Zonal Jets as Meridional Transport Barriers in the Subtropical and Polar Lower Stratosphere

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ABSTRACT

Applications of recent results from dynamical systems theory to the study of transport and mixing in incompressible two-dimensional flows lead to the expectation that, independent of the background potential vorticity (PV) distribution, weakly perturbed zonal jets are associated with barriers that inhibit meridional transport. Here the authors provide evidence in support of this expectation based on the analysis of isentropic winds in the lower stratosphere as produced by the Canadian Middle Atmosphere Model (CMAM), a comprehensive general circulation model. Specifically, barriers to meridional transport are found to be associated with the (eastward) austral polar night jet, for which the meridional gradient of background PV is large, and also for the (westward) boreal summer subtropical jet, for which the background PV gradient is quite small. The identification of the meridional transport barriers is based on the computation of finite-time Lyapunov exponents (FTLEs), which characterize the amount of stretching about fluid particle trajectories. Being composed of regular fluid particle trajectories lying on invariant tori, the meridional transport barriers are identified with topologically circular, local minimizing curves or trenches of the backward-plus-forward FTLE field. Results from explicit passive tracer advection experiments and flux computations are also presented, which are consistent with results inferred using the FTLE diagnostic.

1. Introduction

In level or layerwise two-dimensional potential vorticity (PV)-conserving flows, PV isolines behave as material fluid lines. In the presence of a strong meridional gradient of background PV the meridional displacement of PV isolines is smaller than it would be in the presence of a weak PV gradient as a result of pseudomomentum conservation (see appendix A in Shepherd 1988). The smaller meridional displacement of PV isolines means less likelihood of Rossby wave breaking and hence of meridional transport. This is the basis of the “PV barrier” mechanism, which has traditionally been invoked to explain why an eastward zonal jet, which can be associated with a large meridional gradient of background PV, can behave as a meridional transport barrier (Dritschel and McIntyre 2008; Juckes and McIntyre 1987; McIntyre 2008). The most vivid example of such a behavior is provided by the austral polar night jet at the edge of the polar vortex in the earth’s lower stratosphere, which leads to the required confinement of cold polar air that allows the chemical reactions leading to the development of the Antarctic ozone hole to take place (McIntyre 1989).

The PV barrier argument cannot be applied to westward zonal jets, which, unlike eastward zonal jets, are not associated with steep meridional gradients of background PV. But this does not rule out the possibility that these zonal jets also behave as meridional transport barriers. Examples of westward zonal jets in the lower stratosphere are the boreal and austral summer subtropical jets. There is extensive observational evidence suggesting that these zonal jets act as meridional transport barriers (Grant et al. 1996; McCormick et al. 1995; Minschwaner et al. 1996; Mote et al. 1998; Rosenlof et al. 1997; Trepte and Hitchman 1992; Trepte et al. 1993). The observational evidence comes from measurements of the concentration of water vapor, nitrous oxide, and...
aerosols including those injected into the lower stratosphere by volcanic eruptions, most notably the Mount Pinatubo eruption. A review of stratospheric transport is provided in Shepherd (2007), where some attention is paid to subtropical meridional transport barriers, which constitute key ingredients of the so-called “tropical pipe” model of stratospheric transport (Plumb 1996). Consistent with the observational evidence above are results from studies of stratospheric transport based on the effective diffusivity mixing diagnostic computed using reanalyzed winds (Allen and Nakamura 2001; Haynes and Shuckburgh 2000a,b). The effective diffusivity was introduced by Nakamura (1996) and is a measure of mixing resulting from the stretching and folding of passive tracer contours. The three effective diffusivity studies cited above reveal that the summer subtropical jets coincide with regions of anomalously low effective diffusivity, thereby suggesting that these westward zonal jets act as meridional transport barriers.

That zonal jets, either eastward or westward, can be associated with meridional transport barriers independent of the background PV distribution may be anticipated from fluid mechanical arguments applied to simple examples. An example of recognized relevance to stratospheric dynamics (McIntyre and Palmer 1983) is given by the Stewartson–Warn–Warn Rossby wave critical layer solution (Stewartson 1978; Warn and Warn 1978). In that solution the meridional gradient of PV is uniform, but wave breaking and meridional transport occur within the critical layer. Thus, in this problem, it is the geometry of the velocity field, which depends on the intrinsic phase speed, that controls transport, not the PV gradient (Ngan and Shepherd 1997). As a result, for stationary waves, zonal jets (of either sign) may be expected to inhibit meridional transport because they increase the intrinsic phase speed of Rossby waves, thereby suppressing Rossby wave breaking.

Extending the above fluid mechanical arguments to more complicated situations seems to be very difficult, if not impossible. Fortunately, however, recent development in the area of dynamical systems has led to the development of tools that are appropriate to study transport in any flow regime. Such tools have recently been used to provide both theoretical and numerical support for the expectation that, independent of the background PV distribution, zonal jets can be associated with meridional transport barriers. Specifically, the theoretical support is provided by Rypina et al. (2007a,b) and is rooted in Kolmogorov–Arnold–Moser (KAM) theory (cf., e.g., Arnold et al. 2006). The relevant results of the vast KAM theory are those that relate to the stability of a degenerate 1-degree-of-freedom Hamiltonian system involving an arbitrary number of basic frequencies (Sevryuk 2007). The relevance of those KAM theory results stem from the fact that those results apply to the stability of fluid particle trajectories in an unsteady incompressible two-dimensional flow that accurately approximates the balanced geophysical flow in question. In particular, they apply to the study of fluid particle motion in the presence of zonal jets, which is beyond the scope of traditional KAM theory as noted earlier (del-Castillo-Negrete and Morrison 1993; Morrison 2000). The most recent KAM theory results cited above provide theoretical grounds for expecting that zonal jets should behave as meridional transport barriers, independent of the background PV distribution. Numerical support for this expectation is provided by Beron-Vera et al. (2008) through explicit passive tracer advection experiments using simulated winds. The winds used in that study were dynamically self-consistent, but they were produced by a highly idealized model. Specifically, the winds were sustained by an inviscid unforced reduced-gravity quasigeostrophic model initialized from a slightly perturbed “PV staircase” flow—that is, a flow consisting of a periodic sequence of alternating narrow eastward and broad westward zonal jets with PV piecewise constant between adjacent eastward jets (e.g., Dritschel and McIntyre 2008; Marcus and Lee 1998).

The goal of the present paper is to test the aforementioned dynamical systems–based expectation, namely that meridional transport barriers should be associated with zonal jets independent of the background PV distribution, based on the analysis of isentropic winds in the lower stratosphere as produced by a comprehensive general circulation model. The specific model considered is the Canadian Middle Atmosphere Model (CMAM) (Beagley et al. 1997). The principal motivation for focusing on free-running model winds rather than reanalyzed winds is the fact that the latter are not very reliable in the subtropics and tropics. Indeed, winds from data assimilation systems are known to give excessive dispersion, especially in the subtropics and tropics. Apart from the noise inherent in data assimilation systems, a significant problem is the lack of direct wind observations, and that in the subtropics and tropics winds are not well constrained by temperature. A summary of the vast literature devoted to this topic can be found in Shepherd (2007). It is noteworthy that CMAM has been shown to accurately model the tropical pipe (Sankey and Shepherd 2003), which justifies our particular model choice.

The remainder of the paper is organized as follows. Background information on the methodologies employed in the paper is given in section 2. This includes a review of the main results of the relevant KAM theory,
2. Methods

a. Setup and KAM theory results

As a result of the predominantly balanced nature of motions in the lower stratosphere, fluid particle trajectories near a zonal jet on an isentropic surface obey to a good approximation the same equations as those in an incompressible two-dimensional flow on a Cartesian plane. The equations constitute a nonautonomous 1-degree-of-freedom Hamiltonian system with the streamfunction playing the role of the Hamiltonian, and the fluid particle’s meridional and zonal coordinates playing the roles of the generalized coordinate and conjugate momentum, respectively. An appropriate approach to the study of Lagrangian dynamics near a zonal jet is to decompose the streamfunction into a steady unperturbed part, which corresponds to the zonal jet, plus an unsteady perturbation component, which is composed of a superposition of an arbitrary number of traveling waves (Beron-Vera et al. 2010; Rypina et al. 2007a). In a reference frame uniformly translating with the phase speed of one of those waves, the streamfunction takes the form

$$
\psi(x, y, t) = h(x, y) + r(x, y, \Omega_1 t, \ldots, \Omega_N t). 
$$

(1)

where $x$ is the wave comoving zonal coordinate, $y$ is the meridional coordinate, and $\Omega_j t \in \mathbb{T} = \mathbb{R}/2\pi$, where $\{\Omega_j\}$ are (relative) wave frequencies [for details, see Beron-Vera et al. (2010)]. The most succinct description of the Lagrangian dynamics in a neighborhood of the zonal jet’s axis makes use of action-angle variables $(I, \theta) \in D \times \mathbb{T}$, where $D \subset \mathbb{R}$ is closed and bounded. Such variables are defined implicitly by

$$
I := \frac{1}{2\pi} \int_{-\infty}^{\infty} X(y; h) \, dy, \quad \theta := \partial_I G, \quad G(y, I) := \int_0^y X(\xi; h) \, d\xi,
$$

(2)

where $X$ is the (moving) zonal coordinate of an isoline of $h$. In these variables the background and perturbation Hamiltonians, respectively, take the forms

$$
h(x, y) = H(I),
$$

$$
r(x, y, \Omega_1 t, \ldots, \Omega_N t) = R(I, \theta, \Omega_1 t, \ldots, \Omega_N t).
$$

(3a)

The action-angle variables $(I, \theta)$ evolve according to

$$
\dot{I} = -\partial_\theta R, \quad \dot{\theta} = \omega(I) + \partial_I R,
$$

(3b)

where

$$
\omega(I) := H'(I).
$$

(3c)

When $R = 0$, the Hamiltonian $H$ is a first integral (conserved quantity) and the motion described by (3) is completely integrable. The latter means that (3) can be solved by quadrature. Trajectories are then seen to lie on one-dimensional tori (topologically equivalent to circles) $\{I = \text{const}\}$. The frequency of the periodic motion along a torus is given by $\omega(I)$.

When $R \neq 0$, the Hamiltonian $H + R$ is in general not a first integral. In such a case, the motion obeying (3) is not integrable and trajectories may be irregular (not periodic or quasiperiodic). However, under sufficient smoothness assumptions on $H$ and $R$, if $|R|$ is sufficiently small, the frequencies $\{\Omega_j\}$ are sufficiently irrational, and $\omega(I)$ is not a constant, then system (3) admits a measure theoretically large set of invariant tori close to $\{I = \text{const}\}$ with frequency $\omega(I)$ close to $\omega(I)$ that vibrate with frequencies $\{\Omega_j\}$. The latter is an informal statement of a corollary of the KAM theorem proved in Sevryuk (2007). A few remarks are worth making. First, each admitted torus divides the phase space into a well-defined inside and outside. As a consequence, any trajectory starting in between any two such tori is constrained to remain in between for all time. In other words, the admitted tori act as barriers for the meridional transport of passive tracers. Second, the condition on the frequency mapping, commonly referred to as a nondegeneracy condition, differs from the standard, so-called Kolmogorov nondegeneracy condition, which requires that the frequency mapping’s twist does not vanish [i.e., $\omega'(I) \neq 0$]. The nondegeneracy condition involved in the KAM theorem cited above is a very weak nondegeneracy condition introduced by Rüssmann (1989) and allows one to prove the existence of many invariant tori even when the Kolmogorov nondegeneracy condition is locally violated, [i.e., when $\omega'(I) = 0$ at isolated $I$ values]. The importance of the latter is that such situations are found in the presence of zonal jets: at the core of a zonal jet the frequency of the fluid particle motion has a local extremum, and hence $\omega'(I) = 0$ there. Third, unlike the standard KAM theorem, the nonstandard KAM theorem above does not allow one to relate the frequencies along the admitted perturbed
tori with those along the unperturbed tori. This prevents one from speaking about “persistence” of invariant tori as in standard KAM theory (Sevryuk 1995). However, it should remain clear that the latter is inconsequential for our purposes: what really matters is that invariant tori can be proven to be present, and that such invariant tori serve as meridional transport barriers.

In Rypina et al. (2007a) it was further argued that tori lying in the vicinity of a twistless torus are very resistant to breaking. This particular form of stability has been referred in Rypina et al. (2007b) to as “strong KAM stability.” The basic rationale for the strong KAM stability is as follows. Torus destruction is caused by the excitation of resonances, which occurs when \( \omega(I)/\Omega_n \in \mathbb{Q} \) for some \( n \). The overlapping of resonances leads to the destruction of tori that are captured in the resonance. The width in action \( I \) of a resonance at a twist torus [i.e., for which \( \omega'(I) \neq 0 \)] scales like \( |\omega'(I)|^{-1/2} \), while the distance (in action) between nearby resonances scales like \( |\omega'(I)|^{-1} \) (Chirikov 1979). Consequently, when \( |\omega'(I)| \) is small the distance between nearby resonances is larger than the width of each individual resonance, which reduces the possibility of resonance overlapping and thus torus destruction. In Rypina et al. (2007b) it is shown that at a twistless torus of order \( m \)—that is, for which \( \omega'(I) = \cdots = \omega^{(m-1)}(I) = 0, \omega^{(m)}(I) \neq 0 \), where \( m > 1 \)—the resonance width scales like \( |\omega^{(m)}(I)|^{-1/(m+1)} \), which is smaller than that at a torus with twist. Because resonances are excited at isolated action values and resonance widths are narrower near twistless tori, tori lying in their vicinity are expected to be quite robust. As noted above, \( \omega'(I) = 0 \) at the core of a zonal jet. The strong KAM stability argument then predicts that invariant tori near the core of a zonal jet are very robust, and that these serve as meridional transport barriers.

Additionally, following the initial contribution of del-Castillo-Negrete and Morrison (1993), a large body of work has been dedicated to investigations of the dynamics of twistless tori under the influence of a periodic perturbation (e.g., Apte et al. 2005; Balescu 1998; del-Castillo-Negrete et al. 1996, 1997; Delshams and de la Llave 2000; Gaidashev and Koch 2004; Morrison 2000; Simó 1998).

b. Invariant tori as special classes of Lagrangian coherent structures

The invariant tori discussed in the previous section are material fluid curves that are different from those that are commonly referred to as Lagrangian coherent structures (LCSs) and that have been the main focus of previous studies of stratospheric transport and mixing (Boffetta et al. 2001; de la Cámara et al. 2009; Joseph and Legras 2002; Koh and Legras 2002; Lekien and Ross 2010). As defined in Haller and Yuan (2000), repelling (attracting) LCSs are material fluid curves that repel (attract) nearby fluid particles at the highest local rate over a finite-time interval. So defined, repelling (attracting) LCSs are akin to stable (unstable) manifolds in flows with periodic time dependence (e.g., Ottino 1989; Samelson and Wiggins 2006). Thus, rather than inhibiting transport (as invariant tori do), LCSs understood above facilitate mixing.

Haller (2001b) has introduced an extended definition of the LCS notion that allows invariant tori to be identified as special classes of LCS. Specifically, in Haller (2001b) the LCSs as defined in Haller and Yuan (2000) are referred to as hyperbolic LCSs, and two other classes of LCS are introduced: parabolic LCSs and elliptic LCSs. Parabolic LCSs are defined as material fluid curves that do not repel or attract nearby fluid particles at a noticeable rate over a finite-time interval. Elliptic LCSs are defined as material fluid curves that do not experience exponential stretching or folding over a finite-time interval. As such, elliptic LCSs constitute finite-time generalizations of invariant tori and thus include invariant tori as special classes. The above three types of LCS make up the complete skeleton (Mathur et al. 2007) of the Lagrangian circulation.

The need for a definition of LCS that applies to a finite-time interval is usually justified by the shortness of available velocity field records, which can be either generated numerically or, in the best-case scenario, observed. However, the need for such a definition can also be motivated by an interest in isolating and identifying LCSs that have a finite lifetime. A good example is the problem of isolating and identifying LCSs in the presence of stratospheric zonal jets because the latter have a seasonal dependence. With this in mind, it seems natural in that problem to restrict attention to Lagrangian evolution over time intervals not longer than about 1 month and to consider the velocity field in July as being independent of that in January, for example. Also, studying Lagrangian evolution on isentropic surfaces as done in this paper is physically meaningful only over time scales of up to 1 month (Haynes 2005). This provides further motivation for restricting attention in the Lagrangian calculations presented below to time intervals not exceeding approximately 1 month.

c. Detection of invariant tori

The computation of finite-time Lyapunov exponents (FTLEs) has been identified as one diagnostic tool that is capable of revealing hyperbolic LCSs (Haller 2001a,b, 2002; Shadden et al. 2005).
et al. (2010) the FTLEs can also be used to diagnose the presence of invariant tori and, more relevantly from the applications standpoint, their finite-time generalizations or elliptic LCSs.

The forward (backward) time FTLE characterizes the amount of forward (backward) time stretching about a fluid particle trajectory. Specifically, the (largest) FTLE is given by

$$\lambda^+_t(x_0) \equiv (2\pi)^{-1} \ln \lambda_{\text{max}} C^{t \to t_0}_t(x_0).$$

Here \(\lambda_{\text{max}}\) stands for maximum eigenvalue and

$$C^t_{t_0}(x_0) \equiv \nabla_{t_0} \nabla^T_t(x_0)$$

is the (right) Cauchy–Green tensor, where \(\nabla_t \equiv \nabla x \mapsto x(t, x \cdot t_0)\) is the flow map, which takes the position of a fluid particle at time \(t_0\) to that at time \(t \neq t_0\). Important properties of the FTLE are its objectivity (i.e., independence of the reference frame) and quasi-Lagrangian character. The objectivity of the FTLE readily follows from the invariance of \(C^t_{t_0}(x_0)\) under the reference frame change \(x \mapsto Q(t)x + a(t)\), where \(Q(t)\) is a proper orthogonal matrix (e.g., Ottino 1989). The quasi-Lagrangian character of the FTLE, expressed as \(d\lambda^+_t(x_0)/dt_0 = O(|t|^{-1})\), has been proven in Shadden et al. (2005). Local maximizing curves or ridges of forward (backward) FTLE field correspond with some caveats (Branicki and Wiggins 2010; Haller 2002) to regions of maximal local repulsion (attraction). As such, forward (backward) FTLE ridges have been broadly identified as quite good detectors of repelling (attracting) hyperbolic LCS in a large variety of applications [see Peacock and Dabiri (2010) for a recent review]. Sufficient and necessary conditions for FTLE ridges to mark hyperbolic LCS have been recently derived in Haller (2011).

Indicators of the presence of invariant tori can be expected to be given by topologically circular, local minimizing curves or trenches of the backward-plus-forward FTLE field. This expectation stems from the fact that invariant tori are closed curves on which motion is regular and associated with a regular trajectory is a vanishing infinite-time LE. Beron-Vera et al. (2010) employed topologically circular trenches of the backward-plus-forward FTLE field as indicators of invariant tori but noted a few caveats. The most important caveat is that such topologically circular trenches most easily indicate the presence of twistless tori. But because those tori are very resistant to breaking according to the strong KAM stability argument, that diagnostic tool is well suited to detecting those tori.

Additional diagnostic tools that are capable of revealing hyperbolic LCS include the hyperbolicity time introduced by Haller and Yuan (2000), extensions of this concept proposed by Haller (2001b), and the finite-size Lyapunov exponents or FSLE (Artale et al. 1997). As the FTLE diagnostic, these additional diagnostics could well be employed in detecting elliptic LCSs. In fact, the FSLE diagnostic was applied to reanalyzed stratospheric winds by Joseph and Legras (2002) and Koh and Legras (2002), who noted the appearance of a trench of FSLE field around the austral pole in the winter. Such a trench was associated with the locus of the largest PV contrast at the edge of the austral stratospheric polar vortex. Those authors further speculated that KAM theory may play a role in explaining its maintenance. It has also been argued (Allen and Nakamura 2001; Haynes and Shuckburgh 2000a; Shuckburgh and Haynes 2003) that the effective diffusivity may be useful in identifying transport barriers of the elliptic LCS type. Consistent with this, Joseph and Legras (2002) noted that the FSLE trench they detected coincided well with a region of anomalously low effective diffusivity. However, d’Ovidio et al. (2009) and Shuckburgh et al. (2009) noted difficulties with this association. One reason for such difficulties may be the fact that when effective diffusivity calculations are made, one has to define a reference curve across which diffusivity is estimated. That means that a perfect barrier that is wobbly, such as in the case of the invariant tori predicted by the KAM theory discussed above, will necessarily lead to nonzero diffusivity (or nonzero flux).

3. Results

In this section the FTLE diagnostic is employed in the extraction of estimates of invariant tori from CMAM isentropic winds, or, more precisely, of finite-time generalizations of invariant tori or elliptic LCSs. The legitimacy of the estimated invariant tori, which serve as meridional transport barriers, is investigated by carrying out explicit passive tracer evolution experiments as well as flux computations.

CMAM constitutes an extension into the middle atmosphere of the general circulation models developed by the Canadian Centre for Climate Modeling and Analysis known as CCCma (Beagle et al. 1997; de Grandpré et al. 2000; Scinocca et al. 2008). The model is based on the primitive equations of motion. It has a spectral representation in the horizontal. The vertical coordinate is hybrid, being terrain-following at low levels and approximately pressure-following elsewhere. It includes a fully interactive stratospheric chemistry module, a comprehensive radiation scheme, and a suite
of other parameterizations relevant to physical processes from the earth’s surface up to the model lid at 100-km altitude. Sea surface temperatures and sea ice distributions are prescribed. The particular simulation considered here corresponds to an ensemble of two simulations for the recent past (1950–2004), denoted REF1 in Eyring et al. (2005), which do not include a quasi-biennial oscillation (QBO). These simulations employ a spectral representation in the horizontal with a resolution of T32 (triangular truncation at spherical harmonic index 32, corresponding to a spacing of 6° on the linear transform grid) and a finite difference formulation in the vertical with an approximately 3-km resolution on average (the vertical resolution in the troposphere and lowermost stratosphere is much higher, about 1 km). The time-stepping scheme is semi-implicit, with a time step of 360 s. The passive tracer calculations presented here are based on wind time series in selected periods along year 2000. Winds were archived every 6 h, after interpolating them onto the 550-K isentropic surface, which lies within the lower stratosphere. Transport calculations based on isentropic winds do not account for the effects of exchanges across isentropic surfaces. As noted above, for the earth’s stratosphere this is a good approximation for time scales on the order of 1 month or less.

It has been demonstrated (e.g., Eyring et al. 2006; Waugh and Eyring 2008) that CMAM is capable of adequately representing several aspects of the dynamics of the stratosphere as well as its thermodynamics and chemistry. However, for the purpose of the present work it is sufficient to note that CMAM correctly describes the most salient features of the seasonal circulation in the lower stratosphere. Features of specific interest for the present work are the following. First is the seasonal formation and decay of an intense cyclonic vortex over the winter pole, the associated eastward zonal jet (so-called polar night jet), and the sharp PV contrast across the edge of the vortex. In both hemispheres, a polar vortex forms in the fall, reaches maximum strength in midwinter, and decays in late winter to early spring. Disturbances are more vigorous in the Northern Hemisphere than in the Southern Hemisphere. This is mainly a result of the rougher topography and larger land–sea contrast in the Northern Hemisphere than in the Southern Hemisphere, leading to stronger excitation of stationary planetary Rossby waves in the Northern Hemisphere than in the Southern Hemisphere. Consistent with these observations, the boreal polar vortex is weaker than the austral polar vortex. Second is the seasonal formation of a westward zonal jet in the summer subtropics (here referred to as summer subtropical jet), which is associated with relatively homogeneous background PV distribution. In both hemispheres, a westward zonal jet develops in the subtropics in late spring, reaches maximum strength in midsummer, and decays in late summer to early fall. Consistent with the prevalence of these westward zonal jets, which inhibit the propagation of stationary planetary Rossby waves, the summer subtropics are relatively disturbance free. Figure 1 illustrates these characteristics of the seasonal circulation in the lower stratosphere as described by the CMAM simulation considered. The figure shows in time–latitude space the distribution of zonally averaged PV with contours of zonally averaged zonal velocity overlaid on the 550-K isentropic surface. Note the presence of westward zonal jets in the summer subtropics and eastward zonal jets in the winter polar region. Note also the strong meridional gradients of PV associated with the eastward zonal jets (particularly the austral polar night jet) in marked opposition to the weak meridional gradients of PV associated with the westward zonal jets.

Our analysis will be mainly restricted to the period 1 July 2000–31 July 2000, a period during which the (westward) boreal summer subtropical jet and the (eastward) austral polar night jet are both already well developed. Accordingly, the left panel of Fig. 2 shows selected streamlines for winds averaged over a 30-day interval around 15 July 2000. These averaged streamlines reveal clearly the presence of the two zonal jets in question: the boreal summer subtropical jet, whose axis (indicated by a dashed curve) undulates roughly about 20°N, and the austral polar night jet, whose axis (also indicated by a dashed curve) wobbles around approximately 60°S. The latter can be fairly accurately approximated as a superposition of a purely zonal jet and a wavenumber-2 Rossby wave. The corresponding streamfunction is taken as an estimate for the unperturbed Hamiltonian in the wave-comoving frame $h(x, y)$ in (1) to which the KAM theory results described above can be applied. [To apply such results, it is not necessary to know the structure of the perturbation Hamiltonian $r(x, y, \Omega_1, \ldots, \Omega_M)$ in (1); it is sufficient to note that over a finite time interval it can be approximated with an arbitrary precision with a superposition of a finite number of traveling waves.] A caveat to bear in mind is that the intensity of each jet, the boreal summer subtropical jet and the austral polar night jet, varies with the season. For example, the boreal summer subtropical jet is absent in the boreal winter, while the austral polar night jet is not present in the austral summer. This seasonal variability of the background flow should be incorporated into the unperturbed Hamiltonian. But since such variability occurs over a time scale that is long compared to that of any wave present in the
perturbation field, it is most natural to adopt a parametric description of the unperturbed Hamiltonian where the parameter is a slow (seasonal) time variable. With this caveat in mind, we turn to the description of the right panel of Fig. 2. The plot in this panel shows the estimated structure of $v(I)$, defined in (3c), near the axis of the boreal summer subtropical jet (top) and the austral polar night jet (bottom). The computation of $v(I)$ was carried out using coordinates $x = a(\lambda - \lambda_0) \cos \vartheta_0$ and $y = a(\vartheta - \vartheta_0)$, where $a$ is the mean earth radius. These coordinates are locally Cartesian in the vicinity of a reference latitude $\vartheta_0$, which was chosen to be $+20^\circ$ for the boreal summer subtropical jet and $-60^\circ$ for the austral polar night jet (the choice of reference longitude $\lambda_0$ is unimportant). The estimated $v(I)$ within the vicinity of each of the zonal jets exhibits a global maximum right at the zonal jet’s axis. The twist condition is thus violated on each of the corresponding tori. According to the strong KAM stability argument described above, the tori in the vicinity of each of these twistless tori should be very resistant to breaking under perturbation. In other words, independent of the background PV structure associated with the zonal jets, these are expected to act as barriers to meridional transport.

To test this expectation, we first concentrate on the computation of the FTLE diagnostic. As noted above, topologically circular trenches of the backward-plus-forward FTLE can be identified with invariant tori. The left panel of Fig. 3 shows zonally averaged zonal wind $\bar{v}$ (thick curve) and potential vorticity $\bar{f}$ (thin curve) on 15 July 2000. The second, third, and fourth panels from the left in Fig. 3 show backward, forward, and backward-plus-forward FTLE on 15 July 2000, respectively. The FTLE fields shown were computed by evaluating (4) using finite differences for a set of initial fluid particle positions distributed on a regular grid with 1000 points in longitude and 500 points in latitude. The fluid particle trajectory integrations involved in the FTLE field calculations were carried out using an implementation of the Dormand–Prince 4(5) algorithm. The required interpolations in space and time were performed using a cubic method. The integration time $|\tau| = 7$ days was found to be sufficiently long to carry out the transport barrier identification; other choices do not alter substantially the results. Note the occurrence in Fig. 3 of two trenches in the backward and forward FTLE fields, one located near the core of the boreal summer subtropical jet (which undulates about roughly $20^\circ$N) and the other located near the core of the austral polar night jet (which undulates approximately around $60^\circ$S). A thin (thick) red curve is used to indicate the backward (forward) FTLE trench in the Northern Hemisphere, while a thin (thick) blue curve is employed to mark the backward (forward) FTLE trench in the Southern Hemisphere. These curves were computed as attractors of the gradient vector field $-\nabla A_s(x_0)$. Note that the thin and thick red curves and the thin and thick blue curves nearly overlap in position. In other words,
two backward-plus-forward FTLE trenches are revealed, suggesting the presence of two invariant tori consistent with the KAM theory results discussed above. Importantly, these invariant tori, which serve as barriers for the meridional transport, are associated with the eastward polar night jet, for which the background PV gradient is large, and the westward boreal summer subtropical jet, for which the background PV gradient is quite small.

An explicit test of the transport barrier nature of the identified backward-plus-forward FTLE trenches is now presented. Such a test is based on the execution of explicit passive tracer advection experiments. Specifically, Fig. 4 shows selected snapshots for July 2000 of the evolution of passively advected tracers (dots) superimposed on the backward FTLE field on the corresponding date (grayscale). The thick and thin black curves are zonally averaged zonal velocity and PV, respectively, on each date that the tracer distributions and FTLE fields are shown. The passive tracers, each composed of $5 \times 10^3$ synthetic fluid particles, were released on latitude lines flanking the latitude lines 20°N and 60°S. In the vicinity of each of these latitude lines a backward-plus-forward FTLE trench is seen to undulate about a mean latitude. Such backward-plus-forward FTLE trenches are found to exist (but are not shown here) throughout the period mid-May–mid-October 2000. Consistent with the expectation that the identified backward-plus-forward FTLE trenches indicate the presence of invariant tori, which behave as meridional transport barriers, the passive tracers are not seen to traverse them, even after 30 days of evolution. The behavior of the passive tracers just described is to be contrasted to that of passive tracers in Fig. 5. This figure is similar to Fig. 4, but for January 2000. During that month neither the boreal summer subtropical jet nor the austral polar night jet is present. Thus, meridional transport barriers are not expected to develop near 20°N or near 60°S. Consistent with this expectation, passive tracers near 20°N and 60°S are seen to mix freely.

Next a more ambitious test of the transport barrier nature of the identified backward-plus-forward FTLE trenches is presented. Such a more ambitious test is based on the computation of the instantaneous area flux per unit length across an FTLE trench. Let $\mathbf{x}_0 = \mathbf{X}_0(s)$, where $s$ varies in a certain domain of $\mathbb{R}$, be a local parameterization of the FTLE trench at time $t = t_0$. The instantaneous area flux per unit length across a point on the moving FTLE trench is equal to the projection of the velocity relative to the moving curve in the normal direction to the curve at that point. That is,
\[
\mathbf{u}[X_{t_0}(s),t_0] = \left\{ \mathbf{u}[X_{t_0}(s),t_0] - \frac{dX_{t_0}(s)}{dt_0} \right\} \cdot \mathbf{n}[X_{t_0}(s),t_0],
\]

(6)

which vanishes if the FTLE trench is a material curve. Because at any time \(t = t_0\) each of the FTLE trenches of interest here is describable by an invertible function \(y_0 = \eta(x_0,t_0)\), (6) can be readily computed as follows. First, note that

\[
\frac{dX_{t_0}(s)}{dt_0} = \alpha(x_0,t_0) + \left( \alpha(x_0,t_0) \frac{\partial \eta}{\partial x_0}(x_0,t_0) \right) \mathbf{y},
\]

(7)

where \(\alpha(x_0,t_0)\) is arbitrary. Second, note that

\[
\mathbf{n}[X_{t_0}(s),t_0] = \frac{\mathbf{y} - \partial_{x_0} \eta(x_0,t_0) \mathbf{x}}{\sqrt{1 + \left[ \partial_{x_0} \eta(x_0,t_0) \right]^2}}.
\]

(8)

As a result,

\[
\varphi[X_{t_0}(s),t_0] = \left\{ u[x_0,\eta(x_0,t_0),t_0] - u[x_0,\eta(x_0,t_0),t_0] \right\} \times \partial_{x_0} \eta(x_0,t_0) - \partial_{t_0} \eta(x_0,t_0),
\]

(9)

where \(u (v)\) is the zonal (meridional) velocity component. [Haller (2011) presents a computable formula for the instantaneous flux per unit length across an FTLE ridge that is expressed in terms of the FTLE itself. A similar formula can be derived for an FTLE trench, but its evaluation is computationally more demanding than that of the formula considered here.] Figure 6 presents an evaluation of (9) for each of the trenches revealed in the backward and forward FTLE fields on 15 July 2000, which are depicted in Fig. 3. Specifically, the blue thin (thick) curve in Fig. 6 is the instantaneous area flux per unit length across the backward (forward) FTLE trench associated with the boreal summer subtropical jet, which wobbles about roughly 20°N. The red thin (thick) curve is the flux across the backward (forward) FTLE trench associated with the austral polar night jet, which undulates about roughly 60°S. The dashed curve is shown for reference and corresponds to the flux across the 60°S latitude line. Consistent with the results shown in Fig. 4, the flux across each of the FTLE trenches revealed is very small (in absolute magnitude), particularly compared to that across 60°S. The presence of a small flux across each of the FTLE trenches is explained by the small mismatch in the position of the forward and backward FTLE trenches, which may be attributed to the spatiotemporal coarseness of the velocity field that supplies advection in the Lagrangian calculations. These results prevent one from rigorously concluding that associated with the westward boreal summer subtropical jet and the eastward austral polar jet are invariant tori. However, they do provide support to the idea that both eastward and westward zonal jets behave as barriers for the meridional transport to a good approximation.

Before closing this section we turn our attention back to Fig. 5 to note that a trench is seen to develop in the
backward FTLE field on 31 January 2000 near 20°S.
Such a backward FTLE trench is most evident from
February through early March 2000 (not shown). A
trench (not shown) is also seen to be present in the
forward FTLE field near 20°S in the same period and to
roughly overlap in position with the backward FTLE
trench. This backward-plus-forward FTLE trench near
20°S indicates the presence of an invariant torus, which
behaves as a meridional transport barrier. This feature is
entirely analogous to the transport barrier associated
with the boreal summer subtropical jet that we have
described. We have not found a strong indication of
the presence of a polar meridional transport barrier in
the boreal winter. This is consistent with the observation
that during year 2000 of the CMAM simulation
considered, disturbances on the boreal polar vortex
are quite strong, leading to a poorly-defined (e.g., not
persistent) boreal polar night jet. It should be noted that
this is not a spurious result of the CMAM simulation;
rather, it is a realistic feature that has been observed in
the transport-barrier diagnostics of Sankey and Shepherd
(2003).

4. Summary and discussion

In this paper we have presented support for our hy-
pothesized association of transport barriers with both
eastward and westward zonal jets. Our association of
transport barriers with zonal jets independent of the
background potential vorticity (PV) distribution builds
on recent results relating to Kolmogorov–Arnold–
Moser (KAM) theory, which deals with the stability of
Hamiltonian systems under perturbation. Such KAM
theory results predict the presence of invariant tori (i.e.,
closed material fluid curves that do not experience ex-
ponential stretching or folding) in the vicinity of the
axis of a zonal jet, which serve as barriers for the mer-
idional transport of passive tracers. The basis of such
KAM theory results is the enhanced probability of nonoverlapping resonances as a consequence of the lack of twist of the frequency mapping at a zonal jet axis. Unlike our prior work, which was based on the analysis of a dynamically self-consistent, but highly simplified, flow description, the study presented here is based on the analysis of isentropic winds produced by a comprehensive general circulation model. Such a comprehensive general circulation model is given by the Canadian Middle Atmosphere Model (CMAM), which is capable of describing the features of the seasonal circulation in the lower stratosphere most relevant for the study carried out here. Such features include the seasonal formation and decay of a polar vortex in the winter hemisphere with an associated eastward zonal jet at, and a marked PV contrast across, its perimeter; and the seasonal formation and decay of a westward zonal jet in the summer subtropics with an associated nearly uniform background PV distribution. Consistent with the KAM theory predictions, it was found that invariant tori (finite-time generalizations of invariant tori, or elliptic Lagrangian coherent structures, to be more precise) that serve as transport barriers are associated with the eastward austral polar night jet, the westward boreal summer subtropical jet, and the westward austral summer subtropical jet. No clear evidence of a transport barrier in the boreal winter polar region was found, however, consistent with the absence of a well-developed and persistent boreal polar night jet in the simulation considered. The identification of invariant tori was carried out using the finite-time Lyapunov exponent (FTLE) diagnostic, which is commonly employed to detect LCSs of hyperbolic type (the locally strongest repelling or attracting material fluid curves). Unlike hyperbolic LCS, which are associated with ridges in the backward or forward FTLE field, twistless invariant tori are associated with topologically circular trenches in the backward-plus-forward FTLE field. The transport barrier nature of the identified FTLE trenches was tested by executing explicit passive tracer advection.
calculations. Also, the flux across the identified FTLE trenches was computed and found to be consistently small.

The role played by stratospheric westward zonal jets as meridional transport barriers described here is consistent with the role played by other westward zonal jets occurring in nature. Westward zonal jets are found in the weather layer of gas giants such as Jupiter, whose circulation comprises a sequence of alternating eastward and westward zonal jets (Porco et al. 2003; Vasavada and Showman 2005). The axes of the Jovian zonal jets coincide with the boundary between adjacent belts and zones, which correspond, respectively, to the dark and light zonal bands that characterize visible images of Jupiter's weather layer. Belts and zones have different radiative transfer properties (analyses are not limited to the visible band of the electromagnetic spectrum), which is attributed to differences in chemical composition (Banfield et al. 1998; Carlson et al. 1994; Irwin et al. 2001, 2005; Simon-Miller et al. 2001). Assuming that the weather layer flow is approximately two-dimensional and that chemical species are long-lived, one may infer that both eastward and westward jets act as robust meridional transport barriers. Support for this inference is provided by modeling studies based on a general circulation model of the Jovian atmosphere including condensible clouds (Zuchowski et al. 2009a,b), which show that cloud bands are modulated by both eastward and westward zonal jets. The westward jets are not associated with strong PV gradients. It has also been pointed out that oceanic westward zonal jets can behave as meridional transport barriers in much the same manner as eastward oceanic zonal jets (Thompson 2010). It should be noted, however, that a PV staircase flow model, which has alternating eastward and westward zonal jets with PV piecewise constant between adjacent eastward jets, does not provide a good description of these or other oceanic zonal jets (Berloff et al. 2009). In contrast, a PV staircase flow model with one or two PV steps describes, qualitatively at least, the earth's stratospheric zonal jets (Polvani et al. 1995), while a PV staircase flow model with many PV steps fairly accurately describes Jovian zonal jets (Peltier and Stuhne 2002; Read et al. 2006; Scott and Polvani 2008).

Zonal jets are also observed in the dynamics of magnetized plasmas, which are mathematically analogous to geophysical fluid dynamics (Diamond et al. 2005). Such zonal jets are known to act as barriers that inhibit transport in tokamaks (Thyagaraja et al. 2004). The confinement produced by the transport barriers is of great importance in the design and construction of fusion power plants and is the subject of active research (Boozer 2004). Ideas from dynamical systems theory have been employed (del-Castillo-Negrete 2000; Morrison 2000) to study transport and mixing in magnetized plasma zonal jets, but the connection between the latter and transport barriers has not yet been fully established.

Finally, an open area of research, which deserves to be investigated but is beyond the scope of this paper is the connection between nonlinear stability of PV patches (Dritschel 1988a,b; Sideris and Vega 2009; Tang 1987) and the KAM theory results discussed in this paper, which provide (kinematic) conditions under which such PV patches can be expected to experience weak deformation.

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