On the Height of the Warm Core in Tropical Cyclones

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ABSTRACT

The warm-core structure of tropical cyclones is examined in idealized simulations using the Weather Research and Forecasting (WRF) Model. The maximum perturbation temperature in a control simulation occurs in the midtroposphere (5–6 km), in contrast to the upper-tropospheric (>10 km) warm core that is widely believed to be typical. This conventional view is reassessed and found to be largely based on three case studies, and it is argued that the “typical” warm-core structure is actually not well known. In the control simulation, the height of the warm core is nearly constant over a wide range of intensities. From additional simulations in which either the size of the initial vortex or the microphysics parameterization is varied, it is shown that the warm core is generally found at 4–8 km. A secondary maximum often develops near 13–14 km but is almost always weaker than the primary warm core. It is demonstrated that microwave remote sensing instruments are of insufficient resolution to detect this midlevel warm core, and the conclusions of some studies that have utilized these instruments may not be reliable. Using simple arguments based on thermal wind balance, it is shown that the height of the warm core is not necessarily related to either the height where the vertical shear of the tangential winds is maximized or the height where the radial temperature gradient is maximized. In particular, changes in the height of the warm core need not imply changes in either the intensity of the storm or in the manner in which the winds in the eyewall decay with height.

1. Introduction

One of the most important features distinguishing tropical cyclones from extratropical cyclones is the characteristic warm core, whereby the temperature in the center of the tropical cyclone is warmer than its environment. The existence of the warm core has long been recognized, but surprisingly little is known about its mean structure and variability. Ultimately, the temperature distribution in the hurricane is related to the tangential wind distribution through thermal wind balance, and in tropical cyclones, most (though not all) of the wind field is close to balanced (Willoughby 1990). The two most common variables used to characterize the warm core are its strength (given by the magnitude of the maximum perturbation temperature) and its height (the level where the maximum perturbation is found). Other than the fact that the strength of the warm core generally increases with the intensity of the cyclone, rather little is known about what determines the values of these two variables. Additionally, there are a number of misconceptions that have appeared in the literature over the years regarding the physical interpretation of changes in the strength and height of the warm core, and their relationship to other characteristic structures of the hurricane vortex, such as the slope of the radius of maximum winds (RMW). In this study, we examine the structure of the warm core within idealized numerical simulations and compare the simulated structure to that found in previous observational and numerical studies.

Almost all studies that have assessed the structure of the warm core, be they observational, numerical, or theoretical, compared their results to one or more of three specific case studies: La Seur and Hawkins (1963), Hawkins and Rubsam (1968), and Hawkins and Imbembo (1976). Each of these three studies...
constructed radius–height cross sections of the warm core based on flight-level temperature measurements made at three to five altitudes over roughly a 6-h period. In La Seur and Hawkins, a maximum temperature perturbation of +11°C was found in Hurricane Cleo (1958). There were three flight levels, 800, 560, and 240 mb, and the maximum warm core was found at 240 mb. Hawkins and Rubsam found the strength and height of the warm core in Hurricane Hilda (1964) to be +16°C and 250 mb, respectively, based on flights at 900, 750, 650, 500, and 180 mb (where +12°C was measured). Hawkins and Imbembo found two maxima in the warm core for Hurricane Inez (1966) on 27 September, at 650 and 300 mb, both +9°C. There were again two maxima the next day, +11°C at 600 mb and +16°C at 250 mb. This double maximum, and in particular a maximum as low as 650 mb, was believed to be “rather unusual,” and a warm-core height of 250 mb was thought to be more typical. Note that for the aforementioned studies, the annual mean Caribbean sounding of Jordan (1958) was used as the reference profile, and so the calculated perturbations were not relative to the actual environment. Since the 1960s, there have been very few flights into tropical cyclones at heights above 6 km. Consequently, much of what we think we know about the structure of the warm core is based on these three studies.

In some recent field programs, the high-altitude National Aeronautics and Space Administration (NASA) Earth Resources (ER)-2 aircraft has overflown tropical cyclones at 20-km height, releasing dropsondes that give quasi-vertical profiles of temperature. Halverson et al. (2006) examined the warm core of Hurricane Erin (2001) based on eight ER-2 dropsondes (including one dropped in the “geometric center”), along with 12 dropsondes released at 11–12-km height from a DC-8 aircraft that was circumnavigating the storm at greater than 200-km radius. The perturbation temperature was calculated relative to an environmental profile derived from the composite of a DC-8 sonde dropped 610 km southeast of the center and an ER-2 sonde dropped 340 km northeast of the center. Based on six dropsondes that lay roughly along a line through the center (only two of which were within 200 km of the center), a cross section was constructed, and the warm-core strength and height were +11°C and 500 mb, respectively. This maximum was embedded within a deep layer of nearly constant perturbation temperature of about +10°C from 750 to 300 mb. The top of the warm core (where the perturbation temperature approaches zero) was found at 110 mb. Halverson et al. contrasted the structure of Erin with that of Inez, which had a stronger, higher, and radially narrower warm core than Erin and was also much more intense. Interestingly, they did not mention the fact that Inez also had a second maximum near 600 mb. They attributed the relatively weak warm core above 300 mb in Erin to the fact that the storm itself was weakening, stating that “Without deeply penetrating hot towers, the warm anomaly must weaken” (p. 319). On the other hand, they also attributed the weakening to the removal of the warm core by shear, saying “We believe that the loss of the upper-level warm core and consequent rise of the surface pressure weakened the vortex” (p. 323).

This idea that the height of the warm core is directly related to intensity is widespread, and an apparent rising of the warm core is sometimes cited operationally by the National Hurricane Center (NHC) as evidence for the intensification of a storm (see http://www.nhc.noaa.gov/archive/2010/al11/al112010.discus.040.shtml?) or as the basis for upgrading a subtropical storm to a tropical storm (see http://www.nhc.noaa.gov/archive/2010/al17/al172010.discus.006.shtml?) Furthermore, in discussing his theory of the maximum potential intensity that can be achieved in a given thermodynamic environment, Holland (1997) stated that his results “…emphasize that the maximum height of the warm core and vigor of the convection is a major factor for determining tropical cyclone intensity” (p. 2534). In this study, we show that given current observational and numerical evidence, these arguments are in general not justified.

In section 2, we describe the modeling setup for our numerical simulations and present results from a control simulation. We also present results from simulations with differing quasi-steady state sizes and simulations that utilize different microphysics parameterization schemes. Having shown that the height of the warm core in these simulations is much lower than expected, in section 3 we revisit prior studies and demonstrate that the observational evidence for a preferred upper-tropospheric warm core is actually rather weak. In particular, we challenge the results of recent studies that have utilized microwave remote sensing to characterize the structure of the warm core. In section 4, we clarify the relationship between the height of the warm core and thermal wind balance and demonstrate that there is in general no simple relationship among the height of the warm core, the height where the radial temperature gradient is maximized, and the location where the tangential winds decrease most rapidly with height. A discussion and conclusions are given in section 5.

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1 Hawkins and Rubsam (1968) stated that “soundings” were constructed at 10 nautical mile (~18.5 km) radial intervals, and that the values at heights in between flight levels came from vertical interpolation. As no data existed between 500 and 180 mb, the true maximum perturbation could probably be almost anywhere between these two levels.
2. Structure of the warm core in numerical simulations

a. Description of modeling framework

In this subsection we briefly summarize the modeling framework for this study. A more detailed description can be found in Stern (2010). Additionally, seven of the simulations presented here (including the control simulation) are identical to those discussed in Stern and Nolan (2011, hereafter SN11). We use the Weather Research and Forecasting (WRF) Model v3.1.1 to simulate the development and maintenance of idealized intense hurricanes on a doubly periodic \( f \) plane \((f = 5.0 \times 10^{-5} \text{ s}^{-1})\), starting from weak (maximum 25 m s\(^{-1}\)) tropical cyclone–like vortices embedded within a constant (5 m s\(^{-1}\)) easterly environmental flow, and above a sea surface with a homogenous and fixed temperature (28°C). Each simulated tropical cyclone is initialized in thermal wind balance with the “moist-tropical” Atlantic hurricane season sounding of Dunion (2011). All simulations use 40 vertical levels, and a triply nested grid, with 18-, 6-, and 2-km horizontal resolution, and domains of length 4320, 720, and 360 km on a side, respectively.

The initial radial structure of the tangential wind field is that of a modified Rankine vortex [Eqs. (4.1) and (4.2) of SN11], with solid body rotation inside of the RMW, and with tangential winds decaying outside the RMW as \((r_{\text{max}}/r)^a\), where \(r\) is radius, \(r_{\text{max}}\) is the initial RMW, and \(a\) is a decay parameter. For the control simulation, \(r_{\text{max}} = 90\) km and \(a = 0.5\). The vertical structure is given by a Gaussian decay function [Eq. (4.3) of SN11] above and below the height of the maximum tangential winds (chosen to be 1500 m). All simulations use the Yonsei University (YSU) planetary boundary layer parameterization scheme (Hong et al. 2006), and the control simulation uses the WRF single-moment six-class (WSM6) (Hong and Lim 2006) microphysics parameterization (MP). No convective parameterization is utilized (on any domain), and no longwave or shortwave radiation parameterization is used. As mentioned in SN11, radiation is neglected in the interest of simplicity, and because we believe the impact of radiation to be small on time scales of a week or less. For completeness, an analysis of a simulation that utilizes radiation parameterizations is given in appendix A.

b. Structure of the warm core in the control simulation

Figures 1 and 2 respectively show radius–height plots of the azimuthal mean tangential wind and perturbation temperature for the control simulation, at 0000 UTC of each day (snapshots), starting at 0000 UTC on day 2. In this study we define perturbation temperature as the difference between the local azimuthal mean temperature and the azimuthal mean temperature averaged in the 550–650-km annulus\(^2\) (at each corresponding height,\(^3\) every 250 m). While any choice is at least somewhat arbitrary, we note that ours is consistent with the radii used in the observational studies of Knaff et al. (2004) and Halverson et al. (2006). A further discussion of the choice of reference profiles is given in appendix B.

In the initial condition (not shown), the warm core is centered at about 5-km height, with a strength of \(+4.77^\circ\text{C}\). The strength of the warm core does not change much in the first day (nor do the maximum winds), but the height of the maximum anomaly rises to about 9 km by 0000 UTC on day 2 (Fig. 2a). From day 2 to day 3, the warm core more than doubles in strength, to \(+9.46^\circ\text{C}\), and it lowers to 6 km (Fig. 2b). The warm core has also become much more compact in the lower 9 km, as the RMW has contracted to a quasi-steady size that is about one-third as small as in the initial condition (Fig. 1b; see also Fig. 13a of SN11). Below 9-km height, the RMW is outside of the region with the largest radial temperature gradients, whereas above the RMW is within the region of the largest radial temperature gradients. There is a weak secondary maximum in perturbation temperature at \(-13.5\) km-height. Additionally, there is a ridge of high perturbation temperature that follows the axis of maximum outflow (see Fig. 1b), from 11-km height at 40-km radius to 13-km height at 120-km radius. Consequently, if one were to look at vertical profiles, the maximum perturbation would be found at 6-km height in the inner 20 km, but at 11 km or higher outside of 40-km radius. Finally, the perturbation temperature is negative everywhere in the lowest 1 km, and negative outside of the eyewall below the melting level.

The height of the warm core remains remarkably steady throughout the remainder of the simulation, always near 5 or 6 km. In fact, the evolution of the overall structure of azimuthal mean perturbation temperature is slow enough such that 12-h composites (not shown) look essentially the same as snapshots. By 0000 UTC on day 4, the strength of the warm core has increased another \(-3^\circ\text{C}\) (Fig. 2c), during which time the maximum azimuthal mean tangential wind (hereafter, \(V_{\text{bar, max}}\)) has only increased by 3 m s\(^{-1}\) (Fig. 1c). In contrast, \(V_{\text{bar, max}}\) increased by about 30 m s\(^{-1}\) between 0000 UTC on day 2 and 0000 UTC on day 3, during which time the strength of the warm core increased.

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\(^2\) The base-state profile is determined on the outer domain, while all other calculations are performed on the inner domain.

\(^3\) The azimuthal means are first calculated on the quasi-horizontal model surfaces, and then the fields are interpolated in the vertical to a regular radius–height grid.
by about 5°C. While in general the strength of the warm core increases with intensity, the relationship is clearly not straightforward. While the ridge of high perturbation temperature in the outflow layer remains, the weak local maximum near 13.5 km from day 3 is no longer evident. The warm-core structure at 0000 UTC on day 5 (Fig. 2d) is similar to that of 24 h prior, with the strength increased by another ~2°C. The temperature of the upper troposphere has warmed substantially, with the +4°C contour in the outflow ridge moving outward from 75 to 100 km.

At 0000 UTC on day 6, the upper ridge has only expanded a little farther, and the strength of the primary maximum has increased by 0.3°C. Another ~1°C
warming occurs by 0000 UTC on day 7 (Fig. 2f). The primary maximum is \( \sim 16^\circ C \) and remains at about 6-km height. Throughout the simulation, the perturbation temperature along the RMW remains relatively small in the lower 9 km, less than \( +4^\circ C \). As we will later demonstrate, this fact helps explain why the structure of the warm core can bear little relationship to that of the winds in the eyewall. Finally, while one might expect the height of the warm core to be related in a simple manner to the distribution of eyewall updrafts, this is not the case, as the height of maximum azimuthal mean vertical velocity lowers from 9 to 5 km from 0000 UTC on day 6 to 0000 UTC on day 7 (Figs. 1e,f), while the warm-core height remains steady (Figs. 2e,f).

**Fig. 2.** Perturbation temperature (see text for definition) for the control simulation, every 24 h starting at 0000 UTC on day 2. The contour interval is 0.5°C and the 0°, 4°, 8°, 12°, and 16°C contours are thickened. The RMW is also plotted, as is the 0°C isotherm (dashed).
c. Sensitivity of intensity to microphysics

Ultimately, the warming of the core of tropical cyclones is due to a combination of diabatic heating in the eyewall updraft and dry adiabatic descent within the eye. This descent in the eye is generally believed to be a forced response to the eyewall heating (e.g., Schubert and Hack 1982; Nolan et al. 2007; Vigh and Schubert 2009; Pendergrass and Willoughby 2009). Therefore, it is reasonable to suspect that the structure of the warm core may be sensitive to the distribution of diabatic heating, which in turn may depend on the parameterization of microphysical processes. We performed four additional simulations in which the MP scheme was varied. In addition to WSM6, we consider the five-class (WSM5) and three-class (WSM3) WRF single-moment schemes (Hong et al. 2004). WSM5 is nearly identical to WSM6 but does not include graupel. WSM3 actually predicts all of the species included in WSM5, but rainwater and snow are combined into a single variable, as are cloud water and ice. In WSM3, liquid and solid water cannot coexist, and there is no representation of mixed-phase microphysical processes. Often referred to as a “simple ice” scheme, freezing occurs instantaneously and completely at the first level where the temperature is colder than 273.15 K, and melting occurs instantaneously and completely one level below the freezing level. Both WSM5 and WSM3 have been previously utilized in idealized and real-data simulations (Davis et al. 2008; Nolan et al. 2009a,b; Xiao et al. 2009; Fovell et al. 2009).

The Kessler MP scheme (Kessler 1969) is a warm-rain scheme, in that there is no ice, snow, or graupel. All precipitation is rain and all cloud is liquid, regardless of temperature, and the only microphysical processes are evaporation, condensation, and autoconversion. Kessler is therefore not a realistic MP scheme. However, it is by far the simplest scheme, it is the most consistent with Emanuel’s (1986) potential intensity theory, and the absence of ice allows for the investigation of the sensitivity of structure to such microphysical processes. Finally, we also consider the Lin scheme (Lin et al. 1983), which, like WSM6, is a six-class scheme, including graupel.

For completeness, before further discussing the warm core, we first show the intensity evolution of the simulations with varying MP schemes in Fig. 3 in terms of the minimum sea level pressure ($P_{min}$, Fig. 3a), maximum 10-m wind speed ($V_{max10}$, Fig. 3b), $V_{bar_{max}}$ (Fig. 3c), and the maximum gradient wind speed ($V_{g_{max}}$, Fig. 3d). While development begins at about the same time in all simulations (~1800 UTC on day 1), the rate of intensification is substantially slower in WSM3 compared to the others. For all four measures of intensity, WSM5, WSM6, and Lin are nearly identical through about 0000 UTC on day 3, and Kessler is very similar as well. Lin begins to deepen more rapidly than WSM5 and WSM6 at this point, and Kessler deepens more rapidly beginning after 1200 UTC on day 3. WSM5 and WSM6 separate at this time as well, but always remain close, never differing by more than 8.5 mb. After 0000 UTC on day 5, the intensity of Lin is similar to that of WSM5 and WSM6. Kessler continues to steadily deepen until about 0600 UTC on day 6, and then $P_{min}$ is nearly steady at about 895 mb through 1600 UTC on day 7, after which a rapid weakening occurs. A quasi-steady $P_{min}$ is reached in WSM5 and WSM6 around 1200 UTC on day 6. Lin achieves a minimum $P_{min}$ at 2100 UTC on day 5, followed by a slow weakening for 18 h, and then renewed deepening until 1200 UTC on day 7. WSM3 is 23.4 mb weaker than the control (WSM6) at 0000 UTC on day 6, but since WSM3 continues to intensify for longer than the control, this difference is halved by 1200 UTC on day 7.

Overall, it is clear that WSM3 is weaker than the other MP simulations and that Kessler is stronger than the others, and it is unclear whether there are meaningful differences in intensity among WSM5, WSM6, and Lin. Additionally, while much smoother hour to hour, $P_{min}$ exhibits relatively larger differences among the simulations than $V_{max10}$, as might be expected since the maximum wind speed is related to the square root of the pressure anomaly. More surprisingly, both the short-term variability and the differences between simulations are larger in $V_{bar_{max}}$ (which is an azimuthal mean) than in $V_{max10}$ (which is a point value). In particular, WSM5 has variations in $V_{bar_{max}}$ over 6-h periods of 10–15 m s$^{-1}$.

That the warm-rain scheme (Kessler) intensified more rapidly (from day 3 to day 6) is generally consistent with the results of previous studies (Willoughby et al. 1984; Wang 2002a). While in some studies (such as ours) warm-rain schemes were found to achieve the greatest maximum intensities (Hausman et al. 2006; Li and Pu 2008), in others they were not (Willoughby et al. 1984; Wang 2002a; Fovell et al. 2009). There is some evidence from these studies that the neglect of mixed-phase processes, such as by WSM3, leads to systematically weaker storms, and this appears to be the case for our simulations. We see no convincing evidence that there is a systematic difference in the evolution of intensity or in its maximum among WSM5, WSM6, and Lin.

To demonstrate the lack of robustness of small differences in intensity among MP schemes, we conducted two additional simulations for each of WSM3 and WSM6, and these are shown in Fig. 3 as well. As discussed in
SN11, all of the simulations are initialized with small-amplitude unbalanced random noise added to the initial wind fields. This “random” noise field, which decays in amplitude exponentially away from the storm center, is actually identical among all simulations discussed so far, so there is truly no difference between the initial conditions of simulations with different MP schemes. The additional simulations are each initialized with a different random noise field (but for each pair of WSM3 and WSM6, the noise is the same). The spread in $P_{\text{min}}$ among the three WSM3 simulations is as large as 8.9 mb, and among the three WSM6 simulations it is as large as 15.2 mb. The spread in $V_{\text{max}10}$ and $V_{\text{bar} \text{max}}$ is as large as 9.6 and 10.3 m s$^{-1}$ among WSM3, and as large as 12.6 and 22.4 m s$^{-1}$ among WSM6, respectively. Clearly, the differences among the simulations with different initial random noise are comparable to or larger than those between WSM5 and WSM6. As there is no physical meaning to differences among the random simulations, we must conclude that there is also no systematic difference in either the intensity evolution or in the quasi-steady state intensity between WSM5 and WSM6. It is possible that with a large enough set of simulations, average differences would emerge. Even if that were the case, these differences are likely to be quite small and therefore not particularly meaningful. We wish to emphasize that even in a tightly controlled framework, one cannot simply attribute small changes between simulations to the parameter that was varied.

d. Sensitivity of warm-core structure to microphysics

Returning to our primary focus (the warm core), Fig. 4 shows the radius–height cross sections of perturbation temperature for WSM3 and Kessler at 0000 UTC on days 3, 5, and 7. As in the control simulation (WSM6), the warm core in WSM3 elevates by 3–4 km from the initial condition to 0000 UTC on day 2 (not shown), whereas in Kessler the warm core is centered at the (nonexistent) melting level (not shown). The warm core moves back down to 7 km by 0000 UTC on day 3 in WSM3, similar to (but slightly higher than) WSM6, but about 3°C weaker, consistent with the weaker intensity. A distinctive feature of the perturbation temperature field in WSM3 is that the effect of instantaneous
melting at a single level is clearly visible as an axis of locally low perturbation temperature that lies along or just below the 273.15-K isotherm. This feature is present throughout the simulation, extends inward into the eyewall, and is likely unrealistic. The local maximum at about 13–14 km in the control simulation that is sometimes present after day 3 is also evident in WSM3 after day 4, as is the ridge through the outflow layer. At 0000 UTC on day 4 and thereafter, the height of the primary warm core in WSM3 remains at about 6 km, quite similar to the control simulation.

The warm core in Kessler undergoes a somewhat different evolution than that of WSM3 and WSM6. At 0000 UTC on day 3, the primary warm core is +9.46°C at about 7 km. In addition, there is a local maximum at ~13.5-km height, similar to that which is evident
in WSM6. Only the lowest ~1 km is colder than the environment, whereas in both WSM6 and WSM3 the outer core is colder than the environment below about 4-km height. These differences perhaps reflect the lack of melting in Kessler or the lack of hydrometeors at large radii due to the faster terminal velocity of rain versus snow. Unlike WSM6, in Kessler both the mid- and upper-level warm cores continue to strengthen through 0000 UTC day 5, and the upper core at ~12.5 km is slightly stronger (+13.55°C) than the midlevel core (+13.05°C) at ~4.5 km by this time. The perturbation temperature continues to increase everywhere inside the core, but it changes in such a way that a new warm core forms at 8 km and the original mid- and upper-level warm cores begin to disappear by 0000 UTC on day 6 (not shown). At 0000 UTC on day 7, a single maximum (now at 7 km) continues to strengthen (+18.72°C) and remains the only warm-core center. Finally, note that the ‘‘melting-level’’ isotherm is plotted for Kessler (as in WSM6 and WSM3), despite the fact that it has no meaning in a warm-rain simulation. From this, we can see that while there are clearly differences in warm-core structure between Kessler and the other simulations, the tendency for the maximum perturbation temperature to be found relatively near to the melting level in all of these simulations is not in any obvious manner due to the existence of melting/freezing itself.

e. Vertical structure of the maximum perturbation temperature

To gain further perspective into the structure and variability of the warm core, Fig. 5 shows time–height Hovmöller plots of the perturbation temperature at the storm center, for WSM6, WSM3, and Kessler. For these plots, data are plotted only every 6 h to reduce noise. In both WSM6 and WSM3, the warm core appears to discretely jump from 5 to 9–10 km at 18–24 h into the simulation, followed by a more continuous lowering to 5 (WSM6) or 7 (WSM3) km over the following 12–24 h. After 0000 UTC on day 3, the primary midlevel warm core in WSM6 remains at an essentially constant height. The primary warm core in WSM3 continues to slowly lower to 6-km height by 1200 UTC on day 4, where it remains until the end of the simulation. In contrast to WSM6 and WSM3, the initial warm core in Kessler does not elevate by much in the first day. A slow lowering of the primary warm core from about 6.5 to 4.5 km occurs from 0000 UTC on day 3 to 0000 UTC on day 5. This warm core then disappears in favor of another that lowers from 8 to 6.5 km from 1200 UTC on day 5 to 1200 UTC on day 6, remaining there thereafter. As discussed previously, in all three simulations a secondary maximum in the perturbation temperature forms on

FIG. 5. Time–height Hovmöller plots of the perturbation temperature at the storm center (r = 0) for (a) WSM6, (b) WSM3, and (c) Kessler. To reduce noise, data are plotted only every 6 h. The contouring is identical to Figs. 2 and 4. There are no data plotted after the time at which the center approaches within 648 km of the outer domain boundary.
day 3 around 13–14-km height. While always smaller in magnitude than the midlevel warm core in WSM6 and WSM3, in Kessler the upper-level warm core is comparable to and sometimes stronger than the midlevel core from about 1200 UTC on day 4 until 1200 UTC on day 6.

Figure 6 shows vertical profiles of the perturbation temperature at the storm center, at 0000 UTC of each day, for the MP simulations. The initial profiles of the MP simulations are not plotted since they are all identical to the control simulation. At 0000 UTC on day 2, the warm core is still relatively weak in all simulations, and so it is easy for multiple maxima of nearly equal magnitude to exist at various heights. Nevertheless, there is a clear tendency for the perturbation temperature to be maximized near 9 km in all of the MP simulations (Fig. 6a) except for Kessler, where the maximum is near 6 km. By 0000 UTC on day 4, all of the MP simulations (Fig. 6c) have a midlevel maximum between 5- and 6-km height. All also have an upper-level maxima

![Diagram showing perturbation temperature at center vs height for each MP simulation every 24 h beginning at 0000 UTC on day 2.](image-url)
near 13–14 km, with varying degrees of prominence. By 0000 UTC on day 5 (Fig. 6d), the upper-level maxima in all the MP simulations have become less pronounced, with the notable exception of Kessler, whose upper-level warm core has become the absolute maximum. At this time, Lin has a very broad maximum between 4 and 10 km. At 0000 UTC on day 6 (around the time when a quasi-steady-state intensity is achieved in most simulations) in Lin, this maximum has sharpened somewhat, now centered at 8-km height. Meanwhile, the original midlevel maximum in Kessler has disappeared, having been replaced by a new maximum at 8 km, with the upper-level maximum at 13 km barely remaining distinct. Finally, at 0000 UTC on day 7 (Fig. 6f), there is generally a single midlevel maximum in perturbation temperature for the MP simulations, ranging from −6 km (WSM6) to −8 km (Lin). The separate upper-level maximum is just barely discernible in some simulations (WSM3, WSM6) and essentially absent in others (WSM5, Kessler).

Figure 7 is analogous to Fig. 6 (except here 0000 UTC on day 2 is excluded), but for a set of simulations with varying initial RMW and a. This set of simulations, discussed in more detail in SN11, reaches different quasi-steady-state RMW sizes (Fig. 13a of SN11) while achieving comparable maximum intensities (Fig. 13b of SN11). The control simulation (using WSM6) is labeled R90A50, where the initial RMW follows the letter “R” and 100 times the value of a follows the letter “A,” and the other simulations are labeled analogously. The strength of the warm core in the initial condition increases slightly with decreasing RMW and increases more substantially with decreasing a (Fig. 7a). This is a simple consequence of thermal wind balance, although one that is often neglected in qualitative discussions of warm-core structure. The equation for thermal wind balance in log-pressure height coordinates (Schubert et al. 2007) is

\[
\left( f + \frac{2v}{r} \right) \frac{\partial v}{\partial z} = \frac{g}{T_0} \frac{\partial T_v}{\partial r} \tag{2.1}
\]

For a moist atmosphere, the virtual temperature \(T_v\) is the relevant variable, not \(T\). While the use of \(T_v\) is more accurate when assessing thermal wind balance, in this section we are evaluating the structure of the warm core, which is traditionally defined in terms of \(T\). Therefore, in all discussion and figures that follow, we use \(T\), and as the radial gradients in \(T\) and \(T_v\) are nearly the same, there is no qualitative error in this approximation. Neglecting Coriolis, the radial temperature gradient \(\partial T/\partial r\) is proportional to \((2v/r)(\partial v/\partial z)\), or \((1/r)(\partial^2 v/\partial z^2)\), and so \(\partial T/\partial r\) is inversely proportional to \(r\). All else being equal, the radial temperature gradient increases with decreasing RMW. The strength of the warm core is simply the integrated effect of the radial temperature gradient, and since all else is equal initially between R36A50 and R90A50, the initial warm core is slightly stronger in R36A50. For the same initial RMW (the R90 simulations), the strength of the warm core increases by more than 50% (>2°C) as \(a\) is decreased from 1.0 to 0.25 (i.e., as the outer circulation becomes larger). This is because as \(a\) is decreased, there is increased \(\nu (\partial v/\partial z)\) beyond the RMW, and therefore a larger radial temperature gradient, which when integrated inwards yields a stronger warm core. Note that the height of the warm core is initially identical among all of these simulations. This is because the initial vertical structure of the tangential wind field was specified through multiplication of the radial profile by a function of height alone. Therefore, for a given \(v\), the vertical profile of \(\nu (\partial v/\partial z)\) is initially identical and is minimized at the same height, which is the height of the warm core. The initial height of the warm core would also be identical if we were to change the initial maximum tangential wind speed, as \(\nu (\partial v/\partial z)\) is simply the product of \(\nu\) and some function of height alone. Increasing the maximum wind speed will alter the magnitude of the temperature gradient, but not the shape of its vertical profile, and so the warm-core height remains the same. We of course recognize that our initial vertical structure of tangential wind is arbitrary and unrealistic (as shown in SN11), and therefore so is the structure of the warm core. The point is that it gives us a simple perspective for gaining insight into what controls the strength and height of the warm core more generally, and that in our simulations the initial height of the warm core is solely a function of the chosen vertical structure function, whose coefficients maximize the shear of the squared tangential wind speed at about 5-km height.

By 0000 UTC on day 3 (Fig. 7b), most of the simulations with varying initial RMW and \(a\) have developed prominent midlevel (6–7 km) and upper-level (13–14 km) maxima in the perturbation temperature fields, with the midlevel maximum being somewhat stronger. At 0000 UTC on day 4, the R90 simulations (Fig. 7c) exhibit similar structures as each other, with some variability in the exact heights of the mid- and upper-level maxima. At 0000 UTC on day 6 (Fig. 7e), some of the simulations have a well-defined primary maximum at

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4 Technically this is only true in log-pressure coordinates. In height coordinates, numerical calculations show that the initial warm core becomes lower with increasing intensity (not shown). However, this effect is small, with variations of about 500 m over a realistic range of intensities.
4 or 5 km (R90A25, R90A50, R180A25, and R180A50), while the others have a maximum near 8 km. Finally, at 0000 UTC on day 7 (Fig. 7f), the warm-core heights remain similar to the previous day, ranging from 4.5 km (R180A50) to 7.5 km (R90A75). Interestingly, the two A25 simulations exhibit rather similar shapes to each other, indicating that the perturbation temperature (and therefore tangential wind) structure still has some “memory” of the breadth of the initial wind profile.

There are two important facts that we can glean from the profiles in Figs. 6 and 7. The first is that in all of our simulations (except for a portion of Kessler) after 0000 UTC on day 3, the maximum perturbation temperature (and therefore the height of the warm core) is generally found between 4- and 8-km height. There is frequently a secondary maximum in the upper troposphere (near 13–14 km). However, the strongest maximum is almost always the one at low to midlevels. This
is in contrast to conventional wisdom, which states that the typical height of the warm core is around 250 mb, or about 10.5 km. Therefore, either our simulations have a systematic bias or they are unrepresentative because of the influence of an unrealistic initial condition or the conventional wisdom is wrong. The second fact that is apparent from the vertical profiles of perturbation temperature is that variations in the structure of the warm core do not in any obvious manner reflect variations in either intensity or in the decay of maximum tangential wind with height. Implicit within Halverson et al. (2006) is the idea that the height of the warm core is intimately related to the intensity of the tropical cyclone in terms of either wind speed or Pmin (a higher warm core with increased intensity). An examination of Figs. 1 and 2 reveals that this may not be true, as the height of the warm core lowers from 9 to 6 km in WSM6 as $V_{bar_{max}}$ increases from 28 to 59 m s$^{-1}$ between 0000 UTC on day 2 and 0000 UTC on day 3, and then remains at about 5–6 km as $V_{bar_{max}}$ increases to 91 m s$^{-1}$ by 0000 UTC on day 7. For most of the simulations, the height of the maximum perturbation does not vary much as the intensity rapidly increases.

There is variability with time and between simulations of the height and relative prominence of local maxima in perturbation temperature in the upper troposphere, and for some simulations the height of the primary maximum changes with time as well. As shown in Figs. 11 and 12 of SN11, the vertical structure of the normalized maximum gradient wind speed is quite steady with time at and after 0000 UTC on day 3. Therefore, the structure of the warm core and its variability does not actually tell us much about the structure of the tangential winds in the eyewall. This apparent discrepancy is due to the fact that, as previously discussed, at each level (below ~9 km) the RMW is radially outward of the region of maximum temperature gradient. Therefore, large changes in the radial temperature gradient and in the vertical variation of the gradient can occur without having a noticeable effect on the vertical structure of the maximum wind speed. While the vertical structure of the maximum perturbation temperature can be substantially altered by these temperature gradients, if they are generally confined to the region well inside of the eye, then they will mostly be associated with changes in the structure of the tangential wind speed well inside the eye.

3. Revisiting prior studies on warm-core structure

a. Overview of observations

We now return to the question of whether the structure of the warm core in our simulations is representative of real tropical cyclones. Unfortunately, there are not a lot of quality observations of the structure of the warm core, which is what motivates this discussion. Two of the original case studies on which much of the conventional wisdom of the vertical structure is based found the warm core to be maximized near 250 mb. In Hawkins and Imbembo (1976), however, there were two prominent maxima found on consecutive days in Hurricane Inez (1966), with the lower maximum at 650–600 mb and of equal strength to the upper maximum on one day. The height of this lower maximum is comparable to the 4–6 km found at most times in our simulations. As mentioned previously, the more recent study of Hurricane Erin (2001) by Halverson et al. (2006), based on dropsondes, observed the warm core to be near 500 mb. As Inez was much more intense than Erin, Halverson et al. associated the lower warm core with the weaker storm, whereas in fact there might not be such a relationship. Very recently, Dolling and Barnes (2010) showed from dropsondes that the height of the warm core in Hurricane Humberto (2001) was found between 2 and 3 km on three consecutive days, with the intensity varying from a tropical storm to a category 2 hurricane over this time frame.

b. Can satellite remote sensing really see the warm core?

Knaff et al. (2004) examined the structure of the warm core from remote sensing observations from polar-orbiting satellites. The Advanced Microwave Sounding Unit (AMSU) on the satellites measures the radiance received at frequencies near 57 GHz, from which vertical profiles of temperature can be estimated. This is done through a statistical technique (Knaff et al. 2000) that yields temperature at 40 pressure levels from 1000 to 0.1 mb. Knaff et al. (2004) used 23 pressure levels between 920 and 50 mb in their study and attempted to correct for errors caused by the effects of attenuation.
and scattering of radiation by liquid water and ice, respectively. From a dataset of 186 cases, they constructed the composite warm-core structure for low, medium, and high environmental vertical wind shear, with equal sample sizes. They found that the height of the warm core decreased with increasing shear. Perhaps more importantly, their results show a mean warm core that is maximized near 12-km height, which is even higher than that found in the flight-level studies from the 1960s and 1970s and would imply that our simulations are systematically biased or unrepresentative in some manner.

However, we have serious doubts about the ability of the AMSU temperature retrievals to accurately characterize the structure of the warm core of tropical cyclones. The warm core shown in Knaff et al. is not simply maximized in the upper troposphere; it is confined to that region. There is no substantial warm anomaly below 8-km height, and in fact most of the region below 8 km shows a negative perturbation temperature. Based on this temperature retrieval, Knaff et al. (2004) calculated the azimuthal mean tangential wind field through thermal wind balance. They found that isotachs of various thresholds extended to greater heights when shear was lower. This result may well be confounded by the systematic variation of intensity with vertical wind shear. Furthermore, they found essentially constant gradient wind speed in the lower 8–10 km. This is unrealistic (Shea and Gray 1973; Jorgensen 1984; Stern and Nolan 2009; SN11) and is consistent with the fact that they found no warming below 8 km. Recognizing that intensity variations might have an effect, they created two smaller composites of low and high shear, with all such cases having best-track intensities between 90 and 100 knots (kt; 46–51 m s\(^{-1}\)). Their Fig. 5 (see Knaff et al. 2004) shows the perturbation temperature, gradient wind, and vorticity structure for these composites. They claimed that this figure shows that the height of the warm-core anomaly is higher, and that the tangential winds and vorticity extend “deeper” for the composite with “favorable” vertical wind shear (<7.5 m s\(^{-1}\)). In fact, the composites appear very similar to each other; for example, the 10 m s\(^{-1}\) isotach reaches about 13-km height in the “unfavorable” composite, while the same isotach in the “favorable” composite extends to about 13.5 km. More importantly, the tangential wind structure of both composites are equally unrealistic, as the maximum winds do not decrease with height in the lower and middle troposphere as they should. As Knaff et al. (2004) noted, the horizontal resolution of AMSU is 50 km “at best” and “...will not resolve the details of near-core regions of the TC” (p. 2504). This, of course, is why the RMW of the composite gradient wind field is about 150 km, probably 3–6 times larger than in reality.

Let us assume for the moment that the structure of the warm core in our simulations is an accurate reflection of that in real tropical cyclones, and imagine how it might be seen by AMSU. In our simulations, almost all of the temperature gradient in the lower 10 km is found inside 30-km radius. In essence, the primary warm core is unresolvable by AMSU. Since the warm core becomes much broader in the upper troposphere, it will be at least partially resolved. The ridge of higher perturbation temperatures in the outflow layer is the broadest feature, and this would likely be seen as the location of the maximum perturbation temperature. Perhaps not coincidentally, this ridge is found near 12 km in the simulations, which is the same height found by Knaff et al. To test this conjecture in a simple manner (without simulating the observing system itself), in Fig. 8 we compare the perturbation temperature in the control simulation at 1200 UTC on day 6 to that calculated from the same model output after first coarsening to 54-km horizontal resolution. While the primary warm core is actually at 5-km height, from the coarsened data it would appear to be at 12 km, similar to what is seen in the AMSU data of comparable resolution. It is not completely clear why there is no warm core at all below 8 km in Knaff et al. and not simply a much weaker warm core. It is possible that a region of cooling outside of the inner core outweighs the warming of the core when seen at 50-km resolution. Alternatively, perhaps the correction for attenuation by cloud water is insufficient, which would bias the temperatures cold. In either case, the complete lack of a warm core below 8 km is almost certainly incorrect. The temperature structure in the upper troposphere may well be sufficiently accurate, but the inaccurate structure below likely renders the measurements of the height of the warm core unreliable.

c. Previous numerical simulations

Many previous numerical studies present figures that show the structure of the warm core, although not all comment on the height of the warm core or its significance. One of the earliest such studies was that of Kurihara (1975), who found a maximum perturbation temperature at about 250 mb, using an axisymmetric model with 20-km horizontal grid spacing and 11 vertical levels. In the Rotunno and Emanuel (1987, hereafter RE87) control simulation, there were two maxima in perturbation temperature (defined as the difference from the initial base state sounding), at 6 km (~8°C) and at 15 km (~6°C). The control simulation used Newtonian relaxation to the initial condition as a crude parameterization of radiation. Experiments with either a 2 K day\(^{-1}\) cap on Newtonian cooling or without cooling had
maxima at 5 km (∼9°C) and 18 km (∼2°C), and 11 km (∼10°C) and 17 km (∼4°C), respectively. The authors stated that the warm cores in these two latter simulations were closer to observations, although as they are rather different from each other, this is difficult to see. In the simulation of Persing and Montgomery (2003) (which was identical to the “capped” simulation of RE87, but with 4 times the resolution), there were warm-core centers at 6 and 15 km, in this case with the upper center being stronger (although this was calculated in terms of potential temperature). In their reproduction of the original, lower-resolution RE87 simulation, snapshot perturbation temperature fields at days 12 and 28 actually more closely resemble the RE87 simulation without cooling, with a primary maximum at 10 km

In the control simulation of Hausman et al. (2006), the warm-core height (from a 5-day average) was near 12 km (∼20°C). A simulation without ice microphysics had a broad warm core between 10 and 15 km (∼22°C) but contained an even stronger warm core at 2.5 km (∼24°C). All of the above studies used an axisymmetric model. In his idealized three-dimensional simulation using TCM3, Wang (2002b) found a warm-core height of 275 mb. In a simulation with TCM4, Wang (2008) found a warm-core height of 13 km, which later elevated to 15 km during an annular phase. Overall, past idealized studies show evidence for the occurrence of multiple maxima in perturbation temperature, although the relative dominance of the maxima varies among simulations, and some simulations appear not to exhibit this phenomenon. There is some evidence of a steadiness with time of the height of the warm core (Persing and Montgomery 2003) in some simulations, while not in others (Wang 2008). It is currently unclear what causes all of these differences. In particular, the idea of RE87 that the manner in which radiation is parameterized has a systematic influence on the height of the warm core does not appear robust, as the warm-core structure in the 4X (with a cap on the cooling) simulation of Persing and Montgomery appears similar to the control simulation of RE87 (with no cap).

In a simulation of Hurricane Bob (1991), Braun (2002) showed a cross section of perturbation virtual potential temperature that was maximized at about 5 km. In a footnote, he stated that this was “...much lower than is typically observed...” (p. 1587) and attributed this apparent discrepancy to the use of a domain averaged reference state. He also stated that when using a profile from the environment away from the storm as a reference state, the warm core was maximized at 9–10 km, “...in better agreement with observations within hurricanes” (p. 1587). Zhu et al. (2004) analyzed a simulation of Hurricane Bonnie (1998), using the fifth-generation Pennsylvania State University–National Center for Atmospheric Research Mesoscale Model (MM5) with 4-km grid spacing. At a time corresponding to a hurricane of moderate intensity, they found a broad maximum (8°C) in perturbation temperature from 200 to 400 mb. Comparing this to AMSU-retrieved temperatures at a similar time, they found that AMSU was colder by up to 4°C below 500 mb, which they attributed to cold biases in AMSU caused by precipitation. At two other times, they showed cross sections where the warm core was maximized at either roughly 5 or 8 km.

In Liu et al. (1997), the warm core in the simulated Hurricane Andrew (1992) was found to be near 500 mb, from a snapshot vertical cross section at 54 h. From an
azimuthal mean time-averaged (48–60 h) composite of the same simulation, Liu et al. (1999) found the height of the warm core to be 7 km (also given as 425 mb). They wrote that this was “...6 km lower than that used by Emanuel (1986) and Holland (1997) in their estimating the maximum intensity of tropical cyclones” (p. 2601). That statement implies that the height of the warm core is specified to be in the upper troposphere (~13 km) in Emanuel’s theory, and that this “choice” has an effect on the theoretical maximum intensity. This is not the case, as the structure of the warm core in the theory is actually a consequence of the assumption of slantwise moist neutrality and can only be calculated after knowledge of the winds along the lower boundary. Liu et al. (1999) also implied that the tangential winds decay with height the fastest at the level where the warm core is maximized, and this is incorrect. As seen from Eq. (2.1), the radial temperature gradient is proportional to $(2v/\partial r)$ $(\partial \theta/\partial z)$, not simply $\partial v/\partial z$. For the same temperature gradient at the same radius, the shear will increase linearly with decreasing $v$. Alternatively, for a given shear, the temperature gradient will increase linearly with increasing $v$. Also, as radius is increased, a lesser temperature gradient is required for the same value of shear. The maximum temperature gradient is therefore found where the largest shear of the squared wind speed divided by radius occurs. Note that this still does not guarantee that this is the same as the height of the warm core, as the warm core at the center is the integrated effect of the radial temperature gradient. It is possible for the warm core to be maximized at a different height from that of the maximum temperature gradient.

4. Theoretical aspects of warm-core structure

a. Existing theory

There is very little theoretical guidance regarding warm-core structure. Emanuel (1986, and subsequent modifications) represents the most complete theory for tropical cyclone potential intensity, and as it predicts the complete radius–height structure of the wind and temperature fields, the perturbation temperature can be found. As presented in Fig. 11 of Emanuel (1986), the warm core is maximized at 13 km, and he compared that favorably to observations. Because of the assumption of slantwise moist neutrality, in essence, the vertical profile of perturbation temperature in Emanuel’s theory is given by the difference between two moist adiabats: one originating at the surface at the minimum pressure (which has already been solved for) and the other at the lifting condensation level (LCL) given by the assumed surface relative humidity (80%), temperature, and pressure in the environment. In contrast to the prediction in Emanuel, Holland (1997) essentially parameterized the vertical profile of temperature through an assumption of constant equivalent potential temperature $\theta_e$, along with an arbitrary specification of the vertical profile of relative humidity. From this model, Holland found that the perturbation temperature tends to be maximized around 300–400 mb (~7.5–9.5 km), although he also stated that soundings that yielded the strongest potential intensities had maximum eyewall warming at 150–200 mb (~12–14 km). Holland stated that potential intensity is “...highly sensitive ... to the height of the warm core...” and that “...the major limiting factor on cyclone intensity is the height and amplitude of the warm core that can develop...” (p. 2519).

Several times Holland (1997) cites Eq. (11) of Hirschberg and Fritsch (1993), given below (but modified so as to be diagnostic by removing the time derivatives):

$$\Delta P_s = \frac{P_s}{T_v(P_s)} \int_{P_s}^{P_t} \Delta T_v d \ln p,$$

(4.1)

where $P_s$ is surface pressure and $P_t$ is pressure at some upper level. Holland used this to argue that there is a “...greater impact of upper-level warming on pressure falls...” (p. 2527) at the surface. This argument is then implicit in Holland’s conclusion that “…the height of the warm anomaly that is developed...” is among the factors that “…govern much of the intensity variability of tropical cyclones” (p. 2538). While this argument has intuitive appeal, it is not quite correct for the purpose of understanding the height of the maximum perturbation temperature. To understand why, let us imagine a hypothetical temperature perturbation (of some fixed magnitude) that is vertically localized. If it is in hydrostatic balance, then there is an associated (negative) surface pressure perturbation. Now suppose we were to simply raise this warm “bubble” to a greater height. The balanced surface pressure perturbation associated with this higher warm region must be of greater magnitude than for the lower warm region, consistent with Eq. (4.1).

However, we do not actually have any basis for assuming that the preceding thought experiment represents how the structure of the perturbation temperature changes with intensity. More specifically, there is no reason to assume that a temperature perturbation of constant magnitude rises upward as the surface pressure lowers. In our simulations, we have instead seen that throughout most of the intensification period, a

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8 It is important to note that intensity in Holland’s theory is calculated from the temperature field, whereas in Emanuel’s theory it is the other way around.
midlevel maximum increases in magnitude while maintaining nearly constant height, which is also consistent with a decrease in the surface pressure. More generally, the surface pressure can be consistent with any number of vertical temperature profiles, and its distribution cannot be assumed. As we will show in the next section, the temperature distribution at the center of a tropical cyclone (from a diagnostic perspective) is a complicated nonlinear and nonlocal function of the entire axisymmetric wind field. Therefore, ultimately Eq. (4.1) alone says nothing about either where the maximum perturbation temperature should be found, or whether or not there should be some relationship between its height and intensity.

b. The relationship between the height of the warm core and thermal wind balance

Figure 9 shows an example of how the height of the warm core is related to the radial temperature gradient and the vertical gradient of tangential wind speed, for a composite of 1800 UTC of day 5 to 0600 UTC of day 6 of the control simulation. At this time the maximum warm core is 15.1°C at 5.5-km height (Fig. 9a). The strongest (negative) temperature gradient is −0.778°C km⁻¹ (Fig. 9b) and is found at 2.75-km height, 2.75 km lower than the height of the warm core. The warm core is strongest at 5.5 km because there is a broader region of relatively large gradient at this height, and there is no region of positive radial gradient inside the eye to offset the warming, as there is below. The upper maximum in perturbation temperature near 13 km at this time is the result of a relatively weak radial gradient over a very large radial extent. Note that at low and midlevels, the strongest temperature gradients are found 15–20 km inside of the RMW. Next, we can see that the region of largest radial temperature gradient does not correspond with the region of largest vertical gradient of tangential wind speed. There are three negative extrema in ∂υ/∂z: at about 1-, 10-, and 16-km height (Fig. 9c). The lowest (and strongest) extremum is almost entirely the result of unbalanced flow and so will not be related to the radial temperature gradient.

A more appropriate field to examine is ∂υᵣ/∂z (Fig. 9d), and here there are also three prominent negative extrema: near 9-km height and 35-km radius, near 11-km height and 55-km radius, and near 16-km height and 60-km radius. The largest (negative) shear of the gradient wind does occur near the tropopause, but because υᵣ itself is so small at this height, the associated temperature gradient is also quite small. The other two extrema are associated with large temperature gradients, but they are shifted upward from the extrema in ∂T/∂r, by about 1 km for the upper extremum and by about 6 km for the lower extremum. Note that at the RMW itself (in the lower 9 km), ∂υᵣ/∂z is relatively small compared to the region 5–20 km inside of the RMW. From this, we can now understand why the structure of the perturbation temperature field need not be well related to that of the maximum winds. The field of υᵣ(∂υᵣ/∂z) (Fig. 9e) better resembles that of ∂T/∂r than does ∂υᵣ/∂z. The extremum at 16-km height is no longer prominent, the extremum at 11 km shifts downward to 10 km, and the concentrated extremum at 9 km becomes broadened vertically. Now, the strongest feature is the 10-km extremum, but ∂T/∂r is actually much stronger at 2.75 km. Finally (Fig. 9f), when υᵣ(∂υᵣ/∂z) is divided by the radius and multiplied by 2g/∂z [neglecting Coriolis, this solves Eq. (2.1) for ∂TI/∂r], we are able to get a field that is qualitatively the same as ∂TI/∂r (and with equivalent units). That the low-level shear extremum is located at half the radius of the upper-level extremum is critical in making the low-level shear extremum also the region of strongest (negative) ∂TI/∂r. This is of course, consistent with Eq. (2.1). However, it can be tempting to assume that ∂T/∂r is well represented by ∂υᵣ/∂z itself, and many studies have implicitly made this assumption. What we have just shown is that this assumption is generally not justifiable, and that the relationship between tangential wind and temperature is considerably more subtle.

5. Discussion and conclusions

In this study, we have examined the structure of the warm core in tropical cyclones through a suite of idealized WRF model simulations. As the height of the maximum perturbation temperature in our simulations is consistently much lower than what is widely believed to be the case in real tropical cyclones, this motivated a reexamination of the true state of knowledge regarding this issue. From our assessment of the existing literature, we have seen that conventional wisdom suggests that the warm core is maximized in the upper troposphere near 250 mb (~10.5 km) or even higher. To a large extent, this is based on three case studies from the

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9 Camp and Montgomery (2001) also make this point regarding the assumption of the temperature profile, but in the context of arguing that it is not a useful basis for determining potential intensity. They also note that Holland’s assumption of constant θₑ in the eye is unrealistic.

10 As tangential winds generally decrease with height, the strongest vertical gradients are negative, and so these features are minima, as are the regions of strongest radial temperature gradient. As it can be confusing to refer to the most important features as “minima,” we instead use “extrema.”
1960s and 1970s, using flight-level observations at three to five altitudes. Although there have been very few in situ upper-level observations of the thermal structure of tropical cyclones in the past 40 yr, several such studies have observed the warm core to be substantially lower, near 500 mb (~5.5 km). In these studies (e.g., Halverson et al. 2006), it is often taken for granted that the conventional wisdom is correct, and so causes are sought for the existence of an “anomalous” midlevel maximum, such as weakening by shear. Knaff et al. (2004) claimed to show evidence for the effect of shear from microwave satellite remote sensing. They also claimed to show that the warm core is typically maximized at about 12-km height. This shear effect, if present, is actually very small in their figures. Furthermore, the complete lack of a warm core below 8 km in the satellite “observations”

Fig. 9. For the control simulation at 0000 UTC on day 6, the azimuthal mean 12-h composite (a) perturbation temperature, (b) $\delta T/\delta r$, (c) $\delta T/\delta z$, (d) $\delta y/\delta z$, (e) $v_y(\delta y/\delta z)$, and (f) $(T_0/g)(\delta y/\delta r)(\delta y/\delta z)$. Note that (f) represents Eq. (2.1), solved for $\delta T/\delta r$ neglecting Coriolis, with $T_b = 300$ K. The RMW of the composited wind field is also shown.
renders it impossible to reliably conclude that the maximum is truly in the upper troposphere. From a comparison to our simulations, we showed that the resolution of the satellite measurements is far too coarse to properly resolve the inner core warm-core structure of most tropical cyclones. Indeed, by artificially coarsening the data from our simulation, we showed how the warm core could appear to be 7 km higher than its true location. The inaccuracy of microwave retrievals might be mitigated somewhat in the near future, as the High Altitude Monolithic Microwave Integrated Circuit (MMIC) Sounding Radiometer (HAMSIR) instrument, a high-resolution aircraft-mounted microwave sounder, has shown promising preliminary results (Brown et al. 2007).

In most of our simulations, the maximum perturbation temperature was typically found between 4- and 8-km height. There was often an additional upper-tropospheric maximum, at 13–14-km height. However, in contrast to conventional wisdom, the strongest maximum was generally at midlevels. Where the warm core is most typically maximized in real tropical cyclones and its degree of variability is uncertain, as there are simply an insufficient number of storms well observed in this regard. Based on our above discussion of the literature, there is actually substantial evidence that the warm core is often found in the midtroposphere, from 4- to 7-km height. It is currently unclear how often the warm core is centered in the middle versus the upper troposphere, how often there are multiple maxima, and whether or not there is any dynamical significance to this variability. Nevertheless, we further demonstrated that the height of the warm core plays a less important role in overall tropical cyclone structure than is generally imagined. This is because below about 9 km, the radial temperature gradient in the vicinity of the RMW is actually rather small. Consequently, large changes in the structure of the warm core can occur without obvious corresponding changes in the vertical profile of maximum winds. Finally, we demonstrated that the height of the warm core is not in general at the height where the maximum radial temperature gradient is found, and that both of these regions are well below the height where the vertical gradient of balanced tangential winds is largest. Rather, the fact that the region of strong gradient has an outward slope, combined with the inverse dependence of the radial temperature gradient on radius, works (along with other subtleties of the thermal wind equation) to separate these features, thereby rendering it impossible to relate the structure of the warm core to that of the tangential wind field in a simple way. Therefore, it is rather difficult to connect variability in the height of the warm core to other dynamical features of the inner core, such as the distribution of vertical motion, diabatic heating, or tangential winds.

Given that the height at which the warm core is found may not well relate to other aspects of tropical cyclone structure, it is fair to ask why we should be so concerned with the possibility that the preferred height of the warm core is lower than is generally believed. The answer is that numerous studies have treated the upper-tropospheric warm core maximum as a common characteristic of all tropical cyclones, and have used this belief to either demonstrate consistency with their models or theories (i.e., Holland 1997) or to provide qualitative explanations for other phenomena. Wang (2001) noted that different models exhibited warm cores at different levels, contrasting the idealized simulation of Kurihara and Bender (1982) that had a warm core “…in agreement with observations…” (p. 1371), at 150–250 mb, with the MM5 simulation of Hurricane Andrew of Liu et al. (1997), where the warm core was closer to 500 mb. In Kurihara and Bender, the warm core was actually maximized substantially lower (300 mb) than stated by Wang. Among idealized three-dimensional studies, it is actually only Wang’s simulations that exhibit warm cores so high in the upper troposphere. This is not to say that these simulations are necessarily flawed, only that there are differences among studies that remain unexplained. Wang (2002b) claimed that the upper-level warm core in his simulation was “…consistent with the thermal wind balance required for the outward tilt of the maximum tangential wind with height” (p. 1216). As was shown in Stern and Nolan (2009), this is not quite true, as thermal wind balance alone imposes no constraint on the slope of the RMW. Similar to Wang, Powell et al. (2009) stated that the “…magnitude of RMW tilt is greatest in the upper troposphere, where the warm core is strongest” (p. 869). Even if the warm core were strongest there, the strength and height of the warm core do not control the slope of the RMW. Clearly, the literature is rife with examples of various structural characteristics of tropical cyclones being related to or ascribed to the height of the warm core. There is presently little convincing evidence for any of these relationships.

Finally, we return to the question of the operational significance of the height of the warm core. The height of the warm core is sometimes used by forecasters at NHC to upgrade storms from subtropical to tropical, or as evidence for a change in the intensity of a hurricane (increasing intensity with increasing height). From our results, we can see that it is possible for a hurricane to undergo very large changes in intensity while maintaining a warm core with nearly constant height. It may well be the case that real storms behave differently, but
there is simply insufficient data to currently address this question. Therefore, we feel that the apparent height of the warm core should not be used in operational intensity estimates. We have also shown that the AMSU temperature retrieval products do not accurately reflect the structure of the warm core in the lower and middle troposphere, likely due (in part) to insufficient resolution. This has led to likely erroneous conclusions regarding the height of the warm core, and its relationships with intensity and with vertical wind shear. In spite of this, these temperature retrievals have been used with success in statistical estimation of the intensity of tropical cyclones (Demuth et al. 2004, 2006). This is because the temperature structure in the upper troposphere does indeed relate to the intensity of the storm, whether or not the warm core is maximized in this region. It is telling that one of the potential predictors considered in the development of this technique, the height of the warm core, was not ultimately selected as a predictor.

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APPENDIX A

The Effects of Radiation

In all simulations described previously, no parameterization of the effects of longwave or shortwave radiation has been utilized. In Stern (2010), the sensitivity to radiation was explored, and it was found that the use of radiation schemes had little effect on the intensity evolution, but that the quasi-steady RMW was larger with radiation. That simulation used WSM3 for microphysics and a convective parameterization was active on the 18-km outer domain, and so it cannot be properly compared to our control simulation. Here, we briefly present results from a new simulation (hereafter, WSM6-RA) that is identical to the control simulation, with the exception that longwave and shortwave radiation are parameterized with the Rapid Radiative Transfer Model (RRTM; Mlawer et al. 1997) and Goddard (Chou et al. 1998) schemes, respectively.

FIG. A1. Time series of (a) Pmin, (b) maximum 10-m wind speed, and (c) RMW at 250-m height. This is a comparison of the control and WSM3 simulations to respective simulations that utilized radiation parameterization schemes.
Comparisons of the time evolution of intensity and of the RMW between the control simulation and WSM6-RA are shown in Fig. A1. For completeness, also shown are the original WSM3 simulation and a simulation using WSM3 that respectively utilized the RRTM and the Dudhia (1989) schemes for the parameterization of longwave and shortwave radiation (WSM3-KF-RA), as well as the Kain–Fritsch (Kain and Fritsch 1990) convective parameterization (outer domain only). The quasi-steady RMW is larger in WSM6-RA as compared to the control simulation, as it is in WSM3-KF-RA as compared to WSM3. This is an interesting result and is worth further exploration, but it is beyond the scope of this study. As noted above, there is no obvious significant difference in the intensity evolution of WSM3-KF-RA as compared to WSM3. WSM3-KF-RA is slightly stronger from about 0000 UTC of day 3 through 1200 UTC of day 4 but is slightly weaker after 1200 UTC of day 6. In contrast, WSM6-RA is generally stronger than the control simulation throughout, although there are substantial periods (prior to 1200 UTC of day 3, 1200 UTC of day 5 through 1200 UTC of day 6) when their intensities are essentially the same. The true systematic effect of radiation on intensity cannot really be ascertained without a large number of simulations, and a comparison to WSM6rand2 and WSM6rand3 in Fig. 3 shows that it is likely not as large as is apparent here. Furthermore, as we are not starting from an environment in radiative–convective equilibrium, the direct impact of radiation on tropical cyclone intensity is likely exaggerated. Nevertheless, in the context of our model setup, we might expect a somewhat stronger storm with radiation included because of the cooling tendency of longwave radiation on the upper-level environment (not shown).

A comparison of the time evolution of the warm-core structure is given in Fig. A2, which shows time–height Hovmöller plots of the maximum perturbation temperature at the center of the control simulation and of WSM6-RA. This is similar to Fig. 5, but here we use the azimuthal mean temperature at \( r = 450 \) km (instead of \( r = 558–648 \) km) as the reference profile, as the outer radii of WSM6-RA begin to exit the domain before the end of the simulation. Through 1200 UTC of day 6, the primary warm core maximum is found somewhat lower in WSM6-RA (\(~4\) km) than in the control simulation (\(~5\) km). Around this time, the maximum jumps upward to about 8-km height in WSM6-RA, and then lowers to 6 km on day 7. The secondary maximum in perturbation temperature in the upper troposphere (\(~12–14\) km) is more pronounced and persistent in WSM6-RA, and there is a brief interval around 1200 UTC of day 6 when this maximum is stronger than the midlevel maximum. As with intensity, we cannot necessarily attribute these differences in warm-core structure to a systematic effect of radiation, although it is a possibility. Regardless, this simulation is consistent with our primary conclusion: that the maximum perturbation temperature in numerical simulations is generally found in the midtroposphere.

**APPENDIX B**

**Sensitivity to the Choice of Reference Profile**

The calculation of a perturbation temperature inevitably involves choosing a reference profile to define as the “environment.” A number of different definitions of the base state have been used in various studies that have examined the structure of the warm core. The early
flight-level studies used the annual mean Caribbean sounding of Jordan (1958). The observational study of Halverson et al. (2006) used a combination of two dropsondes at 340 and 610 km from the center. In their AMSU study, Knaff et al. (2004) defined the reference state as the mean in the 500–600-km annulus. In many axisymmetric modeling studies, the temperature at the outer boundary in the initial condition is used as the reference state at all times (e.g., RE87), whereas in others the radially varying initial condition is used (e.g., Persing and Montgomery 2003). In their modeling study of Hurricane Andrew (1992), Liu et al. (1997) used the domain average temperature (on pressure surfaces) as a reference state. Finally, in some studies (e.g., Wang 2008), the reference state is never defined.

While it is not possible for us to determine the effect of the choice of reference profile in these previous studies, we can investigate the impact of our choice in this study. Figure B1 shows time–height Hovmöller plots of the perturbation temperature at the center of the control simulation, for three different choices of reference state: the mean temperature in the annulus from 558 to 648 km (identical to Fig. 5a), the “moist tropical” mean sounding of Dunion (2011), and the average temperature within 150 km of the storm center (slightly smaller than the entire inner domain). Note that the sounding of Dunion (2011) is our initial condition (in the environment) and is nearly identical in terms of temperature to the “Hurricane-Season” sounding of Jordan (1958). The domain-average temperature includes the warm core itself, and so the calculated perturbation temperatures are therefore smaller in magnitude as compared to using the 558–648-km annulus. The height where the maximum perturbation temperature is found using this reference profile is at most times slightly lower than when using the annulus. Overall however, the structures given by the two methods are quite similar.

Using the initial environmental profile as the reference state yields warmer perturbation temperatures at almost all times/heights. This is because the environment warms with time because of surface fluxes and subsequent convection. This environmental warming is greatest at 12-km height (not shown) and, as a result of its height dependence, the locations of the maxima in perturbation temperature can be altered. In particular, the upper tropospheric perturbation temperatures are enhanced relative to the midtroposphere when using the initial condition as the reference state. So after 1200 UTC on day 7, the absolute maximum in perturbation temperature is at about 11 km with this method. Therefore, at times, the choice of reference state can have a large impact on the apparent height of maximum perturbation temperature. However, at most times after 0000 UTC
on day 2, the maximum using this method is found at about 5 km, the same as when using the 558–648-km annulus. The effect of the reference state on the location of maxima depends on both the vertical structure of the differences between two reference states, and on the vertical structure of the inner core temperature field itself. Therefore, it is difficult to generalize the effects of using different definitions, and the effect of the use of a climatological sounding in the early observational studies remains unknown.

While there is no "correct" choice of a reference profile, we believe that some sort of average (at constant height) over an area at least several hundred kilometers away from the center is the most appropriate. A domain average includes the warm core itself, and it can also be misleading because it is often performed on model surfaces, on which height systematically varies with distance from the center. Using the initial condition as the reference is less meaningful because the temperature of the actual storm environment changes with time in most simulations.

REFERENCES


