A Study of the North Atlantic Oscillation Using Conditional Nonlinear Optimal Perturbation

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(Manuscript received 21 May 2012, in final form 26 September 2012)

ABSTRACT
The conditional nonlinear optimal perturbation (CNOP) method is used to explore the optimal precursors that trigger the North Atlantic Oscillation (NAO) anomaly pattern with a triangular T21, three-level, quasi-geostrophic global spectral model based on a viewpoint that the NAO is a nonlinear initial-value problem. With a three-dimensional winter climatological flow as the basic state, initially baroclinic localized optimal precursors on the northward flanks of the climatological Atlantic jet undergo wave breaking during their evolution into the NAO-like anomalies. Accompanied with the formation of the NAO, the north–south variability of the zonal mean westerly anomaly has arisen. Analysis reveals that in the evolution of optimal precursors, the role played by the self-interaction of perturbations (viz., the nonlinear process) in the onset of the negative-phase NAO (NAO\textsuperscript{−}) event is stronger than that in the onset of the positive-phase NAO (NAO\textsuperscript{+}) event. Both the perturbation/basic-state interaction and self-interaction of perturbations determine whether the NAO\textsuperscript{−} event occurs, whereas the nonlinearity process in the NAO\textsuperscript{+} onset only appears to modulate the structure of the perturbation to have a dipole mode over the North Atlantic at the optimization time, and meanwhile cause this dipole mode to become zonally extended. That is to say, the nonlinear process indeed plays an important role during the onset of an NAO event and the CNOP method is a useful tool to identify the dynamics of the onset of NAO teleconnection patterns.

1. Introduction
The atmospheric flow of the Northern Hemisphere in winter exhibits considerable variability on intraseasonal, interannual, and decadal time scales. The North Atlantic Oscillation (NAO) is the most prominent low-frequency dipole mode of atmospheric variability in the mid- to high latitudes of the Northern Hemisphere (Walker and Bliss 1932; Feldstein 2000; Woollings et al. 2008), which has been recognized to have a profound effect not only on regional weather and climate but also the hemispheric-scale circulation (Hurrell 1995; Thompson and Wallace 2000; López-Moreno and Vicente-Serrano 2008).

Although the low-frequency forcings like the ocean, greenhouse gas concentrations, and snow cover can influence the interdecadal and interannual fluctuations of the NAO (Hurrell 1995; Feldstein 2002; Cohen and Entekhabi 1999; Cohen et al. 2005; Dong et al. 2011), the internal atmospheric mechanisms are more important to determine its intrinsic time scale (Hurrell 1995; Feldstein 2003; Luo et al. 2007a; Rivière and Orlanski 2007). Feldstein (2000) showed that the NAO can be viewed as being a stochastic process with an e-folding time scale of about 1 year. Furthermore, Feldstein (2003) suggested that the intrinsic time scale of the NAO is about 2 weeks, and both high-frequency and low-frequency transient eddy fluxes drive the NAO growth.

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DOI: 10.1175/JAS-D-12-0148.1

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Vallis et al. (2004) also emphasized the importance of synoptic eddies for the NAO onset with a barotropic model, and demonstrated that the NAO can be produced by a stochastic stirring that mimics baroclinic eddy development. The crucial role played by the eddy fluxes shown in the above studies illustrates the fact that the NAO life cycle is fundamentally a nonlinear process, and therefore, the NAO life cycle may be closely linked to wave breaking and mixing of potential vorticity, as noted by Benedict et al. (2004), who investigated the synoptic characteristics of individual NAO events, and showed that it is the remnants of synoptic-scale wave breaking that form the physical entity of the NAO. This also has been verified by Franzeke et al. (2004) with a multilevel primitive equation model when the three-dimensional Northern Hemisphere winter climatological flow is used as a basic state. More specially, anticyclonic wave breaking accompanies the occurrence of the positive-phase NAO (NAO+) event, while cyclonic wave breaking is linked with the formation of the negative-phase NAO (NAO−) event. What is more, Franzeke et al. (2004) demonstrated that the essential ingredient determining the NAO phase is the initial latitudinal position of the perturbations, which are located equatorward (poleward) prior to the onset of the positive (negative)-phase NAO event. However, Riviè re and Orlanski (2007) demonstrated that it is the energy and frequencies of eddies that determine the type of wave breaking. The synoptic-scale waves with intermediate frequencies (periods between 5 and 12 days) break anticyclonically, whereas synoptic wave with very high frequencies (periods between 2 and 5 days) will tend to break both cyclonically and anticyclonically with a predominance for cyclonic wave breaking.

Very recently, Woollings et al. (2008) suggested the NAO arises as a result of variations in the occurrence of upper-level Rossby wave–breaking events over the North Atlantic. A positive NAO is envisaged as being a description of periods in which high-latitude blocking episodes are infrequent and can be considered as a basic, unblocked situation. A negative NAO is a description of periods in which these episodes occur frequently. However, in a weakly nonlinear NAO model, Luo et al. (2007a) demonstrated that the NAO is forced by both preexisting planetary-scale and synoptic-scale waves. The eddy forcing arising from the preexisting synoptic-scale waves is shown to be crucial for the growth and decay of the NAO, but the preexisting low-over-high (high over low) dipole planetary-scale wave must be required to match the preexisting positive-over-negative (negative over positive) dipole eddy forcing so as to excite an NAO+ (NAO−) event. Especially, Luo et al. (2008) showed that wave breaking is not a necessary condition for the occurrence of NAO events. To a certain extent, planetary wave breaking, synoptic-scale wave breaking, and jet variability seem to be different descriptions of the NAO phenomenon.

The above studies showed one common fact: that the NAO can be viewed as a nonlinear initial-value problem (Benedict et al. 2004; Franzeke et al. 2004; Luo et al. 2007a,b, 2008). Then, the natural problem to be explored is the following: What perturbations can optimally produce the characteristic NAO anomaly pattern in a Northern Hemisphere winter climatological flow, and therefore, the jet variations? How is the formation of the NAO related to the wave breaking? What is the role played by the nonlinear process? The present study attempts to address these questions by using the concept of conditional nonlinear optimal perturbation (CNOP), which was first proposed by Mu and Duan (2003). At present, this method has been successfully used in the exploration of optimal precursors to ENSO (Duan et al. 2004, 2009; Duan and Mu 2006) and blocking onset (Jiang and Wang 2010; Mu and Jiang 2011).

The outline of this paper is as follows. In section 2, we describe the model and show its ability to simulate the NAO events. Section 3 presents the corresponding nonlinear optimization problem related to the exploration of optimal perturbations triggering the NAO anomaly. The numerical results are presented in section 4. In addition to the relationship between NAO event and jet displacement, wave breaking is discussed here. Further explanations of the perturbation development are described in section 5. Finally, the main results are summarized and discussed in section 6.

2. The T21L3 model and its ability to simulate the NAO events

a. Description of the model

In this study, a T21 quasigeostrophic (QG) global spectral model proposed by Marshall and Molteni (1993) is adopted, which integrates prognostic equations for potential vorticity at 200 (level 1), 500 (level 2), and 800 hPa (level 3). The model equations are as follows:

\[ \frac{\partial Q_i}{\partial t} = -J(\Psi_1, Q_i) - D_i(\Psi_1, \Psi_i) + S_i, \quad (1a) \]

\[ \frac{\partial Q_2}{\partial t} = -J(\Psi_2, Q_2) - D_2(\Psi_1, \Psi_2, \Psi_3) + S_2, \quad (1b) \]

\[ \frac{\partial Q_3}{\partial t} = -J(\Psi_3, Q_3) - D_3(\Psi_2, \Psi_3) + S_3, \quad (1c) \]

where the index \( i = 1, 2, 3 \) refers to 200, 500, and 800 hPa, respectively; \( J \) is the Jacobian of a two-dimension field. The potential vorticity is defined as
\[ Q_1 = \nabla^2 \Psi_1 - R_1^{-2}(\Psi_1 - \Psi_2) + f, \quad (2a) \]
\[ Q_2 = \nabla^2 \Psi_2 + R_1^{-2}(\Psi_1 - \Psi_2) - R_2^{-2}(\Psi_2 - \Psi_3) + f, \quad (2b) \]
\[ Q_3 = \nabla^2 \Psi_3 + R_2^{-2}(\Psi_2 - \Psi_3) + f \left( 1 + \frac{h}{H_0} \right), \quad (2c) \]

where \( f = 2\Omega \sin \phi; R_1 (\approx 700 \text{ km}) \) and \( R_2 (\approx 450 \text{ km}) \) are Rossby radii of deformation appropriate for the 200–500- and the 500–800-hPa layers, respectively; \( h \) is the real orographic height; and \( H_0 \) is a scale height (9 km).

The linear operators \( D_1, D_2, \) and \( D_3 \) represent the effects of Newtonian relaxation of temperature, linear drag on the 800-hPa wind, and horizontal diffusion of vorticity and temperature. \( S_1, S_2, \) and \( S_3 \) are time-independent but spatially varying sources of potential vorticity, which are estimated based on European Centre for Medium-Range Weather Forecasts (ECMWF) analyses of streamfunction at three pressure levels for each day in January and February 1984–89. With this forcing, the model provides good simulations of winter-time midlatitude situations in the Northern Hemisphere. At present, it is successfully used in the investigation of optimal precursors to blocking and its related predictability (Mu and Jiang 2008; Jiang and Wang 2010; Mu and Jiang 2011).

In addition, for our next research, the equation for potential vorticity perturbation of the model is also deduced as follows:

\[ \frac{\partial q_i}{\partial t} = -J(\Psi_i, q_i) - J(\psi_i, Q_i) - J(\psi_i, q_i) - D_i, \quad (3) \]

and \( i = 1, 2, 3. \) The potential vorticity perturbation is as follows:

\[ q_1 = \nabla^2 \psi_1 - R_1^{-2}(\psi_1 - \psi_2), \quad (4a) \]
\[ q_2 = \nabla^2 \psi_2 + R_1^{-2}(\psi_1 - \psi_2) - R_2^{-2}(\psi_2 - \psi_3), \quad (4b) \]
\[ q_3 = \nabla^2 \psi_3 + R_2^{-2}(\psi_2 - \psi_3), \quad (4c) \]

where \( q_i(\psi_i) \) represents the perturbation superimposed on the reference state \( Q_i(\Psi_i). \)

b. Ability to simulate the NAO events

To explore the optimal precursors triggering the NAO anomaly, it is necessary to check whether the T21L3 model has the ability to simulate the NAO events. First, to obtain the NAO teleconnection pattern, we run an 18 000-day integration by using the T21L3 QG model [Eq. (1)] with initial condition of ECMWF analysis of 0000 UTC 1 December 1983. At each grid point, the anomaly field is obtained by subtracting the winter climatological mean, which is the mean state of the above 18 000-day integration, as shown in Fig. 1a. Empirical orthogonal function (EOF) analysis is performed on a 200-hPa streamfunction anomaly field confined to the North Atlantic region between 90°W and 60°E north of 30°N. The NAO corresponds to the first EOF whose variance contribution is about 32.2%. A typical NAO dipole mode is seen, with one negative center over Greenland, and another positive center extending across the midlatitude North Atlantic and Europe, which is shown in Fig. 1b.

We use the term “index amplitude” to refer to the principal component time series of EOF1. Briefly, if the index amplitude is greater than 1.2 standard deviations.
FIG. 2. Temporal evolution of the composite anomalous 200-hPa streamfunction field for the persistent episodes: (a) the negative-phase NAO and (b) the positive-phase NAO. Lag 0 corresponds to the onset day. Contour interval is $3 \times 10^6 \text{ m}^2 \text{ s}^{-1}$. 

(a) lag-10  lag-5

lag0  lag5

lag10  lag15

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(a) lag-10  lag-5

lag0  lag5

lag10  lag15
FIG. 2. (Continued)
for five or more consecutive days, then a persistent episode is defined to have taken place. The first day of a persistent episode is defined as the onset day. When the index amplitude is positive (negative) during a persistent episode, that particular episode is referred to as the positive (negative)-phase NAO. Also, if the time interval of two consecutive persistent episodes is less than some specific days (e.g., 30 days), these two persistent episodes are discarded to avoid the confusion of the former decay with the precursor of the latter one. With these definitions, there are 70 (61) persistent episodes for the positive (negative)-phase NAO. The temporal evolution of the composite anomalous 200-hPa streamfunction field for the above persistent episodes is shown in Fig. 2a for the negative-phase NAO and in Fig. 2b for the positive-phase NAO. Lag 0 corresponds to the onset day of the persistent episodes. The dipole mode over the North Atlantic can be clearly seen from lag 0 to lag 5, which period may correspond to the mature stage of the NAO events. From precursor on lag −5 to decay on lag 10, it reveals that the life cycle of the NAO growth and decay is approximately 2 weeks. This result is also consistent with the studies of reanalysis data by Feldstein (2003) and Franzke and Feldstein (2005). What is more, the amplitude of the composite negative-phase NAO is stronger than that of the composite positive-phase NAO (not shown), which is also consistent with the observation analysis. All of the above analysis reveals that the T21L3 model has the ability to describe an NAO event in the Northern Hemisphere winter situation.

3. Theory regime

a. Identification of the NAO regime

To construct a nonlinear optimization problem to explore the conditional nonlinear optimal perturbations that drive the NAO onset, it is necessary for us to choose an objective criterion to quantify the NAO regime. Here, a blocking index \( B \) introduced by Liu (1994) is modified to an NAO index, in which the typical blocking anomaly pattern is replaced by an NAO anomaly pattern, to measure the resemblance of a particular circulation pattern to the specified NAO anomaly pattern:

\[
B = \frac{\langle \psi_{\text{NAO}} \cdot \psi_d \rangle}{\langle \psi_{\text{NAO}} \cdot \psi_{\text{NAO}} \rangle},
\]

where \( \psi_{\text{NAO}} \) is the specified streamfunction NAO anomaly (e.g., EOF1 for this study) and \( \psi_d = \Psi - \Psi_c \) is the daily streamfunction anomaly field over the climatological mean \( \Psi_c \). The angle brackets denote the Euclidean inner product on a sphere, integrated over height.

\[
\langle \mathbf{x}, \mathbf{y} \rangle = \int \int \mathbf{x} \cdot \mathbf{y} \, dV,
\]

where \( V \) represents the integration over the whole atmosphere. A circulation pattern with \( B \) larger (smaller) than some positive (negative) value is defined as a positive (negative) NAO regime. Note that a larger-amplitude positive (negative) \( B \) corresponds to a more-pronounced positive (negative) NAO flow.

b. Optimal perturbations driving NAO

The optimal perturbations that drive the NAO onset can be defined as a type of CNOP that is superimposed on a zonal flow over the Atlantic region and causes the flow to acquire the largest change toward the NAO anomaly pattern. Then, the nonlinear optimization problem can be constructed according to the above NAO index. If we let \( \mathbf{q}_0 \) denote the initial potential vorticity of the reference state, \( \mathbf{Q}_T = \mathbf{M}_T(\mathbf{q}_0) \) is the solution of the T21L3 model at time \( T \), where \( \mathbf{M}_T \) represents the nonlinear propagator. Then, the conditional nonlinear optimal perturbation triggering the NAO− onset (CNOP-Ne) is the initial perturbation \( \mathbf{q}_{0p}^s \) that satisfies the initial constraint condition \( ||\mathbf{q}_{0p}|| = \sigma \) and makes the objective function \( J \) acquire the minimum:

\[
J(\mathbf{q}_{0p}^s) = \min_{||\mathbf{q}_0|| = \sigma} J(\mathbf{q}_0);
\]

whereas the conditional nonlinear optimal perturbation triggering the NAO+ onset (CNOP-Po) is the initial perturbation \( \mathbf{q}_{0p}^s \) that maximizes the same objective function under the same initial constraint condition:

\[
J(\mathbf{q}_{0p}^s) = \max_{||\mathbf{q}_0|| = \sigma} J(\mathbf{q}_0),
\]

where \( J \) is defined as the difference of the NAO indices between the perturbed basic state and the reference zonal flow:

\[
J(\mathbf{q}_0) = \Delta B = \frac{\langle \mathbf{F}[\mathbf{M}_T(\mathbf{Q}_0 + \mathbf{q}_0)] - \Psi_c \cdot \psi_{\text{NAO}} \rangle}{\langle \psi_{\text{NAO}} \cdot \psi_{\text{NAO}} \rangle} - \frac{\langle \mathbf{F}[\mathbf{M}_T(\mathbf{Q}_0)] - \Psi_c \cdot \psi_{\text{NAO}} \rangle}{\langle \psi_{\text{NAO}} \cdot \psi_{\text{NAO}} \rangle} = \frac{\langle \mathbf{F}[\mathbf{M}_T(\mathbf{Q}_0 + \mathbf{q}_0) - \mathbf{M}_T(\mathbf{Q}_0)] \cdot \psi_{\text{NAO}} \rangle}{\langle \psi_{\text{NAO}} \cdot \psi_{\text{NAO}} \rangle},
\]

where \( \mathbf{F} \) is an operator that transforms the potential vorticity of the basic state to the streamfunction field [see Eq. (2)], and \( \mathbf{q}_0 \) is a randomly assigned initial potential vorticity perturbation that satisfies \( ||\mathbf{q}_0|| \leq \sigma \). A presumed positive constant \( \sigma \) represents an upper bound of the initial perturbation magnitude.
Fig. 3. (a)–(c) The geopotential height of CNOP-Ne (gpm) triggering the negative NAO phase and (d)–(f) its nonlinear evolution at day 5 for an optimization time of 5 days at (top) 200, (middle) 500, and (bottom) 800 hPa.
A numerical method is adopted to solve the above two nonlinear optimization problems, and the optimization algorithm of spectral projected gradient 2 (SPG2) is used (Birgin et al. 2000), which is able to obtain the smallest value (a local or global minimum) of a function with several variables that are subject to box or ball constraints. For the above maximum problem, it is considered to seek the minimum of a new objective function that equals the negative of the original objective function with the same constraint condition. To solve these optimization problems by using SPG2, the objective function and its gradient with respect to the initial condition should be provided. The detailed process for deducing the gradient of the objective function is described by Jiang and Wang (2010), which needs the adjoint version of the T21L3 model. For the problem to be solved here, approximately 200 iterations are generally regarded as the stopping condition for convergence. We have tried many different initial perturbations, and only the global minimum is found.

4. Onset of an eddy-driven NAO event

In this section, we explore the NAO onset by solving the above nonlinear optimization problems. The winter climatological flow is specified as the reference state.
The initial constraint condition $\sigma = 4.0 \times 10^5 \text{ m}^2 \text{s}^{-1}$ is chosen so that the amplitude of the initial geopotential height perturbation at 500 hPa is within 20 gpm. The results with an optimization time of 5 and 8 days are presented respectively.

**a. Onset of an NAO$^-$ event**

1) **OPTIMAL PRECURSOR TRIGGERING THE NAO$^-$ EVENT**

The perturbations optimally triggering the NAO$^-$ onset with an optimization time of 5 days are obtained numerically. The CNOP-Ne and its nonlinear evolution at three levels at day 5 are shown in Fig. 3. It can be seen that CNOP-Ne is mainly located over North America with a westward tilt with height of the geopotential height isolines, on the northward flanks of the Atlantic jet. With time it propagates downstream and grows through the upscale energy transfer. At the optimization time, it develops into an NAO-like pattern, with one strong positive center over Greenland and another weak negative center over the midlatitude North Atlantic.

With the extension of the optimization time, what will happen about the optimal precursors? Fig. 4 presents the CNOP-Ne with an optimization time of 8 days and...
its nonlinear evolution at 500 hPa at day 8. Similarly, CNOP-Ne is mainly located over the North America and the northwestern Pacific, slightly more upstream than that with an optimization time of 5 days. With time it propagates downstream and grows in amplitude. At the optimization time, it develops into an NAO$^+$ pattern that is similar to that observed in Feldstein (2003).

2) LINKAGE WITH JET DISPLACEMENT AND WAVE BREAKING

To see the jet variations, the time mean of westerly anomalies at 500 hPa during the optimization time, which are calculated from the geopotential height fields of CNOP-Ne with an optimization time of 8 days and its nonlinear evolutions, are shown in Fig. 5a. It is shown that there exists a low-over-high zonal wind anomaly in the midlatitude North Atlantic. It is natural to say that the westerly jet anomaly exhibits an equatorward shift during an NAO$^+$ event, which is consistent with other studies (Luo et al. 2007b; Woollings et al. 2010).

To aid the visualization of wave breaking, the latitudinal gradient of the potential vorticity (PV) for the total and planetary-scale part of the perturbed basic state by CNOP-Ne are calculated respectively. The planetary-scale field is obtained by Fourier analysis,
FIG. 8. As in Fig. 3, but for CNOP-Po with an optimization time of 8 days.
which consists of zonal wavenumber 0–4. The wave breaking is usually identified by a sign reversal of the latitudinal gradient of the PV (Benedict et al. 2004; Luo et al. 2008), in which the planetary-scale part (remainder) is defined as the planetary (synoptic) wave breaking. Figure 6 presents the temporal evolution of the perturbed basic state by CNOP-Ne with an optimization time of 8 days and the latitudinal gradient of its planetary-scale PV. It is found that the initial weak ridge over the North Atlantic Ocean surrounded by two troughs at its two sides is intensified by the poleward intrusion of warm air and equatorward advection of cold air, finally forming a negative-phase NAO, which resembles to the schematic depictions in Benedict et al. (2004). This process also bears a striking resemblance to an omega-type blocking (Berggren et al. 1949; Shutts 1983). Besides, we can find that the planetary wave breaking is observed to be weak at the beginning, then grows over the North Atlantic, which is evident around Baffin Bay at days 6 and 8. This conclusion is consistent with the findings by Luo et al. (2008), who indicated that the planetary wave breaking is in phase with the development of the NAO$^-$ event. Similarly, the synoptic wave breaking can be seen in Fig. 7. At the beginning the synoptic wave breaking is weak. When the NAO$^-$ further grows, the synoptic wave breaking seems to be stronger with time. Also note that cyclonic wave breaking at the western side of the ridge is evident along a northwest–southeast axis at days 6 and 8, and meanwhile weak anticyclonic wave breaking can be seen at the eastern side of the ridge.
b. Onset of an \(\text{NAO}^+\) event

1) Optimal Precursor Triggering the \(\text{NAO}^+\) Event

Similarly, the perturbations optimally triggering the \(\text{NAO}^+\) onset with an optimization time of 8 days are obtained numerically. The CNOP-Po and its nonlinear evolution at three levels at day 8 are shown in Fig. 8. It can be seen that the CNOP-Po is mainly located in the northwestern Pacific Ocean and north-central area of North America with a baroclinic vertical structure. Comparing CNOP-Po (Fig. 8b) with CNOP-Ne at 500 hPa with the same optimization time (Fig. 4a), it seems that CNOP-Po is more upstream. This is consistent with the results obtained in Feldstein (2003), which showed that the growth of the negative NAO anomalies appears to be primarily in situ while the positive phase contains wave precursors in the Pacific domain. This has also been confirmed by the results of Benedict et al. (2004). With time, it propagates downstream and develops into a structure distributed over almost the whole zonal direction in the Northern Hemisphere mid- to high latitudes. However, a strong negative center over southern Greenland is evident at the optimization time, with three weak positive centers in its western, southern, and eastern directions, respectively.

2) Linkage with Jet Displacement and Wave Breaking

The time mean of westerly anomalies at 500 hPa during the optimization time, which are calculated from

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**Fig. 10.** As in Fig. 7, but for CNOP-Po.
FIG. 11. The time evolution of (a) $E(J_{P1})$ and (b) $E(J_{P2})$ (m$^2$ s$^{-2}$) for CNOP-Ne with an optimization time of 5 days at 200 hPa.
the geopotential height fields of CNOP-Ne with an optimization time of 8 days and its nonlinear evolutions, are shown in Fig. 5b. For the positive-phase NAO, a high-over-low zonal wind anomaly exists in the midlatitude North Atlantic, which means that the westerly jet anomaly exhibits a poleward shift during a positive-phase NAO event. This further reveals that the north–south variability of the zonal mean westerly anomaly is a well-known characteristic of the NAO, confirming the finding of Luo et al. (2007b) and Vallis and Gerber (2008).

Similarly, the temporal evolution of the perturbed basic states by CNOP-Po with an optimization time of 8 days and the planetary and synoptic wave breaking are shown in Figs. 9 and 10, respectively. It is found that the initial weak ridge over the North Atlantic Ocean is weakened by the equatorward advection of cold air. At the same time, a large-scale high is established over the subtropical Atlantic. Therefore, a positive-phase NAO forms, which also resembles to those in Benedict et al. (2004) and Luo et al. (2008). Besides, from Fig. 9 it is found that the occurrence of the planetary wave breaking along the western coast of Europe into the North Atlantic Ocean is also in phase with the development of the NAO+ event. Figure 10 shows that the climatological flow perturbed by such kind of CNOP-Po involves two branches of anticyclonic wave breaking along a northeast–southwest axis. With time, the wave breaking seems to be stronger and propagates eastward. Finally, one branch extends from southern Greenland along the eastern coast of North America in the southwest direction, and the other branch extends from Europe through the midlatitude and subtropical North Atlantic in the southwest direction, eventually forming an NAO+ pattern.

In general, the above model results show that the CNOPs do not only produce NAO-like anomalies, but the basic states perturbed by them also mimic the key wave-breaking features. In addition, the north–south variability of the zonal mean westerly anomaly can also be revealed. Certainly, CNOP-Ne evolving on the climatological flow can develop into an isolated NAO− event. In contrast, CNOP-Po based on the climatological flow do not only evolve into an NAO+ event but also into large-scale perturbations outside the North Atlantic Ocean at the optimization time.

5. Possible physical mechanism related to the NAO onset

To explore the possible mechanism related to the NAO onset, we use the equation for quasi-geostrophic potential vorticity perturbation of the model, which is shown in Eq. (3). It can be seen that the evolution of the perturbation is governed by three advection terms and one dissipation component. As we know, the dissipation term always displays an anomaly pattern that is opposite to the anomaly of the potential vorticity perturbation. Therefore, much more attention should be paid to the self-interaction of perturbations and perturbation/basic-state interaction terms. To observe the effect of these two terms on large-scale NAO pattern, we focus on the planetary-scale projections of the self-interaction of perturbations $J_{P1}$ and perturbation/basic-state interaction $J_{P2}$, respectively. More specifically, we calculate the streamfunction tendencies of these two terms $E(J_{P1})$ and $E(J_{P2})$ and their projections onto the typical NAO pattern separately, which can be referenced to Feldstein (2003).

$$J_{P1} = -J(q_{p}, q_{i})_{P}, \quad (10)$$

$$J_{P2} = -J(q_{p}, q_{i})_{P} - J(q_{i}, Q_{i})_{P}, \quad (11)$$

where $E$ is a linear operator that transforms the potential vorticity perturbation in Eq. (4) into the streamfunction perturbation. The subscript $P$ represents the planetary-scale part, which consists of the zonal wavenumber 0–4. Note that the perturbation $q_{p}(q_{i})$ includes both the synoptic-scale and planetary-scale waves.

Figure 11 shows the time evolution of $E(J_{P1})$ and $E(J_{P2})$ for CNOP-Ne with an optimization time of 5 days at 200 hPa. It was shown that the $E(J_{P2})$ field evolves into a high/low dipole pattern over the North Atlantic Ocean, especially after day 3, in which the magnitude of the northern positive anomaly center is slightly stronger than that of the southern negative anomaly one. The $E(J_{P1})$ field also presents a high/low pattern over this
Fig. 13. As in Fig. 11, but with an optimization time of 8 days.
area, which contributes to the formation of $\text{NAO}^-$ pattern. The above results can also be clearly illustrated from Fig. 12, which shows that the projection of streamfunction tendency terms of $\mathbf{E}(J_{p1})$ (the nonlinear term) and $\mathbf{E}(J_{p2})$ (the linear term) onto the typical NAO anomaly pattern with time. It is found that the self-interaction of CNOP-Ne and CNOP-Ne/basic-state interaction both promote the development of the $\text{NAO}^-$ dipole anomaly. However, the linear term induced by CNOP-Ne/basic-state interaction plays a more important role than the contribution of the nonlinear term induced by the self-interaction of CNOP-Ne, especially after day 2. Certainly, we cannot omit the role played by the nonlinear process.

Figure 13 shows the time evolution of $\mathbf{E}(J_{p1})$ and $\mathbf{E}(J_{p2})$ for CNOP-Ne with an optimization time of 8 days at 200 hPa. Combined with Fig. 14, which shows the projection of streamfunction tendency terms of $\mathbf{E}(J_{p1})$ and $\mathbf{E}(J_{p2})$ onto the typical NAO anomaly pattern with time, we find that the $\mathbf{E}(J_{p2})$ field evolves into a high-over-low pattern over the North Atlantic Ocean at day 5. Afterward, the northern positive anomaly becomes stronger, which contributes to the formation of the negative NAO phase. The $\mathbf{E}(J_{p1})$ also plays a positive role in the NAO$^-$ event but with a weaker impact. That is to say, both the self-interaction of perturbations and perturbation/basic-state interaction promote the development of the negative-phase NAO event.

The time evolution of $\mathbf{E}(J_{p1})$ and $\mathbf{E}(J_{p2})$ for CNOP-Po with an optimization time of 5 days at 200 hPa is shown in Fig. 15. The $\mathbf{E}(J_{p2})$ field has clearly a low–high pattern over the North Atlantic Ocean, whereas the $\mathbf{E}(J_{p1})$ field presents a high–low pattern over this area. However, the $\mathbf{E}(J_{p2})$ field determines the phase of the NAO event. The projections of these two terms onto the typical NAO anomaly in Fig. 16 also illustrates that the perturbation/basic-state interaction promotes the development of the NAO$^+$ event, whereas the self-interaction of perturbations prohibits the formation of the NAO$^+$ event. Similar results can be found in Fig. 17 for the optimization time of 8 days. In conclusion, the perturbation/basic-state interaction plays a more important role than the self-interaction of perturbations in promoting the formation of the positive-phase NAO event.

And here, to further better illustrate the role played by the nonlinearity, we calculate the linear evolution of CNOPs by integrating Eq. (3) in which the nonlinear term is set to zero [viz., $J(\psi, q) = 0$]. Moreover, we calculate the NAO index of the linear and nonlinear evolution of CNOP at the optimization time of 8 days, respectively. It is found that the difference of NAO indices between the nonlinear and linear evolutions of CNOP-Ne at day 8 is 0.3, whereas the difference of NAO indices between the nonlinear and linear evolutions of CNOP-Po at day 8 is only 0.17. That is to say, the nonlinear process induced by the self-interaction of perturbations may play a more important role in the NAO$^-$ onset than that in the NAO$^+$ onset.

Figure 18 shows the linear evolution of CNOP-Ne and CNOP-Po at the optimization time of 8 days at 500 hPa. From Fig. 18a, it can be clearly seen that in the linear regime, the CNOP-Ne develops into a wave-train structure concentrated over the North Atlantic and over Europe with a positive anomaly located over southern Greenland. Figure 18b illustrates that CNOP-Po evolves into a pattern distributed over almost the whole Northern Hemisphere mid- to high latitudes, in which a strong negative center located over southern Greenland accompanied by three weak positive centers to its west, east, and south directions. Comparing Fig. 18b with Fig. 8e, we find that the difference between them over the North Atlantic mainly lies in the amplitude of the negative and positive centers, whereas for Fig. 18a and Fig. 4b, it is the difference of the patterns. The above results also reveal the fact that the role played by nonlinear process in the onset of the negative-phase NAO is stronger than that in the onset of the positive-phase NAO. More precisely, the nonlinear term in the NAO$^-$ onset determines the low-high dipole mode of NAO, whereas the nonlinear term in the NAO$^+$ onset appears to modulate the structure of perturbation to have a dipole mode over the North Atlantic, and at the same time, it causes this NAO anomaly to become zonally extended. This can be explained in terms of a change in the strength of the mean westerly wind in the mid- to high latitudes associated with the phase of the NAO event over the North Atlantic. For the NAO$^-$ (NAO$^+$) phase, the mean westerly jet in the mid- to high latitudes of north of 40$^\circ$N is weakened (strengthened) owing to the equatorward (poleward) shift of the jet core accompanying the establishment of an NAO$^-$ (NAO$^+$) event. In this case, it is likely that during the NAO$^-$ (NAO$^+$) life cycle the nonlinear advection
FIG. 15. As in Fig. 11, but for CNOP-Po.
term becomes more (less) important because the mean wind advection term becomes relatively weaker (stronger) in the mid- to high latitudes.

6. Summary and conclusions

The onset of the North Atlantic Oscillation (NAO) is explored by using the conditional nonlinear optimal perturbation (CNOP) method from the synoptic view assuming that NAO is an initial-value problem. A triangular T21, three-level, quasi-geostrophic global spectral model and its adjoint version are used.

Considering the three-dimensional winter climatological flow as a basic state, two kinds of CNOPs triggering the negative and positive phases of the NAO anomaly are found, which are both located over the northward flanks of the climatological Atlantic jet with a westward tilt with height. However, the CNOP triggering the NAO$^-$ event is more upstream than that triggering the NAO$^+$ event with the same optimization time, which is consistent with the findings by Feldstein (2003). With time they propagate downstream and grow in amplitude. At the optimization time, they develop into NAO anomaly patterns, with one center over Greenland and another center over the midlatitude North Atlantic. Note that, for the positive-phase NAO there are other large-scale perturbations outside the North Atlantic at the optimization time, which may partly explain why the amplitude of the NAO$^+$ event is weaker than that of the NAO$^-$ event with the same initial constraint condition and optimization time interval.

By exploring the total field triggered by CNOPs, we find that the climatological flow triggered by the CNOP driving the NAO$^-$ onset involves cyclonic wave breaking along a northwest–southeast axis and weak anticyclonic wave breaking to its east, occurring on the poleward side of the climatological jet. The climatological flow

![Fig. 16. As in Fig. 12, but for CNOP-Po.](image)

![Fig. 17. As in Fig. 12, but for CNOP-Po with an optimization time of 8 days.](image)

![Fig. 18. The linear evolution of the CNOPs geopotential height (gpm) at the optimization time of 8 days at 500 hPa: (a) CNOP-Ne and (b) CNOP-Po.](image)
perturbed by the CNOP driving the NAO+ onset involves anticyclonic wave breaking along a northeast–southwest axis. This wave breaking may be due to the fact that the optimal precursors present baroclinic vertical structures, but evolve into the equivalent barotropic NAO structures at the optimization time, which is consistent with Benedict et al. (2004). In addition, from the distribution and evolution of the sign reversal of the latitudinal potential vorticity gradient, it reveals that during the NAO growth the synoptic and planetary wave breaking is both in phase with the development of the NAO− (NAO+) pattern from weak to strong.

A low-over-high zonal wind anomaly exists in the midlatitude of the North Atlantic Ocean for the negative-phase NAO, whereas a high-over-low zonal wind anomaly exists in that area for the positive-phase NAO. This is also consistent with other research (Luo et al. 2007b; Vallis and Gerber 2008; Woollings et al. 2010), which indicates that the north–south variability of the zonal mean westerly anomaly is a well-known characteristic of the NAO.

Analysis reveals that in the onset of the two NAO phases, the intensity of the nonlinearity induced by the self-interaction of perturbations is different in the evolution of perturbations. The role played by the nonlinear process in the onset of the NAO− event is stronger than that in the NAO+ event, which can be revealed from a comparison between linear and nonlinear evolutions of two kinds of conditional nonlinear optimal perturbations. The potential vorticity advection terms induced by the perturbation/basic-state interaction and perturbation self-interaction both determine the negative phase of NAO, whereas the nonlinear process in the positive phase of NAO appears to modulate the structure of the perturbation to have a dipole mode over the North Atlantic, and at the same time, it causes this NAO anomaly to become zonally extended.

The above calculation reveals that NAO is indeed a nonlinear initial-value problem, and CNOP method is a useful tool to identify the dynamics of its onset. It should be pointed out that all of the conclusions presented here are based on an assumption that the winter climatological flow is considered as a basic state in our simple dynamic model. It is known that Franzke et al. (2004) also considered the NAO as an initial-value problem by using a multilevel primitive equation model and revealed that their experiments do not only produce NAO-like anomalies, but they also mimic the key wave-breaking features based on the winter climatological flow with a suitable initial perturbation. This is also uncovered in our experiments. The difference is that we found the spatial pattern of the optimal precursors triggering the NAO onset. In addition, the different role played by the nonlinear process can be well revealed in different NAO phases by using our method. In the next step, we plan to identify precursors that drive the transitions between NAO regimes with the CNOP method.

Acknowledgments. We are grateful to Fabio D’Andrea for providing a code of the T21 QG model and information on its performance and also Prof. Steven Feldstein for his beneficial discussions. In addition, we thank two anonymous reviewers for their helpful comments on this paper. Our study is supported by the National Key Basic Research and Development (973) Project (Grant 2012CB417204) and the National Natural Science Foundation of China (Grants 41230420 and 40905023).

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