Cooling of Entrained Parcels in a Large-Eddy Simulation

TAKANOBU YAMAGUCHI
Cooperative Institute for Research in Environmental Sciences, University of Colorado, and NOAA/Earth System Research Laboratory, Boulder, Colorado

DAVID A. RANDALL
Department of Atmospheric Science, Colorado State University, Fort Collins, Colorado

(Manuscript received 11 March 2011, in final form 24 August 2011)

ABSTRACT

The relative importance, for cloud-top entrainment, of the cooling rates due to longwave radiation, evaporation, and mixing was assessed through analysis of the results produced by a Lagrangian parcel-tracking model (LPTM) incorporated into a large-eddy simulation model. The LPTM predicts each parcel’s trajectory over time, using the resolved velocity simulated by the host model. An LPTM makes it possible to identify entrained parcels: this is almost impossible to do in an observational study.

A nocturnal stratocumulus cloud was simulated over 4 h using a 5-m horizontal grid spacing and a 2.5-m vertical grid spacing. At the start of the last hour of the simulation, over 40 million parcels were placed near the top of the inversion layer and then tracked. Parcel histories were analyzed to identify entrained parcels.

Entrainment occurs in cloud holes, which occur in dry regions of sinking air. Entrainment into the mixed layer is regulated by buoyancy, which requires parcels to be precooled in the inversion layer, prior to entrainment. A mixing fraction analysis was used to separate the cooling due to longwave radiation, evaporation, and mixing. Results show that radiative and evaporative cooling are of comparable importance, but that mixing is by far the dominant cooling mechanism. The radiative cooling rate is strongly inhomogeneous, and only weak radiative cooling is found in regions of entrainment. Therefore, the entrained parcels experience less than the horizontal-mean radiative cooling. Although radiative cooling maintains the boundary layer turbulence, its direct effect on buoyancy of entrained parcels is modest.

1. Introduction

Entrainment at the top of marine stratocumulus cloud layers is a one-way process, in which free-atmospheric air is captured by the turbulent layer below. The entrained air becomes turbulent and cloudy through radiative cooling and interactions with the boundary layer air. Cloud-top entrainment affects cloud-layer properties including cloud cover, cloud-top height, and cloud thickness. These cloud-layer properties are important for global climate as well as local weather (e.g., Randall et al. 2007).

The marine stratocumulus boundary layer consists of a cloudy mixed layer capped by a stable layer, frequently called the inversion layer or entrainment interface layer (EIL). Recently entrained air that is found in the mixed layer has been dragged downward through the EIL by negative buoyancy produced by cooling due to a combination of longwave radiation, evaporation, and mixing.

Attempts to partition the cooling during entrainment have been based on both observations and numerical modeling (e.g., Shao et al. 1997, hereafter SR97; vanZanten and Duynkerke 2002, hereafter VD02). These studies suggest that of the three processes mentioned above, longwave radiative cooling is the dominant contributor to the negative buoyancy. This is plausible in part because the stratocumulus-layer turbulence is known to be radiatively driven (Lilly 1968) and because stratocumulus cloud water amounts are typically small (on the order of 0.5 \( \text{g} \text{kg}^{-1} \)), so the potential for evaporative cooling is limited (Albrecht et al. 1985). The conclusions were obtained indirectly, however, because the histories of entrained air parcels were not directly analyzed.

Corresponding author address: Takanobu Yamaguchi, NOAA/Earth System Research Laboratory (ESRL)/Chemical Science Division (R/CSD2), 325 Broadway, Boulder, CO 80305.
E-mail: tak.yamaguchi@noaa.gov

DOI: 10.1175/JAS-D-11-080.1

© 2012 American Meteorological Society
Recent observational studies suggest that cooling due to mixing may play an important role for entrainment. Gerber et al. (2005) conclude that 1) entrainment occurs at the “cloud holes” (i.e., depleted liquid water content region), 2) thermodynamic properties resemble between the cloud holes and surrounding cloud, 3) the cloud-top detrainment of the cloudy air mixes and then cools and moisten the cloud hole, and 4) entrainment occurs when the buoyancy matches between the cloud hole and cloudy air. Similar conclusions were reached by de Roode and Wang (2007). The formation of cloud holes through entrainment is suggested by a numerical study of Kurowski et al. (2009).

We have attempted to identify individual entrainment events and make direct calculations of the cooling of entrained parcels due to radiation, evaporation, and mixing. To do this, we use a Lagrangian parcel-tracking model (LPTM) coupled with a large-eddy simulation (LES). The LPTM predicts the trajectories of air parcels and diagnoses their velocities and thermodynamic properties by spatial interpolation from the grid of the host model. As Heus et al. (2008) discussed, an LPTM tracks many massless tracer parcels that follow the simulated flow, and these parcels are uniquely identifiable by their trajectories. In this approach, an LPTM can include the effects of mixing as a physical process, which can modify the advected parcel’s properties.

LPTMs have been used previously. For instance, Krueger et al. (1995) used an LPTM to identify convective updrafts and downdrafts in two-dimensional boundary layer simulations. Lin and Arakawa (1997) applied the method to identify the origins of the parcels entrained into deep convective clouds, as simulated by a cloud system–resolving model, and Harrington et al. (2000) studied the effects of radiation on the growth of a population of cloud drops in Arctic stratus clouds. Weil et al. (2004) developed a Lagrangian dispersion model that used LES output to study particle dispersion in the convective boundary layer. Heus et al. (2008) adapted the model of Weil et al. (2004) to show that mixing between a shallow cumulus cloud and its environment happens mainly on the sides of the clouds.

The outline of this paper is as follows. The LES model and LPTM are discussed in section 2. The stratocumulus simulation is described and its results are presented in section 3. Cloud-top entrainment is analyzed in section 4. A summary and conclusions are presented in section 5.

2. LES model and LPTM

a. LES model

Our LES model is the System for Atmospheric Modeling (SAM; Khairoutdinov and Randall 2003). SAM has been widely used (e.g., Caldwell and Bretherton 2009; Cheng and Xu 2009; Khairoutdinov et al. 2009; Yamaguchi et al. 2011), and has participated in several Global Energy and Water Cycle Experiment (GEWEX) Cloud System Study (GCSS) LES intercomparison studies (Moeng et al. 1996; Siebesma et al. 2003; Stevens et al. 2005; Ackerman et al. 2009). SAM is a nonhydrostatic model based on the anelastic equations. It predicts the three velocity components, liquid water static energy, total nonprecipitating water mixing ratio (vapor, cloud water, and cloud ice), total precipitating water mixing ratio (rain, snow, and graupel), and subgrid-scale (SGS) turbulence kinetic energy (TKE). It uses finite differences on a staggered (Arakawa C) grid. A fifth-order scalar advection scheme tested by Yamaguchi et al. (2011) is used. SAM employs a three-phase bulk microphysics parameterization. It utilizes Message Passing Interface with horizontal domain decomposition.

b. LPTM

A parcel’s trajectory is predicted by integrating an ordinary differential equation that governs the parcel’s position:

$$\frac{D\mathbf{x}}{Dt} = \mathbf{u}_r + \mathbf{u}_s.$$  (1)

Here, \(\mathbf{x}\) is the parcel’s position vector, \(\mathbf{u}_r\) is the parcel’s resolved-scale velocity, and \(\mathbf{u}_s\) is the parcel’s SGS velocity. The resolved-scale velocity is determined by interpolation from the resolved-scale velocity field of the host model. The SGS velocity has to be parameterized.

The LPTM determines the resolved-scale parcel displacement by using an iterative method. The resolved-scale velocity halfway through a time step is first estimated as

$$\begin{align*}
\mathbf{u}_r^{t+1/2,n} &= \frac{1}{2}(\mathbf{u}_r^t + \mathbf{u}_r^{t+1,n}) & \text{for } n \geq 1, \\
\mathbf{u}_r^{t+1/2,0} &= \mathbf{u}_r^t & \text{for } n = 0
\end{align*}$$

(2)

where \(t\) is the time index, \(n\) is the index of iteration, and \(\mathbf{u}_r^{t+1,n}\) is the interpolated velocity at the position

$$\mathbf{x}^{t+1,n} = \mathbf{x}^t + \mathbf{u}_r^{t+1/2,n-1} \Delta t,$$

(3)

where \(\Delta t\) is a time step. The parcel position is updated as

$$\mathbf{x}^{t+1} = \mathbf{x}^t + \mathbf{u}_r^{t+1/2} \Delta t,$$  (4)

where the second term on the right-hand side is the parcel displacement, which is the path integral of the
Table 1. List of the simulation setup for the GCSS and FP cases. The FP cases are named as FP-[horizontal grid spacing]-[L or H], where L (H) denotes 5 (2.5)-m vertical grid spacing.

<table>
<thead>
<tr>
<th>Domain transport</th>
<th>Surface flux</th>
<th>Longwave radiation</th>
<th>Domain (m)</th>
<th>Δx (m)</th>
<th>Δz (m)</th>
<th>Δt (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GCSS</td>
<td>Prescribed</td>
<td>Parameterized</td>
<td>3.36</td>
<td>35</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>FP35L</td>
<td>Monin-Obukhov</td>
<td>RRTM 10-s update</td>
<td>5</td>
<td>33.6</td>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>FP35H</td>
<td>Monin-Obukhov</td>
<td>RRTM 5-s update</td>
<td>5</td>
<td>33.6</td>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>FP10L</td>
<td>Monin-Obukhov</td>
<td>RRTM 2.5-s update</td>
<td>5</td>
<td>33.6</td>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>FP10H</td>
<td>Monin-Obukhov</td>
<td>RRTM 2.5-s update</td>
<td>5</td>
<td>33.6</td>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>FP5H</td>
<td>Monin-Obukhov</td>
<td>RRTM 1.25-s update</td>
<td>5</td>
<td>33.6</td>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The implementation of our LPTM into SAM is summarized below:

1. SAM (host model): collect necessary variables in grid arrays.
2. LPTM
   (i) Diagnose \( \mathbf{u}_t^{t+1/2} \) with the iterative Heun scheme.
   (ii) Option: predict \( \mathbf{u}_t^{t+1} \) with Weil et al. (2004).
   (iii) Update position by \( \mathbf{x}_t^{t+1} = \mathbf{x}_t^t + (\mathbf{u}_t^{t+1/2} + \mathbf{u}_t^{t+1}) \Delta t \).
   (iv) Diagnose \( \mathbf{u}_t^{t+1} \) for next time step.

The LPTM is called at the end of the LES time step. Before exiting LPTM, the resolved-scale velocity is diagnosed at the new location to provide an initial velocity [i.e., \( \mathbf{u}_t^x \) in (2)] for the next time step. In this study, parcel advection due to the prescribed large-scale subsidence is neglected.

For thermodynamic scalar output, SAM’s prognostic thermodynamic variables and pressure are spatially interpolated to the parcel locations. Other variables such as temperature and cloud water mixing ratio are not interpolated. They are diagnosed from the interpolated variables, using SAM’s microphysics package, which is described in Khairoutdinov and Randall (2003). In this way, consistent relationships among the variables are preserved (e.g., total water mixing ratio is the sum of water vapor and cloud water mixing ratios). Before writing the output file, a parallel merge sort is used to sort parcels by their tags (i.e., the unique integers associated with each parcel).

c. Evaluation of the LPTM

An LES of the GCSS DYCOMS-II RF01 (Stevens et al. 2005) was performed as a test of the LPTM. The simulation setup is listed as GCSS in Table 1. The duration was three simulated hours. After two simulated hours, the parcels of the LPTM were set into motion, starting from the horizontal coordinates of the scalar points on the C grid, and spaced 15 m apart in the vertical between 352.5 and 1097.5 m. Altogether 460 800 parcels were released. Output was saved once per simulated minute.

With a sufficient number of parcels, the Eulerian (SAM) and Lagrangian (LPTM) mean profiles should be the same (e.g., Heus et al. 2008). Figure 1 shows the
Eulerian and Lagrangian mean profiles of the selected variables at the last time step. Twenty-meter-thick vertical bins were used to compute the Lagrangian mean. Generally between 5000 and 9000 parcels are collected in each bin below the PBL top. There is general agreement between the Eulerian and Lagrangian mean profiles. The Lagrangian mean profiles of the cloud fraction and in-cloud vertical velocity were diagnosed using conditional sampling over all parcels. The agreement of the cloud fraction and in-cloud vertical velocity between the Lagrangian and Eulerian profiles demonstrates that the LPTM produces useful statistics.

An arbitrarily chosen parcel’s evolution is presented, as an example, in Fig. 2. The parcel is initially located above the EIL. The parcel experiences strong cooling and moistening as it passes through the EIL. After the parcel penetrates through the EIL, liquid condenses inside. Then the parcel moves farther downward. The parcel paths shown in Fig. 2b can be compared to those shown schematically in Figs. 14 and 15 of Schubert et al. (1979). Our parcels follow the theoretical paths with some deviation; the parcel paths of the moist static energy and total water mixing ratio are basically circular. The path for the dry static energy is not a figure-eight shape, as in the paper of Schubert et al. (1979), because of surface warming. There is a cooling for the moist static energy and drying for the total water mixing ratio at around the 200-m level in the updraft. The parcel stays at the level for 6 min, possibly because of a mixing event. The parcel’s dry static energy does not change at the 200-m level in the updraft, which means that the dry static energy of the air to be mixed is as warm as the parcel. At the same level in the updraft, the parcel’s total water mixing ratio decreases by approximately 0.07 g kg$^{-1}$, which corresponds to approximately 0.18-K cooling in terms of the moist static energy. Among the parcels tracked, some begin their upward motion at higher levels (e.g., 400 m). These parcels mix with updraft air coming from below.

We conclude that the LPTM produces reasonable results, and that it is a suitable tool for a “direct” study of entrainment.

3. Stratocumulus LES

We performed a 4-h simulation of a stratocumulus cloud using a 5-m horizontal grid spacing and a 2.5-m vertical grid spacing. The case is based on the GCSS DUCOMS-II RF01 (Stevens et al. 2005), which is a non-precipitating nighttime stratocumulus case. While drizzle/sedimentation has been suggested as an important process for marine stratocumulus (e.g., Savic-Jovcic and
Stevens 2008; Hill et al. 2009; Wang and Feingold 2009; Feingold et al. 2010), we exclude it in this study for simplicity and concentrate on the effects of radiative and evaporative cooling and of mixing.

Based on the six-grid-width argument of Bryan et al. (2003), which is discussed later, simulating stratocumulus cloud-top entrainment with LES seems to require a grid spacing of $O(1 \text{ m})$ or smaller. Bretherton et al. (1999) state that entrainment is underresolved with a 5-m vertical grid spacing. A comparably fine horizontal grid spacing should also be used; Gerber et al. (2005) used aircraft data to estimate that the mean width of cloud holes is 5 m. It is prohibitively expensive to use an isotropic 1-m grid spacing at this time.

We modified the GCSS design as follows. The GCSS case uses a prescribed surface flux and parameterized longwave radiation. Our full-physics (FP) configuration computes the surface fluxes based on Monin–Obukhov similarity (Monin and Obukhov 1954) and the longwave radiative flux with the Rapid Radiative Transfer Model (RRTM; Mlawer et al. 1997). An accurate calculation of the longwave radiative flux is required because quantitative assessment of the radiative heating during entrainment is our focus. The GCSS longwave parameterization is not appropriate for our purpose because, for instance, it uses an arbitrarily defined cloud-top height. RRTM is expensive to call every time step. By calling once every few time steps, however, the cost can be reduced to an acceptable level. An analysis of the optimal update period for the radiative flux calculation is presented in appendix A.

The model configurations used in the GCSS and FP cases are listed in Table 1. The configurations for the simulations (FP35L, FP35H, FP10L, and FP10H) were...
used for the resolution sensitivity tests discussed in appendix B, which includes discussions of the vertical profiles and time evolution of FP5H. FP5H is the simulation (with LPTM) used for our cloud-top entrainment study. The vertical grid spacing is stretched above 1.2 km for the FP cases because RRTM requires a domain top near the 30-km level. The FP configuration uses the prescribed geostrophic wind (Table 1), which is independent of height, as a “domain translation velocity” in order to allow a larger time step. The duration of the simulation is four simulated hours.

Figure 3 presents a snapshot of the liquid water path (LWP) and cloud water mixing ratio in a cross-sectional view from FP5H. To aid in understanding the following discussion, an animation corresponding to the figure is provided as supplemental material (available at the Journals Online Web site: http://dx.doi.org/10.1175/2012JAS-D-11-080.s1). There are cloud holes that contain very little condensate. The animation shows that the cloud holes are filled with downdrafts that resemble waterfalls, and the cloudy air is produced like spring water pouring out from multiple updrafts. These updrafts terminate in horizontally divergent outflows near cloud top, which force the cloud holes to merge or break into smaller holes. Clockwise and counterclockwise horizontal rolls occur where the holes merge. The updrafts have lower cloud bases and higher cloud tops than the downdrafts because the downdrafts contain less moisture (due to entrainment drying) and the updrafts contain more moisture (due to surface evaporation). The updrafts frequently overshoot the mean cloud-top height. The animation also shows that very small cloudy parcels separate from the cloud top and evaporate immediately. This can be described as detrainment (Gerber et al. 2005; de Roode and Wang 2007). The cloud holes seem to form streaks, but the narrow, quasi-linear structures are destroyed by the horizontal spreading of the cloud top. It is possible that mesoscale closed cells would form if we used a much larger simulation domain, as demonstrated with coarser grid spacing by Savic-Jovcic and Stevens (2008) and Wang and Feingold (2009).

Power spectra can show whether the turbulence is well represented on the resolved scales. For fully developed turbulence, the variance is expected to decrease with increasing wavenumber \( \kappa = L/\lambda \), where \( L \) is the domain width, which is 3.2 km, and \( \lambda \) is wavelength. The dashed line represents a reference energy cascade line with \( \kappa^{-5/3} \).

FIG. 3. Snapshot picture for FP5H at 225 min, showing (top) liquid water path and (bottom) cloud water mixing ratio in a cross-sectional view.
Figure 4 shows the power spectra of the vertical velocity, for every 20 m between 780 and 880 m averaged over the last hour. The power spectrum for each height was computed with a two-dimensional FFT over the horizontal plane and then binned and averaged with respect to the horizontal wavenumber, $k = \sqrt{k_x^2 + k_y^2}$, as described by Moeng et al. (2010). The 60 instantaneous time power spectra, one simulated minute apart, are averaged to produce the smooth spectra shown. The EIL top (bottom) is at approximately 880–890 (825–835) m. For the 780-m level, the inertial subrange starts near the 600-m wavelength. The power spectrum in the inertial subrange follows the energy cascade line for wavelengths between 600 and 25 m. The longest wavelength of the inertial subrange shortens with increasing height, up to the 840-m level, where it is 70 m. The spectral slopes, for scales less than 25 m (i.e., five grid lengths) are steeper than $k^{-5/3}$. Caution is required for interpreting the power spectra in the EIL (i.e., 860- and 880-m levels) because 1) the turbulence in the EIL is very weak so that the inertial-subrange theory is not expected to hold [note that the variance of vertical velocity is 0.061 (0.025) m$^2$ s$^{-2}$ at 860 (880) m, which gives a rough estimation of the velocity scale of 25 (16) cm s$^{-1}$] and 2) the existence of gravity waves could have strong influence on the computed power spectra. The inertial subrange appears at approximately 25 m for the 860-m level and the spectral slope becomes steeper than $k^{-5/3}$ for scales less than 15 m (i.e., three grid lengths). This result is at least qualitatively consistent with the trend found with the spectra of the lower levels (i.e., the scale for the inertial subrange becomes smaller as height becomes higher). The inertial subrange seems to exist at around 150 m for the 880-m level. The appearances may be misleading, however, because the turbulence is extremely weak. We conclude that the turbulence is well represented in FP5H with the grid spacing used.

4. Cloud-top entrainment

4.1. SAM-LPTM run

The parcels of the LPTM were released at the beginning of the last hour of the FP5H simulation, starting from the horizontal coordinates of the scalar points on the C grid, and spaced 1 m apart in the vertical between 851.25 and 951.25 m. The 850-m level is approximately the middle of the EIL. More than 40 million parcels were tracked. The parcel data were saved once per minute. Saved data included the parcel’s location, velocity components, temperature, water vapor mixing ratio, cloud water mixing ratio, and SGS TKE. Entrained parcels were identified using a simple method. We tested each parcel to see if its initial SGS TKE was less than $10^{-6}$ m$^2$ s$^{-2}$, which means that the parcel was released into nonturbulent air. The parcels satisfying this condition were counted as entrained parcels if their height reached 50 m below the horizontal mean mixed layer height, which is defined below. The choice of 50 m below the mixed layer height is suggested by inspection of the various vertical profiles (e.g., Fig. 5). Approximately 164,000 parcels were identified as entrained during the simulated hour.

The EIL height (level B$^+$) and the mixed layer height (level B) were diagnosed using the method described by Yamaguchi et al. (2011). The method utilizes the vertical profile of the second moment of the liquid water static energy, which always has one peak in the EIL and is close to zero above and below the EIL as shown in Fig. 5. In the EIL, parameters are transported from one homogeneous place (free atmosphere, laminar flow) to another homogeneous place (the well-mixed layer). Above and below the EIL, the variance is much smaller, given the nature of laminar flow above the EIL and turbulent mixing below the EIL. In the upper part of the EIL, the third moment of the liquid water static energy is negatively skewed.
because most of the air still has the properties of the free atmosphere. In the lower EIL, the liquid water static energy is positively skewed because most of the air there has mixed layer properties. The shape of the variance profile is particularly simple and provides an ideal way to detect the top and base of the EIL. It should be noted that the variances of other thermodynamic variables, such as virtual dry static energy and total water mixing ratio, have the same one-peak profile in the EIL, so they could also be used. Using the variance profile of the liquid water static energy, the levels $B_1$ and $B$ are assigned as

$$
\begin{cases}
    z_{B_+} = z & \text{at } 0.05 \max(s_{l}^2) \text{ above } z_{\max}, \\
    z_B = z & \text{at } 0.05 \max(s_{l}^2) \text{ below } z_{\max},
\end{cases}
$$

where the overbar denotes the horizontal mean and

$$
z_{\max} = z \text{ at } \max(s_{l}^2).
$$

A linear interpolation was used to assign both levels $B_+$ and $B$ between the LES grid levels. The estimated levels $B_+$ and $B$ are nicely located in the profiles shown in Fig. 5.

b. Spatial and temporal evolution of entrained parcels

Entrained parcels are found in cloud holes, where sinking motion is dominant. Figure 6 shows the locations of the entrainment events relative to the cloud “geography.” The labeled entrainment events are superimposed over a plot of the LWP. The figure shows the parcels which reached below the level $B$ for the first time at 45 min after the LPTM was initialized (i.e., 225 min into the LES). Approximately 6500 entrainment events are found at 45 min. There are cloud holes where no parcel exists. This is due to the parcel initialization at particular places and particular time. These cloud holes could have entrained parcels with different initialization.

The trajectories of the parcels plotted in Fig. 6 are shown in Fig. 7 as a time sequence. The parcels are initially clustered in particular places. Up until 43 min, their locations are not close to the dry holes. The parcels are quickly positioned for entrainment after 43 min. Many parcels contain cloud water during the last 2 min before their entrainment.

The corresponding time evolution of the two-dimensional histogram of parcel heights and virtual dry static energies is shown in Fig. 8. The buoyancy PDF is broad in the upper EIL, and narrow in the lower EIL. It is a few kelvins wide at 45 min, when the parcels are entrained. The distribution shifts toward negative buoyancy as time progresses. A similar evolution is also observed with other parcels that were entrained at different times. The buoyancies are less than 1.5 K at 45 min, and approximately 4900 parcels are negatively buoyant, which translates to 75% of the all parcels entrained at 45 min. This result supports the concept of buoyancy matching for entrainment, as suggested by Gerber et al. (2005). The parcels with positive buoyancy may have been entrained because of shearing instability, or they may not have been entrained yet because the local mixed layer top is lower than level $B$.

c. Mixing fraction analysis

The mixing fraction $\chi$ has been used to partition the total cooling among longwave radiative cooling, evaporative cooling, and cooling due to mixing (e.g., Albrecht et al. 1985; Kuo and Schubert 1988; SR97; VD02). For a conservative scalar $\phi$, the mixing fraction is defined as

$$
\phi = \phi_{B_+} \chi + \phi_B (1 - \chi),
$$
where $\phi$ denotes the resulting mixture, which consists of a fraction $\chi$ of air from level B+ and the remaining fraction from level B, so that

\[
\begin{align*}
\phi &= \bar{\phi}_{B+} \quad \text{for} \quad \chi = 1 \\
\phi &= \bar{\phi}_B \quad \text{for} \quad \chi = 0.
\end{align*}
\]

Consider the total water mixing ratio $r$, liquid water static energy $s_l$, and virtual dry static energy $s_v$ for parcels in nonprecipitating nighttime stratocumulus:

\[
\begin{align*}
\delta r &= \chi \Delta r + (\delta r)_{LSF} \\
\delta s_l &= \chi \Delta s_l + (\delta s_l)_{RAD} + (\delta s_l)_{LSF} \\
\delta s_v &= \chi \Delta s_v + (\delta s_v)_{EVP} + (\delta s_v)_{RAD} + (\delta s_v)_{LSF}.
\end{align*}
\]

Here $\delta(\cdot) = (\cdot)_{\text{parcel}} - (\cdot)_{B}$ denotes the local fluctuation relative to the mean at level B, and $\Delta(\cdot) = (\cdot)_{B+} - (\cdot)_{B}$ is...
the jump across the inversion. The subscript EVP denotes evaporative heating and RAD denotes longwave radiative heating. The terms with the subscript LSF, which denotes large-scale forcing, represent drying and warming due to large-scale subsidence. The simulation includes the large-scale subsidence that carries the entrained parcels across large thermodynamic gradients, so its effects could be important. The first term on the right-hand side in each of these equations is the contribution due to the mixing, and it is a linear function of the mixing fraction. The magnitude of the mixing term is largest at level $B_1$ and zero at level $B$. Therefore, strictly speaking it is the “potential mixing” that is required for the parcel to pass from the current level to level $B$. These equations include only the processes in the EIL; the influence of the surface fluxes is assumed to be negligible.

Examples of the relationship between mixing fraction and virtual dry static energy are presented in Fig. 9. For

**FIG. 8.** Distributions of the virtual dry static energy and height for the parcels entrained at 45 min. The bin size is $0.12 \, \text{K} \times 1 \, \text{m}$. Each panel shows a different simulation time, up to (bottom-right) the time of entrainment (45 min). The solid line is the horizontal mean, and the dashed lines are the EIL top and base heights.
the pure mixing case, the relationship is linear by definition. When evaporation comes into play, a kink appears at the saturation mixing fraction \( \chi^* \). The saturation mixing fraction is the mixing fraction corresponding to exact saturation of the mixture. For a mixing fraction larger than the saturation value, the air is unsaturated after mixing and evaporation. Negative buoyancy at the saturation mixing fraction is the signature of buoyancy reversal (Siems et al. 1990). Buoyancy reversal has attracted considerable interest because of the possibility that it can enhance the boundary layer turbulence and entrainment through cloud-top entrainment instability (Lilly 1968; Randall 1976, 1980; Deardorff 1980; Yamaguchi and Randall 2008; Lock 2009).

An “established” way to perform a mixing fraction analysis (e.g., SR97 and VD02) is to determine the mixing fraction first, using the total water mixing ratio:

\[
\chi = \frac{\delta r - (\delta r)_{\text{LSF}}}{\Delta r}.
\]

We will discuss how to estimate the large-scale forcing term later. This approach is based on the assumption that there is no loss of total water due to sedimentation/precipitation. From the liquid water static energy equation and the mixing fraction, the radiative heating is obtained by

\[
(\delta s_t)_{\text{RAD}} = \delta s_t - \chi \Delta s_t - (\delta s_t)_{\text{LSF}}.
\]

With the assumption \( (\delta s_t)_{\text{RAD}} = (\delta s_t)_{\text{RAD}} \), the evaporative heating is computed as

\[
(\delta s_t)_{\text{EVP}} = \delta s_t - \chi \Delta s_t - (\delta s_t)_{\text{RAD}} - (\delta s_t)_{\text{LSF}}.
\]

This established approach is simple and straightforward, but its weakness is that it depends on the choice of level \( B^+ \). The jump for the total water mixing ratio is almost the same for any choice of level \( B^+ \) because of the near-zero vertical gradient above the inversion. If the selected level \( B^+ \) is higher than the true level \( B^+ \), the mixing fraction given by (10) between these two levels is 1 because \( \Delta r = r_{B^+} - r_B \) is constant, and \( (\delta r)_{\text{LSF}} \approx 0 \) because \( \partial r/\partial z \approx 0 \). On the other hand, the jump for the liquid water static energy increases as selected level \( B^+ \) becomes higher. For higher choices of level \( B^+ \), the diagnosed radiative cooling becomes stronger, according to (11), because \( \delta s_t = (s_t)_{\text{parcel}} - (s_t)_B \) is independent of level \( B^+ \) and \( \chi = 1 \). Because of the undesirable dependency on the choice of level \( B^+ \), SR97 developed another method based on the assumption that the radiative heating is independent of the mixing fraction. VD02 argued that the assumption is not justified. Our results, presented later, actually support the assumption.

As described in section 4a, the level \( B^+ \) is diagnosed using a method that is consistent with the turbulence statistics in the EIL. In our mixing fraction analysis, the level \( B^+ \) and \( B \) values for each parcel are assigned as

\[
\left\{
\begin{array}{c}
\gamma_{B^+} = \left[\frac{(\gamma^*)_{\text{parcel}}}{\gamma_B}\right]_{\text{LSF}} \\
\gamma_B = \gamma_B
\end{array}
\right.,
\]

where \( t_{B^+} \) (\( t_B \)) denote the time when a parcel is at level \( B^+ \) (level \( B \)). In this way, the heating can be adjusted to exactly zero at the level \( B^+ \) by permitting a little variation of the EIL-top value. The parcel value should not be used for level \( B \), because some parcels are not as cool as the mean.

Since the output was saved every simulated minute, the following method could be and was used to determine \( t_{B^+} \). Straightforwardly, the two consecutive output time levels, \( t \) and \( t + 1 \), before and after a parcel crosses level \( B^+ \) are determined by comparing the time series of the parcel height and level \( B^+ \) as a linear function:

\[
\left\{
\begin{array}{c}
z_{B^+}^{t+1} = z_{\text{parcel}}^{t+1} + \frac{z_{\text{parcel}}^{t+1} - z_{\text{LSF}}^{t+1}}{\Delta t} \delta t \\
z_{B^+}^{t} = z_{B^+}^{t} + \frac{z_{B^+}^{t} - z_{B^+}^{t+1}}{\Delta t} \delta t
\end{array}
\right.,
\]

where \( \delta t \) is an increment time from \( t \), and \( \Delta t \) is the output interval. The time increment is obtained by solving the sequential equations above. The value at \( t_{B^+} \) (e.g., \( r_{B^+}^{\text{LSF}} \)) is computed by a linear interpolation in time. The value at \( t_B \) is obtained in the same way.

Two additional conditions have to be imposed for the entrained parcels in order to perform the mixing fraction analysis. First, a parcel has to be initially above level \( B^+ \),...
otherwise $t_{B+}$ cannot be found. Second, $(s)^{y_{B+}}_{\text{parcel}} \geq (s)^{y_{B+}}$ has to be satisfied in order to avoid small jumps; these parcels may have mixed with overshooting air, which would cool them before they reach the diagnosed EIL height. A total of 144,000 parcels meet these two conditions; this is approximately 88% of the total number of entrained parcels.

The large-scale subsidence terms are estimated as follows. Using the horizontal mean profile, the subsidence tendency term [e.g., $-\frac{\partial \Phi}{\partial z}$] is computed for all levels and then vertically interpolated to the parcel’s height. The subsidence term during output interval is assumed to be the interpolated value multiplied by output interval:

$$
(\delta r)^{y_{B+}}_{\text{LSF}} - (\delta r)^{y_{B+}}_{\text{LSF}}^{-1} = 0.5 \left[ \left( -\frac{\partial \Phi}{\partial z} \right)^{\tau - 1}_{\text{interpolation}} + \left( -\frac{\partial \Phi}{\partial z} \right)^{\tau \text{ interpolation}}_{\text{interpolation}} \right] \Delta t.
$$

Since $(\delta r)^{y_{B+}}_{\text{LSF}} = 0$, the subsidence term at $t$ is given by

$$
(\delta r)^{y_{B+}}_{\text{LSF}} = 0.5 \sum_{\tau = \lfloor t_{B+} \rfloor}^{t - 1} \left[ \left( -\frac{\partial \Phi}{\partial z} \right)^{\tau}_{\text{interpolation}} + \left( -\frac{\partial \Phi}{\partial z} \right)^{\tau + 1}_{\text{interpolation}} \right] \Delta t,
$$

where $\Delta t$ is replaced to $\Delta t - \delta t$ for $\tau = \lfloor t_{B+} \rfloor$ (i.e., the largest previous output time step of $t_{B+}$) and $(\delta r)^{y_{B+}}_{\text{LSF}} < 0$ means drying.

The cooling of the entrained parcels is partitioned over all output time steps from $t_{B+}$ to $t_B$. The mixing fraction diagrams at $t = \lfloor t_B \rfloor$ are shown in Figs. 10 and 11. The total water mixing ratio follows a straight mixing line with small variability for large mixing fractions, but the distribution becomes wider on the left side of the mixing line for small mixing fractions. The shape of the distribution is influenced by the subsidence. In the
absence of subsidence, the total water mixing ratio follows the mixing line because \( \delta r = \chi_0 \Delta r \). The subsidence shifts the distribution toward the left and parallel to the axis of the mixing fraction because, from (10), we have \( \chi = \chi_0 - (\delta r)_{LSF}/\Delta r \), where \( (\delta r)_{LSF}/\Delta r \geq 0 \). A negative mixing fraction means that the parcel is moister than the sum of the level B value and subsidence \([i.e., r_{\text{parcel}} > T_B + (\delta r)_{LSF}]\). The distribution of the liquid water static energy is very similar but inverted.

Some parcels are found in the upper EIL, which suggests that they descended with a speed of at least 1 m s\(^{-1}\), over the next minute, in order to reach level B.

The inversion jump of the virtual dry static energy is approximately 9 K. This is the amount by which a typical parcel at level B+ has to be cooled before it can be entrained. The virtual dry static energy for smaller mixing fraction departs from the theoretical line of mixing and evaporation, which has a kink at a mixing fraction

---

**FIG. 11.** A mixing fraction diagram for the virtual dry static energy, cloud water mixing ratio, partitioned heating components, and subsidence heating at \( t = |t_0| \). The solid line is the theoretical expectation. For the virtual dry static energy, the dashed lines are hourly means. Bin size is \( \frac{1}{100} \) of each axis range.
of 0.054, mainly due to the subsidence. For \( \chi > \chi^* \), the coolest value is at the theoretical line. The theoretical line was constructed using the horizontal averaged profiles and (A.7), (A.9), and (A.10) of SR97.

The radiative heating is nonlinear, with small fluctuations between \(-0.6\) and \(0.3\) K, which is a surprisingly small contribution to the 9-K net cooling. Radiative warming occurs for some parcels. It is possible for local radiative warming to occur in nocturnal stratocumulus (e.g., for parcels that are cooler than the surrounding air). The evaporative heating changes linearly on both sides of the saturation mixing fraction, with stronger subsidence effects appearing for smaller mixing fractions. The warming for the negative mixing fractions is due to condensation that is stronger than the horizontal mean at level B. The strength of the evaporative cooling at the saturation mixing fraction is approximately 0.65 K, which is fairly close to the theoretical value of 0.82 K. The contribution of evaporative cooling to buoyancy reduction is at least as large as that of radiative cooling. The contribution of mixing is, however, the largest of the three: at \( \chi = 0 \) about 7.8 K is due to mixing with cooler air, which accounts for 87% of the total cooling. This is a lower bound because it is based on the largest radiative and evaporative cooling.

The weak radiative cooling discussed above contradicts the results of SR97 and VD02. It is due to the paths of the entrained parcels. Figure 12 shows the radiative heating rate at the level of the maximum cooling rate in terms of the horizontal average, and the corresponding LWP, at 4 h. The radiative heating is relatively weak, only about \( \pm 1 \) K h\(^{-1}\), in the cloud holes, where the parcels are entrained. The corresponding moments of the radiative heating rate presented in Fig. 13 show a highly inhomogeneous and skewed distribution in the EIL. Thus, the entrained parcels are not exposed to a strong radiative cooling at any stage of the entrainment process. Although radiative cooling is the dominant process driving boundary layer turbulence, it does not strongly reduce the buoyancy of the entrained air, contrary to some earlier suggestions (e.g., Randall and Schubert 2004).

5. Summary and conclusions

The goal of this study is a quantitative assessment of the cooling due to longwave radiation, evaporation, and mixing on the course of entrainment. We used an LPTM to perform a direct analysis of entrained parcels, which is not possible in observational studies. Our LPTM was tested using an LES of GCSS DYCOMS-II RF01 and shown to produce satisfactory results; the Lagrangian and Eulerian mean profiles matched closely, and the shapes of the parcel paths are consistent with theory.

A nocturnal marine stratocumulus boundary layer was simulated using a 5-m horizontal grid spacing and a 2.5-m vertical grid spacing. The simulated turbulence was realistic, as judged by the power spectra. The cloud-top detrainment suggested by observational studies (Gerber et al. 2005; de Roode and Wang 2007) was able to be seen in the LES.
Entrained parcels were conditionally sampled over their histories. The horizontal distribution of entrained parcels at the mixed layer top was shown to be clustered in dry cloud holes, where the air is sinking. Entrainment into the mixed layer is regulated by buoyancy. The parcels' virtual dry static energy has to be precooled in the EIL and close to the horizontal mean at the mixed layer top.

A mixing fraction analysis was performed to partition the cooling among radiative cooling, evaporative cooling, and mixing. The results are surprising; the radiative cooling of entrained parcels is small, and comparable to the evaporative cooling. Out of a total cooling of 9 K, the radiative and evaporative cooling only accounted for 1.2 K, and the remaining 7.8 K was due to mixing. Given the spatial distribution of radiative heating rate and entrained parcel paths, the small radiative cooling seems plausible; the entrained parcels did not pass through regions of strong radiative cooling, which were found near cloudy updrafts.

Our analysis has at least three limitations. 1) Numerical diffusion may contribute to the strong mixing. Numerical models generally suffer large errors at a large and sharp gradient such as the inversion layer. 2) RRTM is a plane-parallel model in which only up and down radiative fluxes are computed. The radiative cooling may be underestimated for parcels horizontally close to the cloud edge. Three-dimensional radiative flux calculations may be required for the small grid spacings used in this study. 3) The current microphysics parameterization is an all-or-nothing model in which the cloud fraction of a grid volume is either zero or one. Evaporation may not be realistically simulated because a cloudy grid volume immediately evaporates all cloud water over one time step, which is 0.25 s for our case.

We now summarize a scenario for entrainment, based on the cloud-top detrainment observed in our LES and the large cooling due to mixing. As suggested by Gerber et al. (2005), the source of the air mixed with the entrained parcels is cloud-top detrainment. Wisps of air detrained from the cloud top mix with, moisten, and cool parcels within the EIL. Cooled mainly by such mixing, the parcels descend slowly through the EIL. When they have descended far enough to be influenced by the large-eddy circulations, they are carried toward the downdraft areas by convergent flows near the cloud top. Cooling due to radiation and evaporation weakly accelerates their descent. Final penetration into the mixed layer takes place when the parcel's buoyancy matches that of the surrounding air at level B. In this scenario, although radiative cooling has a strong influence on the boundary layer turbulence, it does not strongly modify parcels as they are entrained. Instead, it has an indirect effect through production of air detrained from radiatively driven turbulence. Further research is needed to evaluate this scenario, especially for the role of cloud-top detrainment in promoting entrainment. The LPTM can be useful for this purpose.

The results of this study apply only to the particular case studied, and it is important to repeat the analysis for a variety of stratocumulus cases to determine to what extent our findings carry over to those cases.

Acknowledgments. The authors thank Steven Krueger for helpful suggestions. The implementation of RRTM into SAM was carried out by Peter Blossey and Robert Pincus. This study has been supported by a grant from the Department of Energy's Scientific Discovery through Advanced Computing (SciDAC) program, Grant DE-FC02-06ER64302; through the National Science Foundation (NSF) support of the Center for Multi-Scale Modeling of Atmospheric Processes (CMMAP), managed by Colorado State University under cooperative agreement...
APPENDIX A

How Often Should Longwave Radiation Be Updated for Stratocumulus LESs?

As pointed out by Xu and Randall (1995), infrequent update of the radiation produces a bias in terms of the correlation of time series. We performed an experiment to determine the optimal update period. First, the case FP35L listed in Table 1 was simulated by updating longwave radiation every time step (i.e., every 0.5 s) for 2 h to spin up the turbulence. The turbulence field was then used as a common initial condition for the tests. The simulations with various update periods were continued for an additional hour. The output was saved every 5 s; thus, 720 samples were collected for each run. We tested the update periods 0.5, 5, 10, 20, 30, 40, 50, 60, 90, and 120 s.

To perform correlation analysis, we focused on the EIL. Since the EIL evolves with time and its evolution differs among the simulations, a common EIL is specified with the lowest (highest) level of the diagnosed EIL base (top) among all cases for the third hour. The correlation was computed for the time series of the horizontal mean EIL profile.

The correlation to the 0.5-s update period for selected variables is plotted in Fig. A1. All of the correlations are high, which means that the change in radiative effect for different update periods is small for a 1-h simulation for the horizontal mean EIL profile. The correlation generally decrease for an update period shorter than 40 s, and the trend is distorted for several variables after 40 s, which means that an update period shorter than 30 s is safer. The numerical cost of RRTM with an update period of 10 s (i.e., once every 20 time steps) is about 1.1 times the numerical cost of the idealized GCSS parameterization called every time step. The timing test suggested that about 10% of the total cost was used by RRTM. The 5-s update period was used for FP35H. The 2.5-s update period was used for FP10L and FP10H.

APPENDIX B

Sensitivity to Model Configuration and Resolution

Changing the configuration from GCSS to FP35L as well as refining the resolution (FP35H, FP10L, FP10H, and FP5H) has a large impact on the simulated fields. As discussed in section 3, each configuration is listed in Table 1.

As shown in Fig. B1, the FP cases are cooler and moister in the mixed layer than GCSS, so that they have more cloud water and thicker cloud fraction. FP35H is coolest and driest among the FP cases. Its cloud water amount and cloud thickness is largest. FP10H is cooler than FP10L while their moisture amount is very similar. FP5H is the second coolest and the moistest. Consider a grid spacing aspect ratio of horizontal and vertical grid spacing (i.e., $\Delta x / \Delta z$, which is summarized in Table B1). The liquid water static energy is cooler for larger aspect ratio, and the total water mixing ratio is drier for larger aspect ratio. The cloud water mixing ratio and cloud fraction do not have systematic relationship to
a grid spacing aspect ratio because they are very sensitive to the vertical grid spacing. For the same vertical grid spacing, however, larger aspect ratio has wetter and thicker cloud layer.

Time series of the selected variables are shown in Fig. B2a. The turbulence of the FP cases is much stronger than GCSS, while they maintain a larger cloud water amount. The entrainment rate of FP35H is smallest; about three-quarters of FP5H. As resolution increases in horizontal and vertical, the entrainment rate tends to increase. The grid spacing aspect ratio influences entrainment rate; smaller aspect ratio results in larger entrainment rate. The surface fluxes are strongly influenced by the grid spacing aspect ratio.

The additional vertical profiles are presented in Fig. B2b. The turbulent activity is weaker for GCSS in terms of the buoyant production of TKE and variance of vertical velocity. Although the maximum buoyant production of TKE matches for the FP cases, the subcloud layer is different, and again the grid spacing aspect ratio influences the profile there. The third moment of vertical velocity of FP5H is well matched with the estimated value by observation shown in Fig. 5 of Stevens et al. (2005), whereas its variance of vertical velocity is stronger. FP10L is closer to FP5H than FP10H for the third moment of vertical velocity, which may also be due to the grid spacing aspect ratio.

The grid spacing aspect ratio is an important factor as well as resolution in vertical and horizontal directions. Our results have similarities of the results presented by Cheng et al. (2010). Their LES tests show that the opposite dependence on horizontal and vertical grid spacings for most cloud-related variables, and changing the grid spacing proportionally in both directions produces compensating effects that minimize the net change.

Determining whether the result of FP5H is converged is a question worth confirming, and we keep this as future work.

**Table B1.** Aspect ratio of the horizontal grid spacing to the vertical grid spacing for the FP cases.

<table>
<thead>
<tr>
<th></th>
<th>FP35L</th>
<th>FP35H</th>
<th>FP10L</th>
<th>FP10H</th>
<th>FP5H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aspect ratio ((\Delta x/\Delta z))</td>
<td>7</td>
<td>14</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>(\Delta x) (m)</td>
<td>35</td>
<td>35</td>
<td>10</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>(\Delta z) (m)</td>
<td>5</td>
<td>2.5</td>
<td>5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>
REFERENCES


—, —, and B. Stevens, 2010: Effects of resolution on the simulation of boundary-layer clouds and the partition of

Fig. B2. (a) Time series of selected variables. (b) Additional vertical profiles.