32</td>
</tr>
<tr>
<td>0.2</td>
<td>2.16 ± 0.041</td>
<td>1.67 ± 0.029</td>
<td>1.54 ± 0.041</td>
<td>1.29</td>
</tr>
</tbody>
</table>

**TABLE 1.** Three fractal dimension estimates and $S$ ratio for aggregates composed of different aspect ratio crystals.
From these data, the mean particle area ratio was determined for particles imaged in all size bins. Only area ratio values for particles larger than 200 μm were used in the calculation of $D_{32D}$, as smaller particles tend less frequently to be aggregates. To estimate the value of $D_{32D}$, $D_{32D}$ was multiplied by $S$ factors determined for each of the datasets based on the aggregate simulation results. For the CRYSTAL-FACE dataset, a value of 1.30 was chosen by calculating a weighted average of the $S$ values shown in Table 2 based on the occurrence of different area ratio values in the dataset. For the ARM dataset, in which more low area ratio particles were present, an $S$ value of 1.25 was chosen. Bullet rosettes, which were common in the ARM dataset, can resemble the simulated aggregates with 5.0 aspect ratio crystals, making it appropriate that the $S$ value should be lower.

Figure 6 shows the mean area ratio by size bin for the two datasets. The CRYSTAL-FACE particles were larger and generally had higher area ratios than the ARM data. The discontinuity in both datasets near the 1000-μm size is where the 2D probe used to calculate the area ratio is changed. The $D_{32D}$ was determined by fitting a curve to the area ratio measurements shown in Fig. 6 to estimate $D_{32D}$, then multiplying by the selected $S$ value for the dataset; $D_{32D}$ for the CRYSTAL-FACE and ARM datasets was calculated to be 2.22 and 2.20, respectively, when the entire datasets were considered. This value agrees reasonably with the 2.05 (CRYSTAL-FACE) and 2.4 (ARM) values calculated empirically by Heymsfield et al. (2004) for the same dataset.

Thus far, the calculation of the prefractal value has been not considered. In the $m-D$ relationship $m = aD^b$, the prefractal value, which is the constant $a$, is critical to scale the mass. In an attempt to maintain the mathematical rigor of this approach, the prefractal value was estimated based on the area measurements. The area–dimensional relationship developed for each measurement was used to determine what size particle would be expected to have an area ratio of 1.0. It is assumed that this particle would be spherical and therefore should have a density of 0.91 g cm$^{-3}$. Once this size has been determined, the size, the mass of the spherical particle of that size, and $D_{32D}$ were input into the $m-D$ relationship and the $a$ value was solved for. For the CRYSTAL-FACE and ARM datasets, the prefractal values are 0.0068 and 0.0028, respectively. Equations (2) and (3) show the $m-D$ relationship determined using the entire CRYSTAL-FACE [Eq. (2)] and ARM [Eq. (3)] datasets:

$$m = 0.0068D^{2.22}(D > 200 \mu m) \quad \text{and} \quad (2)$$

$$m = 0.0028D^{2.20}(D > 200 \mu m), \quad (3)$$

### Table 2. $S$ factor for aggregates of aggregates composed of different Aspect ratio crystals separated by the area ratio (Ar) of the resulting aggregate. Cells marked “x” indicate that there were too few particles in that area ratio range for that shape to be useful.

<table>
<thead>
<tr>
<th>Aspect ratio</th>
<th>Ar &lt; 0.25</th>
<th>0.25 &lt; Ar &lt; 0.35</th>
<th>0.35 &lt; Ar &lt; 0.45</th>
<th>0.45 &lt; Ar &lt; 0.55</th>
<th>Ar &gt; 0.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>1.22</td>
<td>1.21</td>
<td>1.20</td>
<td>1.20</td>
<td>x</td>
</tr>
<tr>
<td>2.0</td>
<td>x</td>
<td>1.28</td>
<td>1.28</td>
<td>1.28</td>
<td>1.28</td>
</tr>
<tr>
<td>1.0</td>
<td>x</td>
<td>1.30</td>
<td>1.30</td>
<td>1.31</td>
<td>1.31</td>
</tr>
<tr>
<td>0.5</td>
<td>x</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
</tr>
<tr>
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<td>x</td>
<td>1.28</td>
<td>1.27</td>
<td>1.27</td>
<td>1.29</td>
</tr>
</tbody>
</table>

**Fig. 6.** The median area ratio values for 2DC and HVPS cloud probe data for the (a) CRYSTAL-FACE and (b) ARM projects. The light lines show ±1 standard deviation for each size bin.
where $D$ and $m$ are in cgs units. The area ratio to $D$ relationships determined by fitting equations to the median values shown in Fig. 6 for CRYSTAL-FACE [Eq. (4)] and ARM [Eq. (5)] are

$$
Ar = 0.20D^{-0.29} (D > 200 \mu m)
$$

and

$$
Ar = 0.22D^{-0.24} (D > 200 \mu m),
$$

where $Ar$ is the particle area ratio.

In addition to calculating $m$–$D$ relationships for the entire datasets, it was possible to calculate $m$–$D$ relationships for 5-s intervals throughout the research flights. To maximize the quality of the results, data points were eliminated from consideration if the correlation coefficient for the area ratio fit was less than 0.8 or if there were fewer than five occupied size bins larger than 200 $\mu m$. Also, data points were removed from consideration if the temperature was warmer than $-5^\circ C$ or if the IWC was less than 0.01 g m$^{-3}$. Data points that did
not meet the criteria outlined were likely to have too few aggregates to expect fractal relationships to reasonably characterize the data. This left 6048 5-s periods for CRYSTAL-FACE and 2976 intervals for ARM. Several useful trends became apparent upon analysis of these results. Figure 7 shows the $D_f$ values and prefractal values calculated for the two datasets plotted with respect to temperature $T$. The lines represent the 10th and 90th percentiles of the data. The scatter in the results is a function of the variability within the cloud as well as sampling statistics. The datasets contained a wide variety of cloud types and at times included riming and different updraft and downdraft strengths, which likely affected the particle properties. The temperature trends led to higher masses being estimated at lower levels within the cloud for a given particle size. The $m-D$ relationships based on fits to the median values are given below for CRYSTAL-FACE [Eq. (6)] and ARM [Eq. (7)]:

$$m = (0.0102 + 0.00013T)D^{(2.4 + 0.0085T)}(D > 200 \mu m)$$

and

$$m = (0.0064 + 0.000095T)D^{(2.1 + 0.0036T)}(D > 200 \mu m).$$

Figure 8. Probability distribution functions of uncertainty (calculated IWC/measured IWC) for IWC calculations using the single equation form and the temperature dependent form of the $m-D$ relationships defined in Eqs. (2) and (5), respectively, for (a),(b) CRYSTAL-FACE and (c),(d) ARM.
To test the relationships in Eqs. (6) and (7), the ice water content was estimated for each of the data points using measured particle size distributions and the $m$-$D$ relationship determined by the temperature and Eq. (6) or (7). Particle mass was limited to the mass of spherical particles. Heymsfield et al. (2004) showed that the contribution of FSSP particles to the total mass was generally less than 10%. The calculated IWC values were compared to the IWC measured by the CVI. Figure 8 shows the uncertainty (calculated/measured) between the calculated IWC values and the CVI IWC for the datasets using both the temperature-independent approach [Eqs. (2) and (3)] and the temperature-dependent approach [Eqs. (6) and (7)]. Note that the temperature-independent approach tends to underestimate the total IWC in a majority of cases. This is because the area ratio data shown in Fig. 6 is the average for all time periods measured that contained particles in each respective size bin. The largest particles were not always present, but including the area ratios from when they were present artificially flattened the area ratio curve, which caused an increase in $D_{123}$. The estimates based on temperature are based on representative particles in each time period, which allows $D_I$ to be high when the large, higher area ratio particles are present.

Temperature-dependent forms of the area ratio equations for CRYSTAL-FACE [Eq. (8)] and ARM [Eq. (9)] are

\[ \text{Ar} = (0.29 + 0.0035T)D^{(-0.19+0.0056T)}(D > 200 \mu m) \]
and

\[ \text{Ar} = (0.21 + 0.0024T)D^{(-0.35+0.0026T)}(D > 200 \mu m). \]

As the $S$ value was determined using aggregate simulations, the sensitivity of the results was tested by introducing slight variations to the $S$ value. A change in the value of the $S$ factor by ±0.02 led to a change in the estimated IWC of ±11%, which is within the stated uncertainty range of the CVI (Twohy et al. 1997). While ±0.02 might not seem like much, 0.02 represents 1/12 of the entire range of $S$ values estimated for all simulated aggregates containing 50 crystals or more.

4. Conclusions

The mass– and area–dimensional properties of cloud ice particles are of fundamental importance to many atmospheric processes. This study has shown that ice crystal aggregates commonly observed in aircraft microphysical data can be treated as fractal particles. The use of fractal geometry to characterize the dimensional properties of cloud particles has many advantages, as particle projected area and particle mass are directly linked over a large size range through fractal geometry.

The fractal properties of ice crystals have been calculated by analyzing two-dimensional images of particle populations. These fractal properties have been used to successfully calculate three-dimensional properties of ice particles. Simulated ice crystal aggregates were used to interpret the relationship between the 2D and 3D fractal properties. The fractal dimension has been shown to vary with temperature. The $m$–$D$ relationships based on the fractal properties inferred from observations have been shown to predict the total cloud IWC with reasonable uncertainty when aggregates are abundant. These results are useful for clouds containing significant populations of aggregate ice crystals. The mass– and area–dimensional relationships should not be used for particles smaller than 200 $\mu m$, which are less likely to be aggregates.

The strength of this technique is that it uses a purely mathematical approach to go from particle projected area directly to the determination of the particle fractal dimension and then to particle mass. This can be used to determine the properties of ice crystal populations over short time periods. Synergistically linking values of particle area and mass will facilitate more reasonable particle terminal velocity calculations across a larger range of particle sizes.

Future research to improve our understanding of the fractal properties of ice crystals would include better measurements of the 3D structure of aggregate ice crystals over a large size range. Aircraft field projects could include optical imaging probes that image particles from the vertical as well as horizontal which, would improve our understanding of the effects of aerodynamics on the aggregation process.

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