Investigating the Linear and Nonlinear Stationary Wave Response to Anomalous North American Snow Cover

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ABSTRACT

Continental-scale snow cover represents a broad thermal forcing on monthly-to-intraseasonal time scales, with the potential to modify local and remote atmospheric circulation. A previous GCM study reported robust transient-eddy responses to prescribed anomalous North American (NA) snow cover. The same set of experiments also indicated a robust upper-level stationary wave response during spring, but the nature of this response was not investigated until now. Here, the authors diagnose a deep, snow-induced, tropospheric cooling over NA and hypothesize that this may represent a pathway linking snow to the stationary wave response. A nonlinear stationary wave model is shown to reproduce the GCM stationary wave response to snow more accurately than a linear model, and results confirm that diabatic cooling is the primary driver of the stationary wave response. In particular, the total nonlinear effects due to cooling, and its interactions with transient eddies and orography, are shown to be essential for faithful reproduction of the GCM response. The nonlinear model results confirm that direct effects due to transients and orography are modest. However, with interactions between forcings included, the total effects due to these terms make important contributions to the total response. Analysis of observed NA snow cover and stationary waves is qualitatively similar to the patterns generated by the GCM and linear/nonlinear stationary wave models, indicating that the snow-induced signal is not simply a modeling artifact. The diagnosis and description of a snow–stationary wave relationship adds to the understanding of stationary waves and their forcing mechanisms, and this relationship suggests that large-scale changes in the land surface state may exert considerable influence on the atmosphere over hemispheric scales and thereby contribute to climate variability.

1. Introduction

a. Potential influences of snow cover on large-scale circulation

Anomalous continental-scale snow cover has the potential to influence both local and downstream climate owing to its radiative and thermal properties, which act to modify the overlying atmosphere (e.g., Barnett et al. 1989; Cohen and Rind 1991; Leathers and Robinson 1993; Cohen and Entekhabi 2001). These influences may occur from regional to hemispheric spatial scales and immediate to seasonal time scales. Because of its considerable spatiotemporal variation, and the striking geographical differences between the Northern Hemisphere (NH) landmasses, the influence of snow on atmospheric circulation is an area of ongoing research. In particular, the nature of the large-scale stationary wave response to anomalous North American (NA) snow cover and its associated mechanisms remains largely unexplored.

Physically based snow–climate teleconnections have been described more often for Eurasia than for NA. This is likely due to Eurasia’s greater landmass, the existence of well-known centers of stationary wave activity over Siberia and the Tibetan Plateau (Plumb 1985), and thus the greater potential for large-scale snow anomalies to influence the overlying atmosphere. Gong et al. (2003a) reported a hemispheric response to anomalous Siberian snow cover that resembled the negative phase of the Arctic Oscillation (AO). This response was constrained...
by the unique topography of the region such that snow-enhanced stationary wave activity propagated upward and poleward (Gong et al. 2004). Others noted potential links between Eurasian snow cover and the North Atlantic Oscillation (NAO; Qian and Saunders 2003; Saunders et al. 2003) or the Indian monsoon (Bamzai and Shukla 1999) and evaluated its potential for climate predictability (Cohen and Entekhabi 2001). Yasunari (1991) described a connection between Eurasian snow cover and large-scale circulation patterns that resembled the positive phase of the Pacific–North American (PNA) pattern.

Relatively little attention has been paid to the dynamical response of the atmosphere to NA snow conditions. Observational studies have generally focused on local-to-continental-scale thermal effects of NA snow on climate (e.g., Karl et al. 1993; Leathers and Robinson 1993; Leathers et al. 2002). However, Leathers et al. (2002) also described a connection between the thermodynamic and radiative effects of snow and circulation by showing that anomalous snow cover leads to geopotential height anomalies over eastern NA, particularly in late winter/early spring when they hypothesize that preexisting circulation anomalies are less prevalent. Modeling studies such as Gong et al. (2003b) described a weak positive AO-like response to a fall snow forcing while Sobolowski et al. (2007) identified a more robust atmospheric response to fall NA snow leading to winter surface temperature anomalies over central Eurasia. Klingaman et al. (2008) described a link between anomalous Great Plains snow cover and wintertime western Eurasian surface air temperatures, facilitated by a snow-induced shift in the NAO toward its positive phase. More recent research has identified and described a physical pathway via which persistent anomalous NA snow cover leads to enhanced downstream transient-eddy activity and surface climate responses over Eurasia (Sobolowski et al. 2010, hereafter referred to as SGT10). SGT10 also identified stationary wave responses to NA snow anomalies throughout the year, but a full investigation was not conducted. A detailed diagnosis of this response, and a description of the physical mechanisms facilitating it, is the aim of the present study.

**b. Stationary wave modeling**

The importance of planetary stationary waves in determining regional climate variations, influencing synoptic-scale patterns, and modifying global patterns of heat and moisture transport is well known (Held and Hoskins 1985; Held et al. 2002). Climatological stationary waves are forced by three mechanisms: diabatic heating/cooling, orography, and transient fluxes. Changes in any of these three fields or the nonlinear interactions between them may result in changes in the stationary wave patterns depending on the strength and location of the perturbation. While GCM studies have been able to explain and reproduce the salient features of the climatological stationary waves seen in the observations, GCMs alone cannot separate the effects of the individual forcing mechanisms. To accomplish this, models of intermediate complexity, which require GCM output or reanalysis data as input, have been developed to explain and understand the forcing and maintenance of stationary waves.

These stationary wave models may be linear (e.g., Ting 1994) or fully nonlinear (e.g., Ting and Yu 1998). We first describe the linear framework. Forcing of the stationary waves in the linear framework is accomplished through diabatic heating/cooling, orography, and transient fluxes (following Ting 1994). Studies using this approach generally conclude that tropical and extratropical heating is the major contributor to observed/modeled stationary wave patterns, with orography anchoring the waves and transients playing a mitigating role (Hoskins and Karoly 1981; Nigam et al. 1986, 1988; Chen and Trenberth 1988a,b; Valdes and Hoskins 1989; Wang and Ting 1999; Held et al. 2002). The linear approach benefits from simplicity and ease of interpretation as the relative importance of each forcing term can be determined by examining the linear model response to each term in succession. As a result of these efforts, linear stationary wave models have been shown to faithfully reproduce the climatological stationary waves in both GCMs and observations (e.g., Ting 1994; Wang and Ting 1999). The linear stationary wave modeling framework has even been applied to diagnose the relative importance of individual forcing terms to the stationary wave response under various climate change scenarios (Joseph et al. 2004).

The linear approach has its limitations as it assumes that each forcing is independent, which is not, strictly speaking, true (Held et al. 2002). To account for nonlinear interactions in the linear modeling framework, the diagnostically stationary nonlinear forcing term has been introduced. Studies incorporating stationary nonlinearity suggest that nonlinear effects play a crucial role in maintaining stationary waves (Valdes and Hoskins 1989; Ting 1994; Wang and Ting 1999). However, it is important to note that this diagnostic term is a gross approximation of the contribution that nonlinear effects make and care must be taken in its interpretation. While the overall magnitude and effect of nonlinearity may be approximated by the linear model, it is impossible to diagnose the nonlinear effects of individual forcings and the nonlinear interactions between forcings with the linear model alone. A fully nonlinear stationary wave model is required to achieve this.

Studies employing fully nonlinear models have generally concluded that there is significant modification of the linear stationary wave response to orography and heating by nonlinearity (Valdes and Hoskins 1991; Ringerl...
and Cook 1997; Ting and Yu 1998; Ting et al. 2001). Ting et al. (2001) showed that, although some aspects of linear and nonlinear responses were roughly comparable (e.g., heating dominates the stationary wave response in the tropics), implicit inclusion of nonlinearity leads to an improvement in the simulation of stationary waves. The fully nonlinear framework also allows diagnosis of important nonlinear interactions in maintaining certain regional features of the climatological stationary waves, such as the anticyclone off the NA northwestern coast in winter.

Employed in sequence, linear and nonlinear stationary wave models allow for a complete diagnosis of the stationary wave response to various forcing mechanisms. The linear model provides diagnostics with respect to direct linear effects and the overall effect of stationary nonlinearity. The nonlinear model then allows a full decomposition of the stationary nonlinear effects of each individual forcing and the interactions between forcings. We employ this approach to study the effects of anomalous NA snow on stationary waves. While the effects of anomalous tropical and extratropical SSTs are well documented (e.g., Ting and Held 1990; Hoerling et al. 1995), there is no previous research that provides a complete diagnosis of the effects of anomalous land surface states such as snow cover on stationary waves.

The specific questions we seek to answer are the following: what are the mechanisms that lead to a stationary wave response to snow? Is the stationary wave response to snow predominantly linear or nonlinear? What are the roles of the individual forcing mechanisms in the linear stationary wave response? If nonlinearity is shown to be important, what are the relative contributions from direct nonlinear effects of individual terms and interactions between terms to the total response?

Through the sequential application of linear and fully nonlinear stationary wave models we investigate the stationary wave response to snow identified in SGT10 and address the questions raised above. Unless otherwise stated, “response” indicates the difference between high-snow and low-snow simulations (this holds for GCM and linear model and nonlinear model simulations). A brief summary of the experiments performed in SGT10, and the relevant results, is presented in the next section. The output from these experiments is used as input to the linear and nonlinear stationary wave models, which are described in section 3. Sections 4 and 5 present the results of the linear and nonlinear stationary wave–modeling simulations, respectively. Results are summarized and discussed in section 6, which also includes a brief comparison of the results to observations. It should be noted, however, that the aim of this study is to isolate the atmospheric response to snow. Therefore, direct comparison to observations is purely qualitative. As Leathers et al. (2002) noted, it is difficult for observation studies to confirm that an apparent change in atmospheric circulation is, in fact, snow-induced. Isolating a purely snow-induced signal from the observations is therefore beyond the scope of the present paper.

2. Background: Previous GCM experiments

The GCM experiments performed in SGT10 provide the input data for the linear and nonlinear stationary wave models. It is therefore useful to provide a brief summary here. SGT10 employed the ECHAM5.4 AGCM at T42 resolution with climatological SST and sea ice conditions (unless otherwise stated, GCM refers to these experiments). High- and low-snow conditions were prescribed over NA for the entire year (see Fig. 1 in SGT10). Here, 40 independent 1-yr integrations were performed for the high- and low-snow scenarios. These ensembles were then averaged and differenced to analyze the response to snow.

The aim of the study was to isolate the response to a realistic but idealized continental-scale snow forcing. Thus, the snow forcing was constructed so that snow extent/depth was always and everywhere larger (smaller) than climatology for a positive (negative) forcing. The data for the snow forcing were obtained from a recently constructed gridded NA snow depth dataset that extends from 1900 to 2000 (Dyer and Mote 2006). Maximum differences in snow water equivalent occurred from late winter through early spring (February–April), whereas the maximum differences in extent occurred during the transition seasons (spring and fall). (Further details with respect to the snow forcing specification appear in SGT10.)

The major results from SGT10 that are relevant to the current study are summarized as follows:

(i) The full-year snow forcing resulted in significant cooling over NA in all seasons. Downstream, significant surface warming occurs over northern Eurasia in spring.

(ii) A hemispheric-scale transient-eddy response was identified through eddy variance statistics and storm-track diagnostcs.

(iii) The transient response was a result of persistent steepened temperature gradients that lead to enhanced baroclinicity and diabatic heating over the storm-track entrance region. The sustained nature of the forcing allowed the signal to strengthen and propagate downstream.

An equivalent barotropic stationary wave response that reached maximum amplitude in the upper troposphere was also noted but not investigated in SGT10. (Unless otherwise noted, all stationary wave patterns
are diagnosed by plotting the zonally asymmetric streamfunction fields in the upper troposphere, $\sigma = 0.257$.) The spring ensemble-mean upper-troposphere stationary wave response to snow is shown in Fig. 1a. There is a significant negative response over much of NA, a positive response directly to the south of NA, and an extensive positive response over northern Eurasia. Although there are noticeable responses in all four seasons (not shown), only spring exhibits a coherent and robust hemispheric pattern that warrants further decomposition with the linear and nonlinear stationary wave models.

We hypothesize that similar physical mechanisms that gave rise to the transient-eddy response are responsible for the spring stationary wave response shown in Fig. 1a. Given the prominent role of the prevailing diabatic heating/cooling patterns in maintaining the climatological stationary waves, snow-induced diabatic heating/cooling is likely to play a central role in the spring stationary wave response. While past research has primarily focused on the effects of anomalous heating (e.g., Ting and Held 1990), Ringler and Cook (1999) used idealized models to show that diabatic cooling in the presence of orography also leads to a significant stationary wave response. In the present case, we argue that it is deep cooling over NA that primarily drives the stationary wave response. An additional contribution from transients may also be expected given the robust transient-eddy response to anomalous snow cover.

We first establish that there is a robust diabatic heating/cooling response to snow in the SGT10 experiments. Figure 2a shows that the NH vertically integrated diabatic heating/cooling response is clearly centered over the NA forcing region and is dominated by cooling. This response confirms overall cooling over the continent and heating over the Atlantic storm-track entrance region and downstream regions, but by itself does not represent a clear pathway by which the surface perturbation affects the upper-level circulation. The physical pathway by which enhanced diabatic cooling influences the circulation is established by examining the vertical profile of the heating response over NA (Fig. 3a). This response indicates that the cooling over NA extends from the surface/boundary layer well into the troposphere. This deep cooling is accompanied by a significant increase in static stability over NA, which then induces changes in the atmospheric circulation and hence stationary waves (Fig. 3b). The generation of these heating/cooling anomalies comes directly from the anomalous snow cover via radiative forcing (likely limited to the lower troposphere) and indirectly via dynamic snow-induced changes in circulation (e.g.,

![Fig. 1. Spring upper-troposphere $(\sigma = 0.257)$ stationary wave streamfunction response to high minus low snow from (a) the GCM experiments, (b) the linear model subjected to all forcings, and (c) the fully nonlinear model subjected to all forcings. Solid (dashed) lines represent positive (negative) responses; contour interval is $0.5 \times 10^5 \text{ m}^2 \text{ s}^{-1}$. Light- (dark) gray shading represents statistical significance at 90% (95%) [relevant to (a) only].]
temperature advection, storm-track changes). Some portion of the diabatic heating/cooling response also comes from transients. Examination of the components of the diabatic heating/cooling response reveals that most of the response below 500 hPa is due to the mean advection terms while most of the response above 500 hPa is due to transients (not shown). Further decomposition of the diabatic heating response to separate the radiatively forced and dynamically forced components is not performed here.

Next, the potential effects of transients are diagnosed via the response of the stationary wave streamfunction tendency to the snow-induced transients described previously (Fig. 2b). If transients are the main forcing behind the stationary wave response we would expect Figs. 1a and 2b to correspond well to each other. However, the pattern in Fig. 2b is quite different from the stationary wave response shown in Fig. 1a and indicates that, while transients may play a role, they are not likely the main driver of the response. These diagnostics are consistent with the stationary wave response shown in Fig. 1a and describe a plausible physical pathway by which anomalous snow cover induces changes to the diabatic heating field, which then leads to an upper-level stationary wave response.

Examination of the GCM output provides some indication of the mechanisms and pathways by which anomalous snow conditions may affect stationary wave patterns. However, we cannot diagnose the relative contributions due to heating/cooling and transients with the GCM alone. Further, the importance of nonlinear interactions between heating/cooling and other forcing mechanisms in maintaining the climatological stationary waves suggests that nonlinear effects may make a substantial contribution. Analysis of the GCM output with linear and nonlinear stationary wave models is undertaken to examine these contributions and interpret their relative importance to the stationary wave response to snow.

3. Methodology

a. Data

Both the linear and nonlinear stationary wave models require basic-state inputs in the form of ensemble-mean zonal-mean $u, v, T, \sigma$ (sigma dot vertical velocity), and $ps$ (surface pressure) fields obtained from GCM output of the high- and low-snow experiments described in the previous section. The forcing term inputs for both models are also computed from the GCM output from the high- and low-snow scenarios and require the horizontal wind vector, vorticity, vertical velocity, pressure, temperature and potential temperature for computation.

b. Linear model

The linear model employed here is a steady state, linear, baroclinic model in sigma ($\sigma$) coordinates and is similar to that employed in Ting and Held (1990), Ting (1994), and Wang and Ting (1999). The horizontal resolution is at R30 (~2.25° × 3.75°) with 14 unevenly spaced vertical sigma levels. The linear model includes equations for vorticity $\zeta$, divergence $D$, temperature $T$, and log of surface pressure (ln $ps$), in addition to mass conservation and hydrostatic equations. The model is essentially the
The dynamical core of the GCM developed by the National Oceanic and Atmospheric Administration (NOAA) Geophysical Fluid Dynamics Laboratory. The model equations are averaged to remove the time tendency terms and then linearized about the zonal-mean basic state. The equations are then solved with prescribed forcing terms by using direct matrix inversion following a numerical method similar to that of Schneider (1989).

In addition to the basic-state input, four forcing components (diabatic heating/cooling, orography, transient fluxes and stationary nonlinearity) are also computed from the high- and low-snow GCM output and input to the model (forcings are computed following Ting 1994). A few points about the forcing terms should be made. Since nonlinear wave–wave interactions are ignored in the linear model formulation, thus the stationary nonlinear term is computed and employed as an additional forcing term. Transients are computed from the high-pass-filtered twice-daily GCM output [high-pass filter follows Trenberth (1991)]. The calculations leading to the diabatic heating response shown in Figs. 2a and 3a include the effects of transients. Since we seek to isolate the effects of the individual forcing terms these transients are removed from the diabatic heating forcing term prior to running the stationary wave models. Inclusion of all four forcings should result in a reasonable reproduction of the GCM stationary wave fields. Since the model is linear the relative contribution of each forcing is determined by straightforward inclusion/exclusion. As with the GCM experiments, the linear model simulations are run for high-snow and low-snow scenarios and the difference to diagnose the response to snow.

Additional damping is required for the linear model to faithfully reproduce observed or modeled stationary waves. These dissipations include Rayleigh friction, Newtonian cooling, and biharmonic diffusion coefficients. The dissipations used here are the same as those employed by Ting et al. (2001). At the lowest three $\sigma$ levels, Rayleigh friction and Newtonian cooling represent the heat and momentum flux in the boundary layer while in the free atmosphere the dissipations partially represent nonlinear effects. The biharmonic diffusion coefficient of $\nu = 1 \times 10^{17} \text{ m}^{-4} \text{s}^{-1}$ smooths out the solutions and is applied to the vorticity, divergence, and temperature equations.

Thus, the part of the total stationary wave response due to any single forcing or set of forcings $f$ may be written as the difference between linear model simulations forced by high-snow and low-snow input obtained from the GCM experiments:

$$\text{LIN}_f = \text{LIN}_{HI} f - \text{LIN}_{LO} f,$$

where $\text{LIN}_{HI}$ ($\text{LIN}_{LO}$) indicates the linear model forced by high- (low-) snow input obtained from the GCM experiments. Using this approach the GCM stationary wave response to snow is decomposed into four parts, each representing the response that is due to an individual forcing mechanism that sum to the total response.

c. Nonlinear model

The fully nonlinear stationary wave model is similar to that employed by Ting and Yu (1998) and Ting et al. (2001). The fully nonlinear model is explicitly time dependent, thus negating the need for a stationary nonlinear forcing term. The time integration scheme is semi-implicit with a half-hour time step [see Hoskins and Simmons (1975) for details]. The spatial resolution is the same as the linear model and the governing equations are essentially the same. Basic-state and forcing inputs, with the exception

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**Fig. 3.** Spring vertical profile response (high minus low snow) of (a) diabatic heating/cooling and (b) static stability $[-\rho g/\theta(\partial \theta/\partial p)]$ over NA (135°W–78°E). Solid (dashed) lines represent positive (negative) responses; contours are drawn at (a) ±0.05, 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6 K and (b) ±0.1, ±0.8, ±0.6, ±0.4, ±0.2, 0.5, 1, 2, 3, 4, and 5 m. Light- (dark) gray shading represents statistical significance at 90% (95%).
of stationary nonlinearity, are identical to those of the linear model. As with the linear model, the equations are solved with the prescribed forcing terms and diagnosis of the stationary wave response due to an individual forcing is obtained through inclusion/exclusion. To maintain consistency with the linear model, transients are damped using the same dissipations described in the previous subsection, with the addition of a 1-day damping on the zonal-mean vorticity, divergence, and temperature to suppress zonal mean variability. A quasi-steady state is reached after about 30 days after which we take the day 31–50 average to approximate the steady-state nonlinear stationary wave solution.

The set of decompositions under the nonlinear framework is somewhat more complex than those of the linear model experiments but follows the same pattern. The presentation of the nonlinear model experiment decomposition follows that of Ting et al. (2001). The nonlinear model is driven by individual forcings and combinations of forcings to obtain the decompositions listed below. Thus, the part of the response due to any single forcing or set of forcings \( f \) may be written as the difference between nonlinear model simulations forced by high- and low-snow input obtained from the GCM experiments:

\[
NLIN_f = NLIN_{HI} - NLIN_{LO},
\]

where NLINHI (NLINLO) represents simulations with high- (low-) snow basic states and forcings obtained from the GCM experiments, and NLIN is the response due to \( f \). We may decompose the responses due to nonlinear interaction effects as follows:

\[
NE_{f_1,f_2} = NLIN_{f_1 + f_2} - NLIN_{f_1} - NLIN_{f_2},
\]

where \( NE_{f_1,f_2} \) represents the part of the response due to the nonlinear interactions between two stationary wave forcings \( f_1 \) and \( f_2 \). Using the above notation, the total nonlinear model response when subjected to diabatic heating/cooling (\( f_1 \)), orography (\( f_2 \)), and transients (\( f_3 \)) may be separated into seven parts,

\[
NLIN_{f_1 + f_2 + f_3} = NLIN_{f_1} + NLIN_{f_2} + NLIN_{f_3} + NE_{f_1,f_2} + NE_{f_1,f_3} + NE_{f_2,f_3} + NE_{f_1,f_2,f_3},
\]

where the first three terms represent the direct nonlinear responses due to individual forcing terms, and the final four represent responses due to interactions between forcings. When all these responses are summed we obtain the total nonlinear response \( NLIN_{f_1 + f_2 + f_3} \) due to all three forcing mechanisms. We evaluate the responses due

the direct nonlinear and interaction effects shown in Eq. (4) in section 5. Furthermore, we also evaluate the part of stationary wave response due to the total effects (TE) of an individual \( f \) as follows:

\[
TE_{f_1} = NLIN_{f_1 + f_2 + f_3} - NLIN_{f_2 + f_3}.
\]

This total effect response includes both the direct nonlinear response due to a specific mechanism and the nonlinear interactions between flows forced by that mechanism and the others.

Since all the inputs to the stationary wave models are ensemble means and the stationary wave models themselves are either steady state or quasi-steady state, statistical significance can only be assessed for the GCM experiments. Significance in the stationary wave modeling framework must be interpreted qualitatively by examining the coherence and magnitude of the streamfunction responses due to individual forcing mechanisms and their interactions. Additional diagnosis of the relative contributions due to individual forcing mechanisms and nonlinear interactions is made from spatial pattern correlations between the linear/nonlinear model responses and the GCM response.

4. Linear model results

Prior to investigating the snow-induced stationary wave response to the individual forcing components, we first confirm that the linear model reproduces the GCM stationary waves and the GCM response to snow. To do this, we compare the zonally asymmetric streamfunction pattern maps and compute an area-weighted spatial pattern correlation \( r \) between the GCM stationary waves and the linear model output over the NH. The spatial pattern correlation between the linear model forced by high- (low-) snow input and the high- (low-) snow GCM simulations indicates that the linear model faithfully reproduces the upper-level stationary waves from the GCM simulations (\( r = 0.71 \), for both scenarios). The linear model subjected to all four forcings also reproduces the main features of the GCM’s stationary wave response to snow reasonably well (cf. Figs. 1b to 1a, \( r = 0.71 \)). The relatively modest level reproducibility is to be expected given the simplifications in moving from the GCM to the linear model and the presence of some nonzero amount of internal variability or noise.

The linear contributions of the individual forcing terms to the GCM response are investigated next. The linear model streamfunction responses due to forcing by heating, orography, transients, and stationary nonlinearity are shown in Fig. 4 and indicate that heating and stationary nonlinearity make significant contributions to the
total GCM response to snow (cf. Figs. 4a,d to Fig. 1a). The linear response to heating is larger and more coherent than the other terms and exhibits robust negative (positive) responses over NA (Eurasia) that correspond to the features seen in the GCM response. Stationary nonlinearity also makes a substantial contribution to the total response, particularly over the PNA region. The linear response to transient forcing is coherent and focused over the PNA region but is more modest than that of heating and stationary nonlinearity (Fig. 4b). The linear response to orographic forcing indicates an enhancement of the lows downstream of the major NH mountain chains but is weaker than the responses due to the other terms (Fig. 4c).

The spatial pattern correlations between the linear model responses due to individual forcings and the GCM response provide information that examination of the streamfunction maps might miss because of the choice of the contour interval and/or magnitude of the response. Table 1 shows these correlations and confirms that heating is the dominant contributor to the total GCM response \( r = 0.49 \). Interestingly, the response to orography, while more modest than those due to transients and stationary nonlinearity when viewed via the streamfunction maps, has a higher spatial pattern correlation than either \( r = 0.32 \). Since the actual orography does not change from simulation to simulation, the response to orography is likely due to changes in the zonal-mean basic state between the high- and low-snow scenarios. Examination of the zonal-mean wind and temperature

**TABLE 1.** Spring spatial pattern correlations between the linear model response due to all forcing terms, the individual forcing terms, and the spring GCM response at \( \sigma = 0.257 \).

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Heating</th>
<th>Transients</th>
<th>Orography</th>
<th>NLIN</th>
</tr>
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<tbody>
<tr>
<td>Correlation</td>
<td>0.71</td>
<td>0.49</td>
<td>0.16</td>
<td>0.32</td>
<td>0.24</td>
</tr>
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</table>
fields from the GCM experiments confirms that significant changes in the basic state occur in response to snow (not shown).

As stated previously, if the GCM response was purely linear than the linear model subjected to all forcings except stationary nonlinearity should closely match the GCM response, given some nonzero amount of climate noise. The substantial linear model streamfunction response due to stationary nonlinearity and the spatial pattern correlation between the stationary nonlinear response and the GCM ($r = 0.24$) are indicators that nonlinear effects may be important. Another diagnosis, which confirms that the linear model does not fully capture the GCM response to snow, is shown via the spatial pattern correlation between the linear model response due to heating, orography and transients and the GCM response. This correlation ($r = 0.63$) is smaller than that of the linear model response to all forcings and the GCM response ($r = 0.71$). Thus, despite showing that the linear responses due to heating and, to a lesser extent, orography and transients, are able to approximate the GCM response to snow—the linear model also indicates that further diagnosis of stationary nonlinearity is needed to complete the picture.

5. Nonlinear model results

a. Reproduction of GCM stationary waves and response to snow

The nonlinear model simulations are undertaken in an effort to better understand the stationary nonlinear response shown in the previous section. While it is not the primary focus of the current study to compare linear and nonlinear models, it is necessary to justify the application of the nonlinear model. In particular, we confirm that the nonlinear model is able to reproduce the stationary nonlinear response shown in Fig. 4d (not shown). The main objective is to employ the nonlinear model as a diagnostic tool to better understand how various direct, interaction, and total effects contribute to the total stationary response to snow. We are also interested in whether employing the nonlinear model results in any improvement over the linear-modeling framework.

Prior to evaluating the direct, interaction, and total effects due to the forcing terms, we confirm that the nonlinear model is capable of reproducing both the GCM stationary waves and the GCM response to snow. The spatial pattern correlation between the nonlinear model forced by high- (low-) snow input and the high- (low-) snow GCM simulations indicates that the nonlinear model faithfully reproduces the upper-level stationary waves from the GCM simulations ($r = 0.85$, for both scenarios). The nonlinear model subjected to all three forcings also reproduces the main features of the GCM’s stationary wave response to snow (i.e., the spatial pattern correlation between Figs. 1a,c, $r = 0.77$). Both these correlations represent an improvement over the linear model. Further, we show that the streamfunction response of the nonlinear model subjected to all three forcings reproduces the robust features of the spring GCM response to snow quite well (Fig. 1c).

b. Direct nonlinear effects

The nonlinear model streamfunction responses due to forcing by the individual mechanisms are shown in Fig. 5. These represent the direct nonlinear effects due to heating, transients, and orography. Like the linear model results, heating exhibits the largest response, in terms of magnitude, with smaller contributions coming from transients and orography. Unlike the linear model, the streamfunction response due to heating bears little resemblance to the GCM response or the direct linear response due to heating (cf. Fig. 5a to Fig. 4a). The direct nonlinear response due to heating is concentrated over NA, features alternating negative (positive) responses over the Northwest (Northeast), and a positive response over northern Eurasia. The responses due to transients and orography closely resemble their linear model counterparts, suggesting that the direct contributions from these two mechanisms to the total response are mainly linear (cf. Figs. 5b,c to Figs. 4b,c).

The top row of Table 2 shows the spatial pattern correlations between the nonlinear responses due to the individual forcing terms (i.e., direct nonlinear effects) and the GCM response. All these correlations are lower than their linear model counterparts with the exception of orography, which is the same. With respect to heating, the spatial pattern correlation under the nonlinear framework is substantially lower than its linear model counterpart ($r = 0.27$ versus $r = 0.49$). These results indicate that individual direct nonlinear effects make a modest contribution to the total response and the improvement over the linear model.

c. Nonlinear interactions

The nonlinear streamfunction responses due to forcing by the various nonlinear interactions are substantial, with the exception of the combination of transients and orography, which is more modest (Fig. 6). The responses due to interactions involving diabatic heating/cooling and orography and diabatic heating/cooling and transients exhibit similar patterns while the response to the three-way interaction shows a similar pattern but of opposite sign. The patterns of these responses are focused over the PNA region and extend across the entire NH. This downstream extension is consistent with the overall spring response, which exhibits a coherent
hemispheric pattern. Furthermore, the large differences in magnitude between the responses due to interaction effects and the total response indicate that substantial cancellation between the various nonlinear effects must occur.

The spatial pattern correlations between the nonlinear response due to interaction effects and the GCM response to snow are negligible (not shown). These are all less than 0.10, indicating that, taken individually, the streamfunction responses due to interaction effects make little contribution to the total response, despite their large magnitude.

d. Total nonlinear effects

Thus far, it is difficult to see how the decomposition of the various nonlinear effects under the nonlinear-modeling framework leads to an improvement over the linear model. However, the diagnoses have not taken into consideration the total effects due to each forcing term. These are expressed by Eq. (5) and represent the nonlinear response due to a particular forcing and all its interactions with the other forcing terms. When viewed in this way, we show that the nonlinear responses due to the total effects of each forcing term make substantial contributions to the total GCM response to snow. The total nonlinear effects of heating/cooling, in particular, dominate the response. Figure 7 shows the nonlinear response due to the total effects of each forcing term. The nonlinear response due to the total effects of heating/cooling closely resembles the GCM response and reproduces all the major features of the stationary wave response to snow (cf. Fig. 1a to Fig. 7a). The response due to the total effects of the transients exhibits a more complex structure, enhancing the total response over northern NA while mitigating the total response over eastern NA and northern Eurasia (Fig. 7b). The total effects due to orography make a positive contribution to the total response and the pattern is similar to that of heating/cooling but of lesser magnitude (Fig. 7c).

The spatial pattern correlations shown in the second row of Table 2 confirm that the responses due to the total effects of each forcing term represent an improvement over their linear model counterparts and, in the case of heating/cooling and orography, make positive contributions to the GCM response. Total effects due to heating/cooling make the dominant contribution to the GCM response to snow ($r = 0.60$). The total effects due to heating/cooling make the dominant contribution to the GCM response to snow ($r = 0.60$).

<table>
<thead>
<tr>
<th>Total</th>
<th>Heating</th>
<th>Transients</th>
<th>Orography</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.77</td>
<td>0.27</td>
<td>0.13</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
<td>0.27</td>
<td>0.45</td>
</tr>
</tbody>
</table>

FIG. 5. Spring stationary wave streamfunction responses (high minus low snow) from the fully nonlinear model forced by (a) diabatic heating/cooling, (b) transients, and (c) orography at $\sigma = 0.257$. Solid (dashed) lines represent positive (negative) responses; contour interval is $0.5 \times 10^6$ m$^2$ s$^{-1}$. The spatial pattern correlations shown in the second row of Table 2 confirm that the responses due to the total effects of each forcing term represent an improvement over their linear model counterparts and, in the case of heating/cooling and orography, make positive contributions to the GCM response. Total effects due to heating/cooling make the dominant contribution to the GCM response to snow ($r = 0.60$). The total effects due to heating/cooling make the dominant contribution to the GCM response to snow ($r = 0.60$).
orography also make a substantial contribution, most likely through the nonlinear interaction between orography and heating ($r = 0.45$). The contribution from the total effects due to transients is more modest and, as shown in the streamfunction response, more complex than the other two forcings ($r = 0.27$). Diagnosis of these pattern correlations combined with the streamfunction maps shown in Fig. 7, suggests that the total nonlinear effects of each forcing are more important than any of the individual direct or interaction effects.

### 6. Summary and discussion

A previous GCM study (SGT10) noted a robust spring stationary wave response to anomalous NA snow conditions but did not propose a physical pathway or investigate the physical mechanisms behind the circulation changes.

The research presented here aims to fill this gap. We investigate the nature of the snow–stationary wave relationship by diagnosing the relative contributions by individual stationary wave forcing components to the GCM response under both linear and nonlinear frameworks. First, the GCM experiments are diagnosed to establish a plausible physical pathway by which deep, snow-induced tropospheric cooling over NA may lead to a stationary wave response. Invoking cooling as the mechanism, which then leads to a stationary wave response, is in contrast to the majority of stationary wave modeling studies that tend to focus on anomalous heating (e.g., Ting and Held 1990; Hoerling and Ting 1994). However, the circulation response in Fig. 1a is qualitatively consistent with previous studies conducted with idealized models and observations that do identify this mechanism (Ringler and Cook 1999; Leathers et al. 2002).
To establish a plausible physical mechanism, the GCM output from high- and low-snow simulations is used to drive linear and nonlinear stationary wave models. The linear model is used to diagnose the purely linear response due to heating, transients, and orography in addition to the overall contribution due to stationary nonlinearity. This stationary nonlinear contribution is deemed large enough to justify application of the fully nonlinear model, which provides a complete diagnosis of the stationary wave response to snow.

The linear stationary wave model is shown to reproduce the patterns of the GCM stationary waves for both the high- and low-snow scenarios and the GCM response to snow (spatial pattern correlations around 0.71). Evaluation of the stationary wave response due to the individual forcing terms reveals that heating/cooling makes the largest contribution to the total response followed by stationary nonlinearity, transients, and orography. The dominant role of heating/cooling is consistent with our initial hypothesis that the snow-induced changes to the diabatic heating/cooling field could drive a stationary wave response. Stationary nonlinearity also makes a substantial contribution, indicating that overall nonlinear effects are important. Despite the large transient-eddy response described in SGT10, transients make a coherent but modest contribution to the total stationary wave response. This result is also consistent with our initial diagnosis presented in section 2 (Fig. 2b). Orography makes a small, but nonnegligible, contribution to the total response, indicating the changes in the zonal-mean background state between the high and low snow scenarios may also play a role in the stationary wave response to snow. While intensification of the zonal-mean circulation should result in a stationary wave response according to stationary wave theory, further research is needed to confirm that these effects play a significant role in the present case.

The fully nonlinear model is also able to reproduce the GCM stationary waves for the individual high- and low-snow scenarios ($r = 0.85$, for both scenarios) and the GCM response to snow ($r = 0.77$). This level of reproduction represents an improvement over the linear model and indicates that, while much of the response to snow may be approximately linear, nonlinear effects and interactions are clearly important. As stated in the introduction, the linear-modeling framework makes an assumption of independence between the forcing terms that we know is not realistic. For example, the diabatic heating term may be influenced by changes in transient activity (e.g., shifting storm tracks) and all terms may modify the flow over mountain ranges, thus altering the

FIG. 7. Spring stationary wave streamfunction responses (high minus low snow) from the fully nonlinear model forced by total effects due to (a) diabatic heating/cooling, (b) transients, and (c) orography at $\sigma = 0.257$. Solid (dashed) lines represent positive (negative) responses; contour interval is $0.5 \times 10^6$ m$^2$ s$^{-1}$. 
mechanical orographic effect. The results of the nonlinear-modeling experiments suggest that these interactions are not only important but indeed necessary to faithfully reproduce and understand the stationary wave response to snow.

When all the interaction effects are included, the nonlinear response due to diabatic heating/cooling closely resembles the total GCM response to snow and exhibits a spatial pattern correlation of 0.60. This represents a substantial improvement over the linear model. Previous research has indicated that nonlinear interactions between heating/cooling and orography, in particular, are important to the maintenance of the climatological stationary waves (e.g., Ting et al. 2001). Similar interactions are important in the present case as the diabatic cooling response due to snow leads to circulation changes that then alter the effect of orographic (mechanical) forcing. Likewise, circulation changes induced by shifts in transient activity interact with heating/cooling circulation changes to contribute to the total effect due to heating/cooling. Similarly, the responses due to the total effects of orography and, to a lesser extent, transients also more closely resemble the total GCM response and contribute to the pattern of the total response. Orography, the direct effects of which exhibit little influence, makes a substantial contribution to the total response when nonlinear interactions, most likely those involving heating, are included. Diagnosis of the stationary wave response to snow using the fully nonlinear model results in improvement over the linear framework and provides additional detail with respect to the nonlinear interactions between heating/cooling and the other forcing terms.

The similarity (dissimilarity) between the responses of the GCM, linear model, and nonlinear models are likely due to model specification and the relative strength of the snow-induced signal (climatological noise). During the spring season, the climatological stationary waves are less active than winter and the snow forcing is at its maximum. Thus, it may be that the level of background variability on which the forcing is applied is important, as Leathers et al. (2002) hypothesize in their observation study. Changes in the distribution of snow during periods of relatively low background variability may have an amplified effect on the large-scale circulation. Given the projected changes in both NH snow cover (e.g., Raisanen 2008) and stationary waves (e.g., Joseph et al. 2004), further research on the month-to-month response of the stationary wave response to snow anomalies, particularly during the spring transition season, is recommended.

A detailed observational analysis to confirm our modeled results is beyond the scope of this paper. However, a first-order look at observations provides qualitative validation of the model results. Correlating spring reanalysis streamfunction anomalies and observed spring NA snow cover results in a pattern similar to the streamfunction response in our simulations (cf. Fig. 8 to Fig. 1a). The resulting pattern is indicative of the negative phase PNA mode and thus suggests a negative relationship between the PNA index and NA snow cover (PNA–NA snow cover correlation is highly significant at $-0.47$). Generally, the PNA is regarded as one of the dominant causative agents of snow anomalies over NA (e.g., Ge et al. 2009, and references therein). We make no argument to the contrary, however, our results indicate that the snow anomalies themselves may have a substantial positive feedback on the background circulation pattern.

The application of models of intermediate complexity allows us to identify the relative contributions due to heating/cooling, orography, and transients to the stationary wave response to NA snow. Much of the stationary wave response is captured under the linear framework; however, the implicit inclusion of nonlinearity adds important information and improves upon the purely linear analysis. Snow-induced changes to the diabatic heating field, most notably the deep cooling over NA, are the primary driver of the stationary wave response. Additionally, nonlinear interactions between heating/cooling and orography and transients are also important contributors. While there is certainly a bit of a chicken or the egg issue with respect to how snow anomalies originate in the first place, our results suggest that existing continental-scale snow anomalies may influence large-scale atmospheric circulation patterns during periods of reduced background variability, thus making an additional contribution to climate variability over mid-latitude regions of the Northern Hemisphere.

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