Midlatitude Eddies, Storm-Track Diffusivity, and Poleward Moisture Transport in Warm Climates

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ABSTRACT

Recent work using both simplified and comprehensive GCMs has shown that poleward moisture transport across midlatitudes follows Clausius–Clapeyron scaling at temperatures close to modern, but that it reaches a maximum at sufficiently elevated temperatures and then decreases with further warming. This study explores the reasons for this nonmonotonic behavior using a sequence of NCAR Community Atmosphere Model, version 3 (CAM3) simulations in an aquaplanet configuration spanning a broad range of climates. No significant change is found in the scale, structure, or organization of midlatitude eddies across these simulations. Instead, the high-temperature decrease in poleward moisture transport is attributed to the combined effect of decreasing eddy velocities and contracting mixing lengths. The contraction in mixing length is, in turn, a consequence of the decreasing eddy velocities in combination with constant eddy decorrelation time scales.

1. Introduction

The net poleward moisture transport by midlatitude atmospheric motion is fundamental to the global hydrological cycle and to the maintenance of the mean equator-to-pole temperature gradient, with about half of the total poleward atmospheric energy flux across midlatitudes carried as latent heat (Trenberth and Stepaniak 2003). At the same time, the release of latent heat has a leading-order effect on atmospheric dynamics, not least on the midlatitude eddies largely responsible for the poleward energy transport (Schneider et al. 2010). Unraveling the interactions among these multiple roles of water vapor is key to a robust understanding of the climate system’s response to increased radiative forcing in both past and future climates.

Given the rapid rise in saturation vapor pressure with temperature, the role of water vapor in climate dynamics is expected to be more significant in warmer climates. Recent proxy data reconstructions indicate that global-mean temperatures during some periods of the deep past—notably the early Eocene climate optimum about 50 million years ago—were much warmer than previously thought (Sluijs et al. 2006; Pearson et al. 2007; Huber 2008). In a recent data-model comparison for the early Eocene (Huber and Caballero 2011), the best-fit model simulation has a global-mean sea surface temperature of about 31°C, over 15°C warmer than today. Climate dynamics at temperatures this high are only beginning to be explored.

Simple theories based on eddy-dominated diffusive transport (Pierrehumbert 2002; Held and Soden 2006) suggest that the total poleward moisture flux \( F \) should scale as

\[ F \sim v_\text{e} q_\text{e}, \]  

where \( v_\text{e} \) is a typical eddy velocity and \( q_\text{e} \) is a reference specific humidity. If \( v_\text{e} \) and the mean relative humidity depend weakly on temperature, then the Clausius–Clapeyron (C-C) relation implies an exponential increase of \( F \) with temperature, a prediction in good agreement with GCM simulations at temperatures close to modern (Held and Soden 2006; O’Gorman and Schneider 2008b).

At higher temperatures the picture becomes more complicated. In simulations using both simplified and comprehensive GCMs, the poleward moisture flux reaches an upper limit at a certain temperature and actually decreases with further warming (Caballero and Langen 2005; O’Gorman and Schneider 2008b). The reasons for
this nonmonotonic behavior have remained unclear. A robust feature seen in both simple and comprehensive models is a sharp drop in midlatitude eddy activity (and thus $v_y$) at high temperatures. This decrease partly explains the deviation from C-C scaling of the moisture flux, but it is not enough by itself to explain the nonmonotonic model results: the flux predicted from (1) using values of $v_y$ and $q_*$ taken directly from the simulations increases monotonically at all temperatures (O’Gorman and Schneider 2008b).

Our main goal here is to explore the mechanisms that determine the behavior of poleward moisture transport in warm climates. A subordinate goal of interest in its own right is to investigate the changes in scale and structure of midlatitude eddies as temperatures increase. In the interest of simplicity, we adopt an aquaplanet model configuration for this exploratory study. The model and simulations are described in section 2. The particular GCM used here differs from those previously shown to exhibit high-temperature moisture flux saturation, and we verify in section 3 that nonmonotonic behavior is also found in this model.

The first hypothesis we pursue is that significant changes in the size and structure of midlatitude eddies in the warm climate regime may limit the eddies’ ability to transport moisture. It is well known that poleward moisture flux in midlatitude cyclones is focused in moist low-level “warm conveyor belts” embedded in the warm sector and rooted in the subtropical boundary layer, while the equatorward return flow descends from the midtroposphere into the cold sector and is therefore much drier (Carlson 1980; Thorncroft et al. 1993; Wernli 1997; Polvani and Esler 2007; Boutle et al. 2010). Since midlatitude static stability increases strongly with temperature (Frierson 2006; Schneider and O’Gorman 2008) and the vertical scale of baroclinically unstable waves is inversely proportional to static stability (Held 1978), it is possible that equatorward trajectories in warm climates become much shallower, leading to reduced moisture contrast between poleward and equatorward flows and weaker net moisture transport. Moreover, cold-sector surface enthalpy fluxes could become strong enough in warm climates to produce deep convection there, in contrast to the shallow convection observed today, providing an additional mechanism for moistening the equatorward flow and reducing net transport. Section 4 investigates changes in eddy characteristics using both spectral and feature-tracking techniques. We find no substantial changes in eddy structure or in the distribution of convection even at the highest temperatures probed.

Next, we consider the bulk moisture transport from a diffusive perspective using both Eulerian (section 5) and Lagrangian (section 6) approaches. We show that the main shortcoming of the scaling (1) is its assumption of a constant mixing length. We find that the mixing length actually changes proportionally to $v_y$ in the simulations, leading to a modified scaling with a quadratic dependence on $v_y$ that successfully captures the nonmonotonic behavior of the modeled moisture flux. Section 7 discusses the reasons for this and summarizes our conclusions.

2. Model and simulations

We use the National Center for Atmospheric Research Community Atmosphere Model, version 3.1 (CAM3) (Collins et al. 2006) coupled to an aquaplanet slab ocean model with prescribed ocean heat transport. The ocean heat transport is a repeating seasonal cycle derived from fully coupled model runs, but it is rendered zonally and hemispherically symmetric, and is the same in all runs. Orbital parameters are kept at their modern values, and insolation has a full diurnal and annual cycle. We perform a sequence of six simulations with successive doublings of atmospheric CO₂ concentration; specifically, $[CO₂] = 280 \times 2^i$ ppm with $i = 0, \ldots, 5$. We refer to these simulations as cases 0–5, respectively. The simulations are all conducted at T42 spectral resolution ($\sim 2.5^\circ \times 2.5^\circ$). A 10-min time step is used to ensure numerical stability in all cases. These are the same simulations examined as part of a previous paper (Caballero and Huber 2010) focused on tropical dynamics. The simulations span 50 yr, with statistics taken over the last 10 yr (when the model is in statistical steady state and shows no appreciable drift.) The global- and annual-mean surface temperature in our simulations ranges from about 295 K in case 0 to about 312 K in case 5, while the equator–pole surface temperature difference decreases from about 32 K in case 0 to about 21 K in case 5.

3. Poleward moisture flux

\( a. \) Net poleward flux

The net poleward moisture flux across a given latitude $\phi$ is given by

$$ F(\phi) = \overline{\nu q}, $$

(2)

where $\nu$ is meridional velocity, $q$ is specific humidity, and the overbar indicates a climatological average and a zonal and vertical mass-weighted integral. Figure 1a shows a snapshot of the instantaneous moisture flux $\nu q$ across 40°N during winter in case 0. Fluxes are primarily driven by cyclonic synoptic-scale systems, with poleward and equatorward flows arranged to the east and west of...
surface lows. The qualitative structure of these fluxes is very similar to what is seen in observations (Ralph et al. 2004) and higher-resolution models (Boutle et al. 2010). Moisture fluxes are sharply peaked in the lowest levels of the troposphere, where moisture is greatest, but some distance above ground where winds are swifter. This bottom confinement of moisture fluxes is further illustrated in Figs. 1b,c, which compares the zonally integrated climatological winter moisture flux at each pressure level in the coldest and warmest runs, and shows that the fluxes in both cases are sharply peaked in the 900–1000-hPa layer.

In steady state, F is most easily calculated indirectly from the surface water flux as follows:

\[
\frac{1}{a} \frac{\partial}{\partial \phi} F(\phi) = E - P,
\]

(3)

where \(P\) and \(E\) are zonal integrals of the climatological precipitation and evaporation rates, respectively; and \(a\) is Earth’s radius. Carrying out the calculation for our simulations shows \(F\) to have a meridional profile very similar in all cases to that in the observed climate (Trenberth and Stepaniak 2003), with a single broad midlatitude maximum. The position of this maximum remains close to about 40\(^\circ\) latitude in all runs (Table 1) despite the substantial poleward shift of the storm track with increasing temperature, discussed below (section 4). While maximum eddy activity shifts poleward with temperature, the maximum meridional moisture gradient moves equatorward, making the location of maximum moisture flux insensitive to temperature.

The value of \(F\) at its midlatitude peak \(F_{\text{max}}\) gives a useful bulk measure of the overall moisture transport across the storm tracks. Figure 2 shows how this quantity changes with temperature and season. In the winter season, the moisture flux shows the nonmonotonic behavior discussed in the introduction, reaching an upper limit at a global mean temperature of about 35\(^\circ\)C. During the summer months, however, the moisture flux is much

<table>
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<th>Case</th>
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<th>Latitude max F</th>
<th>(\mathcal{K})</th>
<th>(L) (km)</th>
<th>(1/\mathcal{F}) (days)</th>
<th>(\tau) (m s(^{-1}))</th>
<th>(\mathcal{U}_{900}) (m s(^{-1}))</th>
<th>(S_{\text{eqwd}}) (hPa deg(^{-1}))</th>
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FIG. 1. (a) Wintertime snapshot of the poleward moisture flux across 40\(^\circ\)N. Red shades indicate poleward flow, and blue shades equatorward flow; L indicates surface lows. (b) Zonally integrated climatological poleward moisture flux on constant pressure levels during winter (December–February in NH, June–August in SH) for the coldest run. (c) As in (b), but for the warmest run. Moisture flux has been multiplied by a fixed latent heat of vaporization of $2.5 \times 10^6$ J kg\(^{-1}\) to give units of PW.
weaker and changes very little with temperature. The annual-mean moisture transport reflects the nonmonotonic behavior of the stronger winter transport.

Returning to the vertical profiles of winter moisture flux shown in Figs. 1b,c, we note that while both profiles have the same overall structure, there is an upward redistribution of flux with temperature: the warm run has stronger fluxes than the cold run above 900 hPa but weaker fluxes in the low-level peak. To understand the drop in total moisture flux occurring at the warmest temperatures (Fig. 2), we must therefore focus on the low-level flow. For this reason the rest of the paper focuses largely on eddies and fluxes in the 900–1000-hPa layer.

b. Poleward and equatorward fluxes

As is apparent from Fig. 1a, equatorward flows carry a nonnegligible amount of moisture. To understand changes in net flux, it is useful to study separately the poleward and equatorward components. Defining flux-weighted mean poleward and equatorward humidities as

\[ q_+ = V^{-1}vqH(v), \]
\[ q_- = -V^{-1}vqH(-v), \]

where \( H \) is the Heaviside function, positive \( v \) implies poleward motion, and

\[ V = vH(v) \]

is the poleward mass flux (which by mass conservation must equal the equatorward mass flux), and then the net poleward flux \( 2 \) can be written as

\[ F = V\Delta q, \]

where \( \Delta q = q_+ - q_- \) can be interpreted as the typical humidity difference between equatorward- and poleward-flowing air.

The temperature responses of \( V, q_+, \) and \( q_- \) are shown in Fig. 3. The mass flux decreases roughly linearly with temperature, dropping about 35% from the coldest to the warmest run. Both flux-weighted humidities increase rapidly with temperature but at different rates: \( q_+ \) at an average of about 6% K\(^{-1}\) and \( q_- \) at about 8% K\(^{-1}\). This differential moistening of poleward and equatorward flows leads to slow, subexponential growth of \( \Delta q \). It is the combination of decreasing \( V \) and slowly increasing \( \Delta q \) that ultimately leads to the nonmonotonic behavior of \( F \).

4. Storm tracks and eddy structure

a. Storm-track location and strength

Figure 4 shows the zonal-mean eddy activity—measured by the rms eddy velocity—in the coldest and warmest runs. We define an “eddy” here as any deviation from the zonal mean. In all cases, eddy activity in the mid-latitude storm tracks peaks near the tropopause but also exhibits a subsidiary maximum in the 900–1000-hPa layer, a feature also observed in the oceanic storm tracks of the
real atmosphere (Lau 1978). It is the presence of these low-level eddy activity maxima combined with the rapid decrease of moisture with height that leads to the sharp low-level moisture flux peaks seen in Fig. 1. Figure 4b also shows a prominent eddy activity maximum near the equatorial tropopause, a feature examined in detail elsewhere (Caballero and Huber 2010).

As temperature increases, the low-level storm tracks migrate poleward and become substantially weaker (Fig. 4; Table 1). Both these responses are familiar from previous work and robust across models (Schneider et al. 2010). The low-level eddy-driven jet (not shown here) also shifts poleward and becomes weaker.

b. Eddy size and phase speed

To determine the typical size and phase speed of the eddies responsible for low-level moisture transport, we perform space–time spectral decomposition of the 900-hPa meridional wind at $\varphi = 37.5^\circ$N, close to where poleward moisture flux peaks in all runs (see section 3). The decomposition can be written as

$$v(x,t) = \sum_{k_f} v_{k_f} \exp \left[ 2\pi i \left( \frac{kx}{2\pi a \cos \varphi} - ft \right) \right] + \text{c.c.}, \quad (8)$$

where $x$ is zonal distance, $t$ is time, $k$ is the zonal wavenumber, $f$ is the frequency, $a$ is the Earth’s radius, $v_{k_f}$ is the Fourier transform of the meridional wind, and c.c. indicates the complex conjugate. The mean wavenumber is computed as

$$\bar{k} = \sum_{k_f} k w_{k_f} \quad (9)$$

FIG. 4. Zonal- and annual-mean climatological eddy velocity scale $\sqrt{2EKE}$ (shading; m s$^{-1}$), where EKE is eddy kinetic energy, and specific humidity (contours; interval = 5 g kg$^{-1}$) for the (a) coldest and (b) warmest runs.
with weights \( w_{k,f} = \frac{|v_{k,j}|^2}{\sum_j |v_{k,j}|^2} \). We also compute mean frequency \( \bar{f} \) using the same weighted averaging over \( f \). The mean phase speed is then

\[
\bar{c} = \frac{2\pi a \cos \phi}{k} \bar{f},
\]

and a typical eddy length scale can be defined as half the mean wavelength as follows:

\[
L = \frac{\pi a \cos \phi}{k}.
\]

The results are displayed in Table 1. The eddy length scale is about 2500 km in the colder runs and increases slightly with temperature, as also found in many other models (Kidston et al. 2010). Phase speeds decrease from 13 to about 9 m s\(^{-1}\) from the coldest to the warmest case, consistently with slower advection by the weakening eddy-driven jet. The 900-hPa zonal-mean wind near 40\(^\circ\) latitude is only slightly smaller than the mean phase speed, consistent with steering levels at around 800 hPa in all cases; this turns out to be an important issue (see section 7).

**c. Eddy structure**

Though the mean horizontal eddy length scale does not vary much across runs (see above), it is possible that the vertical scale of the eddies, as well as the distribution of precipitation within them, could change substantially, with possibly important consequences for moisture flux. To assess such changes, we apply a feature-tracking algorithm to the model simulations, and use it to build a picture of a typical eddy through a compositing procedure similar to that used in much previous work (e.g., Bauer and Del Genio 2006; Field and Wood 2007; Rudeva and Gulev 2011). The feature-tracking method used here (Hanley and Caballero 2012) is a fairly standard cyclone identification and tracking algorithm based on mean sea level pressure (SLP). It pays particular attention to the robust tracking of multicenter cyclones (i.e., cyclones that during some stage of their life cycle feature more than one relative SLP minimum), but these aspects of the method do not play an important role here.

We apply the method to identify and track all the cyclones appearing in the Northern Hemisphere extratropics (30\(^\circ\)–90\(^\circ\)N) during four winter seasons (December–February) in each of the simulations, using 6-hourly SLP model output. The average lifetime of cyclones—the time from when a cyclone is first identified to when it can no longer be tracked—is between 4 and 5 days in all cases, with no trend toward greater or shorter lifetimes as the climate warms.

To create cyclone composites, we first identify the time of maximum intensity for each cyclone, taken as the time step at which the cyclone achieves its lifetime minimum central SLP. We then extract relevant fields from the model output at the time of maximum intensity within a radius of 4000 km from the cyclone center and average the extracted fields across all cyclones. The composites are centered at the cyclone center, and since the model output is on a regular latitude–longitude grid, data for cyclones centered at different latitudes will be on incompatible grids (the true zonal distance between grid points varies as the cosine of latitude). To avoid this problem, we project the data for each cyclone before averaging onto a common azimuthal equidistant grid centered at the cyclone center.

Figures 5a,b show near-surface wind, humidity, and convective precipitation composited in this way. The case 0 composite shows the familiar features associated with mature midlatitude cyclones, including poleward airflow in the warm sector parallel to the cold front and equatorward motion of colder, drier air in a direction roughly orthogonal to the cold front. Convective precipitation is concentrated around the cyclone center and in the region of poleward-flowing moist air along the cold front, as seen in observations (Ralph et al. 2004). There is also some convective precipitation in the cold air upwind of the cold front, presumably due to convective instability arising from low-level warming and moistening as cold, dry air flows over a relatively warmer ocean surface. The overall horizontal scale of the cyclone is roughly 3000 km, consistent with the spectral analysis results discussed above.

No qualitative changes are apparent in the case 5 composite (Fig. 5b). Cold and warm fronts are still recognizable, and though the warm sector is narrower, the overall scale of the cyclone is roughly the same as in the colder case, consistent with the spectral analysis results discussed above. Convective precipitation is considerably stronger and is more concentrated around the cyclone center, with little or no convection in the cold sector. The precipitation resulting from the large-scale condensation scheme (not shown in the figure) is also concentrated in the warm sector and around the cyclone sector, but interestingly it decreases with increasing temperature, dropping by about 30% from the coldest to the warmest run. Near-surface winds are considerably weaker, consistent with the diminished low-level mean eddy amplitudes seen in Fig. 4.

The vertical structure of the composite cyclones is illustrated in Figs. 5c,d. There is again little qualitative difference between the cold and warm cases, both of which feature separate upper- and lower-level meridional wind maxima and a single midtropospheric maximum in vertical wind, suggesting that the mean eddy is a troposphere-filling structure in both cases. The eddies...
are somewhat weaker and deeper in the warmer case, consistent with a raised tropopause, but there is little overall change in the vertical structure; in particular, both cold and warm cases have low-level equatorward flow maxima centered around 900 hPa.

5. Eulerian diffusivity and effective mixing length

Assuming that meridional moisture transport is dominated by eddy fluxes and that a diffusive approximation is applicable, we can write

\[ F = \kappa \frac{\partial}{\partial s} q, \]

where \( \kappa \) is the diffusivity, \( q \) is the mean specific humidity, and

\[ \frac{\partial}{\partial s} = \frac{1}{a} \frac{\partial}{\partial \varphi} + S \frac{\partial}{\partial p}, \]

where \( s \) is the distance along a typical parcel trajectory with slope \( S \) in the meridional–vertical plane (Vallis 2006, section 10.7). Including the vertical derivative on the rhs of (13) is important here because of the very strong humidity stratification. The humidity derivative can also be written as

\[ \frac{\partial}{\partial s} q = \gamma q_{\text{sat}} \]

with

\[ \gamma = r \left( \alpha \frac{\partial}{\partial s} T - \frac{\partial}{\partial s} \ln p + \frac{\partial}{\partial s} \ln r \right), \]

where \( q_{\text{sat}} \) is the saturation humidity at the local temperature \( T \) and pressure \( p \), \( r \) is the relative humidity, and \( \alpha = d \ln e_{\text{sat}}(T) / dT \) with \( e_{\text{sat}} \) as the saturation vapor pressure. Note that \( \gamma^{-1} \) can be interpreted as the characteristic length scale over which \( q_{\text{sat}} \) shows significant variation,
and (14) implies that the meridional moisture gradient will follow C-C scaling so long as this length scale does not change too much.

On dimensional grounds, the diffusivity can be formally written as the product of a velocity and a length as follows:

$$k = \nu_s \ell_e,$$

so that

$$F = \nu_s \ell_e q_{sat} \gamma. \tag{17}$$

Physically $\nu_s$ is interpreted as a typical eddy velocity and $\ell_e$ as an effective mixing length that, as suggested by comparison with (7), can be thought of as the characteristic length scale that maps the meridional humidity gradient into the typical humidity difference between poleward- and equatorward-flowing air. The simplest way to compute $\ell_e$ is directly from (17) as follows:

$$\ell_e = \frac{F}{\nu_s q_{sat} \gamma}. \tag{18}$$

The temperature response of the factors in the rhs of (17) is shown in Fig. 6. Here $\nu_s = \sqrt{2EKE}$, where EKE is the eddy kinetic energy and $\ell_e$ is computed from (18) using $F_{\text{max}}$ as the moisture flux. All quantities are lower-tropospheric averages around the latitude of maximum moisture flux. The mean trajectory slopes needed for the derivatives in $\gamma$ are computed using the Lagrangian back-trajectory algorithm described in section 6 and are listed in Table 1. For reasons to be discussed in section 6, we use the mean slope of equatorward-moving parcels only. There is some trend toward decreasing slopes as the temperature rises: $S$ drops by about 20% from the coldest to the warmest run. The results do not change qualitatively even if a fixed slope is employed in all calculations, confirming that changes in trajectory slopes are not an important issue here.

Two key features are apparent in Fig. 6. First, $\gamma$ changes relatively little across the simulations, so that to a first approximation it can be excluded as an important control on the behavior of poleward moisture flux. Second, both $\nu_s$ and $\ell_e$ decrease with temperature and in fact show very similar scaling, $\ell_e \sim \nu_s$.

These results suggest the following simple scaling for the moisture flux:

$$F \sim \nu_s^2 q_{sat}, \tag{19}$$

where $q_{sat}$ is a lower-tropospheric average of $q_{sat}$. As shown by the dashed lines in Figs. 2, this scaling fits the observed flux very well in winter; in summer the fit is also very good except at the warmest temperatures, where the $\nu_s \sim \ell_e$ scaling fails and $\nu_s^2 q_{sat}$ overestimates the actual flux. The overestimate is strong enough to affect the annual mean.

Note that the diffusive scaling presented here is only designed to capture the eddy moisture flux but it is compared in Fig. 2 with the total flux, which has both eddy and mean components. Direct evaluation of the mean flux $\overline{\nu \pi \sigma}$ in pressure coordinates shows that it is poleward in midlatitudes—as would be expected from a Ferrel cell circulation—and accounts for about 30% of the total flux in the coldest run in winter (somewhat more in summer), dropping to 15% in the warmest run. This varying proportion means that the mean component
scales differently from the eddy component, but nonetheless the scaling (19) is able to at least qualitatively capture the behavior of the total flux.

6. Lagrangian mixing length

An independent definition of the mixing length is obtained by combining the characteristic velocity scale with a characteristic Lagrangian decorrelation time scale (Taylor 1922; Vallis 2006, section 10.2) as shown:

$$\ell_L = \tau_L v_\ast$$  \hspace{1cm} (20)

where

$$\tau_L = \int_0^\infty R(t) \, dt$$  \hspace{1cm} (21)

and \(R(t)\) is the meridional velocity autocorrelation function along Lagrangian trajectories. The underlying physical picture (e.g., Majda and Kramer 1999, section 3) is that \(\tau_L\) gives the typical time for which particle motion remains ballistic, or dominated by advection within an individual, coherent eddy. Such motion is assumed rapid enough that diabatic effects are ineffectual and tracer concentration is conserved. At longer times, the particle is either swept out of the eddy or the eddy itself breaks up or otherwise decays, so that the particle’s Lagrangian velocity decorrelates. Meridional particle dispersion then becomes much slower, diabatic terms have time to act, and tracers mix with the environment. Overall, the motion mixes tracer concentration across a distance of order \(\ell_L\). The Lagrangian and effective mixing lengths can be expected to match in the limit in which diabatic processes are indeed weak. If diabatic processes are rapid enough to significantly alter tracer concentration during ballistic motion, then the result will be a shorter effective mixing length, \(\ell_e < \ell_L\). This is likely to be the case here, since condensation and precipitation can quickly deplete the moisture content of poleward- and upward-moving particles, while relatively dry equatorward-moving parcels can be rapidly moistened when passing through a region of moist convection (O’Gorman and Schneider 2006).

We implement a Lagrangian back-trajectory scheme that follows three-dimensional particle paths within the simulations by integrating the equation set

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x}, t),$$  \hspace{1cm} (22)

where \(\mathbf{x}\) and \(\mathbf{u}\) are three-dimensional position and velocity. The equations are integrated using a standard Runge–Kutta solver with a 1-h time step, using 6-hourly model output. Velocities are linearly interpolated to the particle location in space and time using pressure as a vertical coordinate, with the sign reversed to give backward trajectories. We follow particles with initial positions at 950 hPa and 40°N, released from each longitudinal grid point every 6 h during four simulated winters in each run, giving about 75,000 trajectories per run. Each trajectory is followed for 8 days. We then sort trajectories into two classes—“equatorward,” those whose forward velocity at the start of the back trajectory is equatorward, and “poleward” (the rest)—and treat the statistics of these two classes separately since they turn out to be quite different.

To get a sense of the typical structure of poleward and equatorward trajectories, we consider mean trajectories formed by averaging particle latitude and pressure level at each time step over each class (averaging trajectories rooted at different longitudes is appropriate here given the zonally symmetric statistics of the simulations). Mean trajectories for the coldest and warmest simulations are shown in Fig. 7. In both cold and warm cases, the mean equatorward trajectory is uniformly southeastward and subsiding, consistent with parcel motion within the subsiding branch of midlatitude cyclones embedded in a mean westerly current. Trajectory slopes computed from these mean equatorward trajectories over the last day before crossing 40°N are reported in Table 1.

The early stages of the poleward trajectories are also southeastward and subsiding until they reach the near-surface layer a few degrees south of 40° latitude; the trajectories then turn around sharply and travel poleward almost horizontally, remaining very close to the surface for around a day before crossing 40° again in the poleward direction. The sharp turning of the poleward trajectories can be explained by noting that the starting latitude of 40° (chosen to coincide with the peak moisture flux) is on the equatorward flank of the storm track, so that we are preferentially sampling the equatorward edge of midlatitude eddies.

Meridional velocity autocorrelation functions computed from these trajectories are shown in Fig. 8. For equatorward trajectories the autocorrelation functions are almost identical across runs, with a monotonically decreasing structure and an integral time of about 0.9 days in all cases. This integral time is within the range found in the low-level Pacific storm track of the real atmosphere (Swanson and Pierrehumbert 1997). Expression (20) with constant \(\tau_L\) implies a mixing length proportional to \(v_\ast\), consistent with the behavior of \(\ell_e\) seen in section 5.

Figure 9 compares the Lagrangian mixing length (20) computed for equatorward trajectories with the effective mixing length obtained from (18) in the form...
where $v’q’$ is the mean lower-tropospheric eddy moisture flux. The figure shows $\ell_L$ is about 700 km in the coldest run and declines to about 500 km in the warmest run; these values are comparable to those found in the low-level Pacific storm track of the real atmosphere (Swanson and Pierrehumbert 1997). The Eulerian mixing length $\ell_e$ scales roughly proportionally to $\ell_L$, as expected since both scale roughly as $y^*$. The estimates for $\ell_e$ are smaller than $\ell_L$ at all temperatures, indicating that diabatic effects do play a role in reducing the effective mixing length at all temperatures though they leave the scaling unchanged.

The autocorrelation functions for poleward trajectories (Fig. 8b) are rather different, with a nonmonotonic structure reminiscent of a damped oscillation. The oscillatory component presumably arises because of the rapid turning of poleward trajectories as described above. Unlike the equatorward autocorrelation functions, whose simple structure is adequately described using a single time scale, these functions require two separate time scales to capture the damping and oscillatory components. The presence of two distinct characteristic time scales

\begin{equation}
\ell_e = \frac{v’q’}{v_u q_{sat} y^*},
\end{equation}
means that a single length scale cannot be unequivocally defined, precluding a straightforward application of mixing length theory (Tennekes and Lumley 1972). We can thus draw no conclusions about the behavior of the mixing length for poleward trajectories.

In any event, it is likely that the Lagrangian mixing length for poleward trajectories does not matter much: the thin dotted line in Fig. 3b shows that typical humidity in the poleward branch closely follows C-C scaling, evaluated using surface temperature at the latitude of maximum flux. This suggests that the humidity of poleward-moving parcels is reset to near-surface values through diabatic effects during the day or so when they travel quasi horizontally within the boundary layer prior to crossing the latitude of peak flux. This means that the violation of C-C scaling is concentrated in the equatorward flow, justifying the use of equatorward trajectory slopes when evaluating the effective mixing length in section 5. Put another way, while the effective mixing length appears to be dominated by Lagrangian effects in the equatorward flow, it is likely dominated by diabatic effects in the poleward flow. Testing this hypothesis would require a detailed examination of the diabatic terms and the mean rate of airmass conversion in the poleward flow, which we leave for future work.

7. Controls on $\ell_L$

Summarizing the results of the previous sections, we find that the nonmonotonic behavior of poleward moisture transport has two proximate causes: the constancy of $\tau_L$ which by (20) and (16) implies a quadratic dependence of the diffusivity on $v_\infty$; and the decline of $v_\infty$ with increasing temperature, which is strong enough that the resulting diffusivity drop dominates over the increase in poleward moisture gradient at high temperatures and leads to nonmonotonic behavior.

An overall drop in eddy activity at high temperatures is robustly observed across different GCMs, and several recent papers have addressed the underlying reasons for this behavior (O’Gorman and Schneider 2008a; O’Gorman 2010, 2011). In the model runs studied here, $v_\infty$ measured at the storm-track axis drops about 25% from the coldest to the warmest run. However, as shown in Fig. 6, $v_\infty$ at the latitude of maximum moisture flux drops by about 40%. The difference arises because—as noted in section 3—the distance between the storm-track axis and the peak-flux latitude increases with temperature, so that the location of maximum flux is increasingly farther out on the equatorward flank of the storm track. This geometric effect clearly plays an important—if not dominant—role in explaining the nonmonotonic behavior of storm-track moisture flux.

We turn now to the question of what controls the kinematic quantity $\tau_L$, and how it is related to dynamical eddy time scales. A connection between kinematics and dynamics is explicitly made in the mixing length formula of Ferrari and Nikurashin (2010), who analytically solve a linear quasigeostrophic system stirred by stochastically excited eddies yielding

$$\ell_L = \frac{\tau_d}{1 + \left(\tau_d/\tau_s\right)^2} v_\infty. \quad (24)$$

Here $\tau_d$ is an externally specified linear eddy damping time scale, while $\tau_s = L/(c - U)$, where $L$ is the typical eddy length scale, $c$ is the typical eddy phase speed, and $U$ is a specified background zonal-mean wind; $\tau_s$ is thus the typical time at which the background wind sweeps a particle out of an eddy. A similar expression can be derived through a heuristic scale analysis of the fully nonlinear system (Nakamura and Zhu 2010).

In the presence of strong jets, we expect $U \gg c$ and $(\tau_d/\tau_s)^2 \gg 1$; in this limit, parcels are quickly swept out of eddies, their Lagrangian motion decorrelates on a time scale much shorter than the eddy lifetime, and diffusivity is suppressed. At the other extreme, eddies that are almost stationary with respect to the background flow are likely to have $(\tau_d/\tau_s)^2 \ll 1$, so that $\ell_L \approx \tau_d v_\infty$. Here we are concerned with eddy activity in the lower troposphere and thus close to the steering level, so we expect to be closer to the latter limit.

To test this expectation, we require quantitative estimates of $\tau_d$ and $\tau_s$. For $\tau_s$, we find (see Table 1) that $|c - U|$
increases from about 1 m s\(^{-1}\) in case 0 to about 5 m s\(^{-1}\) in case 5 while \(L' = 2500\) km for all runs, which implies \(t_s \approx 1\) week in all cases. To estimate \(t_d\) we use the cyclone-tracking algorithm described in section 4c to build a composite cyclone intensity life cycle for each run that allows a direct estimate of the typical cyclone damping time scale. As a simple measure of intensity, we take \((\text{SLP} - \text{SLP}_c)/\sin \phi_c\), where SLP\(_c\) and \(\phi_c\) are the SLP and latitude at the cyclone center (taken to be the local SLP minimum), respectively, while SLP is the SLP averaged around a circle of radius 1000 km concentric with the cyclone. This intensity measure is roughly proportional to the magnitude of the near-surface geostrophic wind averaged around the eddy center. The 1000-km averaging radius is chosen to include the area with the strongest winds in a typical eddy (see Fig. 5). We compute intensities over each individual cyclone track and then composite over all tracks, centering at the time of maximum intensity. To facilitate intercomparison, the composite intensity for each run is adimensionalized by its peak value. The results (Fig. 10) show that typical cyclone intensities decay roughly exponentially with a time scale of about 1.8 days in all cases.

These estimates imply \((\tau_d/\tau_L)^2 \lesssim (1.8/7)^2 \ll 1\), confirming that we are in the limit where \(\tau_L \approx \tau_d\). The direct estimates of \(\tau_L\) and \(\tau_d\) given above actually differ by about a factor of 2; but, given the assumptions and approximations made, it would be unreasonable to expect a precise quantitative match. Rather, the relevant and robust result is that \(\tau_L\) in the present problem is chiefly controlled by (and scales as) \(\tau_d\), which stays roughly constant as the global-mean temperature increases.

In a fully turbulent fluid where the main eddy damping mechanism is nonlinear energy exchange with other eddies in a turbulent cascade, the damping time scale should scale as the eddy turnover time, \(\tau_d \sim L/\nu_c\). In the present case, this assumption would predict considerably longer damping time scales at high temperatures, which is not apparent in Fig. 10. Additional damping mechanisms for lower-tropospheric eddies include upward Rossby wave propagation (Thorncroft et al. 1993) and energy exchange with the lower boundary (Swanson and Pierrehumbert 1997), and it appears that these mechanisms dominate over nonlinear energy transfer and vary little across the broad range of climates represented in the simulations.

8. Conclusions

We have studied the changes in midlatitude eddy structure and poleward moisture flux in aquaplanet simulations using NCAR’s CAM3 spanning a broad range of radiative forcings. We summarize our main conclusions as follows:

- During the winter season and in the annual mean, CAM3 exhibits nonmonotonic behavior of the moisture flux as found in previous studies using other models (Caballero and Langen 2005; O’Gorman and Schneider 2008b). The summer moisture flux violates C-C scaling even more strongly, remaining essentially constant throughout the temperature range studied.
- There are no significant changes in the spatial structure and organization of midlatitude cyclones as the temperature increases; the only significant change is a decrease in mean eddy amplitudes.
- Deviations from C-C scaling in both summer and winter are associated with the simultaneous drop in mean eddy velocities and effective mixing lengths as the temperature increases, leading to a large drop in storm-track diffusivity.
- Comparison of Lagrangian and effective mixing lengths implies that the contraction of the effective mixing length is not due to changes in diabatic mixing rates but is of kinematic origin: particles decorrelate in the same time but travel more slowly at higher temperatures, covering smaller distances.
- Ultimately, diffusivity decreases with temperature because mean eddy amplitudes decrease, while eddy damping time scales remain constant as temperature rises.
It is interesting that both the simplified GCM of O’Gorman and Schneider (2008b) and the comprehensive GCM used here reach their maximum poleward moisture transport at the same global-mean temperature of around 35°C, suggesting that this threshold may be a robust emergent feature not particularly tied to specific modeling assumptions or parameterizations, and raising hopes that it may also be a feature of the real atmosphere. However, both models do share some general family characteristics: for instance, convection is parameterized using an upright scheme that ultimately relaxes the lapse rate to a moist adiabat, with no allowance for symmetric instability and slantwise convection. How these assumptions affect the midlatitude mean static stability and eddy amplitudes at high temperatures is not obvious. Furthermore, surface exchange schemes may be important in setting the eddy damping time scale and could strongly impact the decorrelation time and mixing length. Clearly, considerably more work using a hierarchy of models is required to better establish the robustness of the results presented here.

A final point worth making is that the lower-tropospheric mixing lengths found here [and also in the real atmosphere; see Swanson and Pierrehumbert (1997)] are much smaller than the external eddy scale $L$. Moreover, mixing lengths decrease with temperature, while $L$ increases slightly. It is common in theoretical studies to identify $L$ and the mixing length, but this would be obviously inappropriate here. The mixing length can be expected to coincide with $L$ only when $\tau v_a \approx L$, so that particles have ample time to travel the length of the eddy in the cross-stream direction and "feel" the external scale. Where and to what extent this condition is satisfied in the atmosphere remains to be assessed.

REFERENCES


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