The Detection and Significance of Diurnal Pressure and Potential Vorticity Anomalies East of the Rockies

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ABSTRACT

Harmonic analysis of pressure, temperature, and precipitation data from 1000 Automated Surface Observing System (ASOS) stations reveals a mix of stationary and east–west moving disturbances east of the Rockies. Optimization of the pressure data using a “temperature-based tide assumption” separates a strong sun-following continentally enhanced tide from a smaller eastward-propagating wave (EPW). The latter signal moves at a similar speed to the previously discovered eastward-moving precipitation systems. Analysis of ASOS summer precipitation data confirms eastward propagation, but east of 90°W it shows nonpropagating diurnal convection at a fixed local time (i.e., 1800 LST). Analysis of winter days still finds the EPW, suggesting that it is the cause and not the result of the propagating precipitation.

A possible mechanism for the EPW is developed from the linear Bousinesq equations with heating and wind shear. Solutions show eastward-moving diurnal pulses of potential vorticity (PV) generated by imposed heating over the Rockies. Because of the background shear, these pulses produce vertical motion in the lower troposphere.

The PV hypothesis for precipitation propagation was tested with North American Regional Reanalysis (NARR) data. Diurnal drifting thermal and PV anomalies are clearly found near the 500- and 600-hPa levels in both winter and summer. In winter, the PV signal is weaker, moves faster, and does not influence precipitation. The existence of the winter PV signal again suggests that it is the cause, not the effect, of summer propagating precipitation.

1. Introduction

Diurnally modulated precipitation, either stationary or propagating, is observed at many places around the world. Over continents, summer diurnal propagating precipitation systems are found in tropical Africa, southeast China, and in the United States. In the United States, diurnal convective systems propagate eastward from the Rockies over the Great Plains and Midwest (Wallace 1975; Carbone et al. 2002; Carbone and Tuttle 2008; Laing et al. 2008). In coastal areas, such as the rainy western coast of Colombia, strong diurnal convection propagates westward out to sea as far as 700 km from the Andes ridge at a speed of about 15 m s⁻¹ in August and September. The speed and the spatial range cannot be explained by land breeze but may be a remote response to the strong, locally forced diurnal convection over the tropical Andes through gravity waves (Mapes et al. 2003). In a subtropical coast area like Bay of Bengal, the phase of the daily maximum brightness temperature spreads out southeastward for several hundred kilometers parallel to the coast in summertime with a propagation speed of about 15–20 m s⁻¹, which indicates that the significant diurnal variations of precipitation there are probably caused by propagating gravity waves with deep vertical structure that originate from the diurnal cycle over land (Yang and Slingo 2001). In tropical islands, such as Borneo and Sumatra in Indonesia, the peak in diurnal rainfall migrates from the southwestern coastline and heads inland in the daytime and toward offshore in the nighttime under the background westerly wind in the lower troposphere and the easterly wind in the upper troposphere. The rainfall migrates offshore up to 400 km from the coastline, with an average migration speed of approximately 10 m s⁻¹, and the causal mechanisms are probably replicating convection and gravity waves (Mori et al. 2004).

The candidate mechanisms for precipitation propagation over the Great Plains and Midwest include two
broad categories. The first is cold-pool gravity currents (Carbone et al. 1990; Koch et al. 1991; Moncrieff and Liu 1999; Carbone et al. 2002; Davis et al. 2003; Trier et al. 2010) or trapped gravity waves (Crook 1988) in the planetary boundary layer. However, these mechanisms are either inherently dissipative or dispersive, and conditions may often be unfavorable for the generation or maintenance of either. The other is inertia–gravity wave (IGW) excitation and maintenance by the ensemble of latent heating in the free troposphere (Moncrieff and Miller 1976; Tripoli and Cotton 1989a,b). However, the verification of this concept based on observations is costly and logistically difficult, and so far the results are either inconclusive or negative. Other explanations include the solar-driven diurnal and semidiurnal cycles of surface pressure, which result in large-scale convergence over most of the western United States during the day and east of the Rockies at night, thus suppressing daytime convection and favoring nocturnal convection east of the Rockies (Dai et al. 1999); or “remote action” from upstream such as the creation of upper tropospheric potential vorticity (PV) anomalies by convection over the Rockies that are subsequently advected eastward by upper tropospheric winds (Tripoli and Cotton 1989a,b; Carbone et al. 2002). Perhaps the PV anomaly modifies the environment, which influences the convection (Jirak and Cotton 2007).

In this paper, we will study the diurnal signals over the Great Plains and Midwest in Automated Surface Observing System (ASOS) and North American Regional Reanalysis (NARR) datasets. Section 2 describes summertime eastward-propagating precipitation and surface pressure signals in ASOS. In section 3, we introduce a linear model to explain how the PV pulses drifting eastward could produce vertical motions in the lower troposphere in the presence of vertical mean wind shear. In section 4, we investigate the nature of the diurnal heating in North America and detect the PV generation over the Rockies from NARR data.

2. Surface observations (ASOS)

More than 1000 Automated Surface Observing System stations cover the United States. These systems generally report at hourly intervals with parameters including temperature, dewpoint, surface pressure, sea level pressure, precipitation accumulation, wind vector, etc. Here we will analyze 12 years (1996–2007) of monthly or seasonally organized data for a subdomain of the central–eastern United States (Fig. 1).

a. Harmonic analysis of pressure and temperature

To separate the diurnal component from the hourly observations we utilized harmonic analysis (Mass et al. 1991). The diurnal pressure component is represented with an amplitude and phase:
\[ P_n(t) = C_n \cos \frac{2\pi(t - \psi_n)}{24} + \varepsilon. \]  

(1)

Here, \( P_n(t) \) is the surface observation at local solar time (LST) \( t \) (h) at station \( n \); \( \psi_n \) is the calculated phase angle (°) of the diurnal component for \( P_n(t) \). It is related to the time \( t_{\text{max}} \) at which the diurnal part reaches its maximum: \( t_{\text{max}} = \psi_n \times (24/360°) \) (15° is equivalent to 1 h). The quantity \( C_n \) is the amplitude. A similar analysis is done for temperature. Diurnal pressure and temperature amplitude and phase (in LST) are shown in Fig. 2.

The diurnal pressure and temperature amplitudes increase westward from the East Coast and Midwest to the Great Plains and Rocky Mountains because of the higher Bowen ratio there. The phase of the temperature is constant at 220°; the pressure phase varies slightly between 85° and 105°. Some aspects of these variations have been recently studied by the present authors. The small-scale variation in pressure and temperature amplitude reflects local terrain. Our detailed study of Owens Valley, California, during the Terrain-Induced Rotor Experiment (Li et al. 2009) showed that the local surface pressure amplitude and phase are controlled by the diurnal valley circulation. The large-scale pressure and temperature signal in ASOS (Fig. 2) is caused by a sun-following, continentally enhanced thermal tide (Li and Smith 2010). The average pressure phase is similar to that of the diurnal global atmospheric tide, around 105°, but the amplitude is much larger, (i.e., 50–150 Pa compared to 20 Pa) (Chapman and Lindzen 1970; Forbes and Garrett 1979). The wavy variation in pressure phase (Fig. 2a) is due to a small-amplitude eastward-propagating wave (EPW) discussed in section 2c.

### b. Diurnal variations of ASOS precipitation

Carbone et al. (2002) discovered that in the warm season convective precipitation systems typically originate near the Rockies and propagate eastward with speed of 14 m s\(^{-1}\). To compare with their radar-based analysis, a similar analysis of hourly ASOS precipitation rate is presented here (Fig. 3). In June, the precipitation moves eastward at about 14 m s\(^{-1}\) [i.e., matching Carbone et al. (2002)'s speed] often continuous to 82°W. However, east of 92°W, the diurnal precipitation shows a constant phase around 240° (or 1600 LST), which indicates that the local diurnal thermal forcing is indeed dominant east of 92°W. In an overlap region from 95° to 92°W, both traveling and stationary signals are present. As the two signals are out of phase, the local temporal signal appears to have a semidiurnal (i.e., two maxima per day) character. All these diurnal precipitation signals are weaker in the spring and fall and absent in winter (Fig. 3). In section 2c we describe the diurnal harmonic analysis of precipitation.

### c. The temperature-based tidal assumption

The longitudinal phase distribution for surface pressure shows nearly constant phases over the continent, except for a gentle eastward 7° phase lag between 105°W (with pressure phases there around 99°) and 85°W (with pressure phases there around 106°) (Fig. 2a). This small phase lag over a thousand kilometers requires a phase speed of about 230 m s\(^{-1}\), an unphysical value that arises (as we show below) from a mixing of eastward and westward waves.

Since surface pressure is the integral of the air density above the ground, any temperature perturbation in this column will affect the surface pressure variation.
We assume that the observed diurnal surface pressure variation is the sum of the diurnal sun-following tide and some unknown perturbation, probably imposed from aloft. To separate these signals, we introduce a temperature-based tidal assumption. We assume that the tidal pressure amplitude is proportional to the tidal temperature amplitude with a constant phase lag (Li and Smith 2010). We neglect the contribution by the
global diurnal tide, which has no surface temperature component.

d. Optimization method to remove the thermal tide and reveal the EPW

With our temperature-based tidal assumption, we attempt to remove the tide to reveal other signals. We assume that the tide has the form

$$P_{\text{tide}} = C e^{-i(2\pi/360)\theta} T_D^*,$$  \hspace{1cm} (2)

where $P_{\text{tide}}$ and $T_D^*$ are the complex diurnal pressure and temperature amplitudes associated with a fast westward-propagating, sun-following thermal tide. The coefficient $C$ is the conversion ratio describing how surface heating (K) is converted to pressure perturbation (Pa); $\theta$ is the phase lag between pressure and temperature. The starting longitude for the source is assumed to be at 105°W ($-\lambda_0$). Since $A$, $\psi_0$, and $\mu$ are passive variables, they are not shown here.

![FIG. 4. The value of the cost function $P_{\text{RES}}$ of Eq. (4) for summertime (June) surface pressure observations of the stations in the region bounded by 35°–42.5°N, 105°–78°W. The isolines of $P_{\text{RES}}$ are in thick black contours (Pa). Dashed thin gray contours are the phase speed of the slow propagating wave derived from constant $k$. Here $C$ is the conversion ratio for the amount of surface heating (K) that converted to pressure perturbation (Pa); $\theta$ is the phase lag between pressure and temperature. The starting longitude for the source is assumed to be at 105°W ($-\lambda_0$). Since $A$, $\psi_0$, and $\mu$ are passive variables, they are not shown here.](image)

TABLE 1. The value of the parameters for the local minimum from optimization process.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>June (all stations)</th>
<th>January (all stations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{RES}}$</td>
<td>Pa</td>
<td>19.61</td>
<td>16.2</td>
</tr>
<tr>
<td>$C$</td>
<td>Pa K$^{-1}$</td>
<td>14.95</td>
<td>15.03</td>
</tr>
<tr>
<td>$\theta$</td>
<td>°</td>
<td>106.1</td>
<td>106.3</td>
</tr>
<tr>
<td>$A$</td>
<td>Pa</td>
<td>68.47</td>
<td>21.4</td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>°</td>
<td>-52.5</td>
<td>12.2</td>
</tr>
<tr>
<td>$\mu$</td>
<td></td>
<td>0.089</td>
<td>0.066</td>
</tr>
<tr>
<td>$C_p(-k)$</td>
<td>m s$^{-1}$</td>
<td>24.4</td>
<td>12.2</td>
</tr>
</tbody>
</table>
which is positive for an eastward-propagating wave. From $k$, the usual phase speed is found from
\[
C_p = \frac{\sigma}{2\pi k} = \frac{2\pi R \cos \phi}{360^\circ \Delta x} \frac{86400k}{},
\]
where $R$ is the earth’s radius and $\phi$ is latitude. The quantity $\mu$ represents the decay of the unknown wave eastward away from the source.

To separate the two signals ($P_{\text{tide}}$ and $P_{\text{EPW}}$), we utilized the Nelder Mead Simplex optimization method. We seek the optimum values of $C$ and $\theta$, $A$ and $\alpha_0$, and $k$ and $\mu$ to minimize a cost function
\[
P_{\text{RES}} = \sum (P_{\text{obs}}^{\text{tide}} - Ce^{-(2\pi/360)\theta} \bar{T}_D - Ae^{\mu \times (\lambda - \lambda_0)} e^{(2\pi/360)(k(\lambda - \lambda_0) + \phi_0)}).
\]

The quantity $P_{\text{RES}}$ represents the residual pressure after the removal of the tide and the unknown wave. The term $P_{\text{tide}}^{\text{obs}}$ is the diurnal pressure component. There are three complex parameters to be optimized: the complex ratio of the surface tide pressure and temperature ($Ce^{-i \theta}$), the starting phase and amplitude of the unknown wave ($Ae^{i \alpha_0}$), and the phase speed and the decay length of the unknown wave $[e^{-\mu \Delta x} e^{(2\pi/360)k\lambda}]$. 

FIG. 5. (a) The phase of the diurnal precipitation for ASOS stations in the latitudinal band from 35° to 42.5°N for June. Diurnal analysis is poor between 95° and 90°W because of overlapping signals (Fig. 3a). (b),(c) The results of the optimization procedure for observations in June: (b) the longitudinal phase distribution of $P_{\text{EPW}}$ after optimization and (c) the longitudinal amplitude distribution of $P_{\text{EPW}}$. (d) The longitudinal phase distribution of $P_{\text{tide}}$. The black thin line (with phase speed 12 m s$^{-1}$) in (a) is the linear regression for the stations within 105°–90°W. The dashed gray lines in (b) indicate phase speeds of 24 (12) m s$^{-1}$ for the stations within 105°–75°W (95°–78°W). The dashed black line in (c) is the exponential curve fit to the stations within 105°–75°W. Each circle here represents a station. The size of the circle represents the amount of the daily variations contributed by the diurnal harmonic. The error bar is the difference of the diurnal component for the first (1996–2001) and second 6-yr (2002–07) periods. Terrain height and Normalized Difference Vegetation Index (NDVI) value are given without scales.
The eastward-propagating waves: EPW pressure and precipitation

In this section we compare the diurnal harmonic analysis of precipitation and tide-removed pressure. The optimization process for pressure converges (Fig. 4) to a local optimal solution for surface pressure observation in June over the Great Plains and Midwest (35°–42.5°N, 105°–78°W). Another local optimal solution for \(C \rightarrow 0\) is nonphysical. The values of the parameters in Eq. (4) related to this local minimum are listed in Table 1. The pressure results are compared with precipitation results in Fig. 5. The separated tidal pressure has a nearly constant phase near 105° (Fig. 5d) and an amplitude of 70–100 Pa (see Li and Smith 2010). The EPW part shows an eastward increasing phase, with a phase speed about 24 m s\(^{-1}\) between 105° and 75°W and 12 m s\(^{-1}\) between 95° and 78°W (Fig. 5b). The latter speed (12 m s\(^{-1}\)) agrees with our ASOS-derived precipitation speed in Fig. 3 and with Carbone’s radar-derived precipitation speed. The amplitude of the EPW pressure decays eastward (Fig. 5c).

The phase of the diurnal precipitation signal is shown in Fig. 5a. From 105° to 90°W the signal moves eastward. East of 90°W it has a constant phase of about 270°. The signal is very noisy from 98° to 90°W because the signal is not purely diurnal there (see Fig. 3). In our results, the precipitation phase is about 0° at 100°W. According to Carbone et al. (2002), the storms start at midnight east of the Rockies. The amplitude of the precipitation starts from 0.15 mm h\(^{-1}\) at 105°W and decays eastward (Fig. 3b); in Carbone et al. (2002), the observed rainfall rate is about 0.1–2.9 mm h\(^{-1}\).

The same optimization process was also applied to January data to see if we could separate the tide from an
EPW in the winter season. The optimization process again converges to one local optimal solution (Table 1, Fig. 6). The results show that in January the longitudinal phase distribution is flatter both between 105° and 95°W and east of 82°W. The optimization process gives a phase speed around 22 m s⁻¹ between 105° and 75°W and 14 m s⁻¹ between 79° and 84°W (Fig. 6b, Table 1).

However, in January precipitation data there is no clear eastward-propagating signal (Fig. 6a). For most of the stations, the diurnal harmonic contributes less than 40% of the daily precipitation variation (Figs. 3b–d), which means that the diurnal precipitation is not a dominant signal in wintertime.

Two sensitivity tests were done for the optimization process. First, we repeated the optimization only for stations between 95° and 85°W; the results are given in Table 1 also. Second, we assumed that k in Eqs. (3) and (4) is not constant but changes with longitude k(λ). That is, we fit a polynomial curve with power up to 4 for EPW surface pressure phase instead of the linear regression. The results are given in Table 1 for comparison. The minor differences between these analyses indicate that the optimization method is robust. The tide is distinguishable from the EPW. Both the linear regression for only the Great Plains and Midwest region (95°–85°W) and the polynomial curve fit for most of the continent (105°–75°W) give a much slower phase speed for the EPW surface pressure for the region from 95° to 85°W, which is around 12 m s⁻¹ in June and 15 m s⁻¹ in January.

We conclude that the EPW pressure signal exists all year (Figs. 5b and 6b). In summer, the propagation speed of the EPW pressure signal and the precipitation signal are similar (Figs. 5a,b). In wintertime, there is no diurnal eastward-propagating precipitation signal (Fig. 6a). These results suggest that the precipitation might be caused by the pressure anomaly or that they have a common cause. What is the source of this eastward-propagating pressure signal?

### 3. Linear model

A linear model with a heating-generated disturbance (see the appendix) is introduced here for two purposes: to demonstrate how the heating over the Rockies (Banta and Schaaf 1987) can generate an eastward-moving disturbance and to test the optimization method we used with ASOS data. The Boussinesq formulation is standard (e.g., Smith and Lin 1982; Lin and Smith 1986; Raymond 1986; Reisner and Smolarkiewicz 1994; Robinson et al. 2008; Li and Smith 2010), including background static stability, a Coriolis parameter, westerly wind with shear, and specified diurnal heating. Equations are solved in Fourier space and displayed using fast Fourier transforms and asymptotic techniques. Two patterns of heating are considered: isolated elevated heating and continent-wide PBL heating (Fig. 7, Table 2).

#### a. Diurnal elevated local heating with background shear

In the specified configuration, with local diurnal elevated heating, two types of nonlocal disturbances are generated: inertia–gravity waves and drifting PV pulses. In the midlatitudes, with diurnal frequency $\sigma < f$, heating

<table>
<thead>
<tr>
<th>Local elevated diurnal heating</th>
<th>Continental PBL diurnal heating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heating profile</td>
<td>$B_z(z) = e^{-\beta z}$</td>
</tr>
<tr>
<td>Heating region (Fig. 10)</td>
<td>$3B_0e^{-X^2/2750^2}, a_1 = 100$ km</td>
</tr>
<tr>
<td>Heating level</td>
<td>$Z_2 = H = 4$ km</td>
</tr>
<tr>
<td>Vertical heating scale</td>
<td>$h = \frac{1}{\beta} = 1$ km</td>
</tr>
<tr>
<td>Shear level</td>
<td>$Z_1 = 2$ km</td>
</tr>
<tr>
<td>Environmental stability</td>
<td>$N_1 = 0.0025$ s⁻¹, $0 \leq z &lt; Z_1$;</td>
</tr>
<tr>
<td>Background wind</td>
<td>$U_1 = 2$ m s⁻¹, $0 \leq z &lt; Z_1$;</td>
</tr>
<tr>
<td></td>
<td>$U_2 = 10$ m s⁻¹, $Z_1 \leq z$</td>
</tr>
</tbody>
</table>

Table 2. Linear model setup for two cases. Here, $a_1$ is the radius of the heating region, $h$ the heating depth, $N$ the environmental stability, $U$ the background mean wind, $Z_1$ the shear level, and $Z_2$ the elevated heating level; $U_1$ and $U_2$ are the mean wind below and above the shear level, and $N_1$ and $N_2$ are the stability below and above the shear level.
can generate gravity waves using the background wind and upwind phase speed to increase the intrinsic frequency (\(s_1 U_k f\)) (Crook 1988; Lane and Clark 2002). The IGW perturbations are present above and downwind of the heating and they decay away from the source (Fig. 8b).

The second type of nonlocal disturbance is drifting PV pulses. According to \(\frac{DPV}{Dt} = \left(\frac{f}{pC_p}\right)\frac{dQ}{dz}\), heating functions with vertical gradients will generate potential vorticity. Once generated, pulses of PV will drift downstream. In uniform flow, geostrophically balanced eddies will drift without any associated vertical motions. In the presence of shear, however, the drifting pulses are accompanied by vertical motion in the slower-moving lower layer (Fig. 8) (Raymond and Jiang 1990).

The vertical phase profiles of \(w\) (Fig. 9) show upwind and downward-propagating phases above the heating level (4 km), indicating that the momentum is transported downward and the energy is transported upward. Below the shear level, the \(w\) phase lines are vertical (i.e., no vertical propagation) and the phase speed is eastward.

**b. A test of the optimization method**

The linear model gives us an opportunity to test our optimization method (section 2). To the local elevated heating used above, we add a sun-following inhomogeneous PBL heating with doubled strength on the western half of the continent and no heating over the sea (Fig. 7) to simulate a thermal continental tide and mountain–plain circulation (Li and Smith 2010). The combined response is a tide-only surface temperature signal with a constant phase (270°) and a surface pressure signal that feels both forcings (Fig. 10a). The average pressure minimum is in phase with the temperature maximum (i.e., the tidal response) but with oscillations caused by the elevated heating.

To test our optimization method, we assume that we only know the surface pressure and temperature distribution from the combined calculation. Using the same Nelder–Mead optimization procedure as in section 2, the surface pressure is separated into two parts. As shown in Fig. 10b, the EPW part is almost identical to the pure EPW computed without the interfering tide. Only west of the elevated heating and east of the continent coast do the signals differ, as our assumptions about signal composition are invalid there. This test shows that the optimization methods used in section 2 are reliable.
4. Harmonic analysis of NARR data

The good temporal (3 h) and spatial (32 km) resolution of North American Regional Reanalysis data (Mesinger et al. 2006) gives us a chance to test the PV hypothesis developed in section 3. Although NARR data have been available since October 1978, we will use the 12-yr average from 1996 to 2007 corresponding to the ASOS data.

Before commencing our analysis, a comparison was made between NARR surface data and ASOS data in June as a quality test (not shown). The diurnal surface pressure and temperature amplitude and phase agree well between the two datasets whereas precipitation amplitude and phases differ wildly between ASOS and NARR. We conclude the NARR should not be used for diurnal precipitation analysis. Other fields, such as vertical velocity, should be used with caution.

a. The nature of diurnal heating over the continental United States

The surface energy budget from NARR shows that in summertime the Rocky Mountain region is an elevated heat source. The daily mean of sensible heating is about 300 W m\(^{-2}\) and the amplitude of the diurnal component about 180 W m\(^{-2}\). In wintertime, the Rocky Mountains are still a diurnal heat source, but a much weaker one. The daily mean and the diurnal amplitude of the ground sensible heating are about 200 and 50 W m\(^{-2}\). The diurnal heating amplitude is about 30% of that of the summertime.

The diurnal evolution of the planetary boundary layer height shows that in June the diurnal range of the PBL top is between 500 and 5900 m AGL (Fig. 11a), which means that the PBL top is about 500 hPa in the afternoon for the highest part of the Rockies between 106°...
and 104°W. In wintertime (Fig. 11b), the PBL top is about 600 hPa (~3500 m) in the afternoon for the greater part of the Rockies west of 105°W.

b. The detection of PV in midtroposphere

The potential vorticity field is calculated from NARR fields according to the definition (Davis 1992)

\[ PV = (\zeta + f) \left( -g \frac{\partial \theta}{\partial p} \right) \]  

and then subjected to a diurnal harmonic analysis. Figure 12 is the diurnal amplitude and phase distribution of PV at 500 hPa. We choose 500 hPa because it is the top of the daily PBL in the Rocky Mountain source region. Note that 500 hPa (6 km) is also the median steering level for propagating rainfall system in Carbone et al. (2002). The amplitude distribution shows that there is a maximum of about 0.1 PV unit (PVU; 1 PVU = 10^{-6} \text{ km}^2 \text{kg}^{-1} \text{s}^{-1}) around the location of the Rocky Mountains ~40°N, 105°W, with a rapid decay eastward (Figs. 12a,b).

The phase of the PV signal shows a clear eastward movement (Fig. 12c). The projection of the PV phase at 40°N shows an obvious eastward phase tilt (Fig. 12d) with the phase slope indicating an eastward propagation speed of about 12 m s^{-1}. It is consistent with the median zone wind about 14 m s^{-1} at 6-km height in Carbone et al. (2002).

The vertical profile of PV amplitude and its thermal \( f[\zeta - g(\partial \theta/\partial p)] \) in Eq. (5) and dynamical \( \zeta - g(\partial \theta/\partial p) \) in Eq. (5) parts in the Rocky Mountain source area (Fig. 13) shows that the PV is mainly caused by the thermal effect both in June and in January. Thus, our PV signal could as well be described as a thermal signal.

5. Relationship between EPW-pressure and midtroposphere PV

In sections 2 and 4, we described the ASOS-derived EPW-pressure signal and the NARR-derived midtroposphere

FIG. 12. The diurnal (a) amplitude of PV perturbation field at 500 hPa and (b) the amplitude projection at 40°N, (c) the phase distribution of PV at 500 hPa, and (d) the phase projection at 40°N for the summertime (June) 12-yr average. The dashed thin line in (d) is the phase line indicating eastward-propagating speed of 12 m s^{-1}. PV amplitude is in PVU.
PV signal. We now investigate the relationship between these two signals, and their effect on precipitation. In Figs. 14 and 15 we show the phases of four signals for June and January: NARR-PV, NARR-\(w\), ASOS-EPW-pressure (tide-removed) and ASOS-precipitation. This diagram spans the continent. All phases are relative to local solar time.

The PV phases in June (Fig. 14a) show a clear propagation to the east between 106\(^\circ\) and 80\(^\circ\) W. The phase speed is about 12 m s\(^{-1}\). The vertical velocity (Fig. 14b) shows little movement and we suspect that NARR-\(w\) is an artifact of the data assimilation method. Our further examination using Rapid Update Cycle (RUC) data, to which more wind data from a variety of sources are assimilated at higher spatial and temporal resolution than NARR, shows a clear eastward phase tilt in lower troposphere corresponding well with PV at 500 hPa (not shown). EPW pressure shows a fairly constrained eastward movement (Fig. 14c). Between 105\(^\circ\) and 95\(^\circ\) W the speed is fast (~25 m s\(^{-1}\)) whereas from 95\(^\circ\) to 80\(^\circ\) W the speed is slower (~12 m s\(^{-1}\)). We suspect that the former is a mixed signal, as seen in the linear simulation (Fig. 8).

The ASOS precipitation phases have three regional behaviors (Fig. 14d). West of 105\(^\circ\) W and east of 85\(^\circ\) W the phase is flat and nonpropagating, with values near 270\(^\circ\) (or –90\(^\circ\)) indicating local afternoon solar forcing.

Between 105\(^\circ\) and 85\(^\circ\) W there is an eastward propagation at about 12 m s\(^{-1}\). As discussed earlier, the subregion 95\(^\circ\)–85\(^\circ\) W is an overlap region between stationary and propagating precipitation, so our harmonic analysis yields poor results.

A similar comparison of phases is shown for January in Fig. 15. As in June, the January PV propagation is very clear but the speed is a little faster, about 15 m s\(^{-1}\). The NARR-\(w\) shows a tilted phase but the phase speed is irregular and we doubt its validity. The EPW-pressure is similar to summer. The slow region between 95\(^\circ\) and 85\(^\circ\) W has a phase speed near 15 m s\(^{-1}\).

The ASOS precipitation phases are mostly scattered across the whole continent (Fig. 15d). This suggests, as expected, that precipitation in winter is controlled by baroclinic wave systems with little diurnal modulation.

An important issue is the physical relationship between NARR-PV and ASOS-EPW-pressure. In Fig. 16 we plot the phase difference between these two signals as a function of longitude for June and January. West of 95\(^\circ\) W the phase differences changes rapidly with longitude indicating, as shown before, that the two signals have different phase speeds there. Between 95\(^\circ\) and 82\(^\circ\) W, the phase difference is nearly constant. The value is approximately zero with variations of ±50\(^\circ\). This approximate phase lock suggests that the two signals are causally related.
6. Discussion: Cause and effect of multiple diurnal waves

The complex diurnal oscillations of temperature, pressure, and precipitation east of the Rockies were studied above using harmonic analysis of 12 years of ASOS and NARR data. Two additional tools were used. First, using the “temperature-based tidal assumption,” the continentally enhanced thermal tide was removed from the surface pressure data to reveal the EPW. Second, a linear Boussinesq model illustrated the combined effects of the continental thermal tide, inertia–gravity waves, the mountain–plain solenoid (MPS), and drifting PV pulses. We now attempt to determine the cause–effect relationships between these components. In the top part of Fig. 17, we show two aspects of diurnal heating: a continent-wide PBL heating and a localized elevated heating near 103°W. This heating generates three thermal signals: (i) a continent-wide, sun-following tidal response (Li and Smith 2010), (ii) a fast (−24 m s⁻¹) eastward-propagating rapidly decaying IGW east of the Rockies, and (iii) slower (−12 m s⁻¹) eastward drifting PV pulses east of the Rockies. These three thermal signals in turn generate the surface pressure and precipitation signals.

First consider the surface pressure signal detected from ASOS data. The largest contribution to surface pressure is the thermal tide. When the tide is removed, the remaining signal from 105° to 95°W has a mixture of IGW and PV signals. The region east of 95°W has a pure PV signal with little phase lag (0°±50°) between PV and pressure. These relationships hold in winter and summer.

Second, consider the precipitation signal in summer. From 105° to 90°W, the largest contribution to precipitation is PV. While IGW affects pressure in this region, it has little impact on precipitation, perhaps because it generates little vertical motion in the lower atmosphere (Fig. 8). Precipitation in the region from 90° to 85°W is a mixture of PV and local heating. The region east of 85°W has a nearly local PBL heating signal (Parker and Ahijevych, 2007). In winter, the precipitation has no diurnal modulation.
A key issue since Carbone et al. (2002) is the chain of causality for diurnal precipitation east of the Rockies. A strong clue about the cause of the three thermal signals (i.e., tide, IGW, and PV) is that they weaken in winter but do not disappear. In both winter and summer, the eastward-moving PV pulses influence the surface pressure. In winter, however, the diurnal modulation of precipitation completely disappears. Thus, one cannot argue that the precipitation causes the NARR-PV or ASOS-pressure signals. We conclude that the thermal signals cause the diurnal pressure and precipitation signals.

While the above arguments cast some light on mechanisms of diurnal modulation of precipitation, our methods are insufficient to prove any one mechanism. Our preferred hypothesis is the PV idea illustrated in Fig. 18. Diurnal heating over the Rockies generates a train of oscillating PV anomalies that drift eastward with the midtroposphere winds. These PV (or thermal) anomalies generate vertical motion in the lower atmosphere that might trigger convection.

7. Uncertainties and future work

There are number of unresolved issues in the present analysis that deserve further investigation. First is the separation of the large tidal pressure signal from the smaller EPW. While the optimization algorithm seems robust to small changes (section 2) and it performed well with synthetic signals (section 3), further analysis of errors and uniqueness would be useful.

While the linear theory of IGW and PV pulses may be relevant to the summer climate of the eastern United States, the present formulation is overly idealized (section 3). The heating function was chosen more for convenience than realism. Since it is not monotonic in the vertical, it gives a vertical dipole of PV instead of PV of a single sign. In general, the choice of heating function is ad hoc, not driven by physical principles related to surface properties, terrain elevation, or convection physics. An additional problem with the linear theory is the shear. It was formulated as a wind speed discontinuity rather than a smooth gradient.
The selective use of the NARR data for diurnal analysis is questionable (section 4). While a comparison of NARR and ASOS surface pressure and temperature showed good agreement, the precipitation did not compare well. We also doubt the accuracy of vertical motion in NARR because it is not measured but derived during an assimilation process. However, we have put confidence in the temperature field aloft in our PV analysis. Our selective use of NARR fields needs to be further investigated.

While a phase-locked relationship between PV aloft and surface pressure has been found, it is only apparent over a small range of longitude (section 5). Its masking by IGWs and MPS close to the Rockies needs further investigation. In addition, we have presented no dynamical theory for the small phase difference between PV and EPW pressure.

While the PV and EPW-pressure signals seem to move eastward together with the precipitation signal in summer, the convective triggering mechanism is still uncertain. Our mechanism for triggering (i.e., PV- and shear-induced ascent in the lower troposphere; Fig. 18) is plausible, but several other mechanisms could be imagined. Future work utilizing a cloud-resolving configuration of the Weather Research and Forecasting (WRF) model with realistic deep westerly shear will be helpful to detect how the PV mechanism works. This work could be done in two stages. To examine “dry” dynamics, we could see if dry convection over the Rockies will generate the warm pulses, and if the drifting warm pulses cause vertical motion over the plains. We could then add moisture and latent heat to investigate the mechanism and timing of triggered convection.

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FIG. 16. The phase differences between ASOS EPW pressure and NARR PV in (a) June and (b) January.

FIG. 17. The causal relationships between different diurnal signals (NARR-PV, ASOS-EPW-pressure, ASOS-precipitation) in June.

FIG. 18. The schematic plot of PV mechanism: $U$ represents the background wind, $N$ represents the environmental stability, $Z_1$ is the shear level, $H$ is the heating level, and $Q(z)$ is the vertical profile of the diurnal heating over the mountain.
APPENDIX

The Asymptotic Solution for Diurnal Elevated Heating with Background Shear

A linear model with Boussinesq approximation including three momentum, one buoyancy, and one incompressibility condition(s) are solved by the FFT method (Smith and Lin 1982). It is deployed here to demonstrate how gravity waves are modified by environmental parameters such as stability, horizontal and vertical heating scale, the Coriolis effect, and background mean wind.

\[
\begin{aligned}
\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} - f v &= -p' - au, \\
\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + V\frac{\partial v}{\partial y} + fu &= -p' - av, \\
\frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x} + V\frac{\partial w}{\partial y} + w \frac{\partial \phi}{\partial z} &= -p' + b - aw, \\
b_1 + U b_x + V b_y - N^2 w &= B - ab, \\
u_x + v_y + w_z &= 0,
\end{aligned}
\]  
(A1–A5)

where isothermal background atmosphere \( p' = [p - p(z)]/p, \) buoyancy \( b = g(T - T_0 + g z/C_p)/T_0, \) scaled heating rate \( B(x,y,z,t) = B_0 B_H(x,y) B(z) e^{|z|}, \) and \( B_0 = g \bar{Q}/\rho_0 C_p T_0, \) where \( \bar{Q} \) is the heating rate in watts; \( B_0(x,y) \) and \( B_H(z) \) are the horizontal and vertical shape of the heating. A small damping \( (a) \) is added to remove singularities.

Using a Fourier transform \( \tilde{f}(k) = (1/2\pi) \int_{-\infty}^{\infty} f(x)e^{-ikx} \, dx \) (inversion Fourier transform \( f(x) = \int_{-\infty}^{\infty} \tilde{f}(k)e^{ikx} \, dk \)), Eqs. (A1)–(A5) are converted to

\[
\begin{aligned}
\iota k \tilde{u} &= -ik \tilde{p} + \tilde{f} v, \\
\iota k \tilde{v} &= -ik \tilde{p} + \tilde{f} u, \\
\iota k \tilde{w} &= -\tilde{\rho} \tilde{z} + \tilde{b}, \\
\iota k \tilde{b} &= \tilde{B} - N^2 \tilde{w}, \\
\iota k \tilde{u} + il \tilde{v} + \tilde{w} &= 0.
\end{aligned}
\]  
(A6–A10)

with intrinsic frequency

\[
\sigma = \sigma + Uk + Vl - i\alpha.  
\]  
(A11)

From (A6)–(A10), we get a single ODE for vertical velocity,

\[
\ddot{w} + \gamma^2 w = \frac{\dot{B}(k^2 + l^2)}{\sigma^2 - f^2},  
\]  
(A12)

with the vertical wavenumber given by

\[
\gamma^2 = \frac{(N^2 - \sigma^2)(k^2 + l^2)}{\sigma^2 - f^2}.  
\]  
(A13)

The potential vorticity is

\[
\tilde{p} \Gamma = \frac{N_2 T_0}{g} \left( \frac{\tilde{\zeta} + f}{N^2 \tilde{b}} \right) = \frac{f T_0 \dot{B}_{0} \dot{B}_{H}(k,l)}{i\sigma} B_{\nu,z}(z)  
\]  
(A14)

with relative vorticity \( \tilde{\zeta} = ik \tilde{v} - il \tilde{u}. \) PV is determined by the vertical structure of the local heating and the existence of Coriolis force. To simplify this system, we will set \( l = 0 \) for the rest of the analysis.

We assume the diurnal local elevated heating has the vertical structure

\[
B_{\nu}(z) = \begin{cases} 
0 & 0 \leq z < H \\
(z - H) e^{-\beta(z - H)} & z \geq H 
\end{cases}  
\]  
(A15)

with horizontal structure \( B_{\nu}(x) = e^{(-x^2/\alpha^2)}, \) where \( \alpha \) is the horizontal scale of the Gaussian shape heating distribution.

The background atmosphere has a jump in mean wind and stability below the heating level at \( z_1(z_1 < H) \). Note that \( U_1 \) and \( U_2 \) are the mean wind below and above the shear level; \( N_1 \) and \( N_2 \) are the stability below and above the shear level.

The boundary and interface conditions are

- Lower boundary: \( z = 0, (k, 0) = 0; \)
- Interface conditions:

\[
\begin{align*}
&z = z_1: \dot{B}_1(k, z_1) = \dot{B}_2(k, z_1) ; \quad \tilde{\eta}_1(k, z_1) = \tilde{\eta}_2(k, z_1) \\
&z = H: \dot{B}_3(k, H) = \dot{B}_3(k, H) ; \quad \tilde{\eta}_3(k, H) = \tilde{\eta}_3(k, H); \\
\end{align*}
\]

- Upper boundary: decay, \( \tilde{w}\) finite when \( z \to \infty. \)
- The vertical displacement is defined with \( \tilde{\eta} = \dot{w}/i\sigma. \)

The analytical solution for \( w(x, z) \) in Fourier space for the three-layer case is

\[
\begin{align*}
&1. 0 \leq z \leq z_1 \\
&\tilde{w}_1(z) = \frac{\sigma_1}{\sigma_2 (\sigma_1^2 - f^2)} \frac{-ie^{iy_1;1 - iy_2;1}}{y_1(e^{2iy_1;1} + 1) - (\sigma_2^2 - f^2)y_2(e^{2iy_2;1} - 1)(iy_2 - \beta)^2} \frac{B_{0} \dot{B}_{H} k^2}{(e^{iy_1;1} - e^{-iy_1;1})}  
\end{align*}
\]  
(A16)
II. \( z_1 \leq z < H \)

\[
\hat{w}_2(z) = \frac{\gamma_1}{i\gamma_2 (\sigma_1^2 - f^2) \gamma_1 (e^{2i\gamma_1 z_1} + 1) - (\sigma_2^2 - f^2) \gamma_2 (e^{2i\gamma_1 z_1} - 1) (i\gamma_2 - \beta)^2} + \frac{B_0 \hat{B}_0 k^2}{(\sigma_1^2 - f^2) 2i\gamma_2 (i\gamma_2 - \beta)^2} [e^{i\gamma_1 z_1} e^{i\gamma_2 z} + e^{i\gamma_2 z_1} e^{i\gamma_2 z}]
\]

(A17)

III. \( z \geq H \)

\[
\hat{w}_3(z) = \frac{\gamma_1}{i\gamma_2 (\sigma_1^2 - f^2) \gamma_1 (e^{2i\gamma_1 z_1} + 1) - (\sigma_2^2 - f^2) \gamma_2 (e^{2i\gamma_1 z_1} - 1) (i\gamma_2 - \beta)^2} + \frac{B_0 \hat{B}_0 k^2}{(\sigma_1^2 - f^2) 2i\gamma_2 (i\gamma_2 - \beta)^2} e^{i\gamma_2 z}
\]

(A18)

The potential vorticity solution in Fourier space is

\[
\hat{PV}_n &= \frac{f T_0 B_0 \hat{B}_0 (k) B_z(z)}{i\sigma_n} n = 1, 2, 3.
\]

We transform back to physical space using

\[
w_n(x, z, t) = e^{i\omega t} \int \hat{w}_n(k, z) e^{ikx} dk,
\]

\[
PV_n(x, z, t) = e^{i\omega t} \int \hat{PV}_n(k, z) e^{ikx} dk.
\]

We now examine the singularities that control the far field occurrence of vertical velocity and potential vorticity. If the solution of \( \hat{w}_n \) [A16–(A18)] is to be bounded, the eigenvalue \( \gamma_n \) should be finite for all \( k \), which means \( \sigma_n^2 - f^2 \neq 0 \) [A13]. In the midlatitudes (e.g., 45°N), \( \sigma = 7.27 \times 10^{-5} \, \text{s}^{-1} \) is less than the Coriolis parameter \( f = 10^{-4} \), and \( \gamma_n \) becomes a complex number with the imaginary part of \( \gamma_n \geq 0 \), so that \( (i\gamma_n - \beta) < 0 \). Also, the definition of \( \gamma_n \) ensures that \( \gamma_n \neq 0 \). The possible poles for the integral of (A16)–(A18) are \( \sigma_2 = 0 \) or \( (\sigma_1^2 - f^2) \gamma_1 (e^{2i\gamma_1 z_1} + 1) - (\sigma_2^2 - f^2) \gamma_2 (e^{2i\gamma_1 z_1} - 1) = 0 \). The latter condition is equivalent to \( \tan(\gamma_1 z_1) = -i(\sigma_1^2 - f^2)/\sigma_2^2 - f^2 \) of a well-known gravity wave resonance condition. The values of physical parameters in midlatitudes make it impossible to satisfy this resonance condition since an absorption level is hidden in the critical level interface. Other “non-wave” mechanisms must control the far field.

While there is no singularity for \( \hat{w}_3(k, z) \) [Eq. (A17)] and \( \hat{w}_3(k, z) \) [Eq. (A18)], it is still possible for \( \sigma_n = 0 \) in Eq. (A16) for \( \hat{w}_1(k, z) \). When \( \sigma_2 = 0, \sigma + Uzk^* = 0 \), and \( k^* = -(\sigma/U_2) \), it becomes a singularity for \( \hat{w}_1(k, z) \) [Eq. (A16)] and \( \hat{PV}_3(k, z) \) [Eq. (A19)]. According to Riemann–Lebesgue theorem, when \( x \to \infty \), (A20) goes to zero for \( w_2(x, z, t) \) and \( w_3(x, z, t) \) but \( \hat{w}_1(k, z) \) has the advective singularity, \( k^* = -(\sigma/U_2) \), so \( \int_{-\infty}^{\infty} \hat{w}_1(k, z) e^{ikx} dk = \int_{-\infty}^{\infty} f \hat{w}_1(k) [1/(\sigma + Uzk^*)] e^{i\gamma(k)z} e^{-i\gamma(k)z} e^{ikx} dk \). This integral can be evaluated by using contour integration. First, we cut the plane along the \( x \) axis. Then we close the contour with large half circle in the upper plane for \( x > 0 \) and the lower plane for \( x < 0 \) (Jordan lemma). After adding a slight amount of dissipation in the system \( \sigma_2 \) is small, the pole at \( k^* = -(\sigma/U_2) \) shifts its location to \( k^* = -(\sigma/U_2) + i(\sigma/U_2) \). The integration path then runs under this pole. The residue there is \( \text{Res} (k^*) = 2\pi i \hat{w}_1(k^*) e^{i\gamma(k^*)z} e^{-i\gamma(k^*)z} \).

\[
f_{res}(k^*) = \frac{-ie^{i\gamma(k^*)z_1 - i\gamma(k^*)z} \sigma(k^*)}{[\sigma(k^*)^2 - f^2] \gamma_1(k^*) [e^{2i\gamma(k^*)z_1} + 1] - [\sigma(k^*)^2 - f^2] \gamma_2(k^*) [e^{2i\gamma(k^*)z_1} - 1]} B_0 \hat{B}_0 (k^*) k^3
\]

The asymptotic solution for \( w_1(x, z, t) \) when \( x \to +\infty \) is

\[
w_1(x, z, t) = -4\pi f_{res}(k^*) \sin[\gamma_1(k^*)z] \times e^{-i(\sigma(U_2k(z-U_2t))} e^{-(\sigma(U_2k)}.
\]

(A22)

\( f_{res}(k^*) \) represents the horizontal FT of the heat function evaluated at \( k^* \).

The analytic solution for the upper PV in physical space is simpler; it is just advection:

\[
PV_3(x, z, t) = 2\pi f T_0 B_0 B_z(z) e^{-i(\sigma U_2)(x-U_2t)} e^{-i(\sigma U_2)x}.
\]

(A23)
Both $w_1(x, z, t)$ [Eq. (A22)] and $\text{PV}_2(x, z, t)$ [Eq. (A23)] propagate downwind with speed $U_2$. The term $\sin[\gamma_1(k^*)z]$ in Eq. (A22) determines the vertical structure of $w_1$. In the mid or high latitudes, $\gamma_1(k^*)$ is imaginary, and $w_1(x, z, t)$ [Eq. (A22)] will have constant phase below the shear level and increasing amplitude with height ($\gamma_1^2 < 0$).

If there exists no background shear, then $\sigma_1 = \sigma_2$. No matter if $\sigma_1 = 0$, the nominator $\sigma_1$ will cancel with the denominator $\sigma_2$ and singularity will disappear in Eq. (A16). This means that if no shear exists, no vertical motion will be generated below the shear level far away from the heating source. This result is consistent with the general conclusion that vertical motion is invariant under a Galilean transformation of the zonal coordinate. If there is no vertical mean wind shear, then the adding of mean zonal wind is like a Galilean transform, which will not affect the vertical motion field.

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