A microphysical parameterization for shallow cumulus and boundary layer stratocumulus clouds has been developed. Similar to the Khairoutdinov and Kogan parameterization for stratocumulus clouds, the new parameterization is based on an explicit microphysical large-eddy simulation (LES) model as a data source and benchmark for comparison. The predictions of the bulk model using the new parameterization were tested in simulations of shallow cumulus and boundary layer stratocumulus clouds; in both cases the new parameterization matched the predictions of the explicit microphysics LES quite accurately. These results show the importance of the choice of the dataset in parameterization development and the need for it to be balanced by realistic dynamic conditions. The strong sensitivity to representation of rain evaporation is also demonstrated. Accurate formulation of this process, tuned for the case of cumulus convection, has substantially improved precision of rain production.

1. Introduction

The parameterization of clouds in numerical models depends among other things on the model’s grid size. The most accurate parameterization is possible in large-eddy simulation (LES) models that employ high spatial resolution and, therefore, are capable of accurate description of turbulent dynamics. In particular, finescale resolution of individual updrafts/downdrafts allows accurate calculation of local supersaturation and, therefore, a physically grounded representation of nucleation, drop condensational growth, and evaporation, in addition to coagulation and gravitational fallout (sedimentation). These microphysical processes can be formulated in LES models in two ways. In the first approach, referred to as explicit microphysics, cloud drop size distributions (DSD) are described by many size categories and evolve in unconstrained manner according to dynamic and microphysical processes. The computationally less expensive approach is to predict several moments of DSD rather than DSD itself. These moments represent bulk cloud characteristics—for example, liquid water content or cloud drop concentration. Bulk microphysical parameterizations, being relatively simple and computationally efficient, are extensively used in LES models, as well as in mesoscale and even some GCM models. Currently many bulk parameterizations of warm-rain cloud microphysics (Tripoli and Cotton 1980; Beheng 1994; Khairoutdinov and Kogan 2000; Cohard and Pinty 2000; Seifert and Beheng 2001; Liu and Daum 2004), as well as more general liquid- and ice-phase microphysics (Lin et al. 1983; Ferrier 1994; Ferrier et al. 1995; Morrison et al. 2005; Milbrandt and Yau 2005a,b; Song and Zhang 2011), have been proposed and implemented in models of different scales.

The majority of bulk parameterizations are based on partial-moment approach where cloud liquid water is divided into two categories: cloud and rainwater (Kessler 1969). The amount of cloud water can be diagnosed from temperature and water vapor content, assuming that all excess water vapor above the saturation value instantly converts into the cloud water. The initial raindrops form as a result of the autoconversion process. Further growth of the raindrops occurs by the accretion of cloud water droplets. Kessler’s original formulation assumed that the raindrop population followed the Marshall–Palmer drop size distribution (Marshall and Palmer 1948) and considered only one moment of the DSD: liquid water content. Current bulk parameterizations use two- and three-moment formulations and allow more accurate descriptions of microphysical conversion and sedimentation rates.
The threshold radius between the cloud drops and raindrops is cloud-type dependent and not well defined. For example, Berry and Reinhardt (1974) assumed it to be about 50 µm for convective-type clouds, while Cohard and Pinty (2000) defined it at 40 µm. In marine stratocumulus, where precipitation is mostly in the form of drizzle, the threshold has been found to be at about 25 µm (e.g., Khairoutdinov and Kogan 2000). Instead of prescribing the threshold radius an empirical constant, Liu et al. (2004) derived an analytical expression for the threshold radius as a function of the cloud liquid water content and the droplet number concentration.

Different parameterization methods exhibit significant variation in conversion and sedimentation rates (e.g., Wood 2005; Wood et al. 2002). Among the reasons is the use of different datasets to derive a parameterization. As a parameterization is to be employed in simulations of cloud systems under a range of parameters corresponding to a specific distribution of updrafts/downdrafts, it is important to have parameterized conversion rates also derived based on a similar realistic distribution of dynamical parameters. The latter, in turn, will produce a cloud drop spectra dataset weighted according to the distribution of dynamics—that is, a dynamically balanced dataset. Ideally one would want to have this drop spectra dataset obtained in a 3D observational study of a cloud system at various stages of its evolution. However, DSD measurements are currently limited to a single path of an airplane, and, therefore, constitute just a small sample of the whole dataset. Currently the only practical way to obtain the whole dataset is to use an explicit microphysics LES model that represents realistic dynamics most closely. Such an LES-based approach was used by Khairoutdinov and Kogan (2000, hereafter KK2000) for development of a microphysics parameterization for marine boundary layer (BL) stratocumulus clouds (often referred to as KK parameterization).

Our goal is to expand the KK parameterization to represent shallow trade wind cumulus clouds. These clouds are dynamically more vigorous and exhibit a much broader range of cloud properties, relative to marine stratocumulus. Because of their prevalence over much of the subtropical and tropical oceans, shallow trade wind cumuli normally lies well below the 0°C isotherm [in RICO, maximum cloud-top height corresponded to about 4 km (Rauber et al. 2007; Snodgrass et al. 2009)], their properties can be adequately represented by simulations with a warm-rain microphysics model. The measurements from the RICO project supply the initial and forcing conditions for our LES simulations, from which the drop spectra dataset needed for parameterization development is obtained.

The paper is organized as follows. Section 2 describes the model and dataset. Section 3 describes the formulation of the bulk microphysics scheme. Section 4 compares LES simulations obtained from the bulk and explicit microphysics models, and section 5 summarizes the results.

2. Model and dataset

The simulations were based on a new version of the Cooperative Institute for Mesoscale Meteorological Studies (CIMMS) LES (Kogan et al. 1995; Khairoutdinov and Kogan 1999), called the System for Atmospheric Modeling with Explicit Microphysics (SAMEX) (Kogan et al. 2012). The dynamic core consists of the System for Atmospheric Modeling (SAM), developed by M. Khairoutdinov (Khairoutdinov and Randall 2003). SAM is based on nonhydrostatic, anelastic dynamics and uses a monotonic, positive-definite advection scheme for scalars (Smolarkiewicz and Grabowski 1990). The explicit microphysics formulation of SAMEX uses 34 cloud drop bins ranging in size from 1 µm up to 2 mm and 19 cloud condensation nuclei (CCN) bins which range from aerosol dry radius size of 0.076–5.5 µm. SAMEX has been tested on marine stratocumulus case from the Atlantic Stratocumulus Experiment (ASTEX) (Albrecht et al. 1995), and was employed in the Global Energy and Water Cycle Experiment (GEWEX) Cloud System Study (GCSS) model intercomparison of marine trade cumulus based on RICO data (vanZanten et al. 2011).

Initial profiles and simulation forcings corresponding to average conditions over the 3-week period from 16 December 2004 to 8 January 2005 of the RICO field campaign were used for initialization and forcing of the LES simulation [described in detail in the LES model intercomparison study vanZanten et al. (2011)]. The CCN used in the simulation has the total CCN concentration of 104.4 cm$^{-3}$, which is similar to the concentrations measured by the Passive Cavity Aerosol Spectrometer Probe (PCASP) during the RICO flights RF11 on 7 January 2005 (J. Hudson 2010, personal communication). The model horizontal and vertical grid spacing for the simulation was 100 and 40 m,
respectively, with a total of $256 \times 256 \times 100$ grid points ($25.6 \times 25.6 \times 4 \text{ km}^3$ domain). The dynamical time step was 2 s. The cloud drop spectra were selected from nearly 500 clouds developed during the 24-h simulation and provided a dataset of about 52 000 drop spectra obtained using the cloud water mixing ratio $q_c > 0.1 \text{ g m}^{-3}$ threshold.

The main differences in cloud parameters between RICO and ASTEX datasets are illustrated in Fig. 1. Obviously the dynamics of trade wind shallow cumulus clouds is quite different from that of BL stratocumulus. The updrafts in shallow cumulus, not restricted by the strong inversion at the top-of-boundary-layer stratocumulus, are more intense and may result in convective clouds with thickness up to 3 km. The density distribution of trade wind cumulus cloud thickness is shown by the gray bell-shaped curve in the top-left panel in Fig. 1. The average cloud thickness is 1.7 km (note the $x$ axis at the top of the plot) with a standard deviation of 0.7 km. The liquid water content mean value for RICO clouds is $0.71 \pm 0.62 \text{ g m}^{-3}$ compared to the $0.26 \pm 0.16 \text{ g m}^{-3}$ in the ASTEX case. The rainwater for ASTEX clouds is less than $0.2 \text{ g m}^{-3}$ but can reach values up to $1 \text{ g m}^{-3}$ in the RICO case. Correspondingly, trade wind cumulus can produce drop spectra with precipitation rates of up to $100 \text{ mm day}^{-1}$ and radar reflectivities as high as $30 \text{ dBZ}$. As the cloud physics parameterization strongly depends on the dataset, we should expect substantial differences in the parameterized conversion and sedimentation rates.

3. Formulation of the parameterization

The parameterization for cumulus clouds (referred to as Cu parameterization) is based on the same approach
as in KK2000 where it is applied for development of boundary layer stratocumulus parameterization. Following Kessler, the drop size spectrum range is partitioned into two subranges corresponding to cloud droplets and raindrops. To determine the threshold radius between the two water categories, we considered two analyses: 1) the change in drop spectra due to coagulation and 2) the distribution of liquid water content as a function of drop size. As Fig. 2a shows, for drops smaller than certain threshold size coagulation leads to a loss of mass, while for drops larger than the threshold, the mass is gained because of coagulation with smaller drops. KK2000 showed that for marine stratocumulus the threshold drop radius is about 25 \( \mu \text{m} \). For the case of trade wind cumulus convection clouds, the threshold lies in the 35–40-\( \mu \text{m} \) size.

In addition we also analyzed the distribution of water mass over the spectrum. It is generally assumed that spectra with considerable amounts of precipitation exhibit a bimodal distribution. To define the minimum separating the two modes, we computed the percentage of LWC contained in each drop size category for each drop spectrum. The results averaged for the whole dataset are presented in Fig. 2b. One can see that the distribution displays a minimum at the drop radius of 40 \( \mu \text{m} \), which is another indication that for trade wind cumuli this value is reasonable to assume as a threshold between cloud and rainwater.

The parameterization is formulated using the same approach as in KK2000. A set of prognostic microphysical variables includes total CCN count \( n \), \( q_c \), cloud drop concentration \( N_c \), total mean radius of cloud droplets \( R_c \) (defined as mean radius times concentration), rainwater mixing ratio \( q_r \), and raindrop concentration \( N_r \). The corresponding prognostic equations for microphysical variables are as follows:

\[
\frac{\partial n}{\partial t} = \frac{\partial u_i n}{\partial x_i} - \left( \frac{\partial N_c}{\partial t} \right)_{\text{activ}} + \left( \frac{\partial N_c}{\partial t} \right)_{\text{evap}} + \frac{\partial}{\partial x_i} K \frac{\partial n}{\partial x_i}, \quad (1)
\]

\[
\frac{\partial q_c}{\partial t} = -\frac{\partial u_i q_c}{\partial x_i} + \frac{\partial V}{\partial z} N_c + \left( \frac{\partial q_c}{\partial t} \right)_{\text{cond}} - \left( \frac{\partial q_c}{\partial t} \right)_{\text{auto}}
- \left( \frac{\partial q_c}{\partial t} \right)_{\text{accr}} + \frac{\partial}{\partial x_i} K \frac{\partial q_c}{\partial x_i}, \quad (2)
\]

\[
\frac{\partial N_c}{\partial t} = -\frac{\partial u_i N_c}{\partial x_i} + \frac{\partial V}{\partial z} N_c - \left( \frac{\partial N_c}{\partial t} \right)_{\text{activ}} - \left( \frac{\partial N_c}{\partial t} \right)_{\text{evap}}
- \left( \frac{\partial N_c}{\partial t} \right)_{\text{auto}} + \frac{\partial}{\partial x_i} K \frac{\partial N_c}{\partial x_i}, \quad (3)
\]

\[
\frac{\partial R_c}{\partial t} = -\frac{\partial u_i R_c}{\partial x_i} + \frac{\partial V}{\partial z} R_c + \left( \frac{\partial R_c}{\partial t} \right)_{\text{cond}} + \frac{\partial}{\partial x_i} K \frac{\partial R_c}{\partial x_i}, \quad (4)
\]

\[
\frac{\partial q_r}{\partial t} = -\frac{\partial u_i q_r}{\partial x_i} + \frac{\partial V}{\partial z} q_r + \left( \frac{\partial q_r}{\partial t} \right)_{\text{cond}} + \left( \frac{\partial q_r}{\partial t} \right)_{\text{auto}}
+ \left( \frac{\partial q_r}{\partial t} \right)_{\text{accr}} + \frac{\partial}{\partial x_i} K \frac{\partial q_r}{\partial x_i}, \quad (5)
\]

\[
\frac{\partial N_r}{\partial t} = -\frac{\partial u_i N_r}{\partial x_i} + \frac{\partial V}{\partial z} N_r - \left( \frac{\partial N_r}{\partial t} \right)_{\text{evap}} + \left( \frac{\partial N_r}{\partial t} \right)_{\text{auto}}
- \left( \frac{\partial N_r}{\partial t} \right)_{\text{scol}} + \frac{\partial}{\partial x_i} K \frac{\partial N_r}{\partial x_i}. \quad (6)
\]

Here, \( u_i \) is the component of the velocity in the \( x_i \) direction; \( K \) is the turbulence diffusion coefficient. The subscript “cond/evap” refers to the rate of change due to
condensation/evaporation, “activ”—CCN activation, and “auto/accr”—autoconversion and accretion of cloud water by rain. Compared to the system of equations in KK2000, Eq. (6) has the additional term “scol” describing self-collection of raindrops; this term was not included in KK2000 because collection rate between relatively small drizzle drops in boundary layer stratocumulus is negligible. To close the system in Eqs. (1)–(6), the sink/source terms on the right-hand sides are parameterized as a function of the prognostic variables as described below.

a. CCN activation

In the explicit microphysics model the CCN spectrum is explicitly predicted by model equations; as a result, a particular CCN bin is activated when the supersaturation exceeds its critical value defined based on CCN size and chemical composition. In the bulk formulation of microphysics, the total CCN concentration rather than CCN spectrum is predicted. Therefore the drop activation process is formulated using the Twomey’s relationship between the number of activated drops and supersaturation (Twomey 1959):

\[ n = CS^k, \]

where \( S \) is the supersaturation and \( C \) and \( k \) are empirical constants. Let us denote \( S_{\text{thr}} \) as the threshold supersaturation defined as

\[ S_{\text{thr}} = \left( \frac{n + N}{C} \right)^{1/k}, \]

where \( n \) and \( N \) are the current time step values of CCN and cloud drop concentrations. If the supersaturation on the current time step \( S \) exceeds \( S_{\text{thr}} \), then the additional CCN are activated according to the following expression:

\[ \delta N = -\delta n = \max[0, (CS^k - n - N)]. \]

This number is added to \( N \) and subtracted from \( n \). In addition to drop concentration, cloud water content and total mean radius are also modified. We assume that the activated drops have the initial radius \( r_{\text{act}} = 1 \) µm. Therefore, the following expressions from KK2000 are adopted:

\[ \frac{\partial R_c}{\partial t}_{\text{act}} = r_{\text{act}} \frac{\partial N_c}{\partial t}_{\text{act}}, \]

\[ \frac{\partial q_r}{\partial t}_{\text{act}} = \frac{4\pi G(T, P)\rho_w}{\rho_a} S_{\text{act}} \int_0^{r_f} rf(r) \, dr \]

\[ = \frac{4\pi G(T, P)\rho_w}{\rho_a} S_{\text{act}} C_{rr}, \]

where \( f(x, r, t) \) is the drop size distribution function; \( r_f \) is the threshold radius dividing cloud drops and raindrops; \( G(T, P) \) is a coefficient in the simplified form of the drop growth equation, which is a function of temperature \( T \) and pressure \( P \); and \( \rho_w \) and \( \rho_a \) are water and air density, respectively:

\[ \frac{\partial r}{\partial t}_{\text{cond}} = \frac{G(T, P)S}{r}. \]

As in the KK scheme, we assume that drop concentration does not change during condensation or during evaporation as long as liquid water content remains larger than a threshold of 0.01 g m\(^{-3}\). When it falls below this threshold, it is assumed that all drops evaporate instantaneously and the drop concentration is added to the total CCN count. A more accurate and sophisticated treatment of condensation/evaporation process, which allows the entire range of mixing scenarios from homogeneous to extremely inhomogeneous, is considered by Morrison and Grabowski (2008).

Similar to Eq. (12), the rate of change of rainwater mixing ratio due to condensation/evaporation can be written as

\[ \frac{\partial q_r}{\partial t}_{\text{cond}} = \frac{4\pi G(T, P)\rho_w}{\rho_a} S_{\text{act}} \int_0^{r_f} rf(r) \, dr \]

\[ = \frac{4\pi G(T, P)\rho_w}{\rho_a} S_{\text{act}} C_{rr}, \]

where \( C_r \) is the mean geometrical radius of raindrops, which needs to be expressed through prognostic variables \( q_r \) and \( N_r \). The data from the explicit model simulations show that \( C_r \) is rather well correlated with the mean volume radius of raindrops:

\[ C_r = C_{rr} r_{\text{vr}}, \]

where the mean volume radius of raindrops is

\[ r_{\text{vr}} = \left( \frac{4\pi \rho_w}{3\rho_a} \right)^{-1/3} q_r^{1/3} N_r^{-1/3}. \]
In simulations of stratocumulus cloud layers, KK2000 found that for drizzle drops in the \( r_v \) range 25–100 \( \mu m \) \( C_r \) can be approximated by a constant (0.86). Simulations of shallow cumulus clouds show that for raindrops with larger \( r_v \) (40–300-\( \mu m \) range), \( C_r \) is better approximated by a decreasing function of \( r_v \) (Fig. 3):

\[
C_r = 0.45 + 23.0r_v^{-1},
\]

where \( r_v \) is in microns. According to Eq. (17), the values of \( C_r \) decrease from about 0.9 for small drizzle drops to about 0.5 for large raindrops; note that 0.55 is the value of \( C_r \) in case of the Marshall–Palmer distribution of raindrops.

Using Eqs. (15) and (16), the rate of change of rainwater mixing ratio due to condensation/evaporation can be written as

\[
\left( \frac{\partial q_r}{\partial t} \right)_{\text{cond}} = 3C_r G(T, P) S \left( \frac{4\pi \rho_w}{3\rho_d} \right)^{2/3} q_r^{1/3} N_r^{2/3}. \tag{18}
\]

While condensation increases the mass of rainwater, it does not change the raindrop concentration. The latter, however, will change when raindrops evaporate. We assume that the rate of change of raindrop concentration due to evaporation is related to the rate of change of rainwater content as follows:

\[
\left( \frac{\Delta N_r}{N_r} \right)_{\text{evap}} = \left( \frac{\Delta q_r}{q_r} \right)_{\text{evap}}. \tag{19}
\]

The rate of change of the prognostic variable \( R_c \) is given by

\[
\left( \frac{\partial R_c}{\partial t} \right)_{\text{cond}} = G(T, P) S \int_0^\infty \frac{f(r)}{r} dr = G(T, P) S N_c \frac{1}{r}. \tag{20}
\]

To calculate the rate in Eq. (20), we parameterize the mean inverse drop radius following the procedure in KK2000. This parameter is calculated assuming that the cloud drop spectrum can be approximated by the Gamma distribution:

\[
f(r) = \frac{N_c \beta^{\gamma+1}}{\Gamma(\gamma+1)} r^\gamma \exp(-\beta r), \tag{21}
\]

where \( \Gamma(x) \) is the gamma function and \( \gamma \) and \( \beta \) are parameters. It can be shown that for this distribution

\[
\left( \frac{1}{r} \right) = \frac{(\gamma + 1)N_c}{\gamma R_c}, \quad \text{where} \quad \gamma = \frac{5 - 2P + \sqrt{8P + 1}}{2P - 2};
\]

\[
P = \frac{3\rho_g q_c N_c^2}{4\pi \rho_w R_c^3}. \tag{22}
\]

Similar to raindrops, the drop concentration can change during evaporation. Because small cloud drops decrease their size during evaporation rather quickly, we assume, for simplicity, that they evaporate instantaneously, when the cloud liquid water falls below some threshold value, which is set in our model to \( 10^{-5} \) g kg\(^{-1}\). After evaporation, the drop concentration is added to the total CCN count.

c. Sedimentation

The fall velocities of variables describing the cloud drop portion of the spectrum (\( N_c, Q_c, R_c \)) do not normally exceed a few centimeters per second and can be safely set to zero without the appreciable loss of accuracy. The mean terminal velocities for the rain liquid water and drop concentration are

\[
V_{N_r} = \sum_{i, r_i > r} \frac{v(r_i)N_i}{N_i} \quad \text{and} \quad V_{q_r} = \sum_{i, r_i > r} \frac{v(r_i)r_i^3N_i}{r_i^3N_i}, \tag{23}
\]

where \( N_i \) is the drop concentration in the \( i \)th spectrum bin, \( v(r) \) the terminal velocity of a drop of radius \( r \), and summation is done over all spectrum bins larger than the threshold bin. Applying the least squares method, the following approximations are obtained:

\[
V_{N_r} = 0.385 r_{vr} + 5.76 \quad \text{and} \quad V_{q_r} = 2.4 r_{vr} - 62.0, \tag{24}
\]

FIG. 3. Scatterplots of mean geometrical rain radius \( r_r \) vs the mean volume radius of raindrops \( r_v \).
where mean volume radius of the raindrops is in microns, and the terminal velocities in centimeters per second. Corresponding scatterplots (Fig. 4) demonstrate improved correlation of the new cumulus sedimentation rates compared with the old KK approximations. The latter significantly underestimate sedimentation for $q_r$ and overestimate them for $N_r$ in the range of large values of these parameters—that is, exactly where the bias potentially may have the largest effect on precipitation. The new sedimentation rates, however, also display a large spread of data points. Seeking approximation formulas as a function of two parameters, $q_r$ and $N_r$, instead of one, $r_{VR}$, did not improve the approximation, which indicates the need to explore a three-moment formulation of the sedimentation process (Milbrandt and McTaggart-Cowan 2010; Shipway and Hill 2012).

d. Autoconversion

The formulation of autoconversion of cloud water to rain followed the approach of KK2000. We assume that the rate of change of a prognostic variable due to autoconversion can be expressed in the form

$$\frac{\partial x}{\partial t}_{\text{auto}} = c f a c b.$$

Here $x$ denotes a prognostic variable, such as $q_r$ or $N_r$; $f$ and $c$ denote cloud variables $q_c$ and $N_c$; and $a$, $b$, and $c$ are parameters, which have to be determined from least squares minimization procedure as described by Eqs. (25)–(28) in KK2000. The autoconversion rate in the explicit microphysics model is calculated as a total mass change per unit time because of coagulation of droplets with each mass smaller than, but with a coalesced mass larger than the threshold bin mass. Applying the least squares method, the autoconversion rate is approximated as

$$\left( \frac{\partial q_r}{\partial t} \right)_{\text{auto}} = 7.98 \times 10^{10} \times q_r^{4.22} N_c^{-3.01},$$

where $q_r$ and $q_r$ are in kilograms per kilogram and $N_c$ is per centimeters cubed.

The source of the raindrop concentration due to autoconversion is defined by assuming that all new rain
drops have a radius equal to the threshold value $r_t = 40 \mu m$:

$$\left( \frac{\partial N_c}{\partial t} \right)_{auto} = \left( \frac{\partial q_r}{\partial t} \right)_{auto} \left( \frac{4 \pi \rho_w r_t^3}{\rho_a} \right).$$  \hspace{1cm} (27)

The absolute values of the exponent coefficients in Eq. (26) ($a = 4.22$, $b = -3.01$) are larger than in the case of KK ($a = 2.47$, $b = -1.79$). This indicates that in the case of cumulus clouds the dependence of autoconversion rates on $q_c$ and $N_c$ is much stronger. The scatterplot in the top panel of Fig. 5 shows a rather good correlation between the autoconversion rates given by Eq. (26) and the ones calculated from the explicit model. For comparison, the bottom panel shows autoconversion rates using the KK parameterization. Clearly, the autoconversion rates using the new Cu parameterization are better correlated with explicit model rates compared to KK autoconversion rates; the latter significantly overestimate the small and underestimate the large rates. The correlation coefficient $R^2$ of the Cu autoconversion rates is about 0.8 despite the large spread of data points, which, however, decreases as autoconversion rates increase. The large spread of data points indicates a significant degree of freedom left by definition of autoconversion rates only by two parameters, $q_c$ and $N_c$. As was already indicated by researchers in previous studies, the accuracy of approximation can be improved by introducing the third moment in the formulation—for example, the mean cloud drop radius or the dispersion width (e.g., Beheng 1994; Liu and Daum 2004).

e. Accretion

The accretion rate is defined as a total mass increase per unit time corresponding to collisions between a drop smaller and a drop larger than the threshold 40-μm bin. It is evaluated by solving the stochastic coagulation equation for all spectra from the database obtained in RICO simulation. As in KK2000, we assume that the accretion rate depends only on cloud and rainwater content: $q_c$ and $q_r$. The regression analysis similar to one used for the autoconversion rate yields

$$\left( \frac{\partial q_r}{\partial t} \right)_{accr} = 85.5 q_c^{1.05} q_r^{0.98}.$$  \hspace{1cm} (28)

Equation (28) shows near-linear dependence on cloud and rain liquid water ratio; this, as was noted by Beheng (1994), reflects the relevance of the continuing growth model for formulation of the accretion process. A near-perfect agreement between the accretion rate in Eq. (28) and the one calculated from the explicit microphysical model (correlation coefficient of 0.99) is demonstrated by the scatterplots in Fig. 6. Noticeable also is the very narrow spread of the accretion rate data points, indicating that it is rather well defined by two variables, $q_c$ and $q_r$. The difference between the accuracy of definition of accretion versus autoconversion rates stems from the shape of the collision-efficiency curves. For collector drops larger than 30–40 μm, the collision-efficiency coefficients vary less and depend weaker on the size ratio of colliding drops; therefore, the shape of the small drop spectrum is less important and its characterization by two variables is adequate. The KK accretion rates also display a narrow spread of data points; however, they significantly underestimate small to moderate accretion rates (bottom panel in Fig. 6). Note also that the range of accretion rates is about an order of magnitude larger than the autoconversion rate range, which points to the larger role of accretion in the overall rain production in cumulus clouds.
The accretion reduces the cloud water content, but it also affects the cloud drop concentration. The corresponding sink term is approximated assuming that all collected cloud drops have the size of the mean volume radius:

\[
\left( \frac{\partial N_r}{\partial t} \right)_{\text{accr}} = \left( \frac{\partial q_r}{\partial t} \right)_{\text{accr}} \frac{4\pi\rho_w}{\rho_a} r_v^{3/2}
\]

(29)

### f. Self-collection

The rate of self-collection was evaluated only for raindrops, as the collisions between small cloud droplets are less frequent and can be neglected. The self-collection rates for raindrops are quite well approximated by two variables, \( q_r \) and \( N_r \). The scatter of data points is small and is decreasing with the increase of self-collection rates (Fig. 7):

\[
\left( \frac{\partial N_r}{\partial t} \right)_{\text{scol}} = 205 q_r^{1.55} N_r^{0.60}.
\]

(30)

### 4. Testing of parameterization

#### a. Trade wind shallow cumulus case

The performance of parameterization in the case of trade wind cumulus clouds was studied in a simulation initialized with RICO data using the thermodynamic profiles and forcings from a configuration employed in the GCSS model intercomparison (vanZanten et al. 2011). The numerical setup of the SAMEX simulation is described in section 2.

Figures 8 and 9 show the horizontal mean vertical profiles of cloud thermodynamic and microphysical parameters averaged over the last 2 h of the 24-h simulation. The thermodynamic structure [profiles of vertical velocity variance, turbulent kinetic energy (TKE), buoyancy flux, and relative humidity] in the simulation using the new Cu parameterization closely follows that of the explicit model. For comparison we also show profiles produced in simulations using two different implementations of the KK parameterization. The first (referred to as KKSC) is the original stratocumulus parameterization; it employs conversion and sedimentation rates, as well as the formulation of the condensation/evaporation process following KK2000; that is, the coefficient \( C_r \) in Eq. (15) is set to a constant value of 0.86. The KKCU uses the same conversion and sedimentation rates as in KKSC, but, similar to Cu parameterization, defines \( C_r \) according to Eq. (17) as a decreasing function of \( r_v \). Thus, contrasting Cu and KKCU simulations shows the effect of using the new Cu conversion and sedimentation rates compared to the old KK rates.
Both KK simulations produce clouds that are more energetic near the tops than clouds in the Exp and Cu simulation. This is reflected by maxima in profiles of velocity variance, TKE, buoyancy, and moisture (Fig. 8). Microphysical profiles shown in Fig. 9 show that KK parameterization is much less efficient in converting cloud water to rainwater, resulting in a smaller amount of precipitation. The surface precipitation in KKSC is only about 10%–15% of the amount produced in the benchmark explicit simulation. By using in KKCU a formulation of condensation/evaporation tuned for cumulus clouds, the precipitation amount is increased more than twofold; however, the KK conversion and sedimentation rates still produce only about one-third of the explicit model precipitation.

The formation of precipitation in the new Cu parameterization, on the other hand, is more satisfactory, as evidenced by the vertical profiles of precipitation flux during the last 2 h, as well as accumulated precipitation (Fig. 9). The most conspicuous deficiency of all bulk formulations, including the new Cu version, is the underestimate in how quickly precipitation rate diminishes as it falls. The reason may be the difference in treatment of rain evaporation in the bulk and explicit models; however, large errors in the bulk formulation of sedimentation evidenced by the wide spread of data points in Fig. 4 may also be a contributing factor. One of the ways to increase the accuracy of the sedimentation process is to introduce additional prognostic variable for prediction of the rain category—for example, radar reflectivity as in three-moment schemes of Milbrandt and Yau (2005b) and Kogan and Belochitski (2012).

The difference in the precipitation process is also illustrated in Fig. 10, which shows time evolution of accumulated precipitation at the surface for all four simulations. The use of the unaltered KK parameterization (KKSC) results in very low precipitation formation process. When the parameter \( C_r \) is defined according to Eq. (17) instead of a constant \( C_r = 0.86 \) as assumed in
KK2000, then the KK conversion and sedimentation rates result in substantially increased precipitation production. The new Cu conversion/sedimentation rates are more accurate compared to KK rates, and the amount of precipitation over the course of the 24-h simulation resembles most closely the explicit model precipitation formation process. One notable difference is that in the explicit model clouds are producing precipitation in more discreet events, with periods of intense precipitation of about a half hour followed by relatively weak precipitation. In the bulk model the precipitation is more gradual during the course of simulation.

b. Marine boundary layer stratocumulus

ASTEX case

As the Cu parameterization is developed based on the shallow cumulus cloud system dataset, it is interesting to test its applicability to the case of boundary layer stratocumulus. For this we compared simulations of the stratocumulus boundary cloud layer using the Cu and KK parameterization. The latter was rather extensively tested against the explicit model simulations in KK2000 and, therefore, can be used as a benchmark.

The stratocumulus cloud layer case was based on thermodynamic sounding from the ASTEX A209 flight data; its complete description is given in Khairoutdinov and Kogan (1999; KK2000). The SAMEX model horizontal and vertical grid spacing for this simulation was 30 and 12.5 m, respectively, with a total of $256 \times 256 \times 100$ grid points.

Figure 11 shows vertical profiles of boundary layer dynamic and microphysical parameters averaged over the last hour of the 6-h simulation. The profiles of TKE, vertical velocity variance, cloud and rainwater, precipitation flux, and the accumulated precipitation agree remarkably well with the KK parameterization. They show a familiar bimodal stratification in TKE and vertical velocity variance characteristic for decoupled drizzling cloud boundary layer. The agreement between the two cases in prediction of microphysical characteristics is also very good. The rain flux rate, as well as profile of the accumulated precipitation, is nearly identical in both cases.

The time series of the TKE and liquid water path (LWP) averaged over the entire integration domain are very close to each other (Fig. 12). The slight oscillations in the TKE with a period of roughly 2 h are a known artifact of a limited domain in numerical simulations. The phase shift in TKE fluctuations is due to random small perturbations during the initial spinup of the simulation; the average TKE, however, evolves similarly in both cases. The small differences in TKE do have some effect on LWP; however, the effect on accumulated precipitation is insignificant. We can conclude that the Cu parameterization may be used for boundary layer stratocumulus with the accuracy similar to the KK parameterization.

5. Conclusions

We present a microphysical parameterization for cumulus clouds (Cu parameterization). Like the KK parameterization that Khairoutdinov and Kogan (2000) developed for marine boundary layer stratocumulus clouds, it partitions liquid water into a precipitating (rainwater) and a nonprecipitating part (cloud water) and solves equations for six prognostic variables, which include mixing ratio and number concentration of cloud drops and raindrops, total mean radius of cloud drops, as well as concentration of CCN ($q_c$, $q_r$, $N_c$, $N_r$, $R_c$, $n_{ccn}$). These two parameterizations were developed using data from simulations of different cloud types: trade wind shallow Cu versus boundary layer stratocumulus; the former are characterized by a wider range of cloud parameters. The choice of a dataset in parameterization development is important, as signified by the rather large differences in the performance of Cu and KK parameterizations. The dataset should have a parameter range wide enough to cover all possible parameter combinations which may be encountered during a simulation. However, also important for deriving accurate regression parameterizations is to have a correct weight of parameter combinations that realistically represent the balance between heavy, moderate, and weak rain-producing spectra. The most accurate practical tool for obtaining such a dataset is an LES explicit microphysics model. It provides cloud spectra formed under the 3D
realistic thermodynamic conditions, and, therefore, contains dynamically balanced spectra corresponding to realistic distributions of cloud/rain liquid water content, drop concentrations, and precipitation intensities. This dynamically balanced dataset is difficult to reproduce by artificially solving the coagulation equation with a prescribed set of initial conditions.

By applying regression analysis to the LES-derived dataset, we obtained approximations of the conversion and sedimentation rates valid for the case of cumulus convective clouds. The approximations of accretion and self-collection rates were especially precise, while the autoconversion and sedimentation rates showed significant spread of data points. Fortunately, the largest errors for autoconversion and sedimentation rates are when they are small and, therefore, have a lesser effect on precipitation formation. It is speculated that further improvement of autoconversion and sedimentation rates may be possible by applying three-parameter approximations in the framework of three-moment microphysical parameterization schemes (Milbrandt and Yau 2005a; Kogan and Belochitski 2012).

The new bulk microphysics parameterization was incorporated into the dynamic framework of the SAMEX LES model and tested against simulations using the explicit microphysics version of the model. The simulations were based on observations of a shallow cumulus convective cloud system during RICO field campaign and a drizzling stratocumulus cloud boundary layer during ASTEX. The key thermodynamic and microphysical parameters of the cumulus convective system in RICO simulation matched very well the parameters of the benchmark explicit microphysics simulation. The new formulation of autoconversion and accretion rates, and accounting for self-collection process resulted in a significantly enhanced rain production compared to the KK formulation. Although our effort focused mainly on deriving new conversion and sedimentation rates for cumulus convective clouds, we also found strong sensitivity to representation of rain evaporation. The accurate formulation of this process tuned for the case of cumulus convection has substantially improved precision of rain production, even when using old KK conversion/sedimentation rates.

FIG. 11. Vertical profiles of horizontally averaged parameters 6 h into simulation based on the ASTEX A209 case. The solid (dashed) lines correspond to simulations using Cu (KK) parameterization; (a) turbulent kinetic energy, (b) vertical velocity variance, (c) cloud liquid water (black) and rainwater (gray), (d) precipitation flux, and (e) accumulated precipitation.
In a simulation of stratocumulus boundary layer cloud case based on ASTEX data, the new parameterization was shown to match quite accurately the performance of the KK parameterization; thus, the Cu parameterization can be applied for a range of parameters covering both stratocumulus boundary layer and trade wind shallow cumulus convective clouds. The performance of the Cu parameterization in the case of stronger convection (e.g., congestus cumulus clouds) or mixed-phase clouds has not been tested and needs a separate investigation.

As in KK, the new approach includes the total CCN concentration as a separate prognostic variable and predicts the cloud drop concentration based on supersaturation in a manner similar to the explicit microphysical model. As supersaturation is a strong function of local thermodynamic variables, the proposed bulk approach requires detailed representation of individual updrafts; therefore, it is applicable first and foremost in LES and cloud-resolving models. The generalization of the parameterization for meso- and large-scale models, strictly speaking, requires supplementing the conversion and sedimentation rates with probability distribution functions of cloud and rain parameters (Pincus and Klein 2000). This task is beyond the scope of the study and will be the focus of a separate investigation. However, even for cloud-resolving models the formulation of supersaturation can be simplified similar to the approach described in Mechem and Kogan (2003). In contrast to the detailed activation scheme used in KK2000 and in this study, condensation in Mechem and Kogan is based on simple saturation adjustment. The CCN activation process is parameterized by the empirical formulae of Martin et al. (1994) and O’Dowd et al. (1997) that relate bulk CCN and cloud droplet concentrations. Under such formulation, the \( R_c \) variable in not needed, and the total number of predictive variables is reduced to four. Such a much more computationally efficient formulation, however, lacks the ability to accurately predict activation of cloud droplets.

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