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ABSTRACT

The mesoscale kinetic energy (KE) spectra of the mei-yu front system are investigated through idealized numerical simulations. In the mature stage, the upper-tropospheric KE spectrum resembles a $k^{-3}$ power law for wavelengths between 1000 and 400 km and shallows to a slope of approximately $-5/3$ at smaller wavelengths. A similar behavior can be observed in the lower stratosphere. At both levels, the rotational KE spectrum shallows nearly to the same extent as the divergent KE spectrum at smaller wavelengths, accounting for the transition in the total KE spectrum. About 12 h after the latent heating is turned off, the mesoscale KE spectra hardly show the distinct spectral transition, especially in the upper troposphere.

The spectral KE budget for various height ranges is analyzed and compared. In the upper troposphere, the mesoscale KE is deposited through the buoyancy flux and removed by the advective nonlinearity and vertical pressure flux divergence. The buoyancy flux spectrum in the mature phase has a peak at scales of around 300 km and a plateau throughout the mesoscale, which suggests a significant injection of KE in the mesoscale. The negative contribution of the advective nonlinearity demonstrates that to some extent the mesoscale KE derives from a nonlinear upscale cascade, with the buoyancy-produced energy source located at the lower end of mesoscale spectrum. In the lower stratosphere, the mesoscale KE is deposited through the advective nonlinearity and vertical pressure flux divergence and removed by the buoyancy flux. This suggests that the lower-stratospheric KE spectrum is influenced by both the downscale energy cascade and vertically propagating IGWs.

1. Introduction

Observational studies (Nastrom and Gage 1985; Lindborg 1999; Lindborg and Cho 2001) have shown that kinetic energy (KE) spectrum as a function of horizontal wavenumber $k_h$ displays two regions with distinct slopes. At synoptic scales, where the flow is predominantly quasigeostrophic (QG), KE spectrum follows power-law dependence with an approximate $-3$ slope. In the mesoscale range, which is wavenumbers corresponding to wavelengths below around 500 km, the KE spectrum shallows to a slope of approximately $-5/3$. The $k^{-3}$ dependence over the synoptic scales can be well explained by the theory of QG turbulence (Charney 1971; Boer and Shepherd 1983), but the dynamics of the mesoscale portion with $k_h^{-5/3}$ dependence remain under discussion (Lindborg 2005, 2007; Tulloch and Smith 2009).

Mainly two hypotheses have been put forward to explain the $-5/3$ slope of mesoscale KE spectrum: the upscale energy cascade (Gage 1979; Lilly 1983; Falkovich 1992) based on two-dimensional turbulence (Kraichnan 1967) and the downscale energy cascade based on three-dimensional turbulence (Kolmogorov 1941). The 2D turbulence argument for $-5/3$ slope may be problematic owing to some limitations: First, it requires a large energy source at small scales to ensure the formation of upscale energy flux in the mesoscale. However, the nature of such energy source is obscure. Second, the inverse
energy cascade from small scales relies on the assumption of strong stratification (Lilly 1983), but numerical simulations of stratified turbulence have shown that strong stratification alone does favor a downscale cascade (Riley and deBruynKops 2003; Lindborg 2006; Brethouwer et al. 2007). Moreover, a variety of other downscale cascade mechanisms have been identified, including nonlinearly interacting inertia–gravity waves (IGWs; Dewan 1979; VanZandt 1982) and quasigeostrophic turbulence (Tung and Orlando 2003). Despite most recent evidence pointing toward a direct cascade, the nature of the mesoscale cascade is still controversial. Lilly (1983, 1989) suggested that this small-scale source could be generated by decaying convective clouds and thunderstorm anvil outflows. Ricard et al. (2013) argued that diurnal variations of KE suggested an injection of energy at small scales and the mesoscale KE amplitude strongly depended on the presence of the physical processes.

However, each of the above theories, despite having very different mechanisms, implicitly regards the mesoscale as a range where there is neither a significant source nor sink of KE. Thus, the relevance of these theories to the complex atmosphere is clearly in question. In fact, some typical weather systems associated with mesoscale phenomena, such as baroclinic wave system and mei-yu front system, can implement the increase or decrease of KE through the strong convection and other physical processes.

Since the simulations of idealized baroclinic waves were known to provide convenient framework for the study of fundamental mesoscale dynamics, Waite and Snyder (2009, hereafter WS2009) investigated the mesoscale KE spectrum of an idealized dry baroclinic wave life cycle. They found that the lower-stratospheric spectrum was governed not only by a downscale energy cascade but also by the vertical pressure flux divergence associated with vertically propagating IGWs. Subsequently, they further investigated the role of moist processes in the development of the upper-tropospheric KE spectrum and concluded that the moist processes significantly injected KE into the mesoscale with a peak at scales of around 800 km (Waite and Snyder 2013, hereafter WS2013).

Different from baroclinic waves, the mei-yu front system varies both in structural features and development mechanisms. Idealized baroclinic waves can resolve the sloping frontal structures (Snyder et al. 1993), but most of the fronts exhibited in the simulations are close to the polar front characterized by strong thermal gradient, while the mei-yu front system, which significantly affects the precipitation in East Asia during summer, differs a lot from the typical midlatitude fronts. Chen and Chang (1980) found that mei-yu fronts at different sections have different structures and weather patterns. Its eastern section near Japan is similar to baroclinic wave and presents strong thermal gradient as well as a westward tilt toward the upper-level cold center, whereas the western section over southern China is a shallow with strong moisture gradient and low-level horizontal wind shear, but weak temperature gradient. As the mesoscale convective systems (MCSs) develop and propagate along the mei-yu front zone, moist processes may play a more important role in the evolution of the mei-yu front system than baroclinic wave system. On the other hand, because of the different scales along and across front, the mei-yu front system cannot be well explained by the turbulence theories based on isotropic assumption. Therefore, the mesoscale features of the mei-yu front and the corresponding KE spectrum intrigue us.

Does the mei-yu front system develop a mesoscale $-\frac{5}{3}$ KE spectrum? What is the role of the moist processes in the formation and maintenance of such mesoscale KE spectrum? And what are the differences between the quasi-two-dimensional front system and the baroclinic wave life cycle? In this paper, we take a closer look at these questions by investigating the mesoscale spectra and spectral energy budget of an idealized mei-yu front system simulated by the Advanced Research Weather Research and Forecasting model (ARW). Section 2 describes the numerical model and simulation setup. Section 3 presents the results of the control simulation, which successfully reproduced characteristics analogous to that documented by the observations and previous studies. Section 4 presents the mesoscale KE spectra characteristic of the mei-yu front system, and discusses the sensitivity of the mesoscale KE spectra to latent heating. A diagnostic analysis of spectral KE budget is given in section 5. Discussion and conclusion are presented in section 6.

2. Model description and numerical setup

a. The model

The three-dimensional (3D) model used in this study is the ARW, version 3.2 (Skamarock et al. 2008), with some modifications in the governing equations. We assume that the mei-yu front is oriented in the east–west direction and is forced by the meridional geostrophic wind $V_g$, which is only a function of $z$. On the basis of this hypothesis, key modifications in governing equations include (i) the use of Coriolis forcing calculated from meridional wind perturbation in $x$-direction momentum equation and (ii) the introduction of the large-scale
zonal geostrophic advections imposed for the potential temperature in Eq. (4) and water vapor in Eq. (5). The modified governing equations are similar to those described in 3D nonhydrostatic modeling study of precipitation core–gap structure along cold front by Kawashima (2007). Thus, the modified governing equations, expressed in height coordinates, can be written as follows:

\[
\begin{align*}
\frac{du}{dt} &= -\frac{1}{\rho} \frac{\partial p'}{\partial x} + f(v - V_s) + D_u, \\
\frac{dv}{dt} &= -\frac{1}{\rho} \frac{\partial p'}{\partial y} - fu + D_v, \\
\frac{dw}{dt} &= -\frac{1}{\rho} \frac{\partial p'}{\partial z} - g \frac{\rho'}{\rho} + D_w, \\
\frac{d\theta}{dt} &= S_\theta - u \left( \frac{\partial \theta}{\partial x} \right)_{LS} + D_\theta, \\
\frac{dq_v}{dt} &= S_{q_v} - u \left( \frac{\partial q_v}{\partial x} \right)_{LS} + D_{q_v}, \\
\frac{dq_m}{dt} &= S_{q_m} + D_{q_m}, \\
\frac{d\rho_d}{dt} + \rho_d \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) &= 0.
\end{align*}
\]

In the equations above, \(u\) and \(v\) are horizontal velocities along zonal and meridional directions, respectively; \(w\) is the vertical velocity; \(\theta\) is the potential temperature; \(d/dt = \partial_t + u \partial_x + v \partial_y + w \partial_z\) is the material derivative; \(\rho_d\) is the density of the dry air; \(p\) is the pressure; \(g\) is the gravitational acceleration; \(f\) is the Coriolis parameter; \(D_\phi\) denotes the dissipation of \(\phi\) (with \(\phi\) a placeholder for the fields \(u, v, \theta\), etc.); \(q_v\) is the water vapor mixing ratio; \(q_m = q_o, q_e, q_t, \ldots\) represent the mixing ratios for cloud, ice, graupel, rainwater, etc. and \(\rho_d = \rho_d(1 + q_e + q_t + q_r + q_i + \ldots\) is the total mass density. The terms \(S_\theta, S_{q_v}\), and \(S_{q_m}\) represent the diabatic contribution to \(d\theta/dt, dq_v/dt\), and \(dq_m/dt\), respectively. Primes indicate deviations from the time-invariant hydrostatically balanced dry reference states \(\bar{\rho}(z)\) and \(\bar{\rho}_d(z)\).

The terms \((\partial \theta/\partial x)_{LS}\) and \((\partial q_v/\partial x)_{LS}\) are the large-scale alongfront gradients of the geostrophic potential temperature and water vapor, respectively. These terms balance the vertical shear of \(V_s\) according to the thermal wind relation, which represent the large-scale baroclinicity. The detailed procedure to specify these terms is the same as that described by Crook (1987); that is, \((\partial \theta/\partial x)_{LS} = (f/\gamma)(\partial V_s/\partial z)\) and \((\partial q_v/\partial x)_{LS} = \bar{q}_v(z)(L_o \bar{\rho}_d \bar{R}_o \bar{T}_0)/(\partial \theta/\partial x)_{LS}\), where \(\gamma = 0.65\) is the reference potential temperature, \(R_o\) is the gas constant for water vapor, \(L_o\) is the latent heating of vaporization, \(\bar{\rho}_d(z)\) and \(\bar{\rho}_d = (\bar{p}/\bar{\rho}_0)^{R_o/\gamma}p_o\) are the water vapor mixing ratio and the Enxer pressure of base state, and \(T_0 = 273.15\) K is the 0°C temperature.

The model domain is an \(f\) plane of dimensions \(L_x = 1000\) km (alongfront), \(L_y = 2000\) km (across front), and \(L_z = 20\) km, with \(f = 1.0 \times 10^{-4}\) s\(^{-1}\). The lateral boundaries are periodic in \(x\) and open in \(y\). The surface is flat at \(z = 0\). The horizontal grid spacing is \(\Delta x = \Delta y = 5\) km and time step is \(\Delta t = 20\) s. The terrain-following hydrostatic pressure \(\eta\) is used as the vertical coordinate, and the vertical grid employs 41 layers, with a nearly constant vertical spacing of \(\Delta z \approx 500\) m.

In the present idealized simulations, advection is handled with the fifth-order horizontal and third-order vertical upward-biased scheme. Explicit sixth-order numerical diffusion (Knievel et al. 2007) is used in conjunction with the second-order horizontal and vertical diffusion, as it is sufficient to suppress noise at poorly resolved scales. Thus, the diffusion terms \(D_\phi\) are given as \(D_\phi = [K_h(\partial^2 \phi/\partial x^2) + K_v(\partial^2 \phi/\partial z^2)]\phi + 2^{-6}(2\Delta t)^{-1}\beta(\partial^2 \phi/\partial x^2 + \partial^2 \phi/\partial z^2)\phi\), where the eddy viscosities \(K_h\) and \(K_v\) are given by the turbulent kinetic energy closure method, and \(\beta = 0.12\) is a parameter that specifies the amount of diffusion applied in one time step to features with wavelengths of 2\(\Delta\) (Skamarock et al. 2008). The diffusion acts on all the three components of winds, on potential temperature, and on all moisture variables, etc. Additionally, the eddy viscosities used for scalars \(\theta, q_v, q_m\) are divided by a turbulent Prandtl number \(Pr = 0.5\). To minimize gravity wave reflection off the upper boundary, Rayleigh damping is applied to the vertical velocity in the upper 5 km of the model domain (Klemp et al. 2008). As the main focus of this study is to examine the effect of moist microphysical processes on the mesoscale energy cascade, no convective parameterization, surface fluxes, radiation scheme, or boundary layer models are utilized. The Morrison et al. two-moment scheme (Morrison et al. 2009) is used for microphysics parameterization, in which six species of water are included: vapor, cloud droplets, cloud ice, rain, snow, and graupel/hail. The simulations are integrated for 48 h. Fields output are available every 1 h.

b. Initialization

We initialize our simulations with a zonal uniform surface jet following Orlanski and Ross (1977) and Ross and Orlanski (1978), and the initial conditions in the \(y-z\) plane are described as below:

(i) The meridional geostrophic wind is given as

\[V_s(z) = 3.0\, \text{m s}^{-1} - (10.0\, \text{m s}^{-1}) \tanh(z/7000\, \text{m}).\]
As shown in Fig. 1a, the vertical shear of $V_x$ is approximately $-1.4 \times 10^{-3} \text{s}^{-1}$ at low levels (0–3 km), which gives the large-scale zonal gradients of the potential temperature $[\text{K}(100 \text{km})]^{-1}$ and the mixing ratio of water vapor $(\partial q_{w}/\partial x)_L$ of $-0.4 \text{K}(100 \text{km})^{-1}$ and $-0.4 \text{g kg}^{-1}(100 \text{km})^{-1}$, respectively.

(ii) The analytical form of the initial zonal wind $u_0(y, z)$ is given by

$$u_0(y, z) = -\frac{L_y - y}{2y_0} u_m \{\text{tanh}[\beta(L_y - y + \alpha z - y_0)]\},$$

(9)

where $y$ is positive-definite in the model domain and $u_m$ is a jet-intensity parameter. For all the simulations described herein, the parameters in Eq. (9) are set as $u_m = 20 \text{m s}^{-1}$, $y_0 = 800 \text{km}$, $\beta = (50 \text{km})^{-1}$, and $\alpha = 100$. Initial profile of zonal wind $u_0(y, z)$ is shown in Fig. 2a.

(iii) The initial potential temperature and the mixing ratio of water vapor fields are assumed to be of the form

$$\theta_0(y, z) = \bar{\theta}(z) + \theta_0'(y, z) \quad \text{and} \quad q_{e0}(y, z) = \bar{q}_{e}(z) + q_{e0}'(y, z),$$

(10)

(11)

where $\bar{\theta}(z)$ and $\bar{q}_{e}(z)$ are the horizontally homogeneous base sounding, while $\theta_0'(y, z)$ and $q_{e0}'(y, z)$ are perturbations from them, respectively. For the fidelity of the simulated mei-yu front system, the base sounding (Fig. 1b, solid line) onto which the perturbations are added refers to the real sounding profile (Fig. 1b, dotted line) averaged over the area $31^\circ$–$32^\circ$N, $118^\circ$–$121^\circ$E at 0000 UTC 12 July 2010. For the base state in all simulations to be described, the potential temperature at the surface is $300 \text{K}$ and the lapse rate of potential temperature (Fig. 1b, thin solid line) is $4.2 \text{K km}^{-1}$ for $z < 4 \text{km}$, $4 \text{K km}^{-1}$ for $4 \leq z < 14 \text{km}$, and $18 \text{K km}^{-1}$ for $z \geq 14 \text{km}$; the initial relative humidity (Fig. 1b, thick solid line) is given by $\bar{\text{RH}}(z) = 1 - 0.8(z/16 \text{km})^2$ for $0 < z \leq 16 \text{km}$ and $\bar{\text{RH}}(z) = 20\%$ for $z > 16 \text{km}$. To avoid supersaturation in the initialization process, the relative humidity at low levels is limited to a maximum value of 80%; namely, $\bar{\text{RH}}(z) = \min[80\%, \bar{\text{RH}}(z)]$.

Solving the coupled relation for the hydrostatic balance and the equation of state by using the surface pressure $p_0 = 1000 \text{hPa}$ as the lower boundary condition, we can obtain the dry hydrostatic pressure $\bar{p}(z)$ for the base state. Having computed $\bar{p}(z)$, we can further calculate the vapor mixing ratio $\bar{q}_{e}(z)$ for the base state. The initial perturbation potential temperature $\theta_0'(y, z)$ and perturbation vapor mixing ratio $q_{e0}'(y, z)$ are specified so that the horizontal gradients $\partial \theta_0'/\partial y$ and $\partial q_{e0}'/\partial y$ balance the vertical shear of the initial zonal wind $u_0$ according to the moist thermal wind relation:

$$\frac{\partial \theta_0 (1 + 0.61q_{e0})}{\partial y} = -\frac{f \theta_0 (1 + 0.61q_{e0})}{g} \frac{\partial u_0}{\partial z},$$

or (12)

$$\frac{\partial \theta_0'}{\partial y} + \frac{0.61 \theta_0}{1 + 0.61q_{e0}} \frac{\partial q_{e0}}{\partial y} = -\frac{f \theta_0}{g} \frac{\partial u_0}{\partial z}.$$  

(13)

In Eq. (13), it is very clear that this thermal wind relation is maintained not only by the cross-front gradients of
potential temperature but also by the cross-front gradients of water vapor, in contrast to the initialization of cold front in Crook (1987) and Kawashima (2007), where the cross-front water vapor gradient was neglected in the geostrophic balance and the initial relative humidity was set to a uniform value. The synoptic structure of the mei-yu front is different than that of midlatitude polar fronts. The former is generally characterized by a strong moisture gradient and high equivalent potential temperature gradient and not by a strong temperature gradient (Ninomiya 1984, 2000; Ding 1992). Thus, the cross-front water vapor gradient should not be neglected when studying the mei-yu front system. To pinpoint the effect of potential temperature and water vapor, respectively, we can decompose the Eq. (13) into two parts:

\[
\frac{\partial \theta_0}{\partial y} = \gamma \left( -\frac{f \theta_0}{g} \right) \quad \text{and} \quad \frac{\partial q_{0y}}{\partial y} \approx (1 - \gamma) \frac{1 + 0.61 \bar{q}_e(z)}{0.61 \bar{q}(z)} \left( -\frac{f \theta_0}{g} \right),
\]

(14)

where \( \gamma \) is the weight coefficient in the moist thermal wind relation, owing to the potential temperature gradient. Taking the general character of the mei-yu front into account, we set \( \gamma = 0.5 \) in the present paper, and the sensitivity to different \( \gamma \) will be considered in a future paper. Then, \( \theta_0(y, z) \) and \( q_{0y}(y, z) \) are computed by integrating Eqs. (14) and (15) along the \( y \) direction with \( \theta_0(0, z) = 0 \) K and \( q_{0y}(0, z) = 0 \) g kg\(^{-1} \) as the southern boundary condition. The initial profiles of potential temperature and vapor mixing ratio are shown in Fig. 2b.

c. Experimental design

In this paper, a series of numerical experiments are conducted to explore the characteristics of mesoscale kinetic energy spectra of the mei-yu front system and to investigate the role of latent heating/cooling in the formation and maintenance of such KE spectra. The experiment with moist microphysical processes and concomitant latent heating is considered as the control run, referred to as CNTL. The experiment without moist microphysical processes is referred to as NOMP. The experiments HEAT90% and HEAT80% are similar to CNTL, except for the latent heating rate being respectively reduced to 90% and 80% of the magnitude in CNTL. The experiment as in CNTL, except latent heating/cooling feedback turned off at the integral time \( t = 24 \) h is denoted as HEAT24hoff.

3. Overview of the simulated mei-yu front system

a. Evolution of the convection intensity

The convection intensity of the simulated mei-yu front system is illustrated by the time series of mass-weighted average vertical kinetic energy (VKE; Fig. 3),

\[
VKE = \int_{z=0}^{z=15} \int \int \left( \frac{1}{2} \rho w^2 \right) dx dy dz / \int_{z=0}^{z=15} \int \rho dx dy dz, \quad (16)
\]

where the integration is performed from the surface (\( z = 0 \) km) to \( z = 15 \) km over the model horizontal domain. For CNTL, the convection of the simulated mei-yu front system oscillates with a maximum value of 0.029 m s\(^{-1} \) at \( t = 16 \) h. Taking the value of time-averaged VKE over 12 \( \leq t \leq 48 \) h as a reference state, the total integration time can be divided into three distinct stages: (i) 15–25 h, the more stronger vertical convection; (ii) 26–38 h, relatively weak vertical convection; and (iii) 39–48 h, the vertical convection strengthening again. Also plotted in Fig. 3 is the VKE of experiments HEAT90% and HEAT80%. Compared with CNTL, when the latent heating rate is reduced by 10% or 20%, the maximum convection intensity is reduced to 34% or 7%. Therefore,
the latent heating plays a vital role in the development of convection.

b. Frontogenesis and evolution of frontal structure

To verify the reasonable description of the mei-yu front structure in CNTL, the cross-front characteristics are revealed by the physical fields averaged in the x-direction (along the front zone) over $0 \leq x \leq 1000$ km (zonal mean).

Figure 4a presents the time series of the maximum values of zonal mean precipitation intensity at the surface and the cross-front virtual potential temperature gradients in the lower and upper troposphere, respectively.

The cross-front virtual temperature gradient changes little and no precipitation occurs during the first 13 h, which demonstrates that the initial conditions constructed above are relatively balanced. A rapid frontogenesis onsets at $t = 13$ h, and the maximum values of cross-front virtual potential temperature gradient in the lower troposphere increases from 0.06 K km$^{-1}$ at $t = 13$ h to 0.2 K km$^{-1}$ at $t = 16$ h, while that in the upper troposphere increases from 0.01 to 0.23 K km$^{-1}$, which is consistent with the evolution of the precipitation intensity. A positive feedback process occurs between the mei-yu frontogenesis and cumulus latent heating, in which the front provides low-level convergence and helps organize the convection while latent heating by cumuli further enhances the frontogenetic process (Chen et al. 2003). Figure 4b shows vertical profiles of apparent heat source $S_u$, averaged in the main convective region. From $t = 12$ to $t = 14$ h, the convection is shallow and positive heating only occurs in the lower troposphere. From $t = 16$ to $t = 36$ h, when the deep convection forms, positive heating takes place throughout the troposphere and peaks at about 8 km, with a maximum of 8.8 K (6 h)$^{-1}$ at $t = 16$ h. The $S_u$ profiles are consistent with the apparent moisture sink $S_q$ (not shown), and both are typical for deep convection (Johnson and Ciesielski 2002).

The vertical cross sections of zonal mean virtual potential temperature perturbation, cross-front wind perturbation vectors ($u', w$) (here, $u' = u - V_g$), and cloud water mixing ratio are shown in Fig. 5. At $t = 12$ h, the atmosphere in the warm sector first becomes saturated and convective-scale updraft, which develops from the ascending branch of the circulation forced by the mei-yu front, appears at a height of about 1.5 km (Fig. 5a). Then
the updraft results in the convective precipitation and the attendant latent heating release. The leading edge of the front moves southward approximately 150 km and a 100-km-wide cloud area forms by $t = 26$ h (Fig. 5b), when the virtual potential temperature perturbation peaks at about 8 km with a maximum value of 5.4 K. High-level warming will cause the elevation of the high-level constant pressure surface and, in turn, lead to the outflow of the air mass there. By $t = 26$ h, divergent flow above the heavy rain area is already clear (Fig. 5b).

A detailed view of the structure of the mei-yu front system is presented in Fig. 6. The formation of warm-core disturbance reinforces the horizontal potential temperature gradient in the upper troposphere, with the positive gradient lying over the southern side of the rain area and the negative over the other side. According to the thermal wind balance, the upper-level easterly jet forms on the southern side of the mei-yu frontal rainband, and the upper-level westerly jet forms above the mei-yu front, which consists of the upper-level divergence flow pattern of the mei-yu front system (Fig. 6b).

c. Characteristics of precipitation

As the prefrontal convection develops, a typical mei-yu frontal rainband forms (Fig. 7) and collocates well with low-level wind convergence region. After $t = 16$ h, the precipitation intensity exceeds 65 mm h$^{-1}$. The movement of the rainband is nearly quasi stationary relative to the simulated mei-yu front. Incidentally, the distribution of precipitation also shows a “core-gap” structure, which orients nearly parallel to the mei-yu front.

As described above, both the structure and evolution of the simulated mei-yu front system are remarkably in
good agreement with the observations and previous studies. Therefore, it can be concluded that the idealized initial conditions are reasonable and the simulated mei-yu front system for CNTL is trusty. Thus, the results from CNTL provide a convenient framework for the study of the mesoscale dynamics of the mei-yu front and hence allow a closer look at the kinetic energy spectra of the mei-yu front system with the high-resolution output of the simulations above.

4. Mesoscale kinetic energy spectra of the mei-yu front system

a. Horizontal kinetic energy spectra

Horizontal KE spectra are computed by using two-dimensional discrete cosine transform (DCT; Denis et al. 2002) of the horizontal velocity $u = (u, v)$ at each vertical height level. Let $\phi(k)$ be the DCT of field $\phi$, where $k = (k_x, k_y)$ is the horizontal wave vector, $k_x = (\pi/\Delta)(m/N_i)$, $k_y = (\pi/\Delta)(n/N_j)$. $m = 0, 1, 2, 3, \ldots$, $N_i - 1$, $n = 0, 1, 2, 3, \ldots$, $N_j - 1$, $N_i = 200$ is the number of zonal grid points, and $N_j = 400$ is the number of meridional grid points. Then horizontal KE spectrum at a given time and vertical level is

$$E(k) = \frac{1}{2} [\hat{u}(k)\hat{u}(k) + \hat{v}(k)\hat{v}(k)],$$

where the dependence on $z$ and $t$ is suppressed for clarity.

The total horizontal wavenumber is defined as

$$k_h = |k| = \sqrt{k_x^2 + k_y^2}.$$

Spectra as a function of $k_h$ are constructed by angular averaging over wavenumber bands $k_h - \Delta k/2 \leq |k| < k_h + \Delta k/2$ on the $k_x - k_y$ plane, with $k_h$ being the central radius of the bands (as in, e.g., WS2009); that is,

$$E(k_h) = \sum_{k_h - \Delta k/2 \leq |k| < k_h + \Delta k/2} E(k)/\Delta k,$$

where $\Delta k = \pi/\Delta(N)$ and $N = \min(N_i, N_j)$. Wavelength $\lambda = 2\pi/k_h$.

As shown in Fig. 4b, the direct forcing from latent heating occurs below the height of 12 km and mainly in the upper troposphere. According to the vertical profile of the latent heating, the total KE spectra, averaged over $z = 0-5$, 5-10, and 12-15 km (lower troposphere, upper troposphere, and lower stratosphere, respectively) are
In the lower troposphere (Fig. 8a), for $t = 12–14$ h, the KE spectrum grows significantly for $k_h > 2\pi \times 10^{-5}$ rad m$^{-1}$, corresponding to wavelength less than 100 km. At $t = 14$ h, the KE spectrum approaches an approximately $-0.6$ slope over the smaller scales ($40 \leq \lambda \leq 100$ km). During the following 2 h ($t = 14–16$ h), the spectral range where the KE grows is extended to $k_h = \pi \times 10^{-5}$ rad m$^{-1}$, corresponding to wavelength 200 km. By $t = 16$ h, a spectrum saturation appears over the smaller scales ($\lambda \leq 40$ km), which belong to the strong dissipation scales. For $t = 16–18$ h, the KE spectrum grows only for wavenumbers $\pi \times 10^{-5} \leq k_h \leq \pi/2 \times 10^{-4}$ rad m$^{-1}$, corresponding to wavelengths $40 \leq \lambda \leq 200$ km. The whole KE spectrum reaches saturation by $t = 26$ h, when the amplitude and distribution of the simulated KE spectrum are consistent with that of the Nastrom and Gage spectrum (Skamarock 2004, his Fig. 1). For $t = 26–38$ h, owing to the strong mei-yu front in lower layers, the saturated KE spectrum in the lower troposphere does not have a constant slope; rather, it develops a distinct peak around $k_h = 3\pi/5 \times 10^{-3}$ rad m$^{-1}$ and approaches a slope that is steeper than $-3$ over the large-scale end of the mesoscale (500 $\leq \lambda \leq 1000$ km) and approximately $-5/3$ at smaller scales ($40 \leq \lambda \leq 200$ km). For $t = 40–48$ h, this distinct peak gradually disappears as the low-level frontal structure weakens.

The development of the KE spectrum in the upper troposphere (Fig. 8b) is similar to that in the lower troposphere during the initial period ($t = 12–14$ h), when the convection excited by mei-yu front is quite shallow and the attendant latent heating is relatively weak. As the convection intensifies, the effect of the latent heating in the upper troposphere becomes more important, which ultimately results in the differences of KE spectra growth between the upper and lower troposphere. Such difference is prominent during $t = 14–16$ h and is characterized by the development of the upper-troposphere KE spectrum quickly extending through the whole wavenumber range. In addition, the saturated KE spectrum in the upper troposphere shows a distinct spectral transition in the mesoscale: the KE spectrum develops an approximately $-3$ slope for $k_h < 1.57 \times 10^{-5}$ rad m$^{-1}$ and $-5/3$ slope for $k_h > 1.57 \times 10^{-5}$ rad m$^{-1}$, where $k_h = 1.57 \times 10^{-5}$ rad m$^{-1}$ corresponds to wavelength 400 km (transition scale).

In the lower stratosphere (Fig. 8c), the KE spectrum grows slowly before the establishment of deep convection, and it subsequently shows a rapid increase similar to that in the upper troposphere. The distinct spectral transition can also be observed. According to Fig. 8, the growth of the KE spectra in the troposphere and lower stratosphere can be plotted in Fig. 8 for $t = 0–48$ h. Because of the explicit sixth-order numerical diffusion of model, the spectra fall off rapidly for $k_h > 1.57 \times 10^{-4}$ rad m$^{-1}$, corresponding to wavelengths less than 40 km. We focus our attention on the range of well-resolved wavelengths larger than 40 km.

First, we analyze the evolution of the KE spectra for CNTL. During the first 2 h, the KE spectra show a significant adjustment on all the levels. This is mainly due to the effects of spinup in early integration. As such an adjustment accomplishes, the KE spectra grow slowly. A new rapid development of KE spectra onsets at $t = 12$ h, when the convection occurs.
divided into three distinct stages: the slow growth, rapid development, and mature stage. Obviously, the growth of the KE spectra is well in accordance with the evolution of convective systems, and the rapid developing stage of the KE spectra is the period when the convection develops and reaches its largest intensity (Figs. 3–5).

The analysis above clearly demonstrates the importance of deep moist convection in establishing the mesoscale KE spectra. Apparently, in the upper troposphere, the KE spectra growth is due to the direct forcing from latent heating release by deep convection (Fig. 4b), while in the lower stratosphere, where no direct forcing from latent heating occurs, it may be due to the vertical pressure flux divergence associated with vertically propagating IGWs (WS2009) excited by deep convection. These two different mechanisms will be further investigated in section 5.

Since the development of KE spectra is well in accordance with that of the convection, the evolution of the simulated mei-yu front system for CNTL can be divided into three distinct stages: stage 1 \((t = 15–25 \text{ h})\), denoted as the early-mature phase, is characterized by the strong vertical convection and rapid growth of the KE spectra; stage 2 \((t = 26–38 \text{ h})\), denoted as the mature phase, is characterized by approximately time-invariant KE spectra and relatively weak vertical convection; and stage 3 \((t = 39–48 \text{ h})\), the vertical convection strengthening again and indistinctive frontal structure in the lower troposphere, denoted as the late-mature phase. In the following section, we will further investigate the KE spectra characteristics in the mature phase of the mei-yu front.

b. Time-averaged mature phase KE spectra characteristics

Figure 9a shows the time-averaged mature-phase KE spectra for CNTL. As mentioned in section 4a, the lower-tropospheric KE spectrum develops a distinct peak around wavelength 330 km, which is consistent with the meridional scale of the mei-yu front system.

In the upper troposphere, the KE spectrum develops a \(-3.5\) slope over the large-scale end of the mesoscale \((400 \leq \lambda \leq 1000 \text{ km})\) and a transition to a shallower \(-1.7\) slope at smaller scales \((40 \leq \lambda \leq 400 \text{ km})\). (Spectral slopes are computed here and elsewhere by a least squares power fit over a given wavelength range.)

The lower-stratospheric spectrum is similar to the upper-tropospheric spectrum except for its larger (smaller) magnitude at larger (smaller) scales. And it approaches a slope that is approximately \(-4.1\) for the wavelengths \(400 \leq \lambda \leq 1000 \text{ km}\) and \(-2.1\) for the smaller scales \(40 \leq \lambda \leq 400 \text{ km}\).

The KE spectra averaged over the same period for NOMP are also plotted in the Fig. 9b. Without any moist physical processes, the spectra in the upper troposphere and lower stratosphere cannot reach the same magnitude as that of CNTL owing to the absence of energy source. Moreover, the spectra in the lower troposphere have a nearly constant slope of \(-3.4\).

c. Rotational and divergent KE spectra

As the horizontal velocity field can be partitioned into rotational and divergent parts, the KE spectrum can be naturally decomposed into horizontally rotational and
divergent modes (ROT and DIV, respectively), which can be given by

\[ E_R(k) = \frac{1}{2} \frac{\zeta(k)^2}{|k|^2} \quad \text{and} \quad E_D(k) = \frac{1}{2} \frac{\delta(k)^2}{|k|^2}, \quad (20) \]

where \( \zeta = \partial u/\partial x - \partial v/\partial y \) and \( \delta = \partial u/\partial x + \partial v/\partial y \). The horizontal wavenumber spectra \( E_R(k_h) \) and \( E_D(k_h) \) are defined analogously to \( E(k_h) \). Figure 10 shows the time-averaged mature-phase rotational and divergent KE spectra, averaged in the vertical over the upper troposphere and lower stratosphere.

In the upper troposphere (Fig. 10a), the divergent KE is an order of magnitude smaller than the rotational KE for wavelengths larger than 500 km. The ratio of divergent KE to rotational KE increases as the wavelength decreases, and the divergent KE spectrum crosses the rotational KE spectrum at \( \lambda ' = 25 \) km, which belongs to the dissipation scales. For \( 40 \lesssim \lambda \lesssim 400 \) km, the divergent KE and the rotational KE have the same order of magnitude, but the former is slightly larger, accounting for the \(-5/3\) spectrum of the total KE. The rotational KE spectrum also possesses a clear transition in the mesoscale and develops a slope of \(-3.4\) over the large end of the mesoscale (\( 400 \lesssim \lambda \lesssim 1000 \) km) and \(-2.2\) at smaller scales (\( 40 \lesssim \lambda \lesssim 400 \) km). The divergent KE spectrum has a slope of \(-2.1\) at smaller scales.

d. Sensitivity to the latent heating

To further investigate the influence of latent heating on the maintenance of mesoscale KE \(-5/3\) spectra, we have performed another experiment configured exactly the same as CNTL, except for turning off the latent heating at \( t = 24 \) h. As shown in Fig. 11, the KE for wavelengths less than the transition scale in both the upper troposphere and lower stratosphere is quickly reduced relative to CNTL after the latent heating is turned off, especially during the following 2 h. After about 12 h, the mesoscale KE spectra do not show the distinct spectral transition any longer and its slope for wavelength less than 400 km reduces to approximately \(-2.3\), especially in the upper troposphere.

To highlight the impact of the latent heating, we focus on the time series of KE in a specific scale range. The wavenumber band of width \( 2\Delta k \) around \( k_h/\Delta k = 10 \) is chosen, which corresponds to wavelength 200 km (Fig. 12). After the latent heating is turned off at \( t = 24 \) h, the KE continuously decreases. The decay is approximately

\[ E_R(k) = \frac{1}{2} \frac{\zeta(k)^2}{|k|^2} \quad \text{and} \quad E_D(k) = \frac{1}{2} \frac{\delta(k)^2}{|k|^2}, \quad (20) \]

On the other hand, in the lower stratosphere (Fig. 10b), the divergence KE spectrum crosses the rotational KE spectrum at wavelength \( \lambda \approx 400 \) km. However, for \( 40 \lesssim \lambda \lesssim 400 \) km, the divergent KE spectrum and the rotational KE spectrum also have the same order of magnitude, but the former is slightly larger, accounting for the \(-5/3\) spectrum of the total KE. The rotational KE spectrum also possesses a clear transition in the mesoscale and develops a slope of \(-3.4\) over the large end of the mesoscale (\( 400 \lesssim \lambda \lesssim 1000 \) km) and \(-2.2\) at smaller scales (\( 40 \lesssim \lambda \lesssim 400 \) km). The divergent KE spectrum has a slope of \(-2.1\) at smaller scales.
5. Spectral kinetic energy budget

We have shown the importance of deep convection and the attendant latent heating in establishing and maintaining the mesoscale KE spectra in the upper troposphere and lower stratosphere. To quantify the contributions of these physical processes to the simulated KE spectra, spectral KE budget analysis is made in this section. We focus on the early mature phase \((t = 15–25\text{ h})\) and the mature phase \((t = 26–38\text{ h})\) of the mei-yu front system. For clarity, let \(\mathbf{u} = (u, v)\). Following WS2009, the KE budget equation in spectral space is given by

\[
\frac{\partial}{\partial t} E(k) = T(k) + P(k) + D(k),
\]

where \(T(k)\) is the energy transfer due to the total nonlinear advection (horizontal plus vertical);

\[
T(k) = -\mathbf{u} \cdot \text{DCT}(\mathbf{u} \cdot \mathbf{V} + w \partial_z \mathbf{u});
\]

\(P(k)\) is the spectral tendency due to the horizontal pressure gradient force (the pressure term); and

\[
P(k) = -\mathbf{u} \cdot \text{DCT}\left(\frac{1}{\rho} \nabla \mathbf{p}'\right)
= -\mathbf{u} \cdot \text{DCT}(c_p \theta_v \mathbf{V}') ,
\]

where \(\mathbf{p}' = (p/p_0)^{R_d/c_p}\) is the Exner pressure, \(\mathbf{p}' = \mathbf{p} - \mathbf{p}'\), \(c_p\) is the specific heat of dry air at constant pressure.

In Eq. (21) \(D(k)\) is the dissipation term due to the explicit sixth-order numerical diffusion and second-order horizontal and vertical mixing. As its negative contribution to \(E(k)\) for all time, we treat it as the residual term. Unless otherwise specified, the following analysis will be based on the mesoscale subrange \(40 \leq \lambda \leq 400\text{ km}\), where the KE spectra develop a shallower slope.

We have computed \(T(k)\) and \(P(k)\), calculating the terms inside the DCT in Eqs. (22) and (23) with numerical differences based on three-point, Lagrangian interpolation and proceeding as the KE spectrum. Figure 13 shows horizontal wavenumber spectra of KE tendency \(\partial E(k_h)/\partial t\), KE transfer \(T(k_h)\), and pressure term \(P(k_h)\), averaged in the vertical over the upper troposphere and lower stratosphere and in time over \(t = 15–25\) and \(t = 26–38\text{ h}\).

For \(t = 15–25\text{ h}\), the strong vertical convection occurs (Fig. 4b). In the upper troposphere (Fig. 13a), the mesoscale KE is deposited through the pressure term and removed by the nonlinear advective term. In the lower stratosphere (Fig. 13b), the effect of the advective nonlinearity is to deposit mesoscale KE, while the effect of the pressure term is relatively complicated. The lower-stratospheric spectra of \(P(k_h)\) develop a positive value for wavelengths more than around \(220\text{ km}\) and a negative value for wavelengths less than \(90\text{ km}\), and
cross zero several times in the rest wavelength range
90 ≤ λ ≤ 220 km. It is intriguing that the contributions of
$P(k_h)$ and $T(k_h)$ to KE spectrum tendency are appar-
ently opposite to those of WS2009, which can be attrib-
uted to the inclusion of moist physics in current study.

For $t = 26$–38 h, the convection weakens and tends to
be steady. In the upper troposphere (Fig. 13c), the posi-
tive contribution to mesoscale KE from the pressure
term weakens, and so does the negative contribution
from the nonlinear advective term. In the lower strato-
sphere (Fig. 13d), mesoscale KE is deposited through
both the advective nonlinearity and the pressure term—
especially the latter. For $t = 39$–48 h (not shown), as
the convection strengthens again, the situation is similar
to the early-mature phase.

As mentioned in section 4a, the upper-tropospheric
KE spectra growth is associated with the direct forcing
from latent heating, while the lower-stratospheric KE
spectra growth may be associated with vertically prop-
agating IGWs. To gain insight into these two different
mechanisms responsible for the KE spectra growth,
a further diagnostic analysis of the pressure term is given
in the following. Making the pseudoincompressible ap-
proximation (Durran 1989) and considering the diabatic
influences and the large-scale geostrophic forcing in
Eq. (23), we can write (see further details in appendixes)

$$P(k) = c_p \hat{h}_m \hat{\pi} + c_p \hat{G}_m \hat{\pi} = -c_p \frac{\partial \bar{\sigma}_d \bar{w}}{\partial z} + c_p \bar{w} \delta_z \hat{\pi}' .$$

(24)

The first term on the rhs is the diabatic influences in-
cluding the diabatic contributions to potential tem-
perature and water vapor, which represent the direct effects
associated with deep convection; the second term is the
convergence of the vertical pressure flux, which corre-
sponds to the flux of IGW energy; and the fourth term is
the portion of buoyancy flux, which represents the con-
version of potential to horizontal KE. The latter two
terms actually represent the indirect effects associated
with deep convection. Horizontal wavenumber spectra
of these terms are computed as outlined in Eq. (19).

Horizontal wavenumber spectra of the buoyancy flux
and pressure flux divergence, averaged in the vertical
over the upper troposphere and lower stratosphere and
in time over $t = 15$–25 and $t = 26$–38 h, are shown in
Fig. 14. In the upper troposphere (Figs. 14a,c), where di-
rect forcing from latent heating occurs, both the early
mature and mature phase of the mei-yu front system,
the positive contribution to mesoscale KE comes from the
buoyancy flux, which implies a conversion of available
potential energy to horizontal KE; the negative contri-
bution comes from the vertical pressure flux divergence. It
should be emphasized that, in the mature phase (Fig. 14c),
the buoyancy flux spectrum is characterized by a peak
around wavelength 300 km, which is less than the tran-
sition scale. Namely, there is a significant injection of
KE at 300 km, which validates the existence of the
smaller-scale energy source and implies an upscale
energy cascade.

In the lower stratosphere (Figs. 14b,d), where no di-
rect forcing from latent heating occurs, at both times, the
positive contribution to mesoscale KE comes from the
vertical pressure flux divergence; the negative contribu-
tion comes from the buoyancy flux. Moreover, in the
mature phase (Fig. 14d), the pressure flux divergence
has greater positive contribution than the advective
nonlinearity, which suggests the importance of the ver-
tical pressure flux divergence associated with vertically
propagating IGWs in establishing and maintaining the
lower-stratospheric mesoscale KE spectra.

Fig. 12. Times series of KE in the wavenumber range $9 \leq \Lambda \leq 11$, averaged in the vertical over the (a) upper
troposphere and (b) lower stratosphere for CNTL and HEAT24hoff. The dashed reference curves are exponential
decay with an $e$-folding time of (a) 16 and (b) 14 h, respectively.
Figure 15 shows horizontal wavenumber spectra of the diabatic term and the large-scale gradient forcing averaged in the vertical over the upper troposphere and in time over \( t = 15-25 \) h and \( t = 26-38 \) h. In the upper troposphere, the diabatic term weakens the KE for wavelengths \( 60 \leq \lambda \leq 400 \) km and enhances the KE over the rest of the mesoscale. It is worth noting that the effects of the diabatic term and the large-scale gradient forcing are an order of magnitude smaller than that of the buoyancy flux and the vertical pressure flux divergence.

6. Summary and discussion

Mei-yu front system is one of the most important precipitation systems in East Asia during summer. Because of its inhomogeneity, it cannot be well explained by the inertial turbulence theories. The mei-yu front is characterized by strong moisture gradients, and convective precipitation is its main weather phenomenon. The latent heat release associated with deep convection is responsible for the maintenance and development of the frontal structure (Kuo and Anthes 1982) through the conditional instability of the second kind (CISK) mechanism (Cho and Chen 1995). In this study, the idealized initial fields and numerical simulations are designed to reproduce the typical structure and evolution of the mei-yu front system. Based on this framework, we attempt to gain insight into different mechanisms responsible for the mesoscale KE spectral transition in the upper troposphere and lower stratosphere through the analysis of spectral KE budget.

At the mature phase of the mei-yu front system, in the lower troposphere, where a strong front is present, the KE spectrum develops a distinct peak around wavelength...
330 km, which is consistent with the meridional scale of the mei-yu front system; in the upper troposphere, the KE spectrum develops a $-3.5$ slope over the large-scale end of the mesoscale ($400 \leq \lambda \leq 1000$ km) and a transition to a shallower $-1.7$ slope at smaller scales ($40 \leq \lambda \leq 400$ km). As in the lower stratosphere, the KE spectrum also shows a significant spectral transition: it approaches a slope that is approximately $-4.1$ for $400 \leq \lambda \leq 1000$ km and $-2.1$ for the smaller scales ($40 \leq \lambda \leq 400$ km). The spectral transition scale is found to be around 400 km in the mei-yu front system.

For a typical mei-yu front system, the divergent and rotational KE have the same order of magnitude at smaller scales ($40 \leq \lambda \leq 400$ km). In the upper troposphere the former is slightly smaller, as seen in the analysis of data (Lindborg 2007), while in the lower stratosphere, the former is slightly larger. Moreover, the rotational KE spectrum shallows nearly to the same extent as the divergent KE spectrum. Therefore, the mesoscale KE $-5/3$ spectra in the upper troposphere and lower stratosphere for the mei-yu front system are not only explained by an increasing proportion of the divergent component as the wavelength decreases, which shows a slope of approximately $-5/3$, but also by the spectral transition of the rotational KE spectrum. This is significantly different from that of the baroclinic wave system (WS2009) and may be attributed to the weak baroclinicity of the simulated mei-yu front system.

Sensitivity experiments clearly reveal the importance of deep convection and attendant latent heating in establishing and maintaining the mesoscale KE spectra in the upper troposphere and lower stratosphere, especially the mesoscale KE $-5/3$ spectra at smaller scales. The latent heating leads to the rapid increase of KE. About 12 h after the latent heating turned off, the mesoscale KE spectrum does not show the distinct spectral transition any longer and its slope for wavelengths less...
than 400 km reduces to approximately $-3$, especially in the upper troposphere.

To reveal the formation mechanisms of the mesoscale KE spectrum in the upper troposphere and lower stratosphere, spectral KE budget for various height ranges is computed. The results show that contributions to the KE tendency come mainly from the advective nonlinearity and pressure term. Further analysis of the pressure term demonstrates that it is dominated by the buoyancy flux and the vertical pressure flux divergence. In the upper troposphere, where the direct forcing from latent heating occurs, the mesoscale KE is deposited through the buoyancy flux, which implies a conversion of available potential energy to horizontal KE, and is removed by the nonlinear advective term and vertical pressure flux divergence. In the lower troposphere, the mesoscale KE is deposited through the nonlinear advective term and vertical pressure flux divergence, and is removed by the buoyancy flux. In the mature phase, when the mesoscale KE spectrum develops a slope of $-5/3$ at smaller scales, the buoyancy flux spectrum in CNTL has a peak at scales of around 300 km and a plateau throughout the mesoscale, which suggests the significant input of KE in the mesoscale. The opposite contributions of the nonlinear advective term demonstrate that to some extent the mesoscale KE in the upper troposphere derives from a nonlinear upscale cascade and that the energy source is located at the lower end of mesoscale spectrum, which is generated by the buoyancy.

Based on the above analysis, the development and evolution of the mesoscale KE spectrum in an idealized mei-yu front system results from four interacting phenomena: 1) excitation of deep convection through CISK, primarily in the troposphere; 2) conversion of available potential energy to horizontal KE through buoyancy perturbations, primarily in the upper troposphere; 3) enhancement of the lower-stratospheric spectrum through vertically propagating IGWs excited by the latent heating; and 4) filling out of the KE spectra in the upper troposphere and lower stratosphere through nonlinear interaction. These processes can be described in detail as follows. Because of the high moisture of the mei-yu front system, the upward motion forced by the front soon becomes deep convection through the CISK mechanism. The latent heat released by moist convection mainly occurs in the upper troposphere and hence excites IGWs through the buoyancy production, which also leads to the direct conversion of available potential energy to mesoscale KE. The vertically propagating IGWs in turn transport KE upward and bring the enhancement of the spectrum in lower stratosphere. At last, the KE spectra are filled out through the nonlinear interaction.

The present study further demonstrates that the effect of moist physical processes and interaction among the different heights should be taken into account when investigating the energy cascade in the complex atmospheric system. However, a number of unanswered theoretical questions for future work remain. First, this part has focused on the mesoscale KE spectra in the upper troposphere and lower stratosphere. Since the inertial range theories predict the scaling behavior of the total energy spectrum, further discussion on the potential energy spectrum and the spectral budget of available potential energy is of great importance. A subsequent paper for the corresponding study is in review (J. Peng et al. 2013, manuscript submitted to *J. Atmos. Sci.*). Second, the mechanisms responsible for placing the spectral peaks of
these processes at the mesoscales rather than the synoptic scales should be explored further. Third, our simulations find that moist processes can inject sufficient amounts of kinetic energy to establish a visible kinetic energy cascade, while Tung and Orlando (2003) showed that the single energy injection from solar heating could also establish the Nastrom–Gage spectrum. Thus, to ascertain the significance of moist processes, an explicit comparison between these two energy sources should be made.

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APPENDIX A

The Pseudoincompressible Equation for Moist Air

The pseudoincompressible equation for dry air is presented in Durran (1989). Here, we give the derivation of the pseudoincompressible equation for moist air.

Following Klemp et al. (2007), define a modified potential temperature variable

\[ \theta_m = \theta (1 + 1.61 q_v) \]  

Taking the total derivative of Eq. (A1) yields

\[ \frac{d\theta_m}{dt} = (1 + 1.61 q_v) \frac{d\theta}{dt} + 1.61 \frac{dq_v}{dt} \]  

Let \( \dot{\theta}_g = -u(\partial \theta / \partial x)_{LS} \) and \( \dot{q}_{vg} = -u(\partial q_{vg} / \partial x)_{LS} \), and then Eqs. (4) and (5) in section 2a can be written as

\[ \frac{d\theta}{dt} \simeq S_\theta - \dot{\theta}_g \]  

\[ \frac{dq_v}{dt} \simeq S_{q_v} - \dot{q}_{vg} \]  

Substituting Eqs. (A3) and (A4) into Eq. (A2) gives

\[ \frac{d\theta_m}{dt} = H_m + G_m \]  

where \( H_m = (1 + 1.61 q_v) S_\theta + 1.61 \theta S_{q_v} \), which is the diabatic influences including the diabatic contributions to potential temperature and water vapor, and \( G_m = (1 + 1.61 q_v) \theta_q + 1.61 \theta q_{vg} \), which represents the large-scale forcing of the geostrophic potential temperature and water vapor.

The equation of state for moist air may be written as

\[ \pi = \left( \frac{R_d}{\rho_o} \frac{\rho_d \theta_m}{\theta_m} \right) R_o/C_v \]  

where \( R_d \) is the gas constant for dry air and \( C_v \) is the specific heat of dry air at constant volume.

Taking the logarithm and the total derivative of Eq. (A6) yields

\[ \frac{C_v}{R_d} \frac{d\pi}{dt} = \frac{1}{\rho_d} \frac{d\rho_d}{dt} + \frac{1}{\theta_m} \frac{d\theta_m}{dt} \]  

Substitution from Eqs. (7) and (A5) into Eq. (A7) gives

\[ \frac{C_v}{R_d} \frac{d\pi}{dt} + \left( \nabla \cdot \mathbf{u} + \frac{\partial w}{\partial z} \right) = \frac{1}{\theta_m} (H_m + G_m) \]  

Let the total thermodynamic fields be divided into a time-invariant hydrostatically balanced dry reference state and a perturbation as follows:

\[ \pi = \pi(z) + \pi', \quad \rho_d = \bar{\rho}_d(z) + \rho'_d, \quad \theta_m = \bar{\theta}_m(z) + \theta'_m, \quad \theta_v = \bar{\theta}_v(z) + \theta'_v. \]  

Assuming that \( \pi' \ll \pi \), Eq. (A8) is approximated as

\[ \frac{w}{\bar{\rho}_d} \frac{d(\bar{\rho}_d \bar{\theta})}{dz} + \left( \nabla \cdot \mathbf{u} + \frac{\partial w}{\partial z} \right) = \frac{1}{\bar{\theta}} (H_m + G_m) \]  

or

\[ \nabla \cdot \mathbf{u} + \frac{1}{\bar{\rho}_d \bar{\theta}} \frac{\partial \bar{\theta} \bar{w}}{\partial z} = \frac{H_m + G_m}{\bar{\theta}}. \]  

APPENDIX B

Spectral Budget Equation for the Pressure Term

Let \( \hat{\varphi}(k) \) be the Fourier transform (denoted by \( \mathcal{F} \)) of field \( \varphi \). Taking the Fourier transform of Eq. (A11) yields

\[ i \mathbf{k} \cdot \hat{\mathbf{u}} = -\frac{1}{\bar{\rho}_d \bar{\theta}} \frac{\partial \bar{\theta} \bar{w}}{\partial z} + \frac{H_m + G_m}{\bar{\theta}}. \]  

The horizontal wavenumber spectrum of the pressure term is given as
\[
P(\mathbf{k}) = -\bar{\mathbf{u}}^* \cdot \mathcal{F}(c_p \bar{\mathbf{V}}^\tau \mathbf{e}^\tau) + \text{c.c.} = -c_p \bar{\mathbf{u}}^* \cdot \mathcal{F}(\mathbf{k}^\tau \mathbf{e}^\tau) + \text{c.c.}
\]
\[
= c_p \bar{\mathbf{u}}^* \cdot \mathcal{F}(\mathbf{k}^\tau \mathbf{e}^\tau) + \text{c.c.}
\]
\[
= c_p \left( \bar{\mathbf{H}}_m^\tau \mathbf{e}^\tau \right) + \bar{\mathbf{G}}_m^\tau \mathbf{e}^\tau + \frac{c_p \partial \bar{\mathbf{p}}_d \bar{\mathbf{w}}^* \mathbf{e}^\tau}{\partial z} + \text{c.c.}
\]
\[
= \bar{\phi}(\mathbf{k}) + \text{c.c.}
\]

where an asterisk denotes complex conjugate and c. c. is complex conjugate.

To avoid the spectral aliasing results from the aperiodic structure of the atmospheric fields on regional domains, the discrete Fourier transforms are performed after exploiting even symmetry at the boundary to obtain \(y\)-periodic fields with period \((2N_y - 1)\Delta\) and then \(x\)-periodic fields with period \((2N_x - 1)\Delta\). It is exactly equivalent to the DCT of the original fields (Denis et al. 2002, their appendix). So Eq. (B2) can be given by

\[
P(\mathbf{k}) \approx c_p \bar{\mathbf{H}}_m^\tau \mathbf{e}^\tau + c_p \bar{\mathbf{G}}_m^\tau \mathbf{e}^\tau - \frac{c_p \partial \bar{\mathbf{p}}_d \bar{\mathbf{w}}^* \mathbf{e}^\tau}{\partial z} + c_p \bar{\mathbf{\dot{w}}}^\tau \mathbf{e}^\tau ,
\]

where \(\bar{\phi}(\mathbf{k})\) is the DCT of the original field \(\phi\) and \(\hat{\phi}(\mathbf{k})^* = \hat{\phi}(\mathbf{k})\). It is worth noting that all of the spectra are directly performed by DCT in the present paper.

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