Potential Vorticity of the Madden–Julian Oscillation

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ABSTRACT

This study explores the extent to which the dynamical structure of the Madden–Julian oscillation (MJO), its evolution, and its connection to diabatic heating can be described in terms of potential vorticity (PV). The signature PV structure of the MJO is an equatorial quadrupole of cyclonic and anticyclonic PV that tilts westward and poleward. This PV quadrupole is closely related to positive and negative anomalies in precipitation that are in a swallowtail pattern extending eastward along the equator and splitting into off-equatorial branches westward. Two processes dominate the generation of MJO PV. One is linear, involving MJO diabatic heating alone. The other is nonlinear, involving diabatic heating and relative vorticity of perturbations spectrally outside the MJO domain but spatially constrained to the MJO convective envelope. The MJO is thus partially a self-sustaining system and partially a consequence of scale interaction of MJO-constrained stochastic processes. Convective initiation of the MJO over the Indian Ocean features a swallowtail pattern of negative anomalous precipitation and associated anticyclonic PV anomalies at the early stage, and increasing cyclonic PV generation straddling the equator in the midtroposphere due to increasing positive anomalies in precipitation. These lead to the swallowtail pattern in positive anomalous precipitation and the associated PV quadrupole that signifies the fully developed MJO. The equatorial Kelvin and Rossby waves bear PV structures distinct from that of the MJO. They contribute insignificantly to the structure and generation of MJO PV.

1. Introduction

The Madden–Julian oscillation (MJO; Madden and Julian 1971, 1972) presents many challenging problems. One is how atmospheric convective systems are organized on the MJO scale to form a “convective envelope.” This problem is particularly intriguing at the convective initiation stage of the MJO over the Indian Ocean as illustrated in Fig. 1. There was a strong MJO event, starting over the Indian Ocean in late April 2003. Strong convective activities also occurred in early February and early June with strength and scales comparable to those in late April, but they did not lead to any MJO event. An obvious question is, what were the differences between these cases with and without convective initiation of the MJO?

Many, if not all, theories and hypotheses on the MJO are anchored at the paradigm of convection–circulation interaction [see reviews by Wang (2005), Zhang (2005), and Raymond and Fuchs (2009)]. While MJO wind can be related to convective and synoptic momentum transport (Tung and Yanai 2002; Moncrieff 2004; Biello and Majda 2005), convective diabatic heating is the main energy source for the MJO (Yanai et al. 2000). Potential vorticity (PV) is a quantity that relates the nondivergent circulation directly to diabatic heating. Its characteristics associated with the MJO, however, have never been documented diagnostically. It is unknown, for example, how PV signals may evolve with convective activity related to the MJO and whether it may serve to discriminate convective initiation of the MJO from other types of large-scale convective perturbations over the Indian Ocean as illustrated in Fig. 1. This study explores the extent to which we may advance our understanding of the MJO, especially its convection–circulation interaction, from a standpoint of PV.

Hoskins et al. (1985) provided a comprehensive review of PV. PV applications in an isentropic system benefit from two principles: conservation in an adiabatic and frictionless flow, and invertibility (derivability of mass and motion from a known distribution of PV). Hoskins et al. (1985) illustrated the utility of PV in diagnostics and

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prediction of large-scale weather patterns of middle latitudes where advective processes dominate frictional and diabatic ones. PV has also been applied to studies of many tropical phenomena, such as effects of tropical diabatic heating on the tropical and global general circulations (Hoerling 1992; Krishnamurti et al. 2000), the ITCZ (Schubert et al. 1991), tropical or African easterly waves (Burpee 1972; Dickinson and Molinari 2000; Hopsch et al. 2010), the African easterly jet (Thorncroft and Blackburn 1999), and tropical cyclones, including their generation (Davis and Emanuel 1991), structure (Schubert et al. 1999), and tracks (Shapiro 1996). Schubert and Masarik (2006) investigated theoretically the PV aspect of the MJO. They demonstrated that the PV invertibility principle applies to the MJO. Using a prescribed heating source that moves eastward at the observed MJO propagating speed, they showed that simple PV dynamics can describe the flow in the wake of an MJO convective envelope, but not ahead of it.

In the current study, we take PV as a diagnostic tool to characterize the structure and evolution of the MJO using a global reanalysis product. We explore the extent to which the dynamical structure of the MJO, its evolution, and its direct connections to diabatic heating, especially during its convective initiation over the Indian Ocean, can be described from a viewpoint of PV. To help identify the PV characteristics that are unique to the MJO, we first compare three-dimensional structures and generation of MJO PV to those of the convectively coupled equatorial Kelvin and Rossby waves (hereafter, simply Kelvin and Rossby waves). We then focus on the evolution of PV and diabatic heating during convective initiation of the MJO over the Indian Ocean.

The method and data used are described in section 2. Results are presented in section 3. A summary and conclusion are given in section 4, including a discussion on the shortcomings of applying PV to study the MJO.

2. Method and data

In this study, isobaric rather than isentropic PV is diagnosed, mainly for two reasons. First, in the tropics, strong vertical motions and diabatic heating across isentropic surfaces deplete the main advantage of an isentropic PV analysis. Second, results from an isobaric PV analysis can be directly compared to most other MJO diagnostics at pressure levels. Isobaric PV, denoted as \( P \), is defined as

\[
P = -g(\Omega + \nabla_p \times \mathbf{V}) \cdot \nabla_p \theta,
\]

where \( g \) is the gravitational acceleration, \( \Omega = (0, 0, f) \) is the planetary vorticity vector with \( f \) being the Coriolis parameter, \( \mathbf{V} = (u, v, \omega) \) is the wind vector, \( \theta \) is potential temperature, and \( \nabla_p = i(\partial \partial x) + j(\partial \partial y) + k(\partial \partial p) \) is the gradient operator in pressure coordinates, with \( i, j, \) and \( k \) being the unit vectors that point eastward, northward, and downward, respectively. The unit of PV is \( m^2 \ s^{-1} \ K \ kg^{-1} \), but a commonly used measure of PV is PV units (PVU; \( 1 \text{ PVU} = 10^{-6} \text{ m}^2 \text{ s}^{-1} \text{ K kg}^{-1} \)). Practically, PV is dominated by the vertical component of absolutely vorticity \( (\omega + \zeta) \), where \( \zeta = (\partial v/\partial x) - (\partial u/\partial y) \), and the vertical gradient of potential temperature

\[
P \cong -g(\omega + \zeta) \frac{\partial \theta}{\partial p}.
\]

The PV budget equation is

\[
\frac{dP}{dt} = -g \nabla_p \cdot [\mathbf{Q}(\Omega + \nabla_p \times \mathbf{V})] - g \nabla_p \cdot (\nabla_p \theta \times \mathbf{F}),
\]

where the first term on the right-hand side represents PV generation due to diabatic heating \( Q \) and the second term...
term represents PV destruction by friction $\mathbf{F}$. The local rate of change (or tendency) of PV is
\[
\frac{\partial P}{\partial t} = -\mathbf{V} \cdot \nabla_P P - g(f + \zeta) \frac{\partial Q}{\partial p} - g\left(\frac{\partial Q}{\partial x} + \zeta \frac{\partial Q}{\partial y}\right) + g \nabla_P \theta \cdot (\nabla_P \times \mathbf{F}),
\]  
(4)

where $\zeta_x$ and $\zeta_y$ are zonal and meridional components of relative vorticity, respectively. On the right-hand side of Eq. (4), the first term is PV advection, and the second and third terms are PV generation by, respectively, the vertical and horizontal $Q$ gradients and absolute vorticity. The debate on whether PV can be generated (and/or destroyed) or only redistributed by diabatic heating (Haynes and McIntyre 1987, 1990; Danielsen 1990) is not a concern in the tropics because of strong diabatic heating across isentropic surfaces.

The PV invertibility for the MJO (Schubert and Masarik 2006) will not be applied in this study. Nor will moist PV (Schubert et al. 2001). In a concept of the “diabatic source” of PV (Hoerling 1992), effects of diabatic heating $Q$ on PV generation can be discussed through combining the generation terms in Eq. (4) with PV advection by the divergent flow as a direct response to diabatic heating. These and other extended PV analyses can and should be done after we document the gross features of PV related to the MJO, which is the main objective of this study.

PV anomalies can be expressed as
\[
P^* = P - \overline{P} = -g(f + \zeta) \frac{\partial \theta}{\partial p} - \frac{\partial \theta}{\partial p} + \frac{\partial \theta}{\partial p} + \frac{\partial \theta}{\partial p},
\]  
(5)

where bars indicate time means and primes deviations from the means, or anomalies. With the method to be described shortly, the anomalies can be decomposed into MJO components (denoted by curly brackets) and non-MJO components (denoted by asterisks). MJO PV can then be expressed as
\[
\{P\} = -g\frac{\partial \theta}{\partial p} - \frac{\partial \theta}{\partial p} - g\frac{\partial \theta}{\partial p} - g\left\{\frac{\partial \theta}{\partial p}\right\} - g\left\{\frac{\partial \theta}{\partial p}\right\} - g\left\{\frac{\partial \theta}{\partial p}\right\}.
\]  
(6)

The first and fourth terms on the right-hand side of Eq. (6) are related only to the MJO itself. The second and third terms represent contributions to MJO PV from interactions between the MJO and mean state. The fifth and the sixth terms represent contributions from scale interaction between the MJO and non-MJO perturbations. And the last term represents contributions from scale interaction solely by non-MJO perturbations.

In the PV budget Eq. (4), the term with the vertical gradient of heating contributes to PV generation much more than those with horizontal gradients, found previously by Krishnamurti et al. (2000) and confirmed by our calculations. The total PV change ($dP/dt$) is almost identical to $-g(f + \zeta) \partial Q/\partial p$ for all three types of perturbations. This suggests that the contribution of friction-related processes is also negligible. Indeed, Krishnamurti et al. (2000) estimated that the frictional PV destruction is almost one order of magnitude smaller than $-g(f + \zeta) \partial Q/\partial p$. In the rest of this study, PV generation will be represented only by $-g(f + \zeta) \partial Q/\partial p$ and the other terms will be ignored.

Following a procedure of decomposing a variable into its time mean, MJO and non-MJO components (to be described below), the local tendency of MJO PV is
\[
\frac{\partial \{P\}}{\partial t} = -\mathbf{V} \cdot \nabla_P \{P\} - g\frac{\partial \{Q\}}{\partial p} - g\frac{\partial \{Q\}}{\partial p} - g\left\{\zeta \frac{\partial \theta}{\partial p}\right\} - g\left\{\zeta \frac{\partial \theta}{\partial p}\right\} - g\left\{\zeta \frac{\partial \theta}{\partial p}\right\}.
\]  
(7)

In this equation, the generation of MJO PV comes from the MJO alone (the second term on the right-hand side), interaction between the MJO and mean state (the third and fourth terms), and interactions of anomalies (the last term). The generation by anomalies is
\[
-g\left\{\zeta \frac{\partial \theta}{\partial p}\right\} = -g\left\{\zeta \frac{\partial \theta}{\partial p}\right\} - g\left\{\zeta \frac{\partial \theta}{\partial p}\right\} - g\left\{\zeta \frac{\partial \theta}{\partial p}\right\}.
\]  
(8)

where the terms on the right-hand side represent, respectively, MJO self-sustainment (the first), MJO interaction with non-MJO perturbations (the second and third), and interaction among non-MJO perturbations (the last). Except for the first term $-g\{\zeta \partial \{Q\}/\partial p\}$, all other processes involve scale interactions. By diagnosing the amplitude of each term, we hope to reveal some leads to the relative importance of each of the implied dynamical processes.

Signals for the MJO components were extracted by applying spectral bandpass filtering to its frequency and zonal wavenumber domain as in Wheeler and Kiladis (1999). The non-MJO component was calculated as the differences between the total anomalies and MJO component (e.g., $P^* = P - \{P\}$). To obtain further insight
into how much the Kelvin and Rossby waves may contribute to MJO PV through scale interaction, we extracted their signals using the same bandpass filtering method. Potential problems associated with this filtering method will be discussed in section 4. We did not decompose PV advection into different scale-interaction terms as for PV generation. A complete PV budget analysis for the MJO shall be done in a separated study.

Daily data from the global interim European Centre for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (ERA-Interim; Simmons et al. 2007) for the period of 1998–2009 were used to estimate PV and other dynamical terms. The horizontal resolution of the data is 1.5° longitude × 1.5° latitude. The vertical resolution is 25 hPa for 1000–750 hPa, 50 hPa for 750–250 hPa, and 25 hPa for 250–100 hPa. Diabatic heating $Q$ was estimated as a residual of the thermodynamic equation following the approach of Yanai et al. (1973) using the ERA-Interim data. Daily Tropical Rainfall Measuring Mission (TRMM; Kummerow et al. 2000) 3B42 precipitation data (0.25° longitude × 0.25° latitude) based on precipitation radar and microwave measurements (Huffman et al. 1995) were also used.

For a given type of tropical disturbance (the MJO, Kelvin, or Rossby wave), its vertical and horizontal structures are displayed as linear regressions upon its precipitation time series averaged over 15°N–15°S at a specific longitude. Horizontal distributions are plotted at the level near the maxima of displayed fields. Results passing the Student’s $t$ test at the 95% confidence level are shown or highlighted. The equivalent degree of freedom for the significance test is estimated using autocorrelation of 1-day lag (Zwiers and von Storch 1995).

3. Results

a. General features of tropical PV

The sign of temporally and zonally averaged PV is opposite in the two hemispheres (Fig. 2a). Dynamically, negative PV (vorticity) in the Southern Hemisphere is equivalent to positive PV (vorticity) in the Northern Hemisphere. Both are related to cyclonic circulations. In this study, latitudinal averages (15°S–15°N) were made with the sign of PV in the Southern Hemisphere reversed, and “cyclonic PV” was plotted as positive and “anticyclonic PV” was plotted as negative. The same strategy was applied to calculations of PV generation and spectra of equatorially symmetric and antisymmetric PV.

At most tropical latitudes, there is a local maximum cyclonic PV immediately above the 850-hPa level that appears to penetrate into the tropics from higher latitudes. This is due to the stable stratification in the trade wind inversion layer. Relatively large zonally averaged standard deviation of PV exists at the same level (Fig. 2b), with larger values in the Southern Hemisphere than in the Northern Hemisphere. This might be related to the variability in the depth of the trade wind boundary layer (Albrecht et al. 1995). The zonal mean standard deviation of PV exhibits a dome structure with a minimum near the surface at the equator. The vertical walls of the dome are located near 15°S and 15°N. Outside (poleward) of the walls the meridional gradient of PV standard deviation is greater than inside. The walls may be considered the boundaries of the tropics in a PV analysis. In this study, tropical (or latitudinal) averages were made over 15°S–15°N.

The zonal–vertical structures of temporally and latitudinally (15°S–15°N) averaged PV and latitudinally averaged standard deviation of PV are shown in Fig. 3. Topographic effects are visible, especially over the Andes. There are three layers of large cyclonic PV and PV standard deviation: the upper and middle troposphere, and immediately above the trade wind boundary layer. Large PV in the upper troposphere is likely due to air from higher latitudes, judged from their potential temperature. Their maxima are at longitudes where upper-tropospheric mean winds are westerlies, which form “westerly ducts” allowing extratropical large-scale disturbances with high PV to intrude into the tropics (Webster and Holton 1982). In the rest of this study, results are shown only up to 150 hPa to avoid large PV values that may not be directly related to the MJO and clutter the figures. The
largest PV and its variability in the midtroposphere are both in the same region of known large MJO signals (e.g., from the Indian to western Pacific Oceans) (Salby and Hendon 1994; Zhang and Hendon 1997). The lower-tropospheric maxima in PV and its variability are again associated with the trade wind inversion.

Time–space spectra of tropical PV at 550 hPa (the level of maximum midtropospheric PV variance) are shown in Figs. 4a and 4b. For comparison, time–space spectra of tropical precipitation are shown in Figs. 4c and 4d. They were produced following the method of Wheeler and Kiladis (1999) except the length of each segment for calculating the spectra is 256 days instead of 96 days. The symmetric spectrum of PV shows peaks of the Rossby wave and the MJO as seen in precipitation. There is no PV spectral signal of the Kelvin wave, as expected from linear theories (Schubert and Masarik 2006). In the antisymmetric spectrum of PV, there is no signal of the Yanai (mixed Rossby–gravity or MRG) wave, in contrast to precipitation. Instead, there are strong signals of westward-propagating perturbations of zonal wavenumber 3–15 and periods of 3–8 days. These must be related to the synoptic perturbations such as the African easterly waves (Berry and Thorncroft 2005). In the rest of this study, we will focus on the structure and generation of PV for the MJO in comparison to those for the Kelvin and Rossby waves.

The PV of the MJO, Kelvin, and Rossby waves are compared first as regressions upon their perspective precipitation time series at 90°E. The MJO is considered at its mature stage at this longitude. We will later compare the PV structures of the MJO at 90°E to those at 60°E. The latter is taken to represent longitudes of MJO convective initiation. Neither of the locations should bear any specific meaning for the Kelvin and Rossby waves. Results at longitudes farther east (120° and 150°E) are slightly different because of the mean distribution of convection. Slightly different results also emerge for different seasons (November–April vs May–October). These results are not shown to keep the presentation brief. Results shown here are based on the entire data period (1998–2009).

b. PV at 90°E

Vertical–longitudinal distributions of diabatic heating \( Q \), PV, PV generation (PVG; \(-g(f + \varphi)\partial Q/\partial p\)), the local tendency of PV (\(\partial P/\partial t\)), and the longitudinal distribution
of precipitation, all averaged over 15°S–15°N and regressed upon precipitation at 90°E, are given in Fig. 5 for the (top) Kelvin wave, (middle) MJO, and (bottom) Rossby wave. Interestingly, there are weak but significant PV signals for the Kelvin wave in the midtroposphere (Fig. 5a, contours). The theoretical linear Kelvin wave has zero PV (Schubert and Masarik 2006). The PV signals of the Kelvin wave in Fig. 5a may come from nonlinear Kelvin waves and non-Kelvin wave perturbations in the Kelvin wave spectrum domain (further discussed in section 4). PV signals for the Rossby wave (Fig. 5c) are robust. Positive (cyclonic) PV anomalies are near 550 hPa slightly to the west of and below the heating maximum (colors) and negative PV anomalies immediately to the east in the cooling region. The PV anomalies tilt westward in the upper troposphere, with their maxima above the 150-hPa level.

PV anomalies of the MJO (Fig. 5b) exhibit the gravest baroclinic structure: their midtropospheric maxima (positive anomalies slightly west of the heating center and negative anomalies to the east) are immediately below the level of maximum heating. The signs of MJO PV reverse in the upper troposphere where the amplitudes become much larger. These are similar to the PV of the Rossby wave but without an obvious westward shift. There is a fundamental difference between the PV and heating structures of the MJO and those of the Kelvin and Rossby waves: their dominant zonal scales (measured by doubling the distance between positive and negative PV maxima). For the Kelvin and Rossby waves they are about 60°, corresponding to zonal wavenumber 6. In contrast, the zonal scale of the MJO is about 140°, signifying its planetary-scale nature.
For all three types of perturbations, strongest positive (cyclonic) PV generation (right column, contours) occurs in the lower troposphere, centered around 600 hPa, and strongest negative PV generation occurs in the upper troposphere, both in the region of positive heating. The vertical dipole of PV generation has been predicted theoretically for the MJO by Schubert and Masarik (2006). It is common in the tropics (Krishnamurti et al. 2000) because of the predominant vertical profile of diabatic heating with its maximum in the midtroposphere and the signs of its vertical gradients opposite in the lower and upper troposphere. Positive and weaker PV generation is also found in the boundary layer for all three types of perturbations. Large PV anomalies in the upper troposphere for the MJO and Rossby wave (Figs. 5b,c) are not entirely due to PV generation by diabatic heating. They are partially related to PV advection by intruding extratropical perturbations.

Positive PV anomalies and local tendency ($\frac{\partial P}{\partial t}$, right column, colors) are located above the maxima of PV generation for all three types of perturbations. In fact, they are near the levels of zero PV generation and extend upward to levels of negative PV generation. This vertical dislocation among PV generation, local PV tendency, and PV anomalies is due to upward PV advection (not shown).

The horizontal structures of anomalies in PV, precipitation (PR), PV generation, and local tendency ($\frac{\partial P}{\partial t}$) regressed upon 15°N–15°S-averaged precipitation at
90\degree E are shown in Fig. 6 for the three types of perturbations. Precipitation anomalies of the Kelvin wave (Fig. 6a, colors) are centered at the equator with negligible amplitude beyond 10\degree on both sides. The PV signals for the Kelvin wave (Fig. 6a, contours) display a weak quadruple pattern. This quadruple PV pattern is much stronger for the Rossby wave (Fig. 6c). The pair of cyclonic PV centers is collocated near the longitudes of a pair of positive precipitation anomaly center at about 10\degree S and 10\degree N. To the east is a pair of anticyclonic PV centers at longitudes of a pair of negative precipitation anomalies.

In contrast, precipitation anomalies in the MJO (Fig. 6b) are strong both at and off the equator. They exhibit a swallowtail pattern, with the tails pointing to the west. The swallowtail pattern in precipitation and convection has been previously documented in observations (Hendon and Salby 1994; Maloney and Hartmann 1998), theories (Wang and Rui 1990), and numerical simulations (Salby et al. 1994; Wang and Li 1994). It is ambiguous in composites for MJO phases defined by an MJO index (Wheeler and Hendon 2004; Waliser et al. 2009). This can be easily explained. Figure 6 (like other similar plots in this study) is a statistical snapshot when positive precipitation anomalies of the MJO reach a maximum at the given longitude (equivalent to an average in time). An index-based composite for a particular phase of the MJO is an average with anomalous maxima within a range of longitudes (an average in both time and longitude).

There have been two explanations for the swallowtail pattern of precipitation anomalies. One assumes that the equatorial anomalies to the east are associated with the Kelvin wave component of the MJO and the off-equatorial anomalies to the west are associated with the Rossby wave component (Salby et al. 1994; Maloney and Hartmann 1998). The swallowtail pattern shown in Fig. 6b, however, includes neither Kelvin nor Rossby wave signals. The other explanation relies on frictionally induced equatorial convergence and divergence in regions of surface easterlies and westerlies, respectively, with easterlies leading (east of) westerlies (Wang and Li 1994).

Low-level heating (below the 500-hPa level) appears to occur prior to (east of) high-level heating (above the 500-hPa level) for the Kelvin wave (Fig. 5a) and the MJO (Fig. 5b), but not for the Rossby wave (Fig. 5c). Such a vertical evolution in diabatic heating of the MJO has been observed previously (Lin et al. 2004). But its signals are inconsistent among different datasets and different longitudes (Zhang et al. 2010; Ling and Zhang 2011). The low-level heating at the leading edge of the MJO heating center in Fig. 5b appears at and near the equator in the front part of the swallowtail pattern in Fig. 6b, while the high-level heating is in the tail part off the equator.

The horizontal structures of precipitation and PV anomalies in the Rossby wave (Fig. 6b) tilt poleward and eastward. This tilt is the same as that of the MJO from a Lagrangian point of view but in the opposite direction.
from a Cartesian point of view. There is no similarity at all between the eastern part of the MJO and the Kelvin wave and between the western part of the MJO and the Rossby wave from a perspective of PV anomalies. The MJO is often described as consisting of a structure resembling the Rossby wave to the west and the Kelvin wave to the east (e.g., Zhang 2005). This does not apply to its structures of PV and precipitation anomalies.

The eastward propagation of MJO PV cannot be explained in terms of PV generation alone. Maximum PV generation is east of the maximum PV only slightly. The PV advection leads PV more obviously (not shown). The imbalance between PV generation and advection results in maximum $\partial \tilde{\mathcal{Q}} / \partial \tilde{\theta}$ located east of maximum PV (Figs. 5b,e and 6b,e), leading to the eastward propagation of MJO PV. It is interesting to notice how PV advection plays an equally important but very different role for the Rossby wave. The cyclonic circulations associated with positive heating anomalies, similar to those of the MJO, also partially cancel cyclonic PV generation, but the imbalance results in local tendency of cyclonic PV west of the maximum cyclonic PV anomalies, hence westward propagation. While a detailed PV analysis for the Rossby wave is beyond the scope of this study, the different roles of PV advection that lead to the opposite propagation directions indicate the distinct dynamics of the MJO and Rossby waves despite their seemingly similar circulation patterns.

Total MJO PV ($\{P\}$) shown in Figs. 5 and 6 mostly comes from the third term on the right-hand side of Eq. (6) ($-g\{\xi\} \partial \tilde{\theta} / \partial p$). The first term ($-gf \partial \tilde{\theta} / \partial p$) is the second largest contribution to $\{P\}$. The second term ($-g\tilde{Z} \partial \tilde{\theta} / \partial p$) is nonnegligible only near the tropopause (above the 200-hPa level). The other terms, including $-g\{\xi\} \partial \tilde{\theta} / \partial p$ and $-g\{\xi\} \partial \tilde{\theta} / \partial p$, are all negligibly small. This is a quantitative confirmation of an earlier conclusion from visual inspection of Figs. 5 and 6: the Kelvin and Rossby waves do not contribute significantly to the PV structure of the MJO.

The dominant components of PV generation, shown in Figs. 5 and 6, are the linear term ($-gf \partial \tilde{\theta} / \partial p$) and the nonlinear term ($-g\{\xi\} \partial \tilde{\theta} / \partial p$) in Eq. (7). Among the four processes contributing to the nonlinear term in Eq. (8), the one involving non-MJO perturbations ($-g\{\xi\} \partial \tilde{\theta} / \partial p$) is an order of magnitude greater than the other three. MJO vorticity does not contribute much to the generation of MJO PV. Neither does the Kelvin or Rossby wave. The nonlinear term ($-g\{\xi\} \partial \tilde{\theta} / \partial p$) becomes an order of magnitude smaller when $\xi$ or $Q'$ is replaced by its Kelvin or Rossby wave component. This suggests the stochastic nature of the process by which scale interaction involving non-MJO perturbations contributes to the generation of MJO PV. The vertical structures of the two dominant PV generation terms ($-gf \partial \tilde{\theta} / \partial p$ and $-g\{\xi\} \partial \tilde{\theta} / \partial p$) are similar (not shown), mainly because of the prevailing vertical structure of tropical diabatic heating that might be independent of time scales or perturbation types (Zhang and Hagos 2009). They also bear a similar horizontal structure (Fig. 7). This indicates that generation of MJO PV by non-MJO perturbations ($-g\{\xi\} \partial \tilde{\theta} / \partial p$) is effective only within the MJO convective envelope. In other words, stochastic generation of MJO PV is constrained by the MJO itself.

c. PV at 60°E

The central equatorial Indian Ocean with a reference point at 60°E is taken as the region where convective initiation of the MJO takes place. There is no reason for this longitude to be a preferable location for the Rossby and Kelvin waves. Only the MJO will be discussed in this subsection.

Figures 8 and 9 are the same as Figs. 5 and 6 but only for the MJO with time lags in the regression included. Day 0 is when precipitation at 60°E reaches its maximum. The main features at day 4 are similar to those at 90°E (Figs. 5b,e and 6b,e). The evolution of these features is unique at 60°E. At an early stage of MJO convective initiation (e.g., day $-8$), when positive heating anomalies just emerge, there are strong negative heating anomalies to the east (Fig. 8, left column, colors), a manifestation of convective suppression. As positive heating anomalies and their associated cyclonic PV generation (right column, contours) gradually strengthen and slowly move eastward, the negative heating anomalies and their
associated anticyclonic PV generation become weaker. The zonal scale of the MJO, measured by doubling the distance between its positive and negative heating maxima (or PV generation maxima), grows through its convective initiation stages. There is a weaker positive heating anomaly center propagating eastward across Africa, which remains separated from the MJO heating center and weakens before moves into the western Indian Ocean. The main MJO heating center is seen here as instigated locally over the Indian Ocean.

Corresponding to the evolution of diabatic heating and PV generation, cyclonic MJO PV (left column, contours) in the midtroposphere emerges robustly only at a much later time (day 4), while anticyclonic MJO PV remains prominent from the very early stage (day 28). The positive local tendency of PV ($\partial P/\partial t$, right column, colors) increases and leads cyclonic PV in their eastward propagation.

At the early stage of MJO convective initiation (days $-8$ and $-4$), positive anomalies of MJO precipitation are concentrated at the equator and confined in the zonal direction (Fig. 9, left column, colors) and thus resemble more those of the Kelvin waves (Fig. 6a) than the mature MJO (Fig. 6b). But they gradually grow off the equator and lead to cyclonic PV generation (Fig. 9, left column, contours).
on both side of the equator in a westward–poleward tilting pattern typical of the MJO. Meanwhile, prominent negative precipitation anomalies exist to the east in a swallowtail pattern together with their associated anticyclonic PV and PV generation on both sides of the equator. As precipitation anomalies slowly move eastward, cyclonic PV to the west becomes more robust while anticyclonic PV to the east remain strong. At the later stage (day 4) the unique MJO PV quadrupole is nearly established.

PV advection balances PV generation to a lesser degree during convective initiation than at the mature stage of the MJO. Their residual gives rise to $\partial P/\partial t$ (Fig. 9, right column, colors) that is stronger relative to PV generation than for the mature MJO and is responsible for not only the eastward propagation but also the growth of MJO PV. Eastward PV advection from Africa (not shown) is too weak to establish the MJO PV pattern over the Indian Ocean without PV generation due to diabatic heating.

Also as at a mature stage of the MJO, the dominant generation processes of MJO PV (right column, contours) at the convective initiation stage are due mainly to MJO diabatic heating ($-g\partial Q/\partial p$) and constrained stochastic processes of non-MJO perturbations ($-g\xi^*\partial Q^*/\partial p$) within the MJO convective envelope. The amplitudes of the two processes are similar and vary in concert (not shown). Neither diabatic heating $Q^*$ nor relative vorticity $\zeta^*$ alone shows any identifiable pattern.

4. Summary and discussion

The purpose of this study is to explore the extent to which the dynamical structure of the MJO, its evolution, and its connection to diabatic heating can be described from a viewpoint of potential vorticity (PV). We have paid special attention to the distinction between PV structures of the MJO and equatorial Kelvin and Rossby waves and to PV evolution during convective initiation of the MJO.

Several issues regarding the application of PV to the MJO need to be discussed. First, the PV equation [Eq. (4)] describes only effects of diabatic heating on PV. There is no theoretical base of feedback from PV to diabatic heating. Second, PV does not include the divergent flow, an essential component of the MJO (Hendon and Salby 1994; Wang 2005). A PV description of the MJO is thus incomplete, incapable of revealing the full interaction between convection and the circulation, and cannot fully address the question of MJO convective initiation posed at the beginning of this article, despite useful insight it may provide (see discussions below). Third, the traditional PV definition used in this study does not include moisture, which has been proposed to be critical to the MJO (Raymond and Fuchs 2009). Fourth, the filtering method used in this study is known to suffer from several problems. The MJO spectral window (Fig. 4) is subjectively chosen. All signals in it do not belong to the MJO. All
signals outside are not unrelated to the MJO. The same argument applies to the Kelvin and Rossby wave filtering. It is unsettled whether observed convectively coupled equatorial waves should bear the same structures as theoretically predicted dry linear equatorial waves (Yang et al. 2003) or should include everything coherent with convective signals that fall within spectral windows overlapping the theoretically predicted dispersion curves of the dry linear equatorial waves (Wheeler and Kiladis 1999). The spectral filtering method used in this study assures at least that there is little, if any, Kelvin or Rossby wave component in the extracted MJO signals. With these caveats in mind, we have learned the following from this study.

The MJO bears a robust signature in PV as a quadrupole of cyclonic and anticyclonic PV (Fig. 6c). It is a manifestation of the vortex quadrupole of the MJO (Hendon and Salby 1994), which has served as one of the main validation targets for MJO theories (Majda and Stechmann 2009). The PV quadrupole of the MJO is closely related to the positive and negative anomalous dipole in diabatic heating (Lau and Phillips 1986). It cannot be reproduced by positive heating anomalies alone (Gill 1980; Schubert and Masarik 2006). The Matsuno (1966) approach of including both positive and negative heating anomalies is a correct one to study the dynamical structure of the MJO (Salby et al. 1994; Khouider and Majda 2008). The quadruple PV structure of the MJO is distinct from those of the equatorial Kelvin and Rossby waves (Figs. 5 and 6). The notion that the MJO consists of a Kelvin wave pattern to the east and a Rossby wave pattern to the west does not apply to the PV structure.

Two processes dominate the generation of MJO PV: a linear one involving diabatic heating of the MJO alone \((-g f \frac{\partial Q}{\partial p})\) and a nonlinear one \((-g f \xi^* \frac{\partial Q^\theta}{\partial p})\) involving perturbations spectrally outside the main MJO domain while spatially constrained to the MJO convective envelope (Fig. 7). The nonlinear process is not particularly associated with the Kelvin and Rossby waves. In this sense, the MJO is partially self-sustained and partially forced by MJO-constrained stochastic processes.

Research attention has recently been paid to nonlinearity in the MJO (Wedi and Smolarkiewicz 2010; Majda and Stechmann 2012) and scale interaction between the MJO and synoptic perturbations (Biello and Majda 2005; Maloney 2009). Possible roles of stochastic processes have also been studied for the tropical waves (Khouider et al. 2010) and the MJO (Salby and Garcia 1987; Yu and Neelin 1994). The significant contribution to the generation of MJO PV by non-MJO perturbations \((-g f \xi^* \frac{\partial Q^\theta}{\partial p})\) underlines the roles of nonlinear and constrained stochastic processes in the MJO. The potential importance of the mean state to the MJO (Slingo et al. 1996; Inness et al. 2003; Maloney 2009; Ray et al. 2011) is evident in the structure of MJO PV [through \((-g f \xi^* \frac{\partial Q^\theta}{\partial p})\) but not its generation.

Statistically, the Indian Ocean is the birthplace of MJO PV. Eastward PV advection from Africa does exist. This might be a sign of individual MJO events propagating into the Indian Ocean from the west after their circum-equatorial journey (Matthews 2008). But the signature PV quadrupole of the MJO does not emerge until PV generation by diabatic heating localized in the Indian Ocean becomes robust. This suggests that regardless of upstream conditions, initiation of convection over the Indian Ocean is essential for the MJO to fully develop and grow.

This PV analysis suggests possible early indications for convective initiation of the MJO over the Indian Ocean (Figs. 8 and 9). They include strong negative anomalies in precipitation in a swallowtail pattern and an associated pair of anticyclonic PVs straddling the equator at the early stage, increasing positive anomalies in precipitation evolving into the swallowtail pattern together with cyclonic PV generation on both sides of the equator and emerging cyclonic PV straddling the equator in the midtroposphere. All these, while slowly moving eastward, eventually lead to the signature PV quadrupole that declares the full development of the MJO. It would be interesting to explore which part of this evolution of precipitation and PV can be used to benefit MJO prediction. To fully address questions regarding convective initiation of the MJO, we also need to examine PV structure and evolution of non-MJO cases to compare with MJO cases.

PV can also serve as a useful tool to diagnose numerical models that are incapable of reproducing the MJO (Lin et al. 2006). The structure of MJO PV and its connection to diabatic heating documented in this study may suggest different reasons for such incapability, including the failure to produce (i) robust diabatic heating at the equator due to a parameterization problem that creates a false double ITCZ (Sperber 2004), (ii) a realistic vertical structure of diabatic heating (Li et al. 2009; Zhang and Song 2009) that is essential for the self-sustaining process of the MJO \((-g f \xi^* \frac{\partial Q^\theta}{\partial p})\), (iii) a correct convective spectrum for the constrained stochastic process of MJO PV generation \((-g f \xi^* \frac{\partial Q^\theta}{\partial p})\), and (iv) a realistic mean vertical profile of potential temperature that is needed for a correct MJO PV structure \((-g f \xi^* \frac{\partial Q^\theta}{\partial p})\), among others.

Interpretations of the MJO in terms of the Kelvin and/or Rossby waves have been made in theories (Lau and Peng 1987; Chang and Lim 1988; Wang and Rui 1990; Wedi and Smolarkiewicz 2010), observations (Maloney and Hartmann 1998; Roundy 2008), and numerical simulations.
(Salby et al. 1994). Some of these studies explained the swallowtail pattern of precipitation–heating in terms of the Rossby wave component of the MJO. Our PV analysis demonstrated that the swallowtail pattern of MJO exists without Rossby wave signals. The Kelvin and Rossby waves are not dominant components of the generation of MJO PV through nonlinear processes \(-g(\zeta^* \partial Q^*/\partial p)\). The structural differences in PV between the MJO and Rossby wave are evident. Their seemingly similar circulation patterns provide the opposite PV advection. Solely from a standpoint of PV, we hypothesize that the fundamental dynamics of the MJO depends on neither the Kelvin nor Rossby waves.

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