Nonlinear Response of a Tropical Cyclone Vortex to Prescribed Eyewall Heating with and without Surface Friction in TCM4: Implications for Tropical Cyclone Intensification

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ABSTRACT

The recent debate on whether surface friction contributes positively or negatively to tropical cyclone (TC) intensification has been clarified based on two idealized numerical experiments, one without and the other with surface friction, using the fully compressible, nonhydrostatic TC model, version 4 (TCM4), with prescribed eyewall heating. The results show that with surface friction included, the intensification rate of the TC vortex is largely reduced, indicating that surface friction contributes negatively to TC intensification. Results from tangential wind budgets demonstrate that although surface friction largely enhances the boundary layer inflow and the contraction of the radius of maximum wind (RMW), the positive tangential wind tendency resulting from the frictionally induced inward absolute angular momentum (AAM) transport in the boundary layer is not large enough to offset the negative tendency due to the direct frictional loss of AAM to the surface. Results from the Sawyer–Eliassen equation suggest that the balanced response to eyewall heating is the major mechanism for TC intensification and the unbalanced dynamics due to the presence of surface friction seem to spin up tangential wind in the surface layer near the RMW where the flow is strongly subgradient and spin down tangential wind immediately above where the flow is strongly supergradient. Although surface friction shows an overall net negative effect on TC intensification, it plays a critical role in producing the realistic boundary layer structure with enhanced inflow, a low-level jet in tangential wind with supergradient nature, and a shallow outflow layer at the top of the inflow boundary layer.

1. Introduction

It is well accepted that diabatic heating associated with convection in the eyewall is crucial to the intensification and maintenance of a tropical cyclone (TC). Diabatic heating in the eyewall can force a secondary circulation with radial inflow in the lower troposphere, upward motion in the eyewall, and outflow in the upper troposphere (Willoughby 1979; Shapiro and Willoughby 1982). The heating-driven inflow brings large absolute angular momentum (AAM) inward in the lower troposphere. Since the AAM is conserved in the absence of surface friction, this inward AAM transport will lead to a spinup of the tangential wind and thus the intensification of the TC vortex (Willoughby 1979; Shapiro and Willoughby 1982; Schubert and Hack 1982; Pendergrass and Willoughby 2009). As a result, the intensification of a TC can be understood as the response of the secondary circulation to diabatic heating in the eyewall.

This broad view of TC intensification has been extensively studied based on the balanced dynamics (Shapiro and Willoughby 1982; Schubert and Hack 1982). Namely, under the assumption of hydrostatic and gradient wind balance, a TC can be considered as a slowly evolving circular vortex. In this case, the response of the secondary circulation in a balanced vortex to diabatic heating can be derived by solving the...
Sawyer–Eliassen (SE) equation (Shapiro and Willoughby 1982; Schubert and Hack 1982; Pendergrass and Willoughby 2009). Shapiro and Willoughby (1982) found that the tangential wind tendency is significantly larger when diabatic heating lies inside the radius of maximum wind (RMW), leading to eyewall contraction and intensification of the storm. Schubert and Hack (1982) showed that the heating efficiency in the balanced response is proportional to the inner-core inertial stability and thus the intensity of the TC vortex because the Rossby radius of deformation, which determines the horizontal extent of the response to diabatic heating, is inversely proportional to inertial stability. These results were confirmed later by Pendergrass and Willoughby (2009) for prescribed eyewall-like heating with outward tilting.

In addition to being applied to balanced vortices with prescribed tangential wind structure, the SE equation has also been used to diagnose the secondary circulation in TC vortices from numerical simulations (Persing et al. 2002; Möller and Shapiro 2002; Montgomery et al. 2006; Fudeyasu and Wang 2011). Results from these studies have shown that the balanced solution can well capture the secondary circulation in various model simulations if both the diabatic heating rate and momentum forcing are obtained from the full-physics model simulations, even in the boundary layer where the balanced assumption is invalid. These studies thus demonstrate that TC intensification can be largely explained by the balanced dynamics.

One exception was the solution from the SE equation solver first documented and used in Bui et al. (2009) and later used in Abarca and Montgomery (2015). Both studies showed that the balanced solution largely underestimated the inflow in the boundary layer while overestimating the outflow in the upper troposphere [see, e.g., Figs. 5 and 6 in Bui et al. (2009)]. They suggested that the departure in the boundary layer from the balanced assumption plays an important role in driving the boundary layer inflow. The different results might be partially due to the use of the temperature field that is assumed to be in balance with the actual tangential wind field, which gives an artificial cold core in the boundary layer (Stern et al. 2015), and partially due to the difference in the numerical solver and/or unphysical constraints used to solve the SE equation. A direct comparison is beyond the scope of this study since we do not have the code used in Bui et al. (2009). Recently, Stern et al. (2015) quantified the relative importance of diabatic heating and surface friction in driving the secondary circulation in a full-physics simulation of a TC using a linearized vortex model [Three-Dimensional Vortex Perturbation Analysis and Simulation (3DVPAS)] of Nolan and Montgomery (2002).

They showed that 3DVPAS reproduced the secondary circulation quantitatively well and both heating and friction contributed substantially to boundary layer inflow and the contraction of the RMW. They also compared the 3DVPAS to the SE solutions and showed that, for the same idealized vortex and heating profile as that used in the SE solutions in Schubert et al. (2007) and Rozoff et al. (2008), 3DVPAS yields nearly identical results for the secondary circulation as compared to the analytical SE solutions. Since the 3DVPAS model used in Stern et al. (2015) deviates little from the balanced vortex SE model, their results restate that the secondary circulation and intensification of a TC can be explained well by the balanced dynamics.

Nevertheless, motivated by the results of Bui et al. (2009), Smith et al. (2009) proposed that the balanced response of the secondary circulation to eyewall heating mainly explains the TC size expansion above the boundary layer, while the unbalanced dynamics associated with surface friction explains the inner-core spinup in the boundary layer. They argued that although the AAM is not materially conserved in the boundary layer, large tangential wind tendency can be achieved when the radial inflow is sufficiently large to bring the air parcels to small radii with a minimal loss of AAM due to surface friction. In this case, the TC vortex can spin up when the positive AAM tendency due to frictionally induced inflow in the boundary layer is large enough to offset the negative AAM tendency due to surface friction itself. This means that the unbalanced processes associated with surface friction could contribute positively to TC intensification. This contradicts the traditional view that surface friction is an energy sink of the TC system and thus an inhibiting factor to TC intensification (Emanuel 1989; Raymond et al. 1998; Kepert 2010).

The hypothesized positive contribution of surface friction to TC intensification by Smith et al. (2009) has been recently challenged by Stern et al. (2015) as mentioned above using a linearized vortex model. Stern et al. (2015) found that the combined contribution by frictionally induced inflow and surface friction itself to the tangential wind tendency is negative and thus surface friction is negative to TC intensification. They also pointed out that since the net negative tangential wind tendency due to surface friction and its induced boundary layer inflow is maximized outside the RMW, surface friction contributes significantly to the contraction of the RMW and thus largely determines where the maximum boundary layer convergence and vertical motion, and thus the eyewall clouds and heating occur (Kepert and Nolan 2014).

In a more recent paper, Smith and Montgomery (2015) clarified that the two spinup mechanisms they
proposed earlier (Smith et al. 2009) are coupled through boundary layer dynamics and “a spinup of the winds in the boundary layer requires a spinup of the winds above the boundary layer as well.” They further stated that “Clearly, for a vortex to spin up, the convectively induced inflow must be sufficient to outweigh the frictionally induced outflow above the boundary layer.” They also criticized the use of the linearization in Stern et al. (2015) and pointed out that “one simply cannot expect to use a linear model to assess a theory founded on intrinsically nonlinear processes.” They further stated that “For all of the aforementioned reasons, Stern et al.’s use of the linear model to isolate the separate effects of diabatic heating from those of friction on the dynamics within the boundary layer has no theoretical basis, casting doubt on some of the related conclusions in their paper.” The clarification of Smith and Montgomery (2015) seems to suggest that they believe that a full nonlinear model is required to isolate the individual contributions of diabatic heating and surface friction to TC intensification.

The objective of this study is to assess whether surface friction contributes positively or negatively to TC intensification using the nonlinear TC model, version 4 (TCM4) with the heating rate prescribed to mimic convective heating in the eyewall of a TC. We will also evaluate the balanced and unbalanced responses of a TC vortex to prescribed eyewall heating from the nonlinear model simulations with and without surface friction using the SE equation. The rest of the paper is structured as follows. Section 2 briefly describes the model and experimental design. The results from numerical experiments and diagnostic analyses based on tangential wind tendency budget and the SE equation are discussed in section 3. Major conclusions are given in the last section.

2. Model description and experimental design

The model used in this study is the fully compressible, nonhydrostatic TCM4 developed by Wang (2007). Its applications to the studies of TC inner-core dynamics and structure and intensity changes can be found in Wang (2008a,b, 2009), Wang and Xu (2010), Xu and Wang (2010a,b), Fudeyasu and Wang (2011), and Li and Wang (2012a,b). The model is formulated in the mass vertical coordinate $\sigma (=\bar{p}/\bar{p}_s)$, with $\bar{p}$ and $\bar{p}_s$ being unperturbed pressure and surface pressure, respectively. The model atmosphere has 32 $\sigma$ levels from the surface to about 38-km height with relatively higher resolution in the lower troposphere and near the tropopause. The model domain is quadruply nested with two-way interactive nesting. The horizontal grid spacings of 54, 18, 6, and 2 km have $281 \times 241, 181 \times 181, 217 \times 217,$ and $271 \times 271$ grid points for the four meshes, respectively.

Although TCM4 is a full-physics model, in this study simplified model settings were used since our primary interest is in the response of a given initial TC-scale vortex to prescribed eyewall heating. The model was run without any moist processes (dry runs) and diabatic heating associated with eyewall convection was prescribed (see below). The major model physics then include an $E-\epsilon$ turbulence closure scheme (where $E$ is turbulent kinetic energy and $\epsilon$ is viscous dissipation rate) for subgrid-scale vertical turbulent mixing (Wang 2001), a modified Monin–Obukhov scheme for surface momentum flux calculation (Fairall et al. 2003) with the surface roughness length parameterized following Moon et al. (2007), a nonlinear fourth-order horizontal diffusion for all prognostic variables except for that related to the mass conservation equation, and a simple Newtonian cooling term, which is added to the perturbation potential temperature equation to mimic the radiative cooling in the model (Rotunno and Emanuel 1987). Note that because diabatic heating is prescribed, surface sensible and latent heat fluxes were turned off in this study for consistency.

The model was initialized with an axisymmetric cyclonic vortex on an $f$ plane of 18°N in a quiescent environment over the ocean. The initial vortex had a maximum surface tangential wind of 30 m s$^{-1}$ at a radius of 40 km with the maximum wind decreasing sinusoidally with pressure to vanish at 100 hPa (see Fig. 1). The initial thermodynamic structure of the unperturbed model atmosphere was defined as the western Pacific clear-sky environment given by Gray et al. (1975). The mass and thermodynamic fields of the vortex were obtained by solving the nonlinear balance equation described in Wang (2001). As mentioned above, the diabatic heating rate was prescribed to mimic convective heating in the eyewall of a TC in this study. The heating rate with its maximum value of 200 K day$^{-1}$ in the midtroposphere was centered at a radius of 30 km, namely inside the RMW, with the width of about 20 km (Fig. 1). Note that we also tested different heating rate and the initial intensity of the model TC vortex and the overall results and major conclusions are consistent with those that we will discuss in the following sections.

Two numerical experiments were conducted. In the first experiment (NFR), only diabatic heating was included and the effect of surface friction was turned off by simply setting the drag coefficient to be zero. In the second experiment (FR), both the prescribed heating rate and surface friction were activated. Vertical diffusion has been included in both experiments for
subgrid-scale vertical mixing throughout the model atmosphere. Note that the heating rate was identical and kept unchanged with time in both experiments. Therefore, we did not consider any possible feedback to diabatic heating due to circulation changes forced by surface friction. This allows us to isolate the dynamical effect of surface friction from its thermodynamic effect and thus to clarify the recent debate on the role of surface friction in TC intensification. Each experiment was run for 24 h. Our analyses below will focus on the first 3 h during which the storm intensified in both experiments.

3. Results

a. Storm evolution

Figure 2 shows the time evolutions of the maximum azimuthal-mean tangential wind speed at the lowest model level and the RMW in experiments NFR and FR, respectively, together with the maximum wind speed of the low-level jet in experiment FR. Without surface friction, the model storm intensified very rapidly in the first 4 h and then with some small oscillations in the maximum wind speed due to the breakdown and recovery cycles of the potential vorticity (PV) ring structure of the eyewall (not shown), similar to the case studied by Hendricks et al. (2014) with a shallow water model. Nevertheless, the storm kept intensifying slowly after about 24 h (not shown). The reduced intensification rate after 4 h of integration in experiment NFR seems not to be consistent with the earlier balanced dynamics, which predicts a more rapid intensification for a stronger storm because of higher inertial stability in the inner core and thus higher heating efficiency of the balanced response (Schubert and Hack 1982; Pendergrass and Willoughby 2009). This apparent inconsistency results mainly from the contraction of the RMW while the radial location of heating was fixed in the simulation. As we can see from Fig. 2b, the RMW decreased rapidly with time from 40 to about 27.5 km in the first 4 h of simulation. This means that the maximum heating rate came to be located slightly outside the RMW. After 12 h of simulation, the RMW decreased to less than 25 km. This can explain the reduced intensification rate of the storm after the rapid intensification for the initial 4 h. The contraction of the RMW can be easily understood by the balanced dynamics, which predicts the maximum response in tangential wind tendency to occur inside the maximum heating, favoring the inward shift of the RMW (Shapiro and Willoughby 1982).

With surface friction turned on in experiment FR, the storm shows a slow intensification until 6 h of simulation and reaches a steady state afterward or even weakens slightly after 12 h of simulation (Fig. 2a). This is in sharp contrast to the storm in experiment NFR, suggesting that surface friction has a negative dynamical effect on the storm intensification in the case with prescribed diabatic heating in this study. Nevertheless, surface friction contributes to a faster contraction of the RMW although the difference in the RMW between experiments FR and NFR became quite small after about 9 h of simulation (Fig. 2b). This suggests that surface friction–induced contraction of the RMW does not imply a more rapid intensification of the storm in the case without its feedback to diabatic heating. Note that the frictionally induced faster contraction of the RMW becomes not obvious above 2–3-km height (Fig. 2b).

Previous studies have shown that surface friction can lead to the development of supergradient winds (low-level jet) near the RMW near the top of the inflow boundary layer (Kepert and Wang 2001; Smith et al. 2009). We also show in Fig. 2a the maximum wind speed in the low-level jet in experiment FR. In agreement with earlier studies, this internal maximum wind speed in the boundary layer is supergradient, which is about 5%–8% of the gradient wind near the RMW at the top of the inflow boundary layer (not shown), and is about 15%–20% higher than the maximum wind speed at the lowest model level. This maximum wind speed, however, is still considerably smaller than the maximum wind speed at

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Fig. 1. The radius–vertical profiles of tangential wind (m s\(^{-1}\); contours with an interval of 5 m s\(^{-1}\)) of the initial axisymmetric tropical cyclone vortex and the prescribed diabatic heating rate (K h\(^{-1}\); shading) used in the two numerical experiments in this study.
the lowest model level in experiment NFR (Fig. 2a). This demonstrates that the existence of surface friction–induced supergradient wind could not be considered as a net positive contribution to TC intensification as proposed by Smith et al. (2009) and Smith and Montgomery (2015). If the jet intensity is used to infer intensification of the storm in experiment FR, the intensification rate is roughly 3.3 m s\(^{-1}\) h\(^{-1}\) in the first 4 h of simulation while the corresponding intensification rate in experiment NFR is about 7.0 m s\(^{-1}\) h\(^{-1}\), more than twice as large as that of the jet. This strongly suggests that surface friction could not contribute positively to the spinup of a TC vortex at least if diabatic heating in the eyewall is prescribed.

The azimuthal-mean radial wind, tangential wind, and vertical motion averaged during the 2-h period between 1 and 3 h of simulation in the two experiments are compared in Fig. 3. The immediate response of the initially balanced TC vortex to eyewall heating is the development of the secondary circulation with inflow in the mid–lower troposphere, upward motion in the region with diabatic heating, and outflow in the upper troposphere (Figs. 3a,b). Without surface friction in experiment NFR, the heating-induced inflow is maximum (~2.2 m s\(^{-1}\)) at the lowest model level and immediately outside the center of heating near the RMW. The inflow in the boundary layer is substantially strengthened with surface friction included in experiment FR (Fig. 3c), with the maximum inflow reaching 6 m s\(^{-1}\), about 2.7 times of that in experiment NFR. This confirms that surface friction can induce strong inflow in the boundary layer.

A shallow layer of outflow of about 0.5 m s\(^{-1}\) appears between radii of 30 and 110 km just above the inflow boundary layer in experiment FR (Fig. 3c), which results from the supergradient wind as discussed extensively in previous studies (Kepert and Wang 2001; Schwendike
and Kepert 2008; Zhang et al. 2011). This does not occur in experiment NFR because without the frictional effect, the response to the prescribed heating rate is largely controlled by balanced dynamics. The maximum tangential wind in experiment NFR thus appears at the lowest model level (Fig. 3b) while that in experiment FR occurs in the interior region in the boundary layer (Fig. 3d).

Strong upward motion developed in the heating region in both experiments NFR and FR (Figs. 3b,d), with the maximum upward motion occurring between 9- and 12-km height, well above the level of maximum heating shown in Fig. 1. The difference in vertical motion between the two experiments is quite small and appears only in the boundary layer (Fig. 3f). This together with the difference in radial wind shown in Fig. 3e implies a frictionally induced shallow cell in the boundary layer. This shallow cell often contributes to the gradual spin-down of the TC vortex due to its outflow (Eliassen 1971; Eliassen and Lystad 1977; Montgomery et al. 2001). The
difference fields in radial wind and vertical motion between experiments FR and NFR (FR–NFR) in Fig. 3 demonstrate that different from the effect of diabatic heating, which induces a deep secondary circulation across the entire troposphere (Figs. 3a,b), the effect of surface friction is only significant below 2-km height with substantially enhanced boundary layer inflow extending as far as 200 km from the vortex center and an outflow channel immediately above (Figs. 3e,f).

In addition, the frictionally induced inflow penetrates well inside the RMW in experiment FR. This strong inflow enhances the radial gradient of the tangential wind tendency inside the RMW (Stern et al. 2015), which could play a role in an extra contraction of the RMW as discussed above (Fig. 2b). Note that partly because the diabatic heating rate is prescribed in our experiments, the frictionally induced Ekman pumping (upward motion near the top of the inflow boundary layer near the radius of 30 km) could not penetrate into the upper levels. This decoupling with diabatic heating in our experimental design allows us to examine whether the net dynamical effect of surface friction on the intensification of the TC vortex is positive or negative through both tangential wind budget analysis and diagnostics using the SE equation in the next two subsections.

b. Tangential wind tendency budget analysis

To quantify the contributions to the spinup process, we performed the azimuthal-mean tangential wind tendency budget for the two experiments. The tangential wind tendency equation in the cylindrical coordinates can be written as (Xu and Wang 2010a,b)

$$\frac{\partial \mathbf{V}}{\partial t} = -\mathbf{U} \times \mathbf{a} - \mathbf{w} \frac{\partial \mathbf{V}}{\partial z} + \nabla \mathbf{F} - \mathbf{w} \cdot \nabla \mathbf{a} - \mathbf{v} \frac{\partial \mathbf{V}}{\partial z} + \mathbf{D}_{V},$$

(1)

where \( t \) is time and \( z \) is geometric height; \( \mathbf{U}, \mathbf{V}, \) and \( \mathbf{a} \) are azimuthal-mean radial and tangential winds and vertical motion; \( \mathbf{a} \) is the azimuthal-mean vertical absolute vorticity; \( \mathbf{u}', \mathbf{v}', \mathbf{w}', \) and \( \mathbf{v}' \) are deviations of radial, vertical, tangential winds, and vertical relative vorticity from their corresponding azimuthal-mean values; and \( \mathbf{F} \) and \( \mathbf{D}_{V} \) are vertical turbulent mixing (including surface friction) and subgrid horizontal diffusion terms. The contributions to the local tendency of the azimuthal-mean tangential wind on the right-hand side of (1) are the azimuthal-mean radial and vertical advections, the surface friction and vertical diffusion, the eddy radial and vertical advections, and the horizontal diffusion, respectively.

The azimuthal-mean tangential wind tendency budgets are conducted for both experiments NFR and FR during the intensification period between 1 and 2 h of simulations. Since the storms in both experiments are axisymmetric during the budget period, the eddy term is exactly zero. In addition, although the horizontal diffusion term would likely become substantial in the strong storm, it is generally small for the storm in the budget period and will be combined with the vertical diffusion term in the following discussion. In experiment NFR, the total tangential wind tendency shows negative tendency inside the radius of about 25 km and positive tendency outwards, with the maximum positive tendency near the center of the heating source, inside the RMW (Fig. 4a). Without surface friction, the combined subgrid vertical mixing (vertical diffusion) and horizontal diffusion term is quite small (Fig. 4e). As a result, the tangential wind tendency in experiment NFR is predominantly contributed by the mean advection term (Fig. 4c). The distribution in tangential wind tendency favors the inward shift of the RMW (or eyewall contraction) as seen in Fig. 2b. The positive tangential wind tendency around the RMW (compare Figs. 3b and 4a) manifests the intensification of the storm in response to the prescribed eyewall heating.

The total tangential wind tendency in experiment FR (Fig. 4b) shows a distribution very different from that in experiment NFR (Fig. 4a). Now with surface friction, large negative frictional tendencies (with small contribution by horizontal diffusion) appear in the boundary layer outside a radius of about 20 km (Fig. 4f). The tangential wind tendency contributed by the mean advection term is largely enhanced in the boundary layer (Fig. 4d) owing to the frictionally induced inflow (Fig. 3c). Note that some negative tangential wind tendencies in the layer between 1- and 2.5-km heights outside the radius of 45 km in Fig. 4d reflect the spindown of tangential wind due to the outflow associated with the supergradient winds as seen in Fig. 3c. Note that although the frictionally induced inflow greatly enhances the positive tangential wind tendency by mean advection (Fig. 4d) in the boundary layer compared to that in NFR, the net tangential wind tendency becomes negative outside the radius of about 45 km below 2.5-km height (Fig. 4b). Now surface friction, on the one hand, reduces values near the RMW and thus positive tangential wind tendency only appears in a narrow annular area between radii 20 and 40 km in the heating region (Fig. 4b). Surface friction, on the other hand, increases the radial gradient of tangential wind tendency inside the RMW and thus favors the RMW contraction compared with that in NFR (Figs. 4a,b). This explains why there is a faster contraction yet a slower intensification of the storm in FR than in NFR. Therefore, the result demonstrates that the spinup of
tangential wind due to the frictionally induced boundary layer inflow does not exceed the spindown of tangential wind due to surface friction itself in this case. This means that surface friction has a net negative effect on TC intensification. This is consistent with recent results of Stern et al. (2015) but is in sharp contrast to the mechanism proposed by Smith et al. (2009) as mentioned in introduction. Smith et al. (2009) proposed that the inward angular momentum transport enhanced by frictionally induced inflow in the boundary layer can be larger than the friction-induced angular momentum loss to the surface and thus surface friction can have a net positive contribution to the spinup of the inner core in the boundary layer. Our results have shown that this is not so. This can be seen more clearly from Fig. 5, which shows the tangential wind tendency due to radial advection in NFR (Fig. 5a) and FR (Fig. 5b), respectively, and their difference (Fig. 5c), as well as the sum (Fig. 5d) of the difference and the negative tendency due to surface friction itself (Fig. 4f). For a complete comparison, we also show the difference in the tangential wind tendency due to the total advection between FR and NFR in Fig. 5e and its combination with the negative tendency due to surface friction itself in Fig. 5f. These results demonstrate that the net effect of surface friction on TC intensification in the simulation is negative but contributes to the faster contraction of the RMW.

Note that one may argue that in the budget period the boundary layer inflow in experiment FR is still strengthening, namely the boundary layer is still under the development stage. As a result, the contribution of the inward angular momentum transport by the frictionally induced inflow could be underestimated. This is actually
true in the first hour of simulation but is not the case after about 2–3 h of simulation since the boundary layer is well developed after 2–3 h of simulation in experiment FR. This can be seen in Fig. 6, which compares the difference in hourly averaged tangential wind tendency from the budget (Fig. 6, left) and from that directly calculated from the model outputs between experiments FR and NFR in the first 3 h of simulations. As a reference, the hourly averaged tangential wind tendency in experiment NFR is also shown in Fig. 6. Two points need to be mentioned. First, the tangential wind tendency budget shows high accuracy compared with that directly calculated from the model outputs. Second, the negative tangential wind tendency difference outside the radius of 32.5 km in the boundary layer is quite large in the first hour of simulation (Figs. 6a,b). This is mainly because the frictional inflow in the first hour is under the spinup phase and the frictional angular momentum loss to the surface dominates the spinup as a result of the frictionally induced inflow. As the boundary layer inflow spins up, the negative tangential wind tendency difference outside the radius of 30 km is largely reduced but still negative (Figs. 6c,d). Nevertheless, the large negative difference appears between radii of 25 and 30 km and positive difference within the radius of 25 km. This pattern does not change much during 1–2 and 2–3 h of simulations (Figs. 6c–f), indicating that the boundary layer is well developed after about 1 h of simulation. This rapid spinup of the boundary layer is primarily due to high inertial stability in the inner core of the TC vortex (Eliassen and Lystad 1977; Kepert and Nolan 2014).

The overall effects of surface friction on the storm intensification and the inner-core contraction in the boundary layer can be more clearly seen from Fig. 7, which shows the hourly averaged tangential wind and the corresponding tangential wind tendency from budget analysis in both
experiments NFR and FR. In experiment NFR, the tangential wind and its tendency are both maximal at the lowest model level with the maximum positive tangential wind tendency located inside the RMW, while weak negative tangential wind tendencies occur in the eye region. The distributions of tangential wind and its tendency imply the formation of a ring-shaped vorticity structure and indicate the intensification of the storm and the contraction of the RMW in response to diabatic heating centered at the radius of 30 km. This is consistent with earlier studies based on balanced dynamics (Shapiro and Willoughby 1982; Schubert and Hack 1982; Hack and Schubert 1986; Pendergrass and Willoughby 2009).

With surface friction included in experiment FR, a typical structure in the boundary layer, similar to that observed and simulated in idealized boundary layer and
full-physics models (Kepert and Wang 2001; Wang 2001; Kepert 2006a,b; Schwendike and Kepert 2008), is developed very quickly in the simulation. Now, throughout the layer from the surface to about the 3-km height, the tangential wind speed in experiment FR is much weaker than that in experiment NFR (Figs. 3b,d), indicating the overall negative effect of surface friction on intensification of the storm. Nevertheless, the storm shows a tangential wind maximum, often referred to as low-level jet (e.g., Kepert and Wang 2001; Schwendike and Kepert 2008), at about 400-m height in the boundary layer, which is about 4.5%–6.0% supergradient during the first 3 h of simulation (not shown). Note that with surface friction, the maximum positive tangential wind tendency shifts farther

Fig. 7. The radial–vertical cross sections of the hourly averaged tangential wind speed (m s\(^{-1}\); shading), overlaid with the hourly averaged tangential wind tendency derived from the tangential wind tendency equation (m s\(^{-1}\) h\(^{-1}\); contours with an interval of 2 m s\(^{-1}\) h\(^{-1}\)) in experiments (a),(c),(e) NFR and (b),(d),(f) FR with the time average given above each panel.
inward into the eye region. This leads to relatively smaller positive tangential wind tendencies near the RMW and also a faster contraction of the RMW in the lower portion of the boundary layer. The results in Fig. 7 thus demonstrate that even though surface friction plays a role in spinning down a TC, it is important to the more realistic storm structure in the boundary layer and thus is a key to TC dynamics.

c. Balanced and unbalanced contributions

It is our interest in this subsection to further investigate the balanced and unbalanced responses of the vortex to diabatic heating and surface friction discussed above. Following many previous studies (Bui et al. 2009; Smith et al. 2009; Fudeyasu and Wang 2011), the balanced response will be obtained by solving the SE equation and the unbalanced response is defined as the difference between the total response and the balanced response. This latter is a little bit dangerous because the residual might be related to numerical errors. However, we will show below that in the absence of surface friction the balanced solution can recover well the full nonlinear solution from TCM4, suggesting that the definition of the unbalanced contribution would be acceptable. Here we used the SE equation in height coordinates as given in Bui et al. (2009). The diagnostic equation for streamfunction is written as

\[ \frac{\partial}{\partial r} \left[ -g \left( \frac{\partial \chi}{\partial z} \frac{1}{\rho r} \frac{\partial \tilde{\psi}}{\partial r} - \frac{\partial}{\partial z} \left( \chi C \frac{1}{\rho r} \frac{\partial \tilde{\psi}}{\partial z} \right) \right] + \frac{\partial}{\partial z} \left( \xi \chi \tilde{\psi} + C \frac{\partial \chi}{\partial r} \frac{1}{\rho r} \frac{\partial \tilde{\psi}}{\partial z} - \frac{\partial}{\partial z} \left( \chi C \frac{1}{\rho r} \frac{\partial \tilde{\psi}}{\partial z} \right) \right) \]

\[ = g \frac{\partial}{\partial r} (\chi^2 Q) + \frac{\partial}{\partial z} (C \chi^2 Q) - \frac{\partial}{\partial z} (\chi F), \]

where \( r \) and \( z \) are radius and height; \( g \) is the gravitational acceleration; \( \tilde{\psi} \) is the azimuthal-mean transverse streamfunction given by \( \tilde{\psi} = - (1/r \rho) \partial \tilde{u}/\partial z, \tilde{w} = (1/r \rho) \partial \tilde{v}/\partial r; \tilde{\psi} \) is the azimuthal-mean radial wind; \( \tilde{w} \) is the azimuthal-mean vertical velocity; and \( \rho \) is the mean density at height \( z \), \( \chi = 1/\theta \), where \( \theta \) is potential temperature, \( C = \tilde{u}^2/r + f\tilde{v} \) denotes the sum of centrifugal and Coriolis forces, where \( \tilde{\psi} \) is the azimuthal-mean tangential wind and \( f \) is the Coriolis parameter, and \( \xi = 2\pi r + f \) is the local Coriolis parameter. Terms on the rhs of (2) are those due to diabatic heating (first and second terms) and momentum forcing (third term). Diabatic heating includes both the azimuthal-mean eddy heat fluxes and azimuthal-mean diabatic heating rate: \( Q = u' \partial \tilde{\psi}/\partial r - w' \partial \tilde{\psi}/\partial z + \theta \) and the momentum forcing includes the azimuthal-mean eddy momentum fluxes and the azimuthal-mean surface friction, vertical turbulent mixing, and horizontal diffusion of tangential wind: \( F = -u' \tilde{\psi} - w' \partial \tilde{\psi}/\partial z + F_v + D_v. \) Note that the overbar represents the azimuthal mean and the prime indicates the deviation from the azimuthal mean. As we mentioned earlier, in the first 3 h of simulation, the storm is axisymmetric and thus all prime terms are zero.

As in previous studies, the coefficients in the SE equation are obtained from the time mean of the TCM4 outputs with 6-min intervals and the forcing terms are the prescribed diabatic heating rate and the tangential wind tendency due to surface friction and turbulent vertical mixing as well as horizontal diffusion directly from the model simulation. Note that in our calculations, the discriminant condition for ellipticity is not satisfied on some grid points. To assure the ellipticity of the SE equation for converged numerical solutions, we simply set absolute vorticity to be a small positive value (10^{-6} s^{-1}) at the grid points where absolute vorticity is negative. The SE equation is solved numerically using the successive overrelaxation (SOR) method (Press et al. 1992) with radial grid spacing of 2 km and vertical grid spacing of 500 m in a domain extending from the storm center to a radius of 260 km and from the surface to a height of 18 km. The boundary conditions for the SE equation are the Neumann boundary condition \( \tilde{\psi} = 0 \) at \( z = 0 \) and \( z = 18 \) km and the Dirichlet boundary condition \( \partial \tilde{\psi}/\partial r = 0 \) at \( r = 260 \) km. Similar to the tangential wind tendency budget conducted in section 3b, the SE equation diagnostics is applied to the hourly mean vortex structure and forcing during the intensification phase (1–2 h of simulation) in both experiments NFR and FR.

Figure 8 compares the radial wind and vertical motion diagnosed from the SE equation with those from the TCM4 simulation in experiment NFR. Note that in experiment NFR, the vertical and horizontal diffusion term is very small (Fig. 8c) and contributes little to the secondary circulation in the SE solution (not shown). We can see that the balanced SE solution reproduces quite well the upward motion in the heating region, the inflow in the mid–lower troposphere, and outflow in the upper troposphere, respectively (Figs. 8a,b). The tangential wind tendencies obtained from the budget equation [(1)] using the radial wind and vertical motion from the model outputs and those diagnosed from the SE equation show very similar patterns and magnitudes (Figs. 8c,d) with the residual (Fig. 8f) more than an order of magnitude smaller than the total tendency itself (Fig. 8e). This confirms that in the absence of surface friction the response of the TC vortex to the prescribed diabatic heating in the model simulation is largely a balanced response with unbalanced contribution being quite small. Note that the relatively large negative residual around radius of 25 km below 1-km height results mainly from the slightly underestimated outflow in the eye region as inferred from the difference in negative
tangential wind tendencies in the eye region between Figs. 8c and 8d. This residual indeed is balanced by the small term related to horizontal and vertical diffusion (Fig. 8e).

Since the SE equation assumes both gradient wind and hydrostatic balances, it should be used with caution in the boundary layer when surface friction is included. However, linearized models of TC boundary layer still show values (Kepert 2013) and the SE solution has also been previously used to evaluate the balanced contribution to TC intensification in full-physics model simulations (e.g., Hendricks et al. 2004; Bui et al. 2009; Fudeyasu and Wang 2011; Abarca and Montgomery 2015). Here, the SE solution is used to diagnose the balanced response of the TC vortex in experiment FR as a reference only. As done in Bui et al. (2009), we will first evaluate the total balanced response to both diabatic heating and momentum forcing and unbalanced residual and then the individual responses to diabatic heating and momentum forcing, respectively.

Figure 9 shows the radial wind and vertical motion in the model simulation and its balanced component diagnosed from the SE equation in experiment FR. We can see that overall the balanced SE solution reproduces
the radial wind and vertical motion throughout the troposphere reasonably well (Figs. 9a,b), even in the boundary layer as zoomed up for region from the surface to 3-km height shown in Figs. 9e and 9f. This demonstrates that even with surface friction included, the nonlinear response of the TC vortex to both the prescribed diabatic heating and surface friction can still be largely captured by the linear balanced dynamics. This is in sharp contrast to Bui et al. (2009) but consistent with Stern et al. (2015). Since the SE solution can reproduce the nonlinear response well in our case, we can examine the individual contributions of diabatic heating and momentum forcing to the transverse circulation by solving the SE equation twice, respectively, with diabatic heating only and momentum forcing only in experiment FR. (e),(f) The radial wind in the lowest 3 km of the model atmosphere from (a) and (b), respectively, is zoomed in for a close comparison in the boundary layer.
heating in experiment FR is basically similar to that in NFR expect for stronger inflow below 0.5 km. This might be a result of smaller inertial stability due to much weaker tangential winds in the lower part of the boundary layer in the presence of surface friction in FR (Fig. 3d). Nevertheless, the outflow in the upper troposphere in the balanced response to diabatic heating in experiment FR is also very similar to that in experiment NFR. This implies that the frictional effect occurs mainly in the boundary layer in our simulation with enhanced inflow in the boundary layer and a shallow outflow layer immediately above. This is true as we can see from Fig. 9d for the SE solution with momentum forcing only. As expected, surface friction contributes significantly to the boundary layer inflow and the outflow near the top of the boundary layer with negligible effect above the boundary layer (Fig. 9d).

A comparison of Figs. 9e and 9f indicates that the inward penetration of boundary layer inflow into the region of eyewall heating in the SE solution (Fig. 9b) results mainly from the effect of surface friction (Fig. 9d). Furthermore, the maximum boundary layer inflow in the balanced response occurs at around the 40-km radius (Fig. 9f), about 5 km outside that in the nonlinear model response (Fig. 9e), suggesting that the unbalanced dynamics contributes to the inward penetration of the boundary layer inflow. Note that diabatic heating in the eyewall contributes to the boundary layer inflow significantly and thus the spinup of tangential wind in the boundary layer as well as the contract of the RMW as mentioned earlier (Fig. 2b). This is in contrast to Bui et al. (2009), who stated that diabatic heating in the eyewall contributes mainly to the spinup of tangential wind above the boundary layer while the surface friction–induced boundary layer inflow predominantly spins up the tangential wind in the boundary layer.

Similar to that done for experiment NFR, the balanced contribution to the tangential wind budget is estimated using the radial wind and vertical motion diagnosed from the SE equation. Figure 10 shows the tangential wind tendencies due to advection by the balanced flow induced by both diabatic heating and momentum forcing, diabatic heating only, and momentum forcing only, respectively. Overall, the pattern of the tangential wind tendency due to advection by the balanced flow (Fig. 10b) is very similar to that calculated from the nonlinear model outputs (Fig. 10a). The difference between the nonlinear model tendency and the balanced tendency is significantly large only in the inflow boundary layer below 1.5-km height (Fig. 10f). Since the SE solution captures only the balanced response as indicated above, it is reasonable to consider that the difference reflects predominantly the unbalanced contribution to the tangential wind tendency in the presence of surface friction (note that the difference only appears in the total advection term). We can see from Fig. 10f that the unbalanced contribution is important only near and slightly inside the RMW in the inflow boundary layer. It appears that the unbalanced dynamics spins up the tangential wind in the surface layer near the RMW where the flow is strongly subgradient while spins down the tangential wind immediately above where the flow is strongly supergradient. The results are physically consistent with the assumption of gradient wind balance in the balanced SE solution. Note that since the unbalanced contribution to the tangential wind tendency is quite small above the boundary layer, we can consider that the unbalanced dynamics seem not to contribute to the overall intensification of the storm but contributes to the adjustment of the boundary layer to surface friction.

Figures 10c and 10d show the individual balanced contributions to tangential wind tendency due to advection by diabatic heating and momentum forcing in experiment FR, respectively. Diabatic heating contributes to the spinup of tangential wind throughout the troposphere except for the spindown in the eye region in the lower troposphere (Fig. 10c), very similar to that in experiment NFR (Fig. 8d). The frictionally induced flow contributes to the spinup of tangential wind in the boundary layer significantly but the spindown above (Fig. 10d). However the positive tangential wind tendency contributed by the frictionally induced inflow is considerably smaller than the negative tangential wind tendency due to surface friction (and its related vertical mixing) itself (Fig. 4f), giving rise to a net negative tangential wind tendency near the RMW in the boundary layer with a small positive region well inside the RMW (Fig. 10e). This is in contrast to the argument of Bui et al. (2009) and Smith et al. (2009) as mentioned in section 1. Since the unbalanced residual is mainly due to the presence of surface friction (but without the tangential wind tendency from the direct frictional effect), the sum of the unbalanced residual and the tangential wind tendency related to the net frictional effect can be considered as the total frictional effect on the intensification of the storm (Fig. 11). We can see from Fig. 11 that the total frictional effect contributes little to the intensification of the storm while it favors the contraction of the RMW, consistent with the results from the tangential wind budget discussed in section 3b.

Our results above suggest that the unbalanced, frictional effect contributes little to the TC intensification and acts to reduce the final TC intensity. This is true at least in our case where any feedback to diabatic heating due to surface friction is ignored. We argue that even the boundary layer is strongly coupled with eyewall convection, the unbalanced, frictional effect might not
increase the intensification rate or the final intensity of a TC to any significant extent since the frictional dissipation is the only major energy sink to a TC system in the absence of any unfavorable environmental effects. This also suggests that surface friction might not be a spinup mechanism for TC intensification although it is critical to the formation of the observed boundary layer structure in the inner core of real TCs. The unbalanced dynamics seem to contribute to the adjustment of the boundary layer in the inner-core region where significant subgradient and supergradient flows exist. We should point out that our results discussed above were obtained based on experiments with prescribed eyewall heating centered at a fixed radial location and thus any possible role of surface friction in modifying the magnitude and radial location of eyewall heating was ignored. The possible contribution to TC intensification by friction-induced feedback to eyewall heating will be studied in a future work.

4. Conclusions and discussion

In this study, the response of a TC vortex to the prescribed heating rate in the eyewall with and without surface friction has been studied using the nonhydrostatic, fully compressible nonlinear model—TCM4. The main
The purpose of the study was to clarify the dynamical contribution of surface friction to the intensification of a TC. The experimental design with the prescribed heating rate was attempted to exclude any feedback of surface friction to diabatic heating so that the dynamical role of surface friction in TC intensification can be isolated. Two experiments were conducted: one with the prescribed diabatic heating rate only (NFR) and the other with the same prescribed diabatic heating rate and also surface friction (FR).

The results show that the storm without surface friction intensified continuously in 24 h of simulation with higher intensification rate in the first 3 h because the prescribed heating rate was maximum inside the RMW. The RMW contracted rapidly during the first 3 h of simulation. The intensification rate decreased afterward because the prescribed heating rate became slightly outside of the RMW instead of inside the RMW because of the contraction of the RMW. Both the contraction of the RMW and the intensification of the storm can be well explained by the balanced dynamics. Namely, the response of the tangential wind tendency to diabatic heating in the eyewall is maximal inside the RMW and thus a contraction of the RMW. The response of the secondary circulation to the prescribed eyewall heating in this case can be well reproduced by the balanced dynamics using the SE equation.

With surface friction included, the intensification rate of the storm was largely reduced and the storm intensified only in the first 4 h of simulation and quickly reached a near-steady-state evolution with a much weaker intensity than the storm in the experiment without surface friction. This indicates that surface friction contributed negatively to TC intensification and final intensity. Results from the tangential wind tendency budget show that although surface friction contributed to a faster contraction of the RMW, it reduced the tangential wind tendency significantly near and outside the RMW, suggesting that the tangential wind tendency enhanced by the frictionally induced inward transport of angular momentum in the boundary layer was smaller than the loss of angular momentum due to surface friction. As a result, the net effect of surface friction was to spin down the storm in our case with prescribed eyewall heating. This is further confirmed with the diagnosed balanced and unbalanced contributions to the tangential wind budget using the SE equation. Our results suggest that the unbalanced dynamics due to the presence of surface friction has little effect on tangential wind tendency above the boundary layer, while it contributes to a spinup of tangential wind in the surface layer near the RMW where the flow is strongly subgradient and a spin-down immediately above where the flow is strongly supergradient. We also show that the positive tangential wind tendency due to the frictionally induced inflow could not offset the negative tangential wind tendency directly due to surface friction near the RMW, suggesting a net negative effect of surface friction on TC intensification. Since the unbalanced contribution becomes significant only in the presence of surface friction, we thus proposed that contributions by the unbalanced flow and surface friction can be combined as the total frictional effect, which seems to contribute little to TC intensification but contributes to the contraction of the RMW.

Although surface friction contributed negatively to the intensification of the storm, the storm in the simulation with surface friction developed a realistic boundary layer structure, including the much stronger boundary layer inflow and a supergradient low-level tangential wind jet near the top of the inflow boundary layer. This latter is a distinct feature of the boundary layer of a well-developed TC as in observations and in full-physics numerical simulations. The results also showed that with the prescribed eyewall heating, the frictional effect is significant only in the boundary layer and has little penetrative effect into the free atmosphere above while this might not be the case in real TCs, in which eyewall heating and surface friction are coupled.

The negative contribution of surface friction to TC intensification found in this study contradicts the positive contribution hypothesis of Smith et al. (2009). They proposed that the unbalanced dynamics due to surface friction can be considered as a second spinup mechanism of the inner core in the boundary layer of a TC. They argued that although AAM is not materially conserved in the boundary layer, large tangential wind tendency can be achieved if the frictionally induced inflow is...
sufficiently strong to bring the air parcels to small radii with minimal direct loss of AAM due to surface friction. Our results show that this is not the case since the positive tangential wind tendency due to inward transport of AAM by the frictionally induced inflow could not offset the negative tangential wind tendency directly caused by surface friction in our simulations.

The results from this study is in agreement with the recent study of Stern et al. (2015), who also concluded that the positive tangential wind tendency due to the frictionally induced inflow could not offset the negative tangential wind tendency due to surface friction itself. Smith and Montgomery (2015) recently criticized that Stern et al.’s results were based on a linear model where the solution is valid only for small perturbations to the basic vortex in gradient and hydrostatic balance but invalid for highly nonlinear problems in the boundary layer. In this study with the full nonlinearity included, the result demonstrates that surface friction does contribute negatively or little to TC intensification although it plays a critical role in TC inner-core dynamics in the boundary layer.

Finally, we should point out that the possible feedback of the boundary layer processes to diabatic heating in the TC eyewall has not been considered since the eyewall heating is prescribed and centered at a fixed radial ring in this study. In reality, surface friction contributes to faster eyewall contraction and may affect the radial location of eyewall heating. Possible effect of the friction-induced feedback to eyewall heating on TC intensification needs to be examined in a future study. Nevertheless, we consider that although such a feedback may affect the intensification rate of a TC, it may suppress the negative contribution to TC intensification to some degree but could not turn the negative to positive contribution to TC intensification. As we mentioned earlier, since surface friction is the only momentum sink to the TC system, the effect of surface friction and the associated unbalanced dynamics could not be the dominant mode of TC intensification.

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