Secondary Eyewall Formation in Tropical Cyclones by Outflow–Jet Interaction

YI DAI, SHARANYA J. MAJUMDAR, AND DAVID S. NOLAN
Rosenstiel School of Marine and Atmospheric Science, University of Miami, Miami, Florida

(Manuscript received 8 November 2016, in final form 17 March 2017)

ABSTRACT
This study uses idealized numerical simulations to show that the interaction between tropical cyclones and a midlatitude jet can result in secondary eyewall formation. It is argued that the eddy activity by the outflow–jet interaction can enhance the upper-level outflow, thereby creating an asymmetric stratiform region outside of the primary eyewall. Numerous long-lasting deep convective cells are able to form in the stratiform cloud, creating forcing necessary for the secondary eyewall. The low-level inflow and the TC’s primary circulation advect the deep convective cells inward and cyclonically. The secondary eyewall forms after the deep convection has surrounded the TC. In contrast, numerical simulations without the jet do not show secondary eyewall formation. For moderately strong jets of wind speed 15–30 m s$^{-1}$, there is little sensitivity to the jet strength. There is sensitivity to the distance between the jet and the TC, with secondary eyewall formation evident when their separation is 15° latitude but not when the separation exceeds 20°.

1. Introduction
The role of the environment in influencing tropical cyclone (TC) genesis, track, and structure has been an active area of research for several decades. Common examples include the direct influence by vertical wind shear, air–sea interactions, and low-humidity environmental air on the TC (e.g., Emanuel 1986; DeMaria 1996; Thorncroft and Hodges 2001). In addition, and of importance in this paper, the environmental flow can influence the TC structure by first interacting with the TC outflow, which is the dominant upper component of the TC secondary circulation. Because of its low inertial stability, the outflow often expands thousands of kilometers radially, thereby enabling the environmental flow to affect the TC even when it is far from the TC.

Some ideas on outflow structure have been proposed. Chen and Gray (1985) suggested that the outflow exhibits one of three patterns: single channel, double channel, and no channel, depending on the relative location of the TC and its environmental flow. A numerical modeling study by Shi et al. (1990) revealed that the simulated outflow layer was dominated by a narrow outflow jet, which itself was surrounded by secondary circulations. They also found that adding a forcing that accelerates the outflow within 500 km radially might intensify the TC, while accelerating the outflow outside of 500 km will weaken it. More recently, Rappin et al. (2011) argued that the weak inertial stability in the outflow layer was more favorable for the development of the TC secondary circulation, leading to rapid intensification to the maximum potential intensity. They found that TCs at lower latitudes (with lower inertial stability) strengthened more rapidly; however, the recent numerical study of Zhou (2015) argued that the conclusions of Rappin et al. (2011) did not hold when there existed vertical wind shear or different environmental soundings.

The influence of the environment on the outflow structure has also been investigated in a limited manner. Chen and Gray (1985) suggested that the sudden change of the outflow channel was associated with the upper-level environmental flow. Ooyama (1987) used a simple shallow-water model to demonstrate that the single-channel, double-channel, and no-channel outflow patterns could be induced by different configurations of the environmental flow. It has also been suggested that the environmental flow is important not only because the outflow can be shaped by it, but also because outflow–environment interactions can influence the TC structure and intensity. For example, Holland and Merrill (1984) proposed that the structure change of a TC could be ultimately regulated by interactions between the outflow and its external environmental flow. Two studies demonstrated that the outflow–environment interaction is indicated by the eddy flux convergence of angular momentum (EFC). Pfeffer and Challa (1981) used numerical
models to conclude that the inward EFC was correlated with intensifying storms. Molinari and Vollaro (1989) argued that the rapid intensification of Hurricane Elena (1985) was caused by interaction between the outflow and an approaching upper-level trough, which led to inward EFC. While the environmental flow can be beneficial to TC development, it can also be detrimental. In a composite study of the interaction between the TC and upper-level troughs using reanalysis data, Hanley et al. (2001) found that a “good trough” (favorable for TC intensification) can either be related to a small-scale potential vorticity maximum approaching the TC center or, alternatively, a TC center coupled with upward motion of the secondary circulation of the outflow jet (Shi et al. 1990). On the other hand, the “bad trough” is mainly due to the detrimental effects of the enhanced vertical wind shear. However, using a larger sample of reanalysis data and a different metric that normalizes TC intensity, Peirano et al. (2016) found that TC–trough interactions were not as favorable as described by Hanley et al. (2001). Moreover, they found that the trough-induced EFC was not as important as the trough-induced shear in terms of predicting the TC intensity change.

In this paper, we hypothesize that the environmental flow also plays an important role in secondary eyewall formation (SEF). The SEF is often associated with TC intensity and structure change and thus is important to TC evolution (Willoughby et al. 1982; Houze et al. 2007; Sitkowski et al. 2011). We first note that there are several hypotheses suggesting that the SEF can be explained by internal dynamics. Examples of internal dynamics include vortex Rossby waves (Montgomery and Kallenbach 1997), beta-skirt axisymmetrization (Terwey and Montgomery 2008), and unbalanced boundary layer forcing (Huang et al. 2012). Idealized modeling of the SEF in a quiescent f-plane environment, such as by Terwey and Montgomery (2008), Qiu et al. (2010), and Wang et al. (2016), further supported the idea that SEF can be produced without external forcing.

Next, we suggest that even if the SEF is able to arise by internal processes, external forcing can still influence whether SEF may or may not occur. The fact that a real TC always experiences some type of external forcing (such as β effect, upper-level westerly jet, and upper-level ridge or trough) suggests that we should also consider their roles in SEF. Moreover, both observational and modeling studies have found that SEF can be triggered by external forcing under certain circumstances. Molinari and Vollaro (1990) argued that an observed secondary wind maximum located about 120 km from the TC center might be due to the interaction between the TC and an upper-level trough. Using axisymmetric models, Nong and Emanuel (2003) proved that the simulated TC could undergo SEF by upper-level environmental flow through the wind-induced surface heat exchange (WISHE; Emanuel 1991) mechanism. The asymmetric process matters too: both the numerical studies of Fang and Zhang (2012) and Rozoff et al. (2012) illustrated the importance of β shear on SEF. They argued that the β-shear-induced asymmetric stratiform cloud is the key feature to initiate SEF. Interestingly, Rozoff et al. (2012, p. 2642) argued that “any forcing mechanism that produces sufficiently strong and sustained latent heating outside of the primary eyewall will promote SEF.” This argument is supported by numerical simulations of the external forcing mechanism, such as increased humidity (Hill and Lackmann 2009) and enhanced latent heating (Wang 2009). Also, Leroux et al. (2013) used an 8-km-resolution operational model to reveal that SEF was associated with inward EFC and eddy potential vorticity fluxes induced by TC–trough interaction.

Given the above discussion, it is likely that the internal and external processes that result in SEF are closely connected. The external forcing provides a favorable environment for SEF, and the internal forcing makes it happen. In this paper, an asymmetric, three-dimensional numerical model that captures inner-core dynamics is used to study this possibility. We prescribe an upper-tropospheric westerly jet to provide the external forcing to mimic a simplified version of a jet in the real world. To better clarify the specific effect of the TC–jet interaction on SEF, the model is configured to eliminate other possibilities that can affect SEF [such as β shear; Fang and Zhang (2012)].

The rest of the paper is arranged as follows. Sections 2 and 3, respectively, describe the model configuration and an overview of the simulations. In section 4, key processes related to the TC–environment interaction and SEF are illustrated. Results from sensitivity experiments to assess the robustness of the results are shown in section 5. Finally, concluding remarks are given in section 6.

2. Model configuration

The model design is nearly identical to the balanced shear simulations of Nolan (2011). The model used here is the Weather Research and Forecasting (WRF) Model, version 3.4.1 (Skamarock et al. 2008). It uses a vortex-following two-way interactive nesting method. The model contains three domains with horizontal grid spacing of 18-, 6-, and 2-km; domain sizes of 10 800 km × 7200 km, 1152 km × 1152 km, and 778 km × 778 km; and grid points of 600 × 400, 192 × 192, and 384 × 384, respectively. The outermost domain is selected to be sufficiently large to capture the outflow–environment interaction. The innermost domain possesses a high-enough resolution.
to capture TC inner-core dynamics and a sufficiently large size to capture signals of outflow–environment interaction. We chose 40 equally spaced vertical levels in the WRF normalized hydrostatic pressure coordinate between the surface and approximately 20-km height. For lateral boundary conditions, zonally periodic boundary conditions are used, while in the north and south boundary we apply the free-slip walls. The model is integrated for 7 days.

The Yonsei University (YSU) scheme (Hong et al. 2006) is used for the planetary boundary layer. Radiation is neglected to suppress convection on the outer domain, as in Nolan (2007). For the microphysics parameterization, the WRF single-moment 5-class scheme (WSM5) is adopted (Hong et al. 2004). The cumulus parameterization is not included even for the outermost domain, because when the convection scheme is used, anomalously large convection appears along the nested grid boundaries. Furthermore, the idealized westerly jet will be contaminated (not shown) when the convection moves to where the jet is located.

The initial condition contains a modified Rankine vortex with the decay parameter $\alpha = 0.4$. The initial vortex has a peak tangential wind speed of 20 m s$^{-1}$ at the radius of maximum winds (RMW) of 90 km. The vertical profile of the vortex is similar to that of Nolan (2007) and has a Gaussian-like decay with the maximum wind speed at $z = 1500$ m. A zonally straight westerly jet, located to the north of the vortex, is in thermal wind balance with a narrow baroclinic zone. This jet structure is adopted from Simmons and Hoskins (1977), except that we only chose the meridionally sharper case (such as their Fig. 1b). All the simulations are set on an $f$ plane with $f_{cor} = 6.16 \times 10^{-5}$ s$^{-1}$ (Coriolis parameter at 25°N), because on the $\beta$ plane there are two factors that can confuse our results. One is that the TC-jet interaction on the $\beta$ plane can result in downstream development by a planetary Rossby wave (Riemer et al. 2008); the other is the $\beta$-shear effect, which might cause SEF in a quiescent environment (Fang and Zhang 2012). The Dunion (2011) “moist tropical” sounding is used for the initial thermodynamic structure. The sea surface temperature (SST) is first set as 28°C uniformly in the domain. Then it is adjusted by thermal wind balance to have a meridional gradient at the location of the westerly jet. The vertical cross section of the jet and vortex at the initial time can be found in Fig. 1.

The jet is set far enough from the TC in order not to have a detrimental effect on TC intensity (e.g., strong vertical wind shear), but close enough to be able to interact with the outflow. For the simulation that we define as the control, the vortex is located at $\sim$25°N, while the westerly jet is centered at $\sim$40°N with maximum wind speed of 35 m s$^{-1}$ near 200 hPa. Note that the model is in an $f$ plane centered at 25°N, so the westerly jet at 40°N here is only used as an indicator of the distance between the center of TC and the jet (1500 km). As shown in Fig. 1, while shear variance exists meridionally (e.g., 8 m s$^{-1}$ at 500 km north of the TC center between 200 and 850 hPa and 0 m s$^{-1}$ everywhere south of the TC center), the average initial vertical wind shear over the TC is about 1.1 m s$^{-1}$ within 500 km of the TC center and remains small through the entire simulation because the track of the simulated TC is eastward and slightly southward as a result of the effect of the westerly jet. The largest vertical wind shear is 6.5 m s$^{-1}$ at $f = 72$ h, when the enhanced outflow is highly asymmetric. Then the shear decreases quickly to about 2 m s$^{-1}$ until the end of the simulation. Hence, the calculated vertical wind shear here is just a result of the dynamical response of the TC outflow to the westerly jet. In this study, the westerly jet itself can be thought as a type of environmental vertical wind shear, with spatial variance. It is interesting to show that, even though the conventionally defined environmental vertical wind shear is quite low in this study, the spatially varying shear (the westerly jet) can still have a significant impact on both the TC structure and intensity via the jet–outflow interaction. Experiments also showed that the jet is stable in the 7-day model run, except for a local increase of zonal wind where the jet interacts with the TC outflow. If there is no initial TC, the jet can last at least 10 days without changing shape (not shown).

Besides the control simulation, we have made several perturbed simulations in section 3, where a very small
random perturbation is added into the initial wind field. In addition, the sensitivity experiments that will be shown in section 5 have the same model design with the control simulation, except for the varied TC–jet distances or jet strengths (details to be described below).

3. General behavior of the simulated TCs

To illustrate the TC–jet interaction, Fig. 2 shows the domain-scale field of the jet interaction with the TC. The horizontal speed at $z = 12\text{ km}$ is used to represent the TC outflow here. At $t = 66\text{ h}$, the single outflow channel is already greatly enhanced to the north of the TC. Outside the inner core of the TC there is also an enhanced wind speed region to the northwest of the TC that is connected to the main outflow channel (detailed figures will be shown below). The local wind speed of the jet also increases as a result of the TC–jet interaction.

For the control simulation (CTL), the SEF and eye wall replacement cycle (ERC) occur at around 85 h (Fig. 3). Before 73 h, wavenumber-1 asymmetric rainband-type precipitation is persistent in the northwest quadrant of the TC center. The radius of the primary eyewall is about 30 km. This asymmetric precipitation is gradually advected cyclonically after 73 h. At around 81 h, convection has nearly wrapped around the TC center. When the precipitation is mainly symmetric, the secondary eyewall forms, and a concentric eyewall develops at 85 h. After 85 h, the ERC process starts: the secondary eyewall moves inward and the primary eyewall collapses correspondingly. At the same time, the intensity of the secondary eyewall becomes stronger than that of the primary eyewall.

To better describe the SEF and ERC processes, hourly plots of the potential vorticity (PV) at $z = 2\text{ km}$ for $t = 81–89\text{ h}$ are shown in Fig. 4. The PV in the inner eye region decays slowly since $t = 84\text{ h}$, slightly before the secondary eyewall starts to form at around $t = 86\text{ h}$. At $t = 87\text{ h}$, the double-eyewall structure is evident. Later on, the secondary eyewall intensifies and contracts radially, with the primary eyewall further collapsing. Other fields, such as horizontal tangential wind (Figs. 5a–c) and relative vorticity (Figs. 5d–f) also exhibit the double-eyewall and ERC features. Interestingly, as soon as the secondary eyewall forms, the secondary tangential wind maximum already overpasses the primary one (Fig. 5b), which agrees well with the early decay of the inner eye region (see Fig. 4 from $t = 84$ to 86 h). The azimuthal-mean vertical cross-sectional fields at $t = 82, 84, 86\text{ h}$ are shown in Fig. 6 to illustrate the SEF process. Before the SEF, although the secondary azimuthal-mean tangential wind maximum has not appeared yet, there is already a secondary diabatic heating maximum outside the primary eyewall at around 75-km radius (Fig. 6a). Upon the SEF, the strong tangential wind field expands farther radially. The primary eyewall diabatic heating decays (Fig. 6b) compared with $t = 82\text{ h}$. There appear to be two diabatic heating maxima outside the primary eyewall. This phenomenon may be an artifact of the azimuthal-mean technique. The inner-core region is not perfectly circular but slightly elliptical at $t = 84\text{ h}$ (see PV structure at $t = 84\text{ h}$ on Fig. 4). If we account for this fact, the combined strength of diabatic heating outside the primary eyewall is enhanced at $t = 84\text{ h}$, compared with $t = 82\text{ h}$. At $t = 86\text{ h}$, the diabatic heating of the primary eyewall decays greatly compared to the large value of diabatic heating near the secondary eyewall (Fig. 6c). The maximum tangential wind broadens and moves outward radially, though the magnitude of it does not increase. Above the boundary layer, two azimuthal-mean tangential wind maxima are evident (e.g., $z = 3\text{ km}$ in Fig. 6c).

To show that the secondary eyewall is not simply a random outcome, we did several perturbed experiments, where a random noise function is added into the initial wind field in the core of the vortex. The amplitude of the noise function is 0.1 m s$^{-1}$. A total of eight experiments are simulated (including CTL), four of which are with the westerly jet (JET group), and the other four are without the westerly jet (NOJET group). Figure 7 shows the time series of the maximum 10-m wind speed $V_{\text{max}}$ for every experiment. At $t = 60\text{ h}$, the modeled vortex is already strong enough to have a significant interaction with the jet in the JET simulations. In addition, the maximum 10-m wind speed at $t = 70\text{ h}$ is about 100 knots (1 kt = 0.51 m s$^{-1}$), equivalent to a category-3 hurricane. It is clear that there are substantial differences between the members in the NOJET group and the members in the JET group. Before 60 h, $V_{\text{max}}$ is
slightly larger in the JET group than the NOJET group. This may be due to the upper-level jet facilitating outflow in the upper part of the TC secondary circulation compared with the NOJET group. Between 60 and 96 h, all the JET simulations experience an intensity oscillation corresponding to the ERC. In contrast, the intensity of NOJET simulations grows significantly faster before approaching a nearly steady state.

It is instructive to illustrate the SEF and ERC in a Hovmöller diagram of the azimuthal-mean tangential wind above the boundary layer for each JET simulation (Fig. 8). Before the ERC, the RMW is steady at about 30 km. However, the tangential wind field broadens outward, which was also indicated by Wu et al. (2012) and Huang et al. (2012) and identified by Rozoff et al. (2012) as a key process that precedes SEF. At around 87 h, when the ERC finishes, the RMW jumps to around 50 km. For all the JET simulations, the ERC takes place over a very short period (about 6 h), during which the primary eyewall disappears quickly (within 3 h) after the secondary eyewall is formed. Furthermore, although the two distinct radii of local maximum azimuthal-mean tangential wind are evident in many SEF studies, they are only fleetingly evident in our simulations below $z = 3$ km, and this agrees well with the slight decay of the primary eyewall before the secondary eyewall forms (see Fig. 4 at $t = 84$–86 h). The cause of the early decay of the primary eyewall and short ERC period may be related to the persistent upper-level forcing that induces a strong asymmetric boundary layer inflow (shown below) outside of the primary eyewall, cutting off the source necessary for the development of the primary eyewall even before the secondary eyewall forms. Note also that the TC experiences continual strengthening after the ERC. This is probably because of the TC not having yet reached the steady state when the ERC occurs and is consistent with observed intensity changes in real storms with ERC (Sitkowski et al. 2011).

In contrast to the JET simulations, the intensities in the NOJET simulations grow rapidly from 45 to beyond 80 m s$^{-1}$ between 60 and 96 h (Fig. 9). In addition, the RMWs all shrink from about 55 to around 30 km. Another difference between the NOJET and JET simulations is that, although the maximum wind speed increases, the tangential wind field does not expand radially after 80 h, even out to later times, such as 120 h (not shown).

Based on these sensitivity experiments, we suggest that the SEF/ERC is robust and is a consequence of the nearby westerly jet.
4. Mechanisms leading to SEF

a. Eddy flux convergence of angular momentum

The EFC is frequently used as a diagnostic quantity in TC–jet interaction (Molinari and Vollaro 1989, 1990; Hanley et al. 2001; Nong and Emanuel 2003) and is expressed as

$$\text{EFC} = -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \overline{u_L v_L'},$$

(1)

where $r$ is the radius from the TC center, $u_L$ is the storm-relative radial velocity, $v_L$ is the storm-relative tangential velocity, the overbar represents the azimuthal mean, and the prime denotes the deviation from the azimuthal mean.

Following DeMaria et al. (1993) and Hanley et al. (2001), a TC–jet interaction is defined as when the EFC at around 200 hPa exceeds 10 m s$^{-1}$ day$^{-1}$. We use data from model level 35 (approximately 187 hPa in terms of the base state pressure) instead of 200 hPa to compute the EFC. Comparisons (not shown) show little difference between the EFC at model level 35 versus 200 hPa. Since the EFC patterns in all the JET (NOJET) simulations are very similar, the control and a randomly chosen NOJET experiment are plotted to represent all JET (NOJET) experiments in Fig. 10. In the JET simulation, the azimuthal-mean EFC is larger than 10 m s$^{-1}$ day$^{-1}$ between radii of approximately 500 and 1000 km during the 48–96-h period (Fig. 10a). Moreover, the radial extent of this value of EFC increases with time during that period. On the other hand, the EFC is nearly zero outside the inner core at all times in the NOJET simulation (Fig. 10b). It can be foreseen (shown later) that a strong TC–jet interaction can enhance both the TC outflow and part of the jet in the JET simulations.

b. Stratiform clouds before SEF

This subsection illustrates how the jet can provide a favorable environment for SEF. As shown in the previous subsection, the TC–jet interaction brings large eddy activity (large EFC) in the upper troposphere, which will result in enhanced outflow. Because of the relatively large scale of the outflow, the time composites (between 64 and 69 h) of the horizontal wind speed and divergence of CTL at $z = 12$ km are computed on domain 2 (6-km horizontal resolution) and are illustrated.
in Fig. 11a. As a comparison, the result of one of the NOJET simulations is shown in Fig. 11b. For CTL, the outflow is greatly enhanced in the north, where the westerly jet interacts with the outflow (for a better view of the enhanced outflow, see Fig. 2). In addition to the large pattern of the outflow, there is also a small branch of large wind speed to the northwest reaching inward to the TC center. The divergence (solid contours) is located well with this branch, mostly in the northwest quadrant of the TC. Thus, the enhanced wind speed represents an upper-level eddy forcing, which results in divergence of considerably large scale compared to the inner core. On the other hand, the NOJET simulations show neither an enhanced outflow, nor a corresponding asymmetric divergence pattern (Fig. 11b). Because divergence and convergence are usually related to
convection and modeled convection is dependent on horizontal resolution, it is worth comparing the divergence of domain 2 with that of domain 3 in the same region. Comparison indicates (not shown) that the high-resolution fields (domain 3) are qualitatively similar to that of domain 2, except that many small-scale areas of divergence and convergence (which is related to convection) appear in the high-resolution field. Hence, it is safe to say that the relatively large-scale divergence is a result of upper-level eddy forcing, which is indicated by the enhanced upper-level wind speed.

The asymmetric upper-level divergence is very important to SEF, as it creates asymmetric stratiform clouds in the northwest quadrant of the TC. This can be explained as follows. The upper-level divergence is created by enhanced outflow, meaning that the air just below the level of divergence must rise to satisfy continuity. Such relatively large-scale divergence then results in a large-scale, small-amplitude (about 0.5 m s\(^{-1}\)) updraft of air, leading to the condensation of water vapor due to adiabatic cooling. Hence, liquid water then falls out from the mid-to-upper troposphere, creating small-amplitude downdrafts in the lower troposphere due to evaporative cooling and drag of liquid water. Figures 12a and 12b show time composites (between \(t = 64\) and 69 h) of vertical cross sections of azimuthal-mean and northwest-quadrant-mean vertical velocity, divergence, and equivalent potential temperature \(\theta_e\). The eyewall

![Figure 7](image-url)

**FIG. 7.** Time series of the maximum 10-m horizontal wind speed for different experiments. Black curves are experiments with jet included; red curves are experiments without jet. Dashed blue lines show time interval from 60 to 96 h.

![Figure 8](image-url)

**FIG. 8.** Hovmöller diagrams of the azimuthal-mean tangential wind at model level 9 (approximately 2-km altitude) for four JET simulations from 60 to 96 h. The black curve in each panel is the radius of maximum azimuthal-mean tangential wind.
inside 50-km radius has the strongest updraft through the troposphere, while outside the eyewall there only exists a weak updraft in the mid-to-upper troposphere between 100- and 200-km radius (Fig. 12a). In the northwest quadrant (Fig. 12b), the updraft in the eyewall region is similar to that of the azimuthal mean, while outside the 100-km radius the updraft is much stronger and radially larger. There is also an updraft of about 0.8 m s\(^{-1}\) located in the midtroposphere, which might be due to convection embedded in the stratiform cloud. In addition to the stratified upper-tropospheric updraft, the weak downdraft occurs just below the upper-level updraft. The divergence field also exhibits the typical stratiform cloud: divergence in the upper and lower troposphere (there is weak divergence in the lower level of Fig. 12b; not shown for the current contour) and convergence in the middle troposphere. Moreover, because of the evaporative cooling effect of the stratiform rainfall (white dotted contour in Fig. 12b), the lower boundary layer might be destabilized, as suggested by Bister and Emanuel (1997) and Fang and Zhang (2012).

In addition, the secondary circulation outside the primary eyewall in the northwest quadrant (Fig. 12d) is much stronger than that of the azimuthal mean (Fig. 12c), indicating that the upper-level eddy forcing might play a big role in inducing updrafts outside the primary eyewall. In the northwest quadrant, the weak diabatic heating maximum (Fig. 12d) outside the primary eyewall agrees well with the weak updraft (Fig. 12b) within the stratiform region. There is a weak downdraft just below the maximum heating rate (around \(z = 5\) km) outside the primary eyewall, consistent with the idea that the downdraft might be due to the evaporative cooling of the stratiform rainfall.

To amplify the stratiform cloud argument, the horizontal view of stratiform–convection separation, following the algorithm of Braun et al. (2010), is shown in Fig. 13 at \(t = 67\) h. The stratiform–convection separation algorithm is based mainly on surface rainfall rate, together with the vertical velocity and cloud water mixing ratio. In CTL, large asymmetric stratiform clouds are located to the northwest of the TC (Fig. 13a), while the NOJET simulation does not show any typical large stratiform asymmetry (Fig. 13b).

c. Convection resulting from stratiform clouds

Although the concept may appear a bit counterintuitive, the stratiform cloud region can produce deep convection. Bister and Emanuel (1997) argued that transient shallow convection would occur, given that the planetary boundary layer is destabilized by the evaporative cooling effect of the midlevel stratiform clouds.

![Fig. 9](image-url). As in Fig. 8, but for four NOJET simulations.
Furthermore, shallow convection would lead to convergence underneath, resulting in long-lasting deep convection. Induced by β shear, the highly asymmetric stratiform clouds in Fang and Zhang (2012) [similar stratiform clouds could also be found in Rozoff et al. (2012)] were responsible for the tightened u contour, which might be the leading cause for convection before SEF. Qiu and Tan (2013) also argued that asymmetric inflow was induced in the boundary layer of the downwind portion of the outer rainband (consisting of mainly stratiform clouds), creating convergence and thus supporting deep convection. The enhanced convergence might be the reason why divergence is not obvious in the lower level of the stratiform cloud pattern in Fig. 12b.

The asymmetric stratiform region as shown in Fig. 13a induces a large rainband that wraps cyclonically from the north to the south of the center. Thermodynamically, such a large rainband creates a large asymmetric heating source outside the TC eyewall, as indicated by Fig. 14. Scattered convective heating at $t = 67$ h is mainly concentrated on the upwind part of the rainband (positive values in both Figs. 14a and 14b). While the stratiform heating is negative (evaporative cooling) at around

![Graph](image1.png)

**Fig. 10.** Hovmöller diagrams of the eddy flux of angular momentum convergence at model level 35 for (a) the control simulation and (b) one of the NOJET simulations from 0 to 120 h. Black contours indicate where $\text{EFC} > 10 \text{ m s}^{-1} \text{ day}^{-1}$.

![Graph](image2.png)

**Fig. 11.** Time composite over 64–69 h at $z = 12$ km of the horizontal wind speed (shaded) and horizontal divergence (solid black contours; dotted contour indicates convergence) for (a) CTL and (b) one of the NOJET simulations for data from domain 2.
4.5 km and positive at around 9.5 km, it covers a very large area of the rainband, compared with the small area of convective heating. The importance of the heating rate in the rainband is reminiscent of the finding of Moon and Nolan (2010) that both convective heating and stratiform heating are able to induce a secondary horizontal wind maximum (SHWM; Hence and Houze 2008), which is an important character prior to SEF. Furthermore, Moon and Nolan (2010) also showed that diabatic heating by the asymmetric stratiform-type rainband was more effective than a convective-type rainband in propagating a tangential wind maximum entirely around the TC (their Fig. 13). Given that the rainband in our simulation is mainly made up of stratiform clouds (Fig. 13a), it should be able to induce an SHWM (shown below) efficiently. Hence, we suggest that the triggering of SEF in stratiform clouds is plausible thermodynamically.

**Fig. 12.** (a) Time composite over 64–69 h of vertical cross section of the azimuthal-mean vertical velocity (shaded), divergence (black solid and dotted lines with values of −45, −15, 15, 45, and 75 × 10⁻³ m s⁻², respectively), and equivalent potential temperature (red contours, with values between 340 and 430 K). For values below 343 K, the interval is 1 K; for values above 355 K, the interval is 5 K; otherwise, the interval is 2 K. (b) As in (a), but for the northwest-quadrant-mean field. Deviation of the northwest-quadrant-mean absolute temperature from azimuthal mean is shown in white contours, with dotted (solid) contours indicating −0.5 (0.5) K. (c) As in (a), but for the azimuthal-mean diabatic heating rate (shaded; K h⁻¹) and secondary circulation (radial velocity is divided by 10). (d) As in (c), but for the northwest-quadrant-mean field. (e) As in (a), but for azimuthal-mean inertial stability \( I \). (f) As in (e), but for the deviation of the northwest-quadrant-mean \( I \) from azimuthal-mean \( I \). Solid (dotted) contours indicate the value of 0.1 and 0.05 (−0.1 and −0.05) m s⁻², respectively.
The formation of convection under stratiform clouds could also be addressed via a dynamical (perhaps more straightforward) argument. The secondary circulation of the TC can be greatly affected, given the outflow–jet interaction (eddy forcing) in the upper level (Pfeffer and Challa 1981; Holland and Merrill 1984). At $t = 67$ h, the upper-level outflow is highly asymmetric and concentrated from the north cyclonically to the south-southwest quadrant of the TC (Fig. 15a). Consequently the boundary layer inflow also shows a wavenumber-1 asymmetry outside the eyewall in the northwest quadrant, where the inflow near the primary eyewall is so disrupted by the enhanced inflow outside of the eyewall that an outflow from inside the eyewall has to compensate for the loss of air (see the yellow shading near the eyewall in Fig. 15b). A single vertical cross section from west of the TC to the center (see the black straight line of Fig. 15a) is shown in Fig. 15d. The enhanced outflow induces a secondary circulation similar to Holland and Merrill (1984), where a compensating inflow just below the outflow goes inward from outside. There are individual convective updrafts embedded in

Fig. 13. Stratiform (blue) convection (red) separation for (a) CTL and (b) one of the NOJET simulations at $t = 67$ h. The algorithm is similar to that of Braun et al. (2010).

Fig. 14. Horizontal diabatic heating rate (K h$^{-1}$) at $t = 67$ h for (a) model level 18 (about 4.5 km) and (b) model level 30 (about 9.5 km).
the mid-to-upper-level stratiform updraft outside the eyewall. The boundary layer inflow also increases outside the eyewall but gradually slows down inside 100 km radially, where inertial stability $I$ is slightly increased (Figs. 12e,f). Here, $I$ is defined as

$$I^2 = \left( f_{cor} + \frac{2V}{r} \right) \times (f_{cor} + \zeta),$$

(2)

where $V$ is tangential wind, $r$ is distance to the center, and $\zeta$ is the relative vertical vorticity. A detailed comparison (not shown) suggests that the inertial stability pattern is mainly associated with the absolute vorticity pattern [because $(f_{cor} + 2V/r)$ in Eq. (2) is always positive near the TC inner core]. A positive feedback thus might develop: Increased boundary layer inflow increases convergence where the positive inertial stability perturbation is located. The enhanced convergence increases the absolute vorticity by the stretching effect, thus increasing the inertial stability. Then it is even more difficult for the inflow to enter into the eyewall region; the updraft is more favorable; and the system will continue to develop until the lower-level inflow and upper-level outflow are directly connected by a strong updraft or convection outside the primary eyewall.

The induced mid-to-lower-level inflow brings the outside high-value angular momentum inward, increasing the local tangential wind (Figs. 15c,d). In this way an
SHWM occurs before SEF (Fig. 15c). The SHWM can also be explained by thermal wind balance: the decreased temperature due to evaporative cooling in the stratiform region (Figs. 12b, 15c) increases the horizontal temperature gradient radially. Based on thermal wind balance, the related vertical wind shear must increase. If we assume that the upper-level wind is not affected because the cooling effect only occurs in the lower level, the lower-level tangential wind must correspondingly increase (Nong and Emanuel 2003).

Interestingly, Corbosiero and Torn (2016) argued that the SEF-related convection of Hurricane Igor (2010) is associated with a cold pool. We examined this hypothesis and found that the evaporative cooling of stratiform rainfall does produce a squall-line-type rainband (see the enhanced reflectivity in Fig. 15b and large thermal contrast in Fig. 15c, located in the southwest quadrant, far away from the TC center in each figure). Although it lasted for several hours (see \( t = 69 \) and 73 h in Fig. 3), the squall-line-type rainband moved outward radially and cyclonically in our simulations, implying that it did not contribute to SEF (at least for the case here). In other words, thermodynamic processes alone might not be able to produce a secondary eyewall, and dynamical processes may be necessary to organize the convection to move inward radially.

d. SEF process

Because of the persistent upper-level forcing, the asymmetric stratiform cloud continuously produces convection by the process mentioned above. Once created, convection is advected downwind and inward. The track of individual convection outside of the eyewall is quite similar to what was described by Moon and Nolan (2015), who argued that convective cells in the inner rainband of a simulated TC were simply advected by the rapidly rotating TC wind field while being deformed into spiral shapes (their Fig. 20). One difference is that, while the convection moves radially outward in Moon and Nolan (2015), it moves radially inward in this paper. This can be explained by the opposite direction of the secondary circulation: in Moon and Nolan (2015), vertical wind shear induced an indirect secondary circulation that was made of upper-level radial inflow and low-level outflow near the eyewall, while here the enhanced outflow induces the low-level inflow. Thus, it is the low-level inflow that advects the convection inward radially.

FIG. 16. (a) Filtered low-wavenumber (0–4) radial wind at \( z = 125 \) m (shaded) and low-wavenumber (0–4) tangential wind at \( z = 3 \) km (contoured) at \( t = 76 \) h. Thick contours are for tangential wind \( = 30, 35, 40, \) and \( 45 \) m s\(^{-1}\). (b) As in (a), but for \( t = 78 \) h. (c) As in (a), but for \( t = 80 \) h. (d) Vertical cross section of tangential wind (shaded) averaged within half a quadrant, as shown within red dashed lines in (a). Thick contours are for tangential wind \( = 30, 35, 40, \) and \( 45 \) m s\(^{-1}\). Vectors are secondary circulation (radial velocity is divided by 10). (e) As in (d), but for \( t = 78 \) h. (f) As in (d), but for \( t = 80 \) h.
around 81 h, convection surrounds the center outside the eyewall (Fig. 3). In addition to the convection, the advection of SHWM is also related to the inflow outside the primary eyewall region before the SEF period (Fig. 16). From \( t = 76 \) to 80 h, while the SHWM is advected cyclonically and spirally deformed, it also moves inward radially slowly (Figs. 16a–c). Furthermore, the boundary layer inflow correlates well with the SHWM. From the octant-mean height–radius plots (Figs. 16d–f), the inward movement of the SHWM is associated with two types of inflow: the boundary layer inflow and a midlevel inflow that originates from \( z = 7–9 \) km and descends to \( z = 3–4 \) km as it moves through the SHWM. The midlevel inflow is located just below the enhanced outflow and might be induced by it (similar to Holland and Merrill 1984). The two types of inflow are quite similar to that of Didlake and Houze (2013), who also showed the importance of the stratiform band complex on the SHWM or the broadening of rotational wind preceding the SEF.

While the secondary eyewall is due to the axisymmetrization of many convective cells, not every cell becomes a part of the secondary eyewall. As shown in Rozoff et al. (2006), the filamentation time \( T_{\text{fil}} \) can be used to decide whether or not a region is strain dominated. Following Rozoff et al. (2006), the axisymmetric \( T_{\text{fil}} \) is calculated here as

\[
T_{\text{fil}} = \left( -\frac{V}{r} \frac{\partial V}{\partial r} \right)^{-1/2},
\]

where \( V \) is the azimuthal-mean tangential wind, and \( r \) is radial distance to TC center. When \( T_{\text{fil}} < 30 \) min, convection will be strained and suppressed; however, when \( T_{\text{fil}} > 30 \) min, convection is able to develop. Therefore the rapid filamentation zone is defined as a region where \( T_{\text{fil}} < 30 \) min. Figure 17 shows \( T_{\text{fil}} \) at \( z = 3 \) km. It is first noteworthy that the white region outside of the eyewall is where the radial gradient of the tangential wind is positive. This also indicates the azimuthal-mean SHWM. The SHWM moves radially inward with time and corresponds well to the secondary eyewall.

Just outside of the primary eyewall, there is a rapid filamentation zone \( (T_{\text{fil}} < 30 \) min) ranging from about 35 to 55 km (Fig. 17). Beyond 55 km, convection is sustained and is advected spirally inward. Before 80 h, convective cells outside of the eyewall from the stratiform region that are advected spirally inward are sheared apart inside \( r = 55 \) km because of the strong strain effect in the rapid filamentation zone. As a result, convection becomes more concentrated just outside of radius 55 km. After 80 h, the convection outside of radius 55 km has surrounded the center (Fig. 3). After some adjustments, individual convective cells merge with each other, and a secondary eyewall finally forms at 86 h at a location of 55-km radius. It seems that \( T_{\text{fil}} = 30 \) min defines the critical radius where the secondary eyewall forms. We note, however, that different interpretations exist on the rapid filamentation zone concept. Some studies argue that the rapid filamentation zone may not always imply a depression of convection. For example, Wang (2008) suggested that the strain effect in the rapid filamentation zone may damp high-wavenumber asymmetry and may be favorable for the organization of inner rainbands.

5. Sensitivity experiments

To better address the importance of the jet on SEF, additional sensitivity experiments are introduced here. Experiments LAT15 and LAT20, which are designed to test the sensitivity to the distance between the jet and the TC, are configured with a central location of the TC at 15° and 20°N, respectively, compared with 25°N in CTL. The jet is in the same location in all experiments. Similarly, experiments U10, U15, and U20 are designed to test the sensitivity to the strength of the jet with a maximum wind speed of the jet of 10, 15, and 20 m s\(^{-1}\), respectively, compared with 35 m s\(^{-1}\) in CTL. The results of the sensitivity experiments are shown as Hovmöller diagrams of azimuthal-mean tangential wind above boundary layer (Fig. 18).

Neither LAT15 nor LAT20 shows the ERC or SEF (Figs. 18d,e), which would be identifiable by a sudden change of RMW and intensity (as in CTL). Although the wind field of LAT20 expands (especially after 120 h; not shown), the RMW grows slowly and smoothly with time,
without a sudden increase. For the case of LAT15, the tangential wind field only expands at early times, which is related to the deepening of the TC itself. Also, the RMW of LAT15 stays constant around 47 km.

For the sensitivity to jet strength, both U20 and U15 exhibit the ERC (Figs. 18a,b), except that they occur later than in CTL, which is expected because of the weaker jet strength. The U10 simulation does not exhibit SEF (Fig. 18c). Instead, the RMW evolution is similar to that of LAT20, which is smooth compared to U20 and U15. The azimuthal-mean EFC (not shown) of U10 barely exceeds $10 \text{ m s}^{-1} \text{ day}^{-1}$ outside the inner core, explaining the absence of SEF. It is noteworthy that, while U10 does not exhibit SEF, the intensity (maximum azimuthal mean tangential wind at around $z = 2 \text{ km}$) evolution is similar to CTL (not shown), which also shows a large oscillation.

As a summary of the sensitivity experiments, Fig. 19 illustrates the time series of the RMW for each simulation, along with CTL and one of the NOJET simulations for reference. Results support the argument that the SEF arising from jet–TC interaction is not very sensitive to the strength of the jet, because even experiment U15, with a jet $20 \text{ m s}^{-1}$ weaker than CTL, underwent SEF. However, SEF is very sensitive to distance between the jet and TC, as LAT20 does not experience SEF.

6. Conclusions

The effects of TC–jet interaction on secondary eyewall formation (SEF) are investigated using idealized, controlled WRF simulations on an $f$ plane. With a moderately strong ($35 \text{ m s}^{-1}$) upper-level westerly jet to the north of the TC, a robust SEF/ERC occurs. In contrast, the simulations in which no jet is prescribed do not exhibit SEF. Hence, we suggest that such a difference results from the presence of the westerly jet. Eddy momentum flux convergence (EFC) is used to diagnose the importance of the TC–jet interaction on SEF. It is found that EFC is strong ($>10 \text{ m s}^{-1} \text{ day}^{-1}$), starting from the early development of SEF ($\approx 48 \text{ h}$) until the ERC finishes ($\approx 90 \text{ h}$).

During the interaction between the TC and the jet, asymmetric stratiform clouds are persistently produced in the northwest quadrant of the TC before SEF. It is found that deep convection is able to form in the environment of stratiform rainfall by the coupling of dynamic and thermodynamic processes. Thermodynamically, the...
Asymmetric region induces a large rainband outside of the eyewall, creating a substantial heating source to trigger a secondary horizontal wind maximum prior to SEF (Moon and Nolan 2010). Dynamically, the upper-level forcing induces a weak but relatively large-scale updraft, which is able to trigger an asymmetric secondary circulation outside the primary eyewall. Convergence is enhanced in the bottom of this secondary circulation, thereby favoring convection. Moreover, the lower part of the secondary circulation brings the higher angular momentum inward. Consequently, the local tangential wind is increased.

Once formed, deep convection is organized like an outer rainband. Typically, individual convective cells spiral inward via advection by the TC cyclonic wind. Because of the persistent production of convection in the asymmetric stratiform region, convection spreads all around the TC outside of the primary eyewall. The secondary eyewall forms around the radius defined by \( r_0 \), where the strain effect of the TC flow overcomes the self-development of convection.

The sensitivity experiments indicate that SEF is not very sensitive to the strength of the jet (at least for strength equal to or larger than 15 m s\(^{-1}\)). However, the SEF process was found to be sensitive to the distance between the jet and the TC.

Although several important factors are ignored (such as strong vertical wind shear and the \( \beta \) effect), this study is an attempt to clarify and isolate what may be an important process for SEF: outflow–jet interaction. While the main conclusion of this study is that TC structure and intensity change (and in particular SEF) is dependent on the presence of a jet and its interaction with the outflow, it is not yet clear whether the process hypothesized here is evident in real TCs. Data analysis and modeling of outflow–jet interactions in real TCs will be the next step to address this question.

**Acknowledgments.** This study was supported by the Office of Naval Research Grant N000141410115. The authors also acknowledge computational support from the University of Miami Center for Computational Science and the helpful comments by three anonymous reviewers.

**REFERENCES**


