Annular Mode Variability of the Atmospheric Meridional Energy Transport and Circulation

RAY YAMADA
Courant Institute of Mathematical Sciences, New York University, New York, New York

OLIVIER PAULUIS
Courant Institute of Mathematical Sciences, New York University, New York, New York, and NYUAD Institute, New York University Abu Dhabi, Abu Dhabi, United Arab Emirates

(Manuscript received 27 July 2014, in final form 6 January 2015)

ABSTRACT

Month-to-month variability in the meridional atmospheric energy transport is analyzed in the Modern-Era Retrospective Analysis for Research and Applications (MERRA) reanalysis for 1979–2012. The meridional transport of moist static energy (MSE) is composited onto the high and low phases of the northern and southern annular modes (NAM and SAM). While the high phase of the NAM and SAM is known to involve a poleward shift in the midlatitude storm track and jet, it is shown here that the distribution of poleward MSE transport shifts equatorward. This change is explained by examining the variability of the underlying meridional circulation. In particular, changes in the mass transport averaged on dry and moist static energy levels are considered. These circulations have an advantage over the conventional Eulerian circulation for explaining the total energy transport. They are computed using the statistical transformed Eulerian-mean (STEM) formulation, which provides a decomposition of the circulation into Eulerian-mean and eddy-driven components. The equatorward shift in the MSE transport is largely explained by a poleward shift of the Ferrel cell, while changes in the eddy-driven circulation have a comparatively small effect on the energy transport. The changes in the residual circulation and jet are shown to be consistent through momentum balance arguments. Mean-eddy feedback mechanisms that drive and sustain the annular modes are discussed at the end as a possible explanation for why the changes in the eddy-driven circulation are weak compared to the changes in the Eulerian circulation.

1. Introduction

Midlatitude storms make up an essential part of the climatological atmospheric circulation. They are responsible for most of the poleward transport of energy and water and maintain the surface westerlies against friction (e.g., Peixoto and Oort 1992; Vallis 2006). Understanding their variability is important for assessing how the distribution of wind, temperature, water, and other atmospheric tracers may change over time. The impact of their variability on the large-scale climate is typically captured by an empirical orthogonal function (EOF) analysis of the extratropical geopotential height or zonal wind fields. The leading EOF in both hemispheres is nearly zonally symmetric and is referred to as the northern and southern annular modes, or NAM and SAM, respectively (Limpasuvan and Hartmann 1999; Thompson and Wallace 2000). The annular mode is associated with north–south oscillations of the eddy-driven jet about its mean position (e.g., Hartmann and Lo 1998; Eichelberger and Hartmann 2007), where, by convention, the jet is displaced anomalously poleward (equatorward) when the annular mode is in its positive (negative) phase.

Although the natural variability in the atmosphere arises predominantly from synoptic storms spanning a few days, the annular modes vary on much longer intra-seasonal time scales. This has led to the idea that the zonal wind anomalies persist from a mean-eddy feedback, in which a meridionally displaced jet is supported by the changes it induces in the storm track and eddy forcing. A potential feedback mechanism is proposed in Robinson (2000): anomalously strong westerlies, with vertical shear induced by surface drag, reside over a region of enhanced

Corresponding author address: Ray Yamada, Courant Institute of Mathematical Sciences, New York University, 251 Mercer St., New York, NY 10012.
E-mail: yamada@cims.nyu.edu

DOI: 10.1175/JAS-D-14-0219.1

© 2015 American Meteorological Society

Unauthenticated | Downloaded 07/31/22 09:15 PM UTC
baroclinicity from thermal wind balance. The increase in baroclinicity facilitates the generation of baroclinic eddies, which propagate away from and converge momentum into the jet, thereby strengthening the original westerly anomaly. In the case of the annular modes, this feedback would imply that a shift of the jet will lead to a shift in the baroclinicity and region of baroclinic wave generation.

The baroclinicity must shift with the jet from thermal wind balance, but as Robinson points out, this feedback mechanism depends on the assumption that an increase in baroclinicity strengthens the source of wave activity. Lorenz and Hartmann (2001, 2003) use time lag regression to show that, in response to both the NAM and SAM, the source of synoptic waves shifts with the baroclinicity and that the change in the synoptic eddy forcing supports the zonal wind anomaly. These results indicate that, at least for synoptic eddies, the feedback mechanism appears robust.

The importance of the feedback mechanism has been debated however, as for example, Feldstein and Lee (1998) show that a positive feedback is supported by synoptic eddies, but the total eddy contribution does not appear to increase the persistence of the jet anomaly. The 2D barotropic model study by Vallis et al. (2004) uses a stochastic eddy forcing, which produces low-frequency variability without any mean feedback on the eddy forcing. Whether the low-frequency variability of the jet is due to a positive feedback or is simply forced at onset (Feldstein and Lee 1998), the long-term (e.g., monthly averaged) zonal momentum balance requires that the eddy forcing supports the jet anomaly. That is, for the barotropic wind anomaly to withstand a prolonged shift against surface drag, a sufficient change in the eddy momentum flux convergence must take place within the averaging period. Such coherent changes between the jet and eddy forcing have been observed in various observational (Lau 1988; Karoly 1990; Hartmann and Lo 1998; Limpasuvan and Hartmann 2000) and model studies (Robinson 1991; Yu and Hartmann 1993; Limpasuvan and Hartmann 1999, 2000).

The aforementioned studies have largely focused on understanding the dynamics between the zonal jet and its eddy forcing. Moreover, the mean- eddy dynamics are often studied using vertically integrated (e.g., Feldstein and Lee 1998; Lorenz and Hartmann 2001, 2003) or stochastically forced 2D barotropic (Vallis et al. 2004) dynamics. In such cases, the zonal jet is coupled to only the eddy momentum fluxes. In fully 3D dynamics, both the eddy fluxes of heat and momentum and the meridional circulation are important for momentum balance (Andrews and McIntyre 1976; Edmon et al. 1980). Hence, a systematic change in the storm track should be tied to a change in the meridional mass transport. This would consequently impact the poleward transport of heat, water, and other atmospheric tracers in the large-scale circulation. Understanding the internal variability of the meridional circulation associated with the annular modes is the primary goal of this work.

In this study, we first examine the monthly annular mode variability of the meridional transport of atmospheric energy. Our results are based on the MERRA reanalysis dataset from 1979 to 2012. The atmospheric energy transport includes the contribution from the sensible heat and geopotential energy, which together are referred to as the dry static energy (DSE), the latent heat (LH), and to a lesser extent the kinetic energy. The moist static energy (MSE) is defined to be the sum of the DSE and LH, and its transport gives a close approximation of the total atmospheric energy transport (e.g., Peixoto and Oort 1992). The poleward advection of MSE by the circulation is essential for maintaining global thermal equilibrium by compensating radiative imbalances at the top of the atmosphere. Anomalies from the annual cycle for the monthly averaged transports of MSE, DSE, and LH are composited onto the low and high phases of the NAM and SAM. Since MSE is primarily advected poleward by midlatitude eddies (e.g., Peixoto and Oort 1992), it might seem that the distribution of poleward MSE transport would shift together with the midlatitude storm track and jet. But, to the contrary, we find that it shifts in the opposite direction of the jet in both hemispheres. Namely, in the positive phase of the NAM and SAM, the distribution of poleward MSE transport shifts equatorward as the jet shifts poleward, and similarly for the DSE transport.

The change in the energy transports reflects a change in the underlying meridional mass transport. We show that the equatorward shifts of the DSE and MSE transports are largely explained by the change in the Eulerian circulation. However, the Eulerian circulation does not include the eddy contribution to the energy transport. To understand how the total energy transport is affected by the meridional circulation, we consider the mass transport averaged on surfaces of constant DSE and MSE (Czaja and Marshall 2006; Döös and Nilsson 2011). The circulations that result are similar to the circulations on dry and moist isentropes (Pauluis et al. 2008, 2010) and include contributions from both the Hadley circulation in the tropics and synoptic-scale eddies in the midlatitudes. They offer a better approximation to the mean Lagrangian trajectories of air parcels, since static energy is almost conserved for adiabatic processes. The change in the circulation determines the change in the total energy transport and also has implications for the variability of other atmospheric tracer transports. Additionally, it
connects the change in the energy transport to the change in the jet, as the circulation and jet are related through the zonal momentum budget.

The section overview for the paper is as follows. Details about the dataset and the annular modes can be found in section 2. The results for the annular mode variability of the energy transports are presented in section 3. Section 4 begins with a discussion of the meridional circulation computed on surfaces of constant energy and its relationship to the energy transport. The annular mode variability of the circulation is then presented. The change in the circulation is shown to be dynamically consistent with the jet shift through an analysis of the zonal momentum budget. We then discuss mean-eddy feedbacks for the annular modes and a possible explanation for why the change in the energy transport driven by the eddies is relatively weak compared to the change driven by the Eulerian-mean circulation. We conclude with a summary and discussion in section 5.

2. Dataset and annular modes

a. Dataset

The results in this study are based on monthly and zonally averaged quantities from the MERRA reanalysis dataset from 1979 to 2012 (Rienecker et al. 2011). The MERRA data are output 8× daily (every 3 h from 0000 UTC) on a 1.25° × 1.25° latitude–longitude grid at 42 pressure levels. Zonal averages are first computed at 8× daily resolution and then monthly averages are taken. The monthly and zonal average of a field \( X \) will be denoted by \( \bar{X} \). The deviation from \( \bar{X} \) will be denoted by \( X' \). The anomaly of \( X \) will refer to the departure of \( X \) from its annual cycle, where the annual cycle is computed as the climatological average of \( X \) for each calendar month.

b. Annular modes

The NAM and SAM were computed as the leading EOF of zonally and monthly averaged geopotential height anomaly at 850 hPa from 20° to 80° in their respective hemisphere. The anomaly was first weighted by the square root of the cosine of latitude to weight each point by its spatial area. The NAM and SAM account for 58% and 73%, respectively, of the total variance in the monthly- and zonal-mean 850-hPa height anomaly. The annular mode index is taken to be the standardized (i.e., taken to have unit variance and zero mean) first principal component (PC) time series from the EOF calculation.

The index is defined to be high (low) when its value lies above (below) 1.25 standard deviations. Between 1979 and 2012 (408 months) there were 33 (40) NAM high (low) events and 42 (41) SAM high (low) events that were used for compositing. The variance of the NAM is greatest in the winter, with the months of December–February (DJF) accounting for 67% (50%) of the high (low) events. The seasonality of the SAM is less pronounced, as JJA accounts for only 26% (32%) of the high (low) events. When computing the high (low) annular mode composite of a field, we first remove the field’s annual cycle to subtract out seasonal variability. The resulting anomaly field is then averaged over the months when the annular mode index is high (low). The high and low composites are based on the anomaly field, rather than the full field, and reflect changes associated with extreme annular mode events without confounding the effects of seasonality. When plotting the high or low composite, we add back the field’s annual mean to visualize the effect of the annular mode on the full field. Note that adding back the annual mean does not affect the high minus low composite difference.

The high minus low composite differences of the zonal-mean zonal winds onto the SAM (left) and NAM (right) are shown in shading in Fig. 1. The 1979–2012 annual-mean jet profile is drawn in black contours for comparison. In both hemispheres, the composite difference consists of an equivalent barotropic dipole. The dipole is centered about the midlatitude jet maximum and results from a poleward (equatorward) shift of the midlatitude jet from its mean position during the high (low) phase of the annular mode. This is more clearly seen in the Southern Hemisphere, where the subtropical and midlatitude jets are clearly separated. In the Northern Hemisphere, the jets in the Atlantic are well separated, but in the Pacific the jets are more closely collocated (Eichelberger and Hartmann 2007). The lack of separation throughout the Northern Hemisphere is apparent in the annual-mean jet profile, but the variability described by the NAM is still strongly dipolar about the approximate midlatitude jet position. The dashed, black, vertical lines mark the positive and negative centers of the dipole in the lower troposphere and will be drawn in later figures for reference. They are located at 57° and 30° in the Northern Hemisphere and at 60° and 37° in the Southern Hemisphere. The positive (negative) center position is computed as the average of the latitudes of maximum (minimum) values in the composite difference for pressure levels greater than 600 hPa. Lower-level winds were used to track the changes in the eddy-driven jet since they are less affected by the subtropical jet.

3. Variability of the energy transport

a. Energy transport climatology

The MSE is equal to the sum of the DSE and LH, where the DSE equals \( c_p T + gZ \) and the LH equals \( L_v Q \). Here
the specific heat at constant pressure, \( \epsilon_p \); the latent heat of vaporization, \( L_v \); and the gravitational acceleration, \( g \), are taken to be constants \((\epsilon_p = 1004 \text{ J K}^{-1} \text{ kg}^{-1},
\quad L_v = 2.50 \times 10^6 \text{ J kg}^{-1}, g = 9.8 \text{ m s}^{-2})\). The variables \( T, Z \), and \( Q \) are the temperature, geopotential height, and specific humidity, respectively.

Let \( j(\lambda, \phi, p) \) represent the MSE, DSE, or LH, and \( y(\lambda, \phi, p) \) be the meridional velocity, where \( j \) and \( y \) vary over longitude \( \lambda \), latitude \( \phi \), and pressure \( p \). The total transport of \( j \) across latitude \( \phi \) computed for each month is given by

\[
M_j(\phi) = \frac{2\pi \alpha \cos \phi}{g} \int_{0}^{p_{\text{sc}}} \bar{v}j(\phi, p) \, dp,
\]

where \( \alpha \) the radius of the earth and \( p_{\text{sc}} \) the surface pressure. The quantity \( M_j \) is the zonally and vertically integrated meridional flux of \( j \) at \( \phi \) and can be decomposed into the sum of the \( j \) transport by the mean flow and the eddies

\[
M_j = M_{j,\text{mean}} + M_{j,\text{eddy}},
\]

where

\[
M_{j,\text{mean}}(\phi) = \frac{2\pi \alpha \cos \phi}{g} \int_{0}^{p_{\text{sc}}} \bar{v}^j(\phi, p) \, dp
\]

and

\[
M_{j,\text{eddy}}(\phi) = \frac{2\pi \alpha \cos \phi}{g} \int_{0}^{p_{\text{sc}}} \bar{v}^j(\phi, p) \, dp.
\]

We will refer to \( M_j \) as the total \( j \) transport, \( M_{j,\text{mean}} \) as the mean-flow \( j \) transport, and \( M_{j,\text{eddy}} \) as the eddy \( j \) transport.

Figure 2 shows the climatological annual mean for the MSE (top), DSE (middle), and LH (bottom) transports in the reanalysis. The energy transports are decomposed into the total (solid line), mean-flow (dashed line), and eddy (dotted line) transports. Since northward fluxes are taken to be positive by convention, a poleward energy transport by the circulation is positive in the Northern Hemisphere and negative in the Southern Hemisphere. The total MSE transport is poleward at all latitudes and attains a maximum poleward transport of around 4 petawatts (PW) near 40° in both hemispheres. Most of the poleward energy transport is accomplished by the eddies along the midlatitude storm track, where the maximums are attained in the three eddy energy transports (the LH transport peak occurs further equatorward where there is a higher moisture concentration).

The MSE transport by the mean flow is small in comparison to the eddy MSE transport and exhibits a tripolar structure with a poleward transport at low and high latitudes and an equatorward transport in the midlatitudes. The mean-flow DSE and LH transports are also tripolar but are out of phase, unlike their eddy transports, which weakens the overall MSE transport. The tripolar structure can be explained by understanding how mass is advected by the Eulerian-mean circulation, which is computed from the time and zonally averaged meridional mass transport on pressure surfaces. An Eulerian streamfunction, denoted by \( \Psi_p \), where subscript \( p \) designates that the mass flow is isobarically averaged, is defined as
and consists of three cells: the tropical Hadley cell, the midlatitude Ferrel cell, and the polar cell (Fig. 3). Positive (negative) values of the circulation denote anticlockwise (clockwise) rotation in the figure.\(^1\) While eddies transport energy through quasi-horizontal mixing of high- and low-energy air parcels, the Eulerian circulation transports energy through overturning cells. In the time mean, the cells have no net meridional mass transport but do yield a net poleward energy transport. Since DSE increases with height, the net transport of DSE is in the direction of the upper branch of the cell. Similarly, moisture is concentrated in the lower troposphere and so the direction of LH advection is determined by the lower branch. The DSE transport by the circulation is greater than the LH transport and so the overall MSE transport is in the same direction as the DSE. The Hadley and polar cells are therefore thermally direct (i.e., they provide a net poleward energy transport by bringing high-energy parcels to higher latitudes where they lose energy through radiative cooling), whereas the Ferrel cell is thermally indirect. The alternating signs in the cells of the streamfunction explain the tripole structure that was observed in the mean-flow energy transport in Fig. 2.

### b. Observed changes in the energy transport

The total, mean-flow, and eddy energy transports \( (M_{\text{MSE}}, M_{\text{MSE,mean}}, M_{\text{MSE,eddy}}; M_{\text{DSE}}, M_{\text{DSE,mean}}, M_{\text{DSE,eddy}}; M_{\text{LH}}, M_{\text{LH,mean}}, M_{\text{LH,eddy}}) \) were composited onto the high and low phases of the NAM in the Northern Hemisphere and SAM in the Southern Hemisphere. The high minus low composite differences are shown in Fig. 4 for NAM (right column) and the SAM (left column). The changes in the MSE (black), DSE (teal), and LH (magenta) transports are broken down into the total (top row), mean-flow (middle row), and eddy (bottom row) contributions. The composites were qualitatively similar when just the first or second half of the time series was used instead of the full time series.

The extratropical response to the annular modes is qualitatively symmetric between the two hemispheres, although there are considerable differences in the tropics. In the extratropics, the changes in the total energy

\[\Psi_p(\phi, p) = \frac{2\pi a \cos \phi}{g} \int_0^\beta \nabla \phi(\phi, \tilde{\rho}) \, d\tilde{\rho}\]  

\[\text{(2)}\]

\(^1\) Since pressure decreases with height, its vertical axis is typically plotted in reverse. For this reason, we plot \(-\Psi_p\), rather than \(\Psi_p\), to maintain the sign convention.
transports (top row of Fig. 4) are dipolar and are centered about a nodal latitude near 36° in the Northern Hemisphere and 44° in the Southern Hemisphere. On the poleward (equatorward) side of the node, there is a decrease (increase) in the total poleward transport of DSE. The change in the total LH transport are similarly dipolar but of opposite sign. The dipolar change in the DSE transport acts to shift the distribution of the total poleward MSE transport from Fig. 2 equatorward, while the LH transport change acts to shift it poleward. Equatorward of the node, the changes in the DSE and LH transports largely compensate each other, such that the net change of the total MSE transport is small. Poleward of the node, the change in the MSE transport is dominated by the change in the DSE transport and reaches a maximum reduction of around 0.3 PW near 47°N and 0.25 PW near 56°S.

The dipole structure indicates that the annular mode variability of the midlatitude poleward energy transports are described by north–south vacillations, similar to the eddy-driven jet (the dotted vertical lines mark the jet dipole axes as in Fig. 1). However, while the jet shifts poleward in the high annular mode phase, the distributions of the poleward DSE and MSE transport shift equatorward. The equatorward shift in the MSE transport is characterized by a dipolar anomaly, whereas in the case of the MSE, the change occurs mainly on the poleward flank owing to the latent heat compensation in the subtropics. Since MSE is primarily transported poleward by midlatitude eddies (Fig. 2), an equatorward shift in the total MSE transport appears at odds with a poleward shift in the storm track and eddy-driven jet. To explain these apparently incongruous changes, we first decompose the change in the total energy transport into its mean-flow and eddy components.

Comparing the panels in Fig. 4, the annular mode variability of the total MSE, DSE, and LH transports are noticeably dominated by the change in their mean-flow components, except near 60° where the eddy changes are comparable. The mean-flow changes are similarly dipolar and reflect a change in the Eulerian circulation. Figure 5 shows in shading the high minus low composites of the Eulerian circulation onto the NAM (Fig. 5b) and SAM (Fig. 5a). The black contours show the annual-mean Eulerian circulation for comparison. From the low to the high phase, there is a noticeable poleward shift of the Ferrel cell in both hemispheres. This is indicated by a dipolar anomaly, centered roughly about the mean position of the Ferrel cell, that consists of two anomalous circulation cells: a thermally direct cell in the subtropics and an indirect cell at higher latitudes. The Eulerian circulation dipole aligns well with the mean-flow DSE and LH transport dipoles from Fig. 4 (middle row). The changes in the mean-flow energy transports reflect the energy advected by the anomalous circulation cells. The anomalous direct cell transports DSE poleward and LH equatorward in similar amounts, such that there is a small change in the MSE transport equatorward of around 40° in both hemispheres. The anomalous indirect cell is located at higher latitudes, where there is less moisture, and its net MSE transport is composed mostly of an equatorward DSE transport. This anomalous equatorward MSE transport is equivalent to a decrease in the overall poleward MSE transport. A poleward shift of the Ferrel cell in the high annular mode phase therefore induces an equatorward shift in the midlatitude transports of DSE and MSE.

As has been noted in past studies (Limpasuvan and Hartmann 1999; Thompson and Wallace 2000), the annular mode changes in the midlatitude Eulerian circulation
are consistent with the poleward shift of the jet. There is anomalous warming (cooling) on the equatorward (poleward) side of the anomalous indirect cell, as can be seen in the composite difference of the temperature field (bottom row of Fig. 5). These temperature anomalies arise from the adiabatic warming and cooling of air advected by the modified mean flow. The change in the Eulerian circulation adjusts the background baroclinicity to restore thermal wind balance with the vacillating zonal jet. It should be noted that the jet dipole (denoted by the dashed vertical lines) is not in exact alignment with the Eulerian circulation dipole, especially for the equatorward node. This is because, in addition to the Ferrel cell shift, the change in the Eulerian circulation also involves significant strengthening of the Hadley circulation, which extends the equatorward dipole node farther equatorward.

The composite differences for the eddy energy transports are shown in the bottom row of Fig. 4. The most significant changes are described by a monopolar increase in the poleward eddy energy transport around 60° in both hemispheres but are noticeably stronger in the Northern Hemisphere than in the Southern Hemisphere. Except near the center of the monopole, the responses of the eddy energy transports to the annular modes are small in comparison to those of the mean flow. This is in sharp contrast to the climatological energy transports in Fig. 2, in which the eddies dominated the mean flow. The annular modes clearly do not describe a uniform poleward shift of the midlatitude circulation. For if this were the case, then the equatorward shift in the mean-flow energy transport would be compensated by a poleward shift of the eddy energy transport. The change in the eddy energy transport is markedly different from the change in the eddy momentum flux convergence of the upper troposphere, which has a strong dipolar response that is coherent with the jet shift [e.g., Limpasuvan and Hartmann (2000) and discussed next].
4. Variability of the dry and moist circulations

While the jet shifts poleward in the high annular mode phase, the DSE and MSE transports shift equatorward. To reconcile these rather counterintuitive changes, it is important to understand the variability of the meridional circulation. On the one hand, the circulation determines the energy transport, while on the other hand, the variability of the circulation and jet are related through momentum balance constraints. In this section, we first discuss the connection between the total energy transport and the circulation, and then we examine the annular mode composites of the circulation. We show that the changes in the jet and circulation are dynamically consistent by considering the momentum budget. At the end, we discuss mean-eddy feedbacks, which may explain why the change in the Eulerian circulation is more pronounced than the change in the eddy-driven circulation in the monthly composites.

a. Relationship between the meridional circulation and energy transport

The changes observed in the Eulerian circulation in the previous section helped explain the changes in the mean-flow energy transport. However, the Eulerian circulation does not account for the total energy transport, especially in the midlatitudes where eddies dominate the circulation. The indirect overturning implied by the Ferrel cell gives a strikingly misleading impression of the mid-latitude circulation, which is actually thermally direct (Fig. 2) and largely comprises quasi-horizontal eddy transports of heat and moisture. To understand the variability in the meridional transport of energy and other tracers, it is necessary to consider a more complete description of the meridional circulation that accounts for both the Eulerian-mean and eddy transports.

One alternative description of the circulation, which better captures the total midlatitude energy transport, relies on using a quasi-Lagrangian vertical coordinate, such as entropy or static energy, instead of pressure. We consider the circulation averaged on the latter—surfaces of constant DSE and MSE—and will refer to these as the dry and moist circulations. The motivation here is twofold. First, as static energy is almost conserved for reversible adiabatic processes, averaging the flow on surfaces of constant static energy offers a better approximation of the Lagrangian trajectories than an average on pressure surfaces. The streamfunctions that result consist of a single thermally direct cell that extends from equator to pole in both hemispheres (Czaja and Marshall 2006; Döös and Nilsson 2011). Second, in such framework, the mean circulation accounts for the total energy transport.
without any eddy contribution (as by definition, there is no fluctuation of energy content on surfaces of constant static energy). A streamfunction for the mass transport integrated on levels of constant \( \xi \) can be computed as follows (e.g., Pauluis et al. 2008; Döös and Nilsson 2011):

\[
\Psi_\xi(\phi, \xi_0) = \int_0^{2\pi} \int_0^{\infty} H[\xi_0 - \xi(\lambda, \phi, p)] \nu(\lambda, \phi, p) a \cos \phi \frac{dp}{g} d\lambda, \tag{3}
\]

where \( H(x) \) denotes the Heaviside function, defined \( H(x) = 1 \) for \( x \geq 0 \) and \( H(x) = 0 \) for \( x < 0 \). The zonally integrated meridional mass transport at latitude \( \phi \) between \( \xi \) and \( \xi + d\xi \) is given by \( \partial \Psi_\xi/\partial \xi d\xi \). The total transport of \( \xi \) across latitude \( \phi \) can then be computed as

\[
M_\xi(\phi) = \int_0^\infty \frac{\partial \Psi_\xi}{\partial \xi} d\xi = -\int_0^\infty \Psi_\xi d\xi, \tag{4}
\]

where the second equality is obtained using integration by parts and the fact that \( \Psi_\xi \) vanishes at infinity. Hence, the total transport of DSE and MSE is given by the negative of the integral over energy of the dry and moist streamfunctions, respectively.

Computing the streamfunction as in (3) requires daily and zonally varying data. Instead, we compute the dry and moist streamfunctions using the statistical transformed Eulerian-mean (STEM) approximation (Pauluis et al. 2011). The STEM streamfunction is a statistical generalization of the transformed Eulerian-mean (TEM) streamfunction (Andrews and McIntyre 1976; Edmon et al. 1980) that can be applied to an arbitrary, unstratified, vertical coordinate using only zonal- and monthly-mean statistics output on pressure. It is based on the assumption that the joint probability density function of the meridional velocity, \( v \), and the vertical coordinate, \( \xi \), is approximately bivariate Gaussian. The STEM streamfunction only requires knowledge of the first- and second-order moments: \( \overline{v}, \overline{\xi}, \overline{v^2}, \) and \( \overline{\xi^2} \). Moreover, analogous to TEM, STEM provides a decomposition of the total streamfunction, \( \Psi_{STEM, \xi}(\Psi_\xi \text{ for short}) \), into an Eulerian-mean and eddy-driven streamfunction given by

\[
\Psi_\xi(\phi, \xi) = \Psi_{\xi, \text{mean}}(\phi, \xi) + \Psi_{\xi, \text{eddy}}(\phi, \xi), \tag{5}
\]

where

\[
\Psi_{\xi, \text{mean}}(\phi, \xi) = \frac{2\pi a \cos \phi}{g} \int_{-\infty}^{\xi} \frac{v}{\sqrt{2\pi \xi^2}} \exp \left[ \frac{-(\xi - \xi')^2}{2\xi^2} \right] dp d\xi \quad \text{and} \tag{6}
\]

\[
\Psi_{\xi, \text{eddy}}(\phi, \xi) = \frac{2\pi a \cos \phi}{g} \int_{-\infty}^{\xi} \frac{\overline{v^2}(\xi - \xi')}{\sqrt{2\pi \xi^2}} \exp \left[ \frac{-(\xi - \xi')^2}{2\xi^2} \right] dp d\xi. \tag{7}
\]

The relationship between the energy transport and streamfunction (4) also holds for STEM (derivations are given in appendix A). Additionally, the STEM decomposition (5) allows for the mean-flow and eddy energy transports to be related to their respective components of the STEM circulation:

\[
M_\xi(\phi) = -\int_{-\infty}^{\infty} \Psi_\xi(\phi, d\xi), \tag{8}
\]

\[
M_{\xi, \text{mean}}(\phi) = -\int_{-\infty}^{\infty} \Psi_{\xi, \text{mean}}(\phi, d\xi), \tag{9}
\]

\[
M_{\xi, \text{eddy}}(\phi) = -\int_{-\infty}^{\infty} \Psi_{\xi, \text{eddy}}(\phi, d\xi). \tag{10}
\]

For example, the connection between the changes in the mean-flow energy transport and those in the Eulerian circulation, discussed in section 3, can be made explicit by analyzing the change in the mean-flow component of the streamfunction (9).

Figure 6 shows the 1979–2012 annual-mean climatology of the dry and moist streamfunctions computed from (5), (6), and (7) with \( \xi \) taken to be the DSE (left column) and MSE (right column). The total streamfunction (Figs. 6a,b) is given by the sum of a three-celled Eulerian-mean circulation (Figs. 6c,d) and an eddy-driven circulation (Figs. 6e,f). In both the dry and moist cases, the total circulation consists of a single thermally direct cell. The Eulerian-mean circulation has a three-celled structure as before, but in the total circulation the indirect Ferrel cell is dominated by a direct eddy-driven circulation. The dry circulation has two distinct cores, arising from a strong Hadley circulation in the tropics and an eddy-driven circulation in the mid-latitudes. In contrast, the moist circulation has a single core with a stronger extratropical circulation than that of the dry. These differences are due, in part, to the fact...
that the overturning cells of the Eulerian streamfunction are weaker in the moist case than in the dry case (compare Figs. 6c and 6d). The “upper” and “lower” branches of the circulation represent the mass flow of high- and low-energy parcels, respectively. In the moist case, since low-level air parcels can carry energy in the form of latent heat, high energy does not necessarily reflect high altitude. Consequently, there is more cancellation between the upper and lower branches in the moist representation of the Eulerian circulation than in the dry. This is especially true for the Hadley cell, since in the tropical troposphere MSE is well mixed by convection (Xu and Emanuel 1989; Czaja and Marshall 2006). The stronger extratropical core in the moist circulation can also be attributed to an increase in the eddy mass transport on MSE levels as compared to that on DSE levels (Figs. 6e and 6f). Pauluis et al. (2008, 2010) show that this additional mass flux on moist isentropes arises from a low-level flow of warm moist air parcels that are advected from the subtropics into the storm track by midlatitude eddies.

b. Observed changes in the dry and moist circulations

The dry and moist circulations ($\Psi_{DSE}$, $\Psi_{MSE}$) and their mean-flow ($\Psi_{DSE,mean}$, $\Psi_{MSE,mean}$) and eddy-driven ($\Psi_{DSE,eddy}$, $\Psi_{MSE,eddy}$) components were composited onto the high and low phases of the NAM and SAM. The total, mean-flow, and eddy-driven circulation composite differences are shown in Figs. 7 and 8 for the dry and moist circulations, respectively. The results were again robust when just the first or second half of the time series was used instead of the full time series. First we will discuss the changes in the dry circulation.

The annular mode changes in the extratropical dry circulation (Figs. 7a,b) are dipolar in both hemispheres, with strengthening near the subtropics and weakening in the midlatitude core. This dipole indicates an equatorward shift within the dry circulation during the high phase. The subtropical strengthening of the circulation, centered near 25°N and 30°S, occurs near the joint connecting the subtropical and midlatitude cores. In the high phase, the circulation is more uniformly distributed.
from equator to pole, whereas in the low phase, the circulation is more clearly separated into two distinct cores. From relationship (8), the dry circulation dipole corresponds to the dipole observed in the total DSE transport (Fig. 4, top row). Since the changes in the streamfunction are for the most part equivalent barotropic, there is little cancellation when the streamfunction is integrated in the vertical. Hence, latitudes at which the dry circulation is stronger (weaker) correspond to locations at which the total DSE transport is greater (smaller).

The changes in the total dry circulation can be decomposed into the individual changes in the Eulerian-mean and eddy-driven circulations using the STEM decomposition (5). The composite difference for the Eulerian circulation is shown in the middle row of Fig. 7. The changes are qualitatively the same as those discussed in section 3 and involve a poleward shift of the Ferrel cell in the high annular mode phase. Since the Ferrel cell is thermally indirect, this induces an equatorward shift within the total dry circulation.

The low-to-high changes in the eddy-driven circulation are shown in the bottom row of Fig. 7. The eddy-driven circulation was computed using the contributions from both the transient and stationary eddies. Despite the significant contrast in the land–sea distribution between the two hemispheres, there is a clear symmetry in the response of the eddy-driven circulations. The changes in the eddy-driven cell are marked by three centers: two centers of opposite signs between the jet dipole axes and a third center just poleward of 60°. The two centers between the jet dipole axes form a vertical dipole and are indicative of an upward shift (i.e., to higher energy) in the eddy-driven circulation. The upper branch of the...
Eddy-driven circulation is more intense, while the lower branch is weaker. This change in sign in the vertical results in strong cancellation when the integral is taken in (10). Hence, there is little change in the eddy DSE transport between the jet dipole axes. The center poleward of 60° accounts for the monopolar intensification of the eddy DSE transport that was seen in Fig. 4.

The upward shift in the eddy-driven circulation between the jet dipole axes is consistent with the anomalous warming that occurs at the same latitudes in the high annular mode phase (Fig. 5). In a warmer background state, the transport of eddy energy occurs relative to a higher mean energy. Similarly, the center located poleward of 60° is in a region of anomalous cooling. Although a vertical dipole is not observed here (except for the Southern Hemisphere in the moist case), the sign of the center is consistent with a strengthening of the lower branch that would be induced by a downward shift in the circulation.

The steepening of the temperature gradient (increase in baroclinicity) about the poleward jet dipole axis increases the tilt of the eddy-driven circulation but does not lead to a strong increase in the poleward eddy energy transport. Even near 60°N, where the increase in poleward eddy energy transport is largest, the change in the eddy energy transport is mostly compensated by a decrease in the mean-flow energy transport.

In the case of the moist circulation (Fig. 8), the changes are qualitatively similar to the dry case in that the Ferrel cell is observed to shift poleward and the eddy-driven cell is more steeply tilted in the high phase. However, the extratropical changes in the total circulation are rather tripolar instead of dipolar. The midlatitude changes in the eddy-driven circulation are comparable in magnitude and spatial scale to the changes in the Eulerian circulation. In comparison to the dry case, the dipolar anomaly arising from shift of the Ferrel cell is not as pronounced and the response of the eddy-driven circulation is also stronger.

As discussed earlier, this is largely because the inclusion

---

**Fig. 8.** As in Fig. 7, but for the moist circulation. The contours for the annual-mean circulations are drawn at \([-10, -6, \ldots, 6, 10] \times 10^{10} \text{ kg s}^{-1}\) for the total and eddy-driven circulations and at \([-3, -2, \ldots, 2, 3] \times 10^{10} \text{ kg s}^{-1}\) for the Eulerian-mean circulation. Zero contours are not drawn.
of latent heat accounts for the mass transport of low-level moist air parcels, which weakens the overturning cells of the Eulerian circulation and strengthens the eddy-driven circulation.

c. Momentum balance in annular mode composites

To relate the poleward shift of the jet to the equatorward shift within the dry circulation, we turn here to an analysis of the zonal momentum budget, which under quasigeostrophic scaling (e.g., Peixoto and Oort 1992) can be written as follows:

\[
\frac{\partial \mathbf{v}}{\partial t} = f \mathbf{v} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (\frac{1}{\sin \phi} \mathbf{v} \cdot \sin \phi) + D. \tag{11}
\]

The forcing terms on the right-hand side are the Coriolis force on the Eulerian-mean flow, eddy momentum flux convergence, and friction, respectively, where \( f \) denotes the Coriolis parameter. For studying the dynamics of the dry circulation, it is more useful to consider the TEM formulation of (11) as in Edmon et al. (1980):

\[
\frac{\partial \mathbf{v}}{\partial t} = f \mathbf{v} + \frac{1}{a \cos \phi} \mathbf{v} \cdot \mathbf{F} + D. \tag{12}
\]

Here the eddy forcing is given by the divergence of the Eliassen–Palm (EP) flux vector \( \mathbf{F} \), where

\[
\mathbf{F} = \left( -a \cos \phi \mathbf{u} \cdot \mathbf{F} - a \cos \phi \frac{\partial \mathbf{v}}{\partial \phi} \right)
\]

and \( \theta \) is the potential temperature. The subscript \( p \) denotes the partial derivative with respect to pressure. The Coriolis force is determined by the residual meridional velocity, \( \frac{\partial \mathbf{v}}{\partial \phi} = \frac{1}{\sin \phi} \frac{\partial}{\partial \phi} (\sin \phi \mathbf{v} \cdot \sin \phi) \approx \frac{1}{\sin \phi} \frac{\partial}{\partial \phi} \mathbf{v} \cdot \sin \phi \). The residual circulation represents the mean meridional circulation driven by diabatic heating and cooling and approximates the isentropic circulation (e.g., Haynes and McInrey 1987; McIntosh and McDougall 1996). Its streamfunction, referred to as the TEM streamfunction, is defined as the integrated mass flux by the residual velocity in pressure:

\[
\Psi_{\text{TEM}}(\phi, p) = \frac{2\pi a \cos \phi}{g} \int_{0}^{\phi} \mathbf{v} \cdot \mathbf{F} \, dp. \tag{13}
\]

The residual circulation is closely related to the STEM circulation on DSE used earlier, as Pauluis et al. (2011) have shown that the STEM streamfunction converges toward the TEM streamfunction in the limit of vanishing variance. It is also shown in appendix B that the use of DSE rather than potential temperature does not affect the result. The primary advantage of using the TEM circulation here lies in the explicit formulation of the momentum balance (12).

Above the boundary layer and on monthly time scales, the dominant balance in (12) is between the Coriolis force acting on the residual circulation and the eddy forcing:

\[
f \mathbf{v} + \frac{1}{a \cos \phi} \mathbf{v} \cdot \mathbf{F} \approx 0. \tag{14}
\]

The high minus low composite difference of the eddy forcing (shading) and of the zonal wind (contours) are shown in the middle row of Fig. 9. The change in the eddy forcing consists of an upper-tropospheric dipole that is nearly coincident with the jet dipole, as has been observed by Hartmann and Lo (1998), Limpasuvan and Hartmann (2000), and others. This suggests that the eddy forcing anomaly acts to drive and sustain the jet anomalies. Equation (14) indicates that a positive anomaly in the eddy forcing must be balanced by an easterly Coriolis force and, hence, an equatorward anomaly in the residual velocity. From mass conservation, there must be a poleward anomaly in the residual velocity below, which provides a westerly Coriolis force that supports the surface westerlies against friction (Limpasuvan and Hartmann 2000; Thompson and Wallace 2000). Hence, a positive anomaly in the eddy forcing aloft induces an anomalous indirect circulation. Similarly, a negative anomaly in the eddy forcing induces an anomalous direct circulation. The poleward shift of the jet and eddy forcing must therefore induce an equatorward shift within the residual circulation. This can be seen in the bottom row of Fig. 9, which shows the composite difference of the residual circulation. The balance in (14) can be written explicitly in terms of the residual streamfunction by dividing by the Coriolis parameter and the gravitational constant and then zonally and vertically integrating the expression

\[
\Psi_{\text{TEM}} \approx \frac{2\pi}{fg} \int_{0}^{\phi} \mathbf{F} \cdot \mathbf{d} \rho.
\]

The composite difference shows the approximate change in the circulation that would be induced by the change in the eddy forcing. Note that we are plotting the negative of the residual streamfunction (see earlier footnote).

The change in the residual circulation reflects the change in the Eulerian circulation to the degree in which the eddy forcing in (12) is driven by the changes in the eddy momentum fluxes. From the momentum balance written as in (11), a change in the eddy momentum flux convergence must be balanced by a change in the Eulerian circulation. The top row of Fig. 9 shows the change in the momentum flux convergence in
shading. As compared with the eddy forcing (middle row), the eddy momentum fluxes largely explain the meridional shift in the eddy forcing. The part of the eddy forcing driven by the eddy heat fluxes [not shown; see Limpasuvan and Hartmann (2000)] accounts for the vertical tilt in the eddy forcing.

d. Feedback between the changes in the mean flow and eddies

A strong poleward shift in the eddy forcing aloft may seem to suggest that the region of baroclinic eddy generation near the surface should also shift poleward in the high annular mode phase. Figure 10 shows the high minus low composite difference of the EP flux vectors [arrows, plotted as in Edmon et al. (1980)], which indicate the change in the propagation of wave activity. In climatology, the EP flux vectors are largest near the lower boundary where they point vertically upward and indicate a strong poleward flux of DSE by the eddies; that is, \( \vec{\nu} \cdot \vec{DSE} \) is positive (negative) in the Northern (Southern) Hemisphere (shown in black contours). The composite difference for the eddy DSE flux (shading) indicates that there is some poleward shift in the source of wave activity. Where there is a strong (weak) anomaly in the poleward eddy DSE flux, the anomalous EP flux vectors tend to point upward (downward). However, the most significant changes in the EP flux vectors occur in the upper troposphere in the horizontal direction and are related to changes in the eddy momentum fluxes. This suggests that the region of eddy generation has not shifted very much in the monthly composites. Moreover, a strong shift in the source of wave activity would imply a strong shift in the poleward eddy DSE transport. This is not the case, as we showed earlier that the response...
of the eddy-driven circulation mainly involves tilting of the eddy-driven cell, which has little effect on the poleward eddy energy transport. The most significant change in the eddy DSE transport occurred in the Northern Hemisphere near 60°N but was not large enough in magnitude and spatial extent to shift the total DSE transport poleward. This raises the question, why is there a strong shift in the eddy forcing of the wave activity has not shifted very much? Equivalently, why does the Eulerian circulation respond more strongly to the annular modes than the eddy-driven circulation? Part of the answer lies in the fact that monthly composites conflate the changes in both the buildup and decay phase of the annular mode anomalies. In the studies by Lorenz and Hartmann (2001, 2003) using daily reanalysis, they show that the eddy forcing anomaly supports the zonal wind anomaly at both positive and negative lags (where positive lag means that the mean flow leads the eddy forcing). This indicates that while the eddy forcing anomaly at first drives the buildup of the jet anomaly, it is further sustained as the mean-flow anomaly tends to reinforce the eddy momentum fluxes. At positive lags, their results support the feedback described in the introduction; that is, a poleward shift of the jet leads to a poleward shift of the baroclinicity and wave activity source. This feedback might explain the poleward shift in the wave activity source observed in Fig. 10 and accounts for part of the changes in the eddy forcing in Fig. 9.

At negative lags there is also a strong change in the eddy momentum fluxes, which drives the jet shift and makes up the bulk of the changes observed in Fig. 9. The feedback mechanism described in Robinson (2000) does not apply before the growth of the zonal wind anomaly and so a large shift in the wave activity source would not be expected at negative lags. This may explain why the changes in the vertical component of the EP flux vectors are relatively weak compared to those in the horizontal component in the monthly composites in Fig. 10. While the initial eddy forcing anomaly may be stochastically driven, it continues to grow from as much as 30 days prior to the maximum wind anomaly [e.g., Fig. 5 in both Lorenz and Hartmann (2001, 2003)]. This suggests that the eddy momentum fluxes are organized by another feedback during the growth stage of the zonal wind anomalies that is independent of a shift in the source of wave activity. It is this strong initial growth in the eddy forcing that drives the bulk of the changes in the Eulerian circulation and energy transport.

Here we consider a feedback mechanism based on ideas behind jet formation. The eddy forcing term in (12) can be interpreted in terms of potential vorticity (PV) mixing, since the EP flux divergence can be rewritten in terms of the quasigeostrophic eddy flux of PV,

$$\mathbf{V} \cdot \mathbf{F} = a \cos \phi \partial q^2,$$

where \( q \) denotes PV. In climatology, the eddy PV flux is downgradient (negative) and indicates regions of irreversible mixing from Rossby wave breaking. Eddy mixing is strongest on the flanks of the jet and attenuated near the core of the jet in the upper troposphere and lower stratosphere (Haynes and Shuckburgh 2000). Inhomogeneous PV mixing creates persistent jets through a positive feedback mechanism [e.g., see the review paper by Dritschel and McIntyre 2008]. Rossby waves are less likely to break near the core of the jet, since...
their phase speed is much less than the mean flow (Andrews et al. 1987; Randel and Held 1991). The waves propagate meridionally away from the jet before eventually breaking near critical latitudes on the flanks of the jet, thereby converging momentum into the jet region and decelerating the mean-flow and mixing PV on the sides of the jet. This further sharpens the jet and steepens the PV gradient in the jet region. In this manner, the jet acts as an eddy mixing barrier in the upper troposphere, which reinforces itself as waves are forced to propagate outside the jet region before breaking.

The above feedback mechanism provides a possible explanation for the growth of the annular mode jet anomalies. The shift of the jet corresponds to a shift of the upper-tropospheric mixing barrier, which then leads to a shift in the region of wave breaking. This supports the growing jet anomalies, as there is more wave propagation out of the region of anomalous westerlies and more wave breaking in the region of anomalous easterlies. This mechanism does not require a significant shift in the source of wave activity for there to be a shift in the eddy forcing aloft. The path of wave energy propagation from the existing wave source is sufficiently modified by the changes in the mean flow.

This mechanism is supported by the observed shift in the eddy forcing (i.e., the eddy PV flux) in the middle row of Fig. 9. In the region where the jet is anomalously strong, there is a positive anomaly in the eddy PV flux, which indicates less mixing. This suggests that the anomalous westerlies strengthen the mixing barrier, which reduces the wave drag on the jet in this region and therefore reinforces the original westerly anomaly. Similarly, where the jet is anomalously weak, there is a negative eddy PV flux anomaly, which corresponds to a region of enhanced wave breaking and further deceleration of the mean flow. As the jet is shifted poleward in the high phase, the waves must propagate farther equatorward before breaking. This can be seen by the arrows in Fig. 10, in which equatorward propagation is noticeably stronger in the high phase—an observation that was also noted in Hartmann and Lo (1998) and Limpasuvan and Hartmann (2000). This is also consistent with the index of refraction arguments used in Limpasuvan and Hartmann (2000) and Lorenz and Hartmann (2003) for Northern Hemisphere stationary waves, in which they show that more wave activity is absorbed at high latitudes when the jet is shifted equatorward during the low phase of the NAM.

The right column of Fig. 11 shows the individual NAM high and low composites for the eddy PV flux and jet. The results for the SAM are not shown but are qualitatively similar. In the low phase (Fig. 11d), the jet core is centered about 35°N and 200 hPa and flanked on both sides by stirring regions, as indicated by downgradient eddy PV fluxes. In the high phase (Fig. 11b), since the midlatitude jet shifts poleward away from the subtropical jet, the jet core is less intense and mixing is strengthened in the region near 35°N and 200 hPa as compared to the low phase.
The effect of the change in PV mixing on the residual circulation is shown in the left column of Fig. 11. The circulation is weaker where the jet is present, since the jet impedes PV mixing. For example, in the low phase, the jet is anomalously strong between 25° and 40°N, whereas in this same region, the circulation is weaker than in its high phase when the jet has shifted poleward. It is important to distinguish that the jet acts as a mixing barrier predominantly to the upper-level flow and not the lower-level flow. The midlatitude circulation is reduced in the jet region, not because low-level mixing is impeded, but rather indirectly from the reduction in eddy PV mixing aloft (i.e., more momentum flux convergence) that drives an indirect Eulerian circulation.

5. Summary and discussion

The large-scale variability of the atmosphere is often described by the changes in the annular modes. In particular, the midlatitude jet and storm track are known to shift poleward during the high phase. In this study, we have shown that a simple “poleward shift” interpretation of the annular modes does not apply to the meridional mass and energy transports. In the high phase of both the NAM and SAM, there is an equatorward shift in the total poleward transports of DSE and MSE. This seems inconsistent with a poleward shift of the eddy-driven jet, since most of the poleward energy transport is accomplished by midlatitude eddies in the annual-mean climatology.

To reconcile the changes in the jet and energy transport, the variability in the dry and moist circulations was considered. On the one hand, these circulations explain the total poleward transport of DSE and MSE, respectively. While on the other hand, the circulation is dynamically constrained to the jet. The dynamics of the dry circulation was considered, since the dry circulation is closely related to the residual circulation, which appears explicitly in the TEM formulation of the zonal momentum equation. From the dominant balance in the zonal momentum [Eq. (14)], it can be deduced that a positive anomaly in the eddy forcing aloft drives an anomalous indirect circulation. Since the annular mode in the high phase induces a poleward shift in the upper-tropospheric eddy forcing, there must be an equatorward shift within the residual circulation for momentum balance on monthly time scales.

The poleward shift in the eddy forcing is largely due to a change in the upper-tropospheric eddy momentum flux, which both drives and sustains the shift of the jet. At positive lags, the eddy momentum flux anomaly is supported by a mean-eddy feedback, which involves a poleward shift of the region of baroclinic eddy generation (Robinson 2000; Lorenz and Hartmann 2001, 2003). However, this feedback does not apply at negative lags when the bulk of the change in the eddy momentum flux occurs, which may explain why the eddy heat flux shows little change in the monthly annular mode composites. The initial growth of the eddy momentum flux anomaly arises without a strong change in the source of wave activity. It drives not only the shift of the jet but also the changes in the Eulerian circulation and energy transport. Although the eddy momentum flux anomaly may be initially stochastically driven, the long time scale over which it grows suggests that it is driven by another feedback, such as the mean-eddy feedback related to jet formation. The jet acts as a mixing barrier that alters the path of wave activity propagation from the existing wave source. As the jet shifts poleward in the high phase, Rossby waves tend to break equatorward away from the positive jet anomaly. This further strengthens the positive jet anomaly on the poleward side and further decelerates the jet where the waves break on the equatorward side.

Although the changes in the eddy transports of mass and energy project weakly onto the annular modes, that is not to say that their variability is insignificant. Their primary mode(s) of variability may be independent from the annular modes. Recent work by Thompson and collaborators (Thompson and Woodworth 2014; Thompson and Barnes 2014; Thompson and Li 2015) supports the idea that the annular modes alone are insufficient at capturing the large-scale atmospheric variability. They identify a new mode of variability in each hemisphere: the northern and southern baroclinic annular modes (NBAM and SBAM), which are defined as the first EOF of extratropical eddy kinetic energy. The baroclinic annular modes are similarly annular and hemispheric in scale, but the variability they describe is largely independent of the NAM and SAM. The baroclinic annular modes are associated with variations in the growth of baroclinic eddies and have a strong signature in the eddy heat fluxes (Thompson and Woodworth 2014; Thompson and Li 2015). In contrast, the annular modes describe the shift of the zonal jet and Eulerian circulation that is tied to variations in the eddy momentum fluxes but only weakly reflected in the eddy heat flux. Since the midlatitude circulation largely comprises baroclinic eddies, which account for most of the poleward heat transport, a mode such as the baroclinic annular mode that is directly related to the eddy fluxes of sensible and latent heat could be more useful for capturing the variability of the midlatitude circulation than the annular modes. Identifying a primary mode of variability for the meridional circulation and its implications on climate change is a subject for future study.
Acknowledgments. The authors thank Edwin Gerber and Yutian Wu for helpful comments and discussions. This work was supported by the National Science Foundation under Grant AGS-0944058. This material is based upon work supported by the NYU Abu Dhabi Institute under Grant G1102.

APPENDIX A

Relationship between the Energy Transport and Circulation

The identities (8), (9), and (10) will be proved here. Note that by (1) and (5) it suffices to just prove (9) and (10). It will be shown below that

\[ M_{\xi,\text{mean}} = \int_{-\infty}^{\infty} \xi \frac{\partial \Psi_{\xi,\text{mean}}}{\partial \xi} d\xi \quad \text{and} \quad (A1) \]

\[
\int_{-\infty}^{\infty} \xi \frac{\partial \Psi_{\xi,\text{mean}}}{\partial \xi} d\xi = \int_{-\infty}^{\infty} \xi \frac{\partial}{\partial \xi} \int_{-\infty}^{\xi} \int_{0}^{m_{\text{mean}}(\phi, \hat{\xi}, \xi)} d\hat{\xi} d\xi d\xi = \int_{-\infty}^{\infty} \xi \int_{0}^{m_{\text{mean}}(\phi, \hat{\xi}, \xi)} d\hat{\xi} d\xi \]

\[
= \frac{2\pi a \cos \phi}{g} \int_{0}^{\infty} \nu f(\xi) d\xi d\hat{\xi} = \frac{2\pi a \cos \phi}{g} \int_{0}^{\nu} \xi f(\xi) d\xi d\hat{\xi}.
\]

The last line is by definition \( M_{\xi,\text{mean}} \). The second line follows by the fundamental theorem of calculus and the last line from the definition of expectation.

Now to prove (A2):

\[
\int_{-\infty}^{\infty} \xi \frac{\partial \Psi_{\xi,\text{edd}}}{\partial \xi} d\xi = \int_{-\infty}^{\infty} \xi \frac{\partial}{\partial \xi} \int_{-\infty}^{\xi} \int_{0}^{m_{\text{edd}}(\phi, \hat{\xi}, \xi)} d\hat{\xi} d\xi d\xi = \int_{-\infty}^{\infty} \xi \int_{0}^{m_{\text{edd}}(\phi, \hat{\xi}, \xi)} d\hat{\xi} d\xi
\]

\[
= \frac{2\pi a \cos \phi}{g} \int_{0}^{\nu} f(\xi) d\xi d\hat{\xi} = \frac{2\pi a \cos \phi}{g} \int_{0}^{\nu} \xi f(\xi) d\xi d\hat{\xi} = \frac{2\pi a \cos \phi}{g} \int_{0}^{\nu} \xi f(\xi) d\xi d\hat{\xi}.
\]

The steps are similar to before. The last line follows from the fact that the variance is equal to the expectation of the square minus the square of the expectation.

APPENDIX B

Comparing the Residual and Dry Circulations

Here we show that the residual circulation is a numerically close approximation to the dry circulation averaged on levels of constant DSE.

Let \( \nu \xi = \nabla - \partial(\nu / \xi) / \partial p \) and define the streamfunction such that the usual residual circulation is recovered when \( \xi = \theta \). The circulation computed using \( \xi \) equal to DSE gives a close approximation to the residual circulation (i.e., \( \Psi_{\text{TEM},\theta} = \Psi_{\text{TEM,DSE}} \)), as shown below. Pauluis et al. (2011) show that the TEM circulation is recovered from the STEM circulation in the limit of small eddy variance; that is, \( \lim_{\xi \to \theta} \Psi_{\text{STEM},\xi}(\phi, \xi(p)) = \text{sgn}(\partial\xi / \partial p) \Psi_{\text{TEM},\xi}(\phi, p) \). Thus, when \( \xi \) is equal to DSE, \( \Psi_{\text{STEM},\xi}(\phi, \xi(p)) \approx -\Psi_{\text{TEM},\xi} \approx -\Psi_{\text{TEM},\theta}(\phi, p) \), which shows that the residual circulation is a close approximation to the dry circulation. Here, \( \text{sgn}(\cdot) \) returns the
sign of its argument. There is a minus sign since $\xi$ decreases with pressure. It is assumed that the vertical coordinate $\xi$ is monotonically stratified, such that there is an invertible relationship between $\xi$ and $p$. This is an accurate assumption for the DSE and $\theta$ but not MSE.

It remains to show that $\Psi_{\text{TEM},\theta} \approx \Psi_{\text{TEM,DSE}}$. The two circulations differ only in their eddy-driven parts. In particular, the difference lies between the terms $\mathbf{v}_{\theta} \mathbf{\nabla} p$ and $\mathbf{v}_{\xi} \mathbf{\nabla} \xi$, where $\xi = \text{DSE}$. Expanding the DSE in terms of temperature $T$ and geopotential height $Z$, it can be shown that

$$\frac{\mathbf{v}_{\xi}}{\xi} \mathbf{\nabla} (T - \frac{\alpha}{c_p} p) \approx \frac{\mathbf{v}_{\xi}}{\xi} \mathbf{\nabla} (T' + \frac{\alpha}{c_p} Z')$$

since the eddy flux of geopotential height scaled by the dry adiabatic lapse rate, $g/c_p \approx 10^{-2}$, is negligible compared to the eddy heat flux. The specific volume is denoted by $\alpha$.

**REFERENCES**


