Energy Spectra and Inertia–Gravity Waves in Global Analyses

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ABSTRACT

Several decades after E. Dewan predicted that the shallowing of the atmospheric energy spectrum in mesoscale is produced by the inertia–gravity (IG) waves, global analyses have reached the resolution at which the IG waves across many scales are resolved. The authors discuss the spatial filtering method based on the Hough harmonics that provides the temperature and wind perturbations associated with the IG waves in global analysis data. The method is applied to the ECMWF interim reanalysis and the operational 2014–16 analysis fields. The derived spectrum of IG wave energy is divided into three regimes: a part associated with the large-scale unbalanced circulations that has a slope close to \( \frac{2}{5} \) for zonal wavenumbers \( 1 \leq k \leq 6 \), a synoptic-scale range between 3000 and around 500 km \( (7 \leq k \leq 35) \) that is characterized by a nearly \( -\frac{5}{3} \) slope, and a mesoscale range below 500 km where the slope of the IG energy spectrum in the 2015/16 analyses is steeper. In contrast, the energy spectrum of the Rossby waves has a \( -3 \) slope for all zonal wavenumbers \( k > 6 \). Presented results suggest that energy associated with the IG modes exceeds the level of energy associated with the Rossby waves around zonal wavenumber 35. The exact wavenumber depends on the season and considered atmospheric depth and it is suggested as a cutoff scale for studies of gravity waves.

1. Introduction

The majority of atmospheric energy is in the zonal-mean state. The distribution of the remaining energy as a function of the horizontal scale—the wave energy spectrum—is one of the fundamental characteristics of atmospheric dynamics. The energy spectrum is usually represented as a function of the dimensional horizontal wavenumber and it is characterized by a steeper-sloped region at the scales of baroclinic systems in between the two shallower-sloped regions at the global scales and at the mesoscale (e.g., Hoskins and James 2014, chapter 11). The spectrum is usually presented for kinetic energy on individual horizontal levels or layers as derived from observations (e.g., Nastrom and Gage 1985) or numerical models that can be global (e.g., Koshyk and Hamilton 2001) or limited-area models (e.g., Blažica et al. 2013).

The dynamical processes generating the various regions of the spectrum in nature and in numerical weather prediction (NWP) models are a subject of intense research especially with regard to the mechanisms that produce a shallow-sloped region of the mesoscale spectrum with a slope close to \( -\frac{5}{3} \). The idea that shallower energy spectrum at mesoscale results from a forward energy cascade exists for several decades (Dewan 1979; VanZandt 1982; Bartello 1995; Lindborg and Cho 2001; Lindborg 2006). In particular, Dewan (1979) associated a shallower energy spectrum at mesoscale with the quasi-linear inertia–gravity waves.

This idea is supported by energy spectra from high-resolution numerical simulations that diagnose inertia–gravity waves (Kitamura and Matsuda 2010; Terasaki et al. 2011) or use divergence as a proxy of inertia–gravity waves in the midlatitudes (Waite and Snyder 2009, 2013). The present paper extends these results by discussing the global energy spectra of inertia–gravity waves across many scales and the energy partition between the Rossby and inertia–gravity waves based on the state-of-the-art global analysis and reanalysis data from the European Centre for Medium-Range Weather Forecasts (ECMWF).
The inertia–gravity (IG) waves are considered a part of the overall spectrum of atmospheric response to unbalanced flow including physical parameterizations and boundary forcing (Uccellini and Koch 1987). Their frequencies range between the Brunt–Väisälä frequency $N$ (pure gravity waves) and the inertial frequency $f$ (pure inertial waves) (e.g., Fritts and Alexander 2003). Surface pressure data suggest that in the midlatitudes the mesoscale IG waves are present around one-third of the time during winter (Koch and Siedlarz 1999). The generation mechanism of the IG waves include orography, convection, jets, and fronts as increasingly reported by successful numerical model simulations (e.g., Durrant 1986; Sato et al. 2012; Mirzaei et al. 2014; Plougonven and Zhang 2014).

In recent years, the global analyses have reached the resolution at which the IG waves across many scales are being resolved. For example, Plougonven and Teitelbaum (2003) performed a case study of the stratospheric large-scale IG wave in the ECMWF analyses in late 1990s in comparison with radiosondes. Yamashita et al. (2010) showed that the ECMWF model with resolution T799 represents IG waves with scales around 100 km reasonably similar to a much-higher-resolution simulation with a nonhydrostatic mesoscale model, the Weather Research and Forecasting (WRF) Model. Preusse et al. (2014) analyzed sources of IG waves in the ECMWF analyses produced with T799 resolution. A more recent version of ECMWF analyses with a T1279-resolution model was analyzed by Jewtoukoff et al. (2015), who reported that the model resolves the stratospheric IG wave spectrum reasonably well in comparison with the measurements by stratospheric balloons. Resolved IG waves may be contributed both by increased model resolution and improved parameterizations that provide more realistic (background) forecasts (e.g., Bechtold et al. 2008) as well as by advances in data assimilation methods and an increase in assimilated satellite data.

In many studies, velocity perturbations associated with the IG waves are defined by a cutoff scale of the zonal wavenumber. For example, Jewtoukoff et al. (2015) applied the cutoff at the zonal wavenumber 15 for the stratospheric gravity wave analysis, which corresponds to around 1300 km at the equator. Preusse et al. (2014) interpreted waves with the zonal wavenumbers greater than 6 in the ECMWF analyses produced with the T799 model as IG waves. Clearly, the choice of the cutoff scale can be decided based on the purpose. A more objective definition of the separation scale between the regime dominated by the Rossby wave dynamics and the regime of inertia–gravity waves can be based on the comparison between their corresponding energy spectra. Such comparison is carried out in this study.

A separation of the global energy spectrum in the Rossby and inertia–gravity components is performed by using the Hough harmonics that are eigensolutions of the linearized primitive equations (Hough 1898; Siebert 1961). In spite of a long history of the Hough harmonics in atmospheric research (e.g., Dickinson and Williamson 1972; Kasahara 1976; Phillips 1990), their application for the representation of atmospheric energy distribution has been limited. A more commonly used method for the computation of the global energy spectra is based on the expansion of data in terms of spherical harmonics. Its application is straightforward for global spectral models such as the ECMWF model, and it provides a decomposition of kinetic energy into the rotational and divergent components (e.g., Koshyk et al. 1999; Wedi 2014). In contrast, the Hough harmonics provide energy spectra that can be decomposed into components associated with balanced and unbalanced dynamics. In other words, being eigensolutions of the global linearized equations, the Hough harmonics separate the Rossby and IG components of the horizontal motions on the sphere. Furthermore, in contrast to spherical harmonics that provide a kinetic energy spectrum at a given horizontal level, the Hough harmonics provide the spectrum of kinetic and available potential energy of horizontal motions associated with a prescribed equivalent depth, that is, with a certain vertical mode or a range of modes.

The distribution of energy in planetary scales is a particularly useful diagnostic quantity obtained from the decomposition based on the Hough harmonics since it provides kinetic and available potential energy at the corresponding wavenumber (e.g., Zagar et al. 2015). By contrast, the computation of rotational and divergent components of kinetic energy for an individual wavenumber from the spherical harmonics involves contributions from neighboring wavenumbers (e.g., Koshyk et al. 1999) that makes such spectra hard to use for large scales. Although the Hough harmonics are more difficult to apply, their application is becoming more attractive nowadays with the global models capable to represent the tropical and midlatitudes mesoscale circulations more reliably.

This paper presents the scale-dependent filtering method that supplies global wind and temperature perturbations associated with the IG waves. The method is applied to the analysis and reanalysis data of ECMWF and the global energy spectrum is computed for the Rossby waves (quasi-geostrophic dynamics) and for the IG waves. We present various regimes of the Rossby and IG wave energy spectra, discuss the impact of the
vertical model depth on the spectra, and provide a recommendation for the cutoff scale of the IG waves.

The outline of the paper is as follows. The next section describes the computation of the total energy spectrum by using the Hough harmonics and presents the ECMWF model-level data. In section 3, we demonstrate the spatial filtering of the IG waves on a case study in July 2014—a period analyzed with the highest resolution. We show that a time sequence of filtered wind and temperature perturbations evolves as a linear IG wave. Section 4 discusses properties of average spectra in reanalysis and recent analysis fields. Discussion and conclusions are presented in sections 5 and 6, respectively.

2. Energy spectra and inertia–gravity waves

Here we first discuss the computation of the total energy spectra of the 3D global ECMWF databases on terrain-following levels. Then we present the computation of temperature perturbations associated with individual waves or a range of waves. The 3D wave decomposition is carried out at every time step independently. The time series of the Hough harmonics expansion coefficients are then analyzed in frequency space to show that the wave features are propagated in time through the applied ECMWF data.

a. Computation of the global total energy spectrum

The decomposition of the dynamical fields in scale-dependent wave perturbations associated with the Rossby and inertia–gravity waves relies on the representation of the global linearized equations of motions in terms of \( M \) global shallow-water systems, each characterized by its own average fluid depth that is known as the equivalent depth (Taylor 1936). This parameter, denoted \( D \), is a constant that couples the non-dimensional oscillations of the horizontal wind and geopotential height fields with the vertical structure equation discretized for terrain-following levels (Kasahara and Puri 1981).

The decomposition of data on \( J \) terrain-following (\( \sigma \)) levels into the vertically and horizontally dependent parts is described by the following equation:

\[
X_m(\lambda, \phi) = S_m^{-1} \sum_{j=1}^{J} (u, v, h)^T G_m(j).
\]  

Here, parameters \( \lambda \) and \( \phi \) stand for the geographical longitude and latitude, respectively, whereas \( \mu = \sin(\phi) \). The vector of input 3D data is denoted \( X(\lambda, \phi, \sigma) = (u, v, h)^T \), whereas \( X_m(\lambda, \phi) = (\bar{u}_m, \bar{v}_m, \bar{h}_m)^T \) is the vector of non-dimensional horizontal oscillations associated with the vertical mode \( m \). The geopotential height \( h \) is a sum of hydrostatic height and the surface pressure term that has been introduced by Kasahara and Puri (1981) to combine the thermodynamic equation and the continuity equation. The diagonal \( 3 \times 3 \) matrix \( S_m \) normalizes the input data so that the horizontal expansion is nondimensional. The vertical structure functions \( G_m(j) \) are orthogonal and solved numerically. The increasing value of \( m \) is associated with the reduction of the equivalent depth \( D_m \). The discussion of the vertical structure equation and the impact of the model top has been presented in Zagar et al. (2009a) and Zagar et al. (2015).

The horizontal motions for a given \( m \) are represented by a series of Hough harmonics. The Hough harmonics consist of the Hough vector functions in the meridional direction and waves in the longitudinal direction. The horizontal expansion of the data in terms of the Hough harmonics is performed for each \( m \) as

\[
X^H_m = \frac{1}{2\pi} \int_0^{2\pi} \int_{-1}^{1} X_m(\lambda, \phi) \cdot [H^H_m(m)]^* d\mu d\lambda.
\]

The Hough harmonics are denoted by \( H^H_k(m) \) and are defined by the two indices for the zonal wavenumber \( k \) and meridional mode \( n \) for each \( m \). The complex expansion coefficient \( \chi^k_n(m) \) describes both the two wind components \((u, v)\) and the geopotential height \( h \).

A discrete solution is obtained by replacing the integration by a finite series of the Hough harmonic functions including the zonally averaged state, \( K \) zonal waves, and \( R \) meridional modes so that the horizontal projection reads as

\[
X_m(\lambda, \phi) = \sum_{n=1}^{R} \sum_{k=-K}^{K} \chi^k_n(m) H^H_k(m).
\]  

The general algorithm for solving (3) is detailed in Swarztrauber and Kasahara (1985). It involves the replacement of the wind components by velocity potential and streamfunction variables by using the Helmholtz theorem and the assumption that new dependent variables are proportional to harmonic functions in longitude and time with zonal wavenumber and frequency, respectively. The meridionally dependent variables are expressed in terms of series of the associated Legendre polynomials of order \( n \) and rank \( k \). Their combination builds the Hough vector functions for each \( m \). Resulting equations for the three expansion coefficients as functions of \( k \) and \( n \) contain two independent systems. The nondimensional frequency is obtained as the eigenvalue of the matrix problem. Two dispersion relationships describe the frequency of two kinds of solutions: the eastward- and

The maximal number of meridional modes $R$ combines $N_R$ Rossby modes and $N_{IG}$ inertia–Gravity modes, which includes $N_E$ eastward-propagating inertia–Gravity modes and $N_W$ westward-propagating inertia–Gravity modes: $R = N_R + N_{IG} = N_R + N_E + N_W$. In other words, for each $D_m$ and each zonal wavenumber $k$, there exists a set of $3 \times N_w$ frequencies—one per wave type $w = (R, E, W)$. As shown below, these frequencies are used solely for the formulation of the projection basis and not for studying wave propagation properties. Namely, the frequencies differ depending on whether the linearization is performed around the nonzero mean zonal flow [see corrigendum for Kasahara (1980)]. It is therefore suitable to use the Hough functions constructed with reference to the basic state at rest as a basis for the projection.

The application of (1) and (2) provides global total energy, the sum of kinetic and available potential energies, as a function of scale and motion type. The partition of total energy into the kinetic and available potential energies is calculated from the complex coefficients $\chi_n^k(m)$ as the following scalar product, denoted $I_k^R(m)$:

$$I_k^R(m) = \frac{1}{2} gD_m \chi_n^k(m)[\chi_n^k(m)]^*,$$

(4)

where $[\chi_n^k]^*$ is the complex conjugate of $\chi_n^k$. Depending on the value of the index $n$, (4) refers to the energy component of the Rossby or IG spectrum. The global energy product of the $m$th vertical mode is defined as

$$I_m = I_{m,R} + I_{m,IG} = \frac{1}{2} gD_m \sum_{n=1}^{K} \sum_{k=-K}^{K} \chi_n^k(m)[\chi_n^k(m)]^*.$$

(5)

The summation over $n$ includes both Rossby and IG waves and the two components are obtained by splitting the summation between $n = 1 - N_R$ and $n = 1 - N_{IG}$. It can be shown that the scalar product defined by (5) is equal

$$I_m = \frac{1}{2} gD_m \int_{-1}^{1} \int_{-1}^{1} \left( u_m^2 + v_m^2 + \hat{h}_m^2 \right) d\lambda d\mu$$

$$= \int_{-1}^{1} \int_{-1}^{1} \left( K_m + P_m \right) d\mu d\lambda,$$

(6)

where

$$K_m = \frac{1}{2} (u_m^2 + v_m^2) \quad \text{and} \quad P_m = \frac{1}{2} D_m \hat{h}_m^2,$$

(7)

denote the specific kinetic energy and available potential energy, respectively, of the $m$th vertical mode. Likewise, the energy spectrum with respect to the $k$th zonal wavenumber can be calculated as

$$I_k = I_{k,R} + I_{k,IG} = \frac{1}{2} \sum_{m=1}^{M} gD_m \sum_{n=1}^{K} \chi_n^k(m)[\chi_n^k(m)]^*.$$

(8)

The 2D horizontal energy spectra can be computed for a vertical mode $m$ or as a vertically integrated quantity representing the average horizontal energy distribution as

$$I_m^R = \frac{1}{2} \sum_{m=1}^{M} gD_m \chi_n^k(m)[\chi_n^k(m)]^*.$$

(9)

The maximal number of vertical modes is denoted by $M$ and it can be no greater than $J$. A detailed derivation of the global energy equation in normal mode space from the linearized atmospheric model has been presented in Kasahara and Puri (1981) and revived by Zagar et al. (2015).

Like any linear analysis, the Hough harmonics decomposition relies on the assumption of small-amplitude motions superimposed on a background state (e.g., Kasahara 1976). The background state, the zonally averaged state $k = 0$ is nearly 100% balanced and all modes with $k = 0$ have zero frequency. The $k = 0$ contains over 90% of the global atmospheric energy and it is not discussed here. The presented spectra thus represent energy of the finite-amplitude waves superimposed on the background state.

The energy decomposition in Rossby and IG modes has been published in several papers. For example, Tanaka (1985) and Tanaka and Kung (1988) showed such spectra for the analysis data from late 1970s, whereas Tanaka and Kimura (1996) presented them for the analyses datasets from early 1990s. The applied datasets were low-resolution analyses and the estimated level of wave energy associated with IG modes was smaller than in more recent data (Zagar et al. 2012). Apart from a single high-resolution simulation in Terasaki et al. (2011), the energy spectra presented in this paper are the highest-resolution spectra of the Rossby and IG waves available from NWP and data assimilation systems.

Figure 1 displays the global energy spectrum according to the operational ECMWF analysis data at three
subsequent days in July 2014. The spectrum comprises the whole model depth, that is, 137 layers from the surface to 1 Pa (around 80 km). Similar to previous studies, Fig. 1 suggests that the IG spectrum is much shallower than the spectrum associated with the Rossby waves. Their sum, the total energy spectrum, is shallower on planetary scales than on the synoptic scales in relation to the inverse energy cascade from the scales of baroclinic waves. The bump at \( k = 4-5 \) is associated with intense baroclinic developments at the time. As discussed in the introduction, such features are not available from the spectra based on spherical harmonics. Figure 1 suggests that the Rossby wave spectrum follows the \( 2^3 \) law down to the smallest presented scale around 100 km. In contrast, the IG component of energy is characterized by a shallower spectrum, especially at the largest scales. As the zonal wavenumber increases, the IG spectrum becomes first comparable and then dominant over the Rossby spectrum. The rest of this paper deals with the average properties of this spectrum and its characteristics in physical space.

b. Temperature and wind perturbations of inertia–gravity waves

The decomposition in (1) and (2) is applied on global fields. In the preprocessing step, temperature is used to compute geopotential height, which serves as input to (1).

To produce temperature perturbations associated with the IG waves, the expansion coefficients \( \chi_{m,k}^n(\lambda, \varphi) \) are filtered back to physical space for the part of the spectrum associated with waves and scales of interest. The filtering takes place in two steps. First, we compute the inverse of (2) as

\[
X_{m,IG}(\lambda, \varphi) = \sum_{n=1}^{N_{m,IG}} \sum_{k=K_1}^{K_2} \chi_{m,k}^n(\lambda, \varphi) H_n(\lambda, \varphi; m),
\]

where the indices \( K_1 \) and \( K_2 \) denote minimal and maximal wavenumbers that are filtered to physical space.

Then the vector \( X_{IG}(\lambda, \varphi, \sigma) = (u_{IG}, v_{IG}, z_{IG})^T \) with wind and geopotential height perturbations associated with IG waves is obtained by applying an inverse of (1):

\[
X_{IG}(\lambda, \varphi, \sigma) = \sum_{m=1}^{M} S_m X_{m,IG}(\lambda, \varphi) G_m(j).
\]

Temperature perturbations are computed from the geopotential height \( z_{IG} \) by using the hydrostatic relation in the \( \sigma \) system:

\[
T_{IG} = -\frac{g \sigma}{R} \frac{\partial z_{IG}}{\partial \sigma},
\]

where \( R \) is the gas constant and \( g \) denotes gravity. The expression for the computation of the total temperature field from the geopotential height includes also the surface pressure term.

The filtering procedure in (10)–(12) is carried out at every time step with the data available, just like the forward projection in (1) and (2). The decomposition is partly filtering small scales and the lower troposphere because the applied number of vertical modes is smaller than the number of model levels [for discussion see Zagar et al. (2015)]. Because of relatively little energy in these scales, the spectra show only a small sensitivity.
c. ECMWF datasets

Operational ECMWF analyses are produced by the four-dimensional variational data assimilation (4D-Var) (Rabier et al. 2000) using a 12-h assimilation window twice per day. At 0000 and 1200 UTC the analyses are used as initial conditions for the 10-day high-resolution forecasts. In ECMWF 4D-Var, about 50 million observations of the wind field and mass (temperature, pressure, humidity) field are assimilated in each assimilation cycle. Observations are combined with the first-guess field, which is the 3-h forecast from the previous cycle. In addition to the explicitly resolved part of the gravity wave spectrum, the ECMWF model includes the parameterized gravity waves (Orr et al. 2010). The forecast model T1279 changed its computation grid in 2016 from a linear grid (~16 km at the equator) to a cubic grid (~9 km at the equator). We do not analyze data at the full resolution but at lower resolutions that in the best case resolve the processes with scales around 70 km. The vertical scale of the resolved waves can be expected to be a few kilometers.

In the vertical direction, the ECMWF model currently includes 137 levels between the surface and 1 Pa (at approximately 80 km). The hybrid vertical coordinate in a single point of physical space \((\lambda, \phi, j)\) is defined as \(\eta(\lambda, \phi, j, t) = A(j) + B(j)p_s(\lambda, \phi, t)\), where \(p_s\) is the surface pressure and \(j = 1, \ldots, 137\) is the model-level index. Distribution of model levels over the southern part of New Zealand at 0000 UTC 4 July 2014 is shown in Fig. 2 for the lowest 20 km. The horizontal model surfaces follow the orography in the lower troposphere (\(\sigma\) levels) and become flat (\(p\) levels) in the upper troposphere and higher. The model levels are distributed unevenly with 49 levels above 50 hPa, 20 levels between 50 and 150 hPa, 27 levels under 800 hPa, and 40 remaining levels in the free troposphere between 800 and 150 hPa.

Properties of the datasets are summarized in Table 1. We present the Rossby and IG spectra of the operational ECMWF analyses during the last 2.5 yr. The input data are on the regular Gaussian N128 grid \((512 \times 256\) grid points\) with \(K = 200\) waves \((70\)-km scale in the midlatitudes\). Such spectra are available on a daily basis since November 2014 at http://meteo.fmf.uni-lj.si/MODES for 0000 UTC analyses and high-resolution 10-day forecasts. Since December 2016 the decomposition excludes the top three model levels where some numerical artifacts may occasionally contaminate the IG wave signal. Three 2016/17 winter months, denoted L134, are thus representing the state-of-the-art global Rossby and IG spectra.

A higher-resolution dataset is analyzed for July 2014. In this case, we used N200 grid that consists of \(800 \times 400\) grid points equivalent to a grid spacing of about 50 km at the equator—that is, about 35 km in the midlatitudes. By applying the horizontal truncation of \(K = 300\) we resolve waves with the horizontal scales down to \(70\) km at the equator and around \(50\) km in the midlatitudes, which is close to the scales represented by the model. The whole model depth is projected and thus the label “L137 2014.” Compared to the L137 data for the 2015/16 period, July 2014 data include different numbers of vertical and meridional modes in the expansion in (1) and (2). The projection is performed every 6 h whereas other analyses are decomposed once per day. The motivation for the modal decomposition of this particular month was the DEEPWAVE experiment that took place over New Zealand (Fritts et al. 2016). DEEPWAVE measured the IG waves in the region and the goal of our high-resolution decomposition is to offer to the involved researchers a new method for the IG diagnosis and comparison between the gravity wave observations and their representation in global models.

The Rossby and IG spectra of recent analyses are compared with the average spectra from ERA-Interim (Dee et al. 2011) over 35 yr between 1981 and 2015, ERA-Interim data are analyzed once per day at 1200 UTC on the grid N64 including \(256 \times 128\) data points in the zonal and meridional directions, respectively. Two types
of expansion are applied: one including levels up to about 0.8 hPa (denoted L57) and another involving only the levels under 95 hPa, that is, 36 levels out of 60 levels that are denoted L36 ERA-Interim. The goal of the two separate expansions is to compare climatological spectra representing both troposphere and stratosphere (L57) and mainly troposphere (denoted L36). Some features of the ERA-Interim dataset have previously been presented in Zagar et al. (2015) that also included the vertical structure functions for the ERA-Interim vertical discretization. ERA-Interim dataset is analyzed on a lower resolution including K = 128 waves for L36 data and K = 100 waves for L57 dataset.

All results will be presented for the spectra defined by (8) and (9) that integrate energy over all vertical modes. As discussed in Zagar et al. (2015), the vertical modes are obtained numerically and energy distribution in terms of vertical modes is more complex.

The distribution of energy as a function of the zonal wavenumber is most relevant.

3. Diagnosis of the Rossby and inertia–gravity waves

The decomposition in (1) and (2)—and, when applicable, its inverse in (10) and (11)—is carried out at every analyzed time independently for the prescribed projection basis. As explained above, a time-independent projection means that the frequencies of Rossby and IG eigensolutions of the linearized global equations are not considered.

Without temporal filtering involved, nonlinear dynamics between adjacent time steps may lead to a loss of coherence between the diagnosed Rossby and IG waves at subsequent times. We show examples of IG waves that can be tracked in the concatenated time series of diagnosed linear waves. This is important given the fact that forcings by physical parameterizations greatly affect the energy spectra and the nonlinear spectral fluxes between analyzed times with data (Malardel and Wedi 2016).

a. Time series and variability

Figure 3 shows the energy evolution in two large-scale modes, the n = 3 Rossby mode and the Kelvin mode (n = 0 EIG mode), over a few-month period in 2015. We choose to show the Rossby mode n = 3 as it contributes the largest percentage to the global variance. The presented two zonal wavenumbers, k = 1 and k = 7, are known to contain the largest portion of the standing and traveling variance, respectively (e.g., Watt-Meyer and Kushner 2015). The symmetric Rossby modes with the small meridional index have been extensively studied (e.g., Hirota and Hirooka 1984). The Kelvin wave is a major mode of tropical variability and the most energetic large-scale IG wave (Zagar et al. 2009b). Figure 3 shows that the large-scale Kelvin and Rossby waves are

![Figure 3. Global energy levels in several modes according to the ECMWF analyses during January–June 2015. The Kelvin mode (n = 0 EIG) and (n = 3, k = 1) Rossby mode are multiplied by factors of 0.5 and 0.05, respectively, to be comparable with the (n = 3, k = 7) Rossby mode.](image-url)
maintained in the data and have propagation properties in agreement with the observations and numerical simulations. In particular, there is a dominant signal with period around 5 days in the $k = 7$ Rossby wave and an occasional strong appearance of the 15–60-day-long oscillations in the Rossby $k = 1$ wavenumber. Similarly, the Kelvin wave signal contains periods of about 2 weeks on which 2–3-day subsynoptic-scale variability is superposed.

In addition to the evolution of wave energy shown in Fig. 3, energy associated with various components of variability can be quantified directly from the time series of the complex Hough expansion coefficients $X_k^n(m; t)$. This is performed by a complex fast Fourier transformation of the time series of each mode $(k, n, m)$ (e.g., Swarztrauber 1984). The results based on 2015 data are shown in Fig. 4 for $n = 1$ and $n = 3$ Rossby modes after the summation over all zonal wavenumbers and vertical modes. The spectra have been smoothed by taking the Gaussian-shaped moving averages over the raw spectra by using a Daniell kernel three times (Shumway and Stoffer 2010). The Rossby $n = 1$ mode peaks at 15-day periods as well as at periods between 5 and 7 days; both properties have been known since the satellite data era (e.g., Hirota and Hirooka 1984; Hirooka and Hirota 1985), though not well understood to this day. In comparison to $n = 1$, periods between 10 and 5 days contain much less energy in $n = 3$ Rossby mode. In contrast, periods longer than 35 days are more energetic in $n = 3$ than in $n = 1$ Rossby mode. Although Fig. 4b cannot be directly compared to Fig. 3, which presented a single zonal wavenumber, they agree about largest variability in the Rossby $n = 3$ mode in periods of about 2 weeks and longer.

**b. A case study of inertia–gravity waves**

A case study focuses on the period 4–6 July 2014 and the region between Australia and New Zealand. As mentioned, the motivation is the DEEPWAVE campaign, which collected a large observation dataset that can be used to estimate the gravity wave fluxes (Fritts et al. 2016). The application of the DEEPWAVE observations is beyond the scope of the present paper; we wish to demonstrate how our decomposition method can be used for the validation of the global weather and climate models with such data.

We compare the IG wave packet in the ECMWF deterministic forecast initialized at 0000 UTC 4 July with the analyses available every 6h. Figures 5 and 6 show the IG wave packet associated with the westerly jet south of Australia in terms of the temperature perturbations superposed on the background wind. The perturbations are obtained by applying (10) and (11) with the cutoff scale $K_1 = 15$ (around 950 km at 45°N) that has usually been used as a cutoff scale between the synoptic range and mesoscale range. The highest wavenumber is $K_2 = 300$. First, Fig. 5 compares the wave features in the forecasts and analyses at three time steps on 5 July. The comparison of the four consecutive analysis times shows a gradual increase of the jet strength close to 200 hPa along with its eastward movement. During the day the core of the jet has moved some 158°–208° eastward with an average speed of
around 20 m s\(^{-1}\). At the leading edge of the jet the vertically tilted phase lines of the IG waves are clearly visible with the wave amplitude increasing during the day. The wave is propagating upward with its amplitude increasing. The difference between the analyses and forecast fields is small. The wave packet appears somewhat more coherent in the forecast, especially at 0600 UTC (cf. Figs. 6e and 6a).

The similarity between the operational forecast and analyses in this case suggests that the observations assimilated on 5 July 2014 did not make a significant impact on the developing IG wave. We speculate that the radiance observations, which could provide analysis increments in the temperature field over the study region, were possibly smoothed in the assimilation step. Furthermore, the wind increments, even if produced through the multivariate couplings in the background-error covariance matrix, are not expected to be coupled with the temperature increments via the gravity wave dispersion relationship. It thus appears that the IG waves in the analyses result primarily from the first guess. In agreement with other studies...
verifying the IG waves in the ECMWF analyses, Figs. 5 and 6 suggest that the model represents well the IG waves that are generated by the synoptic-scale features (e.g., jet).

Further properties of the IG wave packet in Fig. 6 are provided in Figs. 7 and 8. Figure 7 shows the vertical profile of the zonal wind perturbation in the upper troposphere and the lower stratosphere at four subsequent hours of the forecast. Shown are steps +30, +31, +32, and +33 h of the forecast started at 0000 UTC 4 July 2014. The location of the profile at 40°S, 130°E is denoted by a black triangle in Fig. 5d. The figure shows that the phase of the wave is moving downward, which is associated with the upward energy propagation. In Fig. 8 we show the IG winds at 1200 UTC 5 July at nearly the same location (40°S, 129°E). Two different presentations of the wave are shown. One is a classical hodograph (e.g., Hamilton 1991) showing winds at different vertical levels between level 76 (around 216 hPa) and level 59 (around 93 hPa). As one moves upward, a counterclockwise rotating wind vector produces the elliptically shaped hodograph shown in Fig. 8a. Such hodographs have been shown very useful to analyze observations of single vertical profiles (e.g., rockets) (e.g., Gubenko et al. 2008; Fritts and Alexander 2003). It is compared with the simulated evolution of the IG wave shown in Fig. 8b. Different points in Fig. 8b correspond to the hourly values of the forecasted winds starting from 0600 UTC 5 July (30-h forecast).

Without the temporal information about the IG wave evolution, the hodograph in Fig. 8a and the dispersion and polarization relationships for IG waves provide estimates of the intrinsic wave properties such as the frequency, phase, and group speeds. From the polarization relation, it follows that the ratio of the major and the minor semiaxes of the polarization ellipse equals $f/v$, where $v$ is the intrinsic wave frequency. The vector lengths of the major and minor axes are estimated at 7.1 and 3.7 m s$^{-1}$, respectively, which results in $\omega = 1.8 \times 10^{-4}$ rad s$^{-1}$ and a wave period of 9.7 h. The wave period suggested by Fig. 8b is somewhat larger, around 12 h. The zonal and vertical group speeds
estimated from the hodograph are 5.93 and 0.13 m s\(^{-1}\), respectively.

4. Energy spectra of the Rossby and inertia–gravity waves

a. Two-dimensional energy distribution in the Rossby and IG waves

Typically the global energy spectra are shown as a function of the total wavenumber that is in the global spectral models defined by the triangular truncation (e.g., Koshyk and Hamilton 2001). In the case of the spectra based on the Hough harmonics, the meridional and zonal dependencies are separated and the truncations are selected to provide the best representativeness of the input data [see discussion in Zagar et al. (2009a)]. On large scales, the global energy spectra derived from spherical harmonics do not represent energy in a single wavenumber and thus large scales cannot be discussed using this kind of decomposition. The missing insight is provided by the Hough harmonics decomposition. Although the present study focuses on the one-dimensional spectra, we first show the 2D energy distribution in one of our datasets to illuminate large-scale circulations due to Rossby and IG modes.

Figures 9 and 10 display vertically integrated, total wave energy in the latest operational ECMWF analyses, the 3-month average from December 2016 to February 2017 (dataset L134). In Fig. 9, global energy is split between the Rossby and IG components. In Fig. 10, only the vertical mode \(m = 1\), the so-called barotropic component, is shown for the Rossby modes and separately for the eastward- and westward-propagating IG modes. The two figures share the main features. They both illustrate that the average energy distribution on planetary scales has a different dependence in the zonal and meridional directions. This is largely due to differences between the tropical and midlatitude circulations. For \(k > 40\), there is no dependence on the meridional modes, which is related to the fact that associated mesoscale processes are not affected by Earth’s curvature. Furthermore, for \(k < 40\) the balanced energy (Fig. 9b) exceeds the IG energy (Fig. 9c). The energy distribution for the \(m = 1\) Rossby modes (Fig. 10a), which well represent average troposphere conditions in the midlatitude, appear isotropic in synoptic scales, especially if one disregards the \(n = 0\) Rossby mode that is in our decomposition the mixed Rossby–gravity mode. The distribution of the total and Rossby wave energy during the Northern Hemisphere (NH) winter is characterized by a maximum in meridional modes \(n = 3–6\) with the absolute maximum in \(n = 4\) at the largest zonal scale (\(k = 1\)). For comparison, distribution in July 2014 (not shown) has a maximum in \(n = 3\). In each season, the most energetic \(n\) corresponds to balanced circulation associated with the midlatitude westerly winds (not shown) that is in NH winter more asymmetric (i.e., \(n = 4\)) than in NH summer (i.e., \(n = 3\)).

In contrast, the maximal global IG energy is found in the lowest meridional modes associated with the large-scale components of the unbalanced tropical circulation. For example, \(n = 0\) is largely contributed by the Kelvin mode, the eastward IG \(n = 0\) mode, which is the single most studied IG mode of the global atmosphere. The largest energy in Kelvin waves is in the zonal wavenumber \(k = 1\) throughout the model atmosphere (Fig. 10c).

Results for other datasets are very similar. A noticeable difference between recent operational analyses and ERA-Interim data concerns the mixed Rossby–gravity wave (figure not shown). Its energy is significantly larger in the operational analyses, which have a better representation of the higher stratosphere and the lower
mesosphere where these waves are regularly observed (e.g., Garcia et al. 2005).

b. One-dimensional global energy spectra

Integrating the energy in Fig. 9 meridionally, we obtain the global energy during 2016/17 NH winter season as a function of the zonal wavenumber (Fig. 11a). Similarly, the averaged energy in operational L137 analyses in July 2014 is displayed in Fig. 11b. Differences between the two spectra are due to differences in season: a somewhat deeper layer of the atmosphere analyzed in July 2014 and the different horizontal resolutions. In addition, the 2016/17 dataset it also included upgrades to the numerics (increased accuracy of the semi-Lagrangian advection solver, the omission of spectral degrading, and a reduction in the diffusion coefficients in the sponge layer affecting vertical levels above 0.5 hPa).

The high-resolution spectrum for the operational analyses in July 2014 can be compared with the results for July in ERA-Interim in Fig. 12. We show the results for L57 and L36 datasets (Table 1) to illustrate the differences between the stratosphere and troposphere in the global spectra. There is more energy in large scales in L57 data, which is contributed by stratospheric circulation. Furthermore, a local minimum in the zonal wavenumber $k = 2$ in L36 data in Fig. 12b is not present in the L57 data, which includes significant stratospheric variability in $k = 2$.

Spectra of the Rossby waves in operational analyses in Fig. 11 have a slope close to $-3$ for a wide range of scales between zonal wavenumber $k = 7$ and the smallest resolvable scale. In contrast, the IG wave part of the spectrum continuously follows a shallower line that appears relatively close to $-5/3$. When the two spectra are summed up, the total global energy spectrum is somewhat shallower than the $-3$ slope. Overall the IG spectrum appears shallower than $-5/3$ in planetary scales and steeper than $-5/3$ on small scales. In between there is a range of scales for which a $-5/3$ slope fits the data relatively well.

The ERA-Interim data is also characterized by a clear shift from a shallower to steeper spectrum of the balanced energy at $k = 7$. However, even a visual inspection of Fig. 12 suggests that the Rossby wave part of the spectrum is steeper than $-3$. The total slope of reanalysis data is close to $-3$. The best linear estimates for various ranges of the zonal wavenumbers are collected in Table 2, but before going into further discussion, we discuss flow regimes producing these spectra.

Similar to Fig. 1, Figs. 11 and 12 display a crossing wavenumber $k_c$ at which the IG wave energy becomes dominant over the Rossby wave energy. Values of $k_c$ are collected in Table 3 along the crossing scales $L_c$ defined at the equator as $L_c = \pi R_e/k_c$, where $R_e$ is the average radius of Earth that is equal 6371 km. In July 2014 L137 data, the crossing takes place at wavenumber $k_c = 35$ (about 570 km at the equator and 400 km in the mid-latitudes) (Fig. 12b). The latest operational analysis shares this property (Fig. 11a and Table 3). Throughout the 2015/16 period, the value $L_c$ deviates up to 100 km from the average depending on the flow, season, and the analyzed model depth. In particular, the vertical model depth (i.e., analyzed layer) is an important factor for the location of the crossing point as illustrated in Fig. 13, which compares the Rossby and IG spectra for several depths of the L137 data presented in Fig. 1. The number of levels reduces from 134 (top at 6 Pa) to 123, 108, and 89 levels (with a top level at 53 hPa); the corresponding crossing scale $k_c$ increases from 39 to 41, 52, and 58, respectively. The change in $k_c$ is due to the IG spectrum since there is no variability in Rossby waves in these wavenumbers in the stratosphere and higher. In addition to the mean zonal state ($k = 0$), which is not discussed, only the planetary waves $1 \leq k \leq 3$ of the
Rossby spectrum are affected when the lower mesosphere and the upper stratosphere are excluded from the consideration, whereas the IG spectra are affected at all scales. (Notice that these larger values of $k_c$ than in Figs. 11 and 12 are from a 3-day period in NH summer.)

In general, both ERA-Interim and operational analysis data suggest that $k_c$ is marginally greater in July than in December, which is most likely associated with more intense unbalanced circulation during NH summers. The ERA-Interim data, which are based on a decade-older forecast model with a lower resolution, less advanced physical parameterizations, and a lower model top, are expected to be less reliable for the IG spectrum and to contain less variability in small scales. Correspondingly, the crossing scale in Fig. 12 is found at larger wavenumbers around $k_c = 50$, which corresponds to zonal scales around 400 km.

c. Energy spectrum in ERA-Interim

The global energy spectrum based on the L57 ERA-Interim data can be considered representative for the current state-of-the-art reanalysis products spanning both troposphere and stratosphere. Presented in Fig. 14, the climatological spectrum for the Rossby and IG waves is divided into three distinctive dynamical regimes:

1) The large-scale regime dominated by the Rossby waves (balanced dynamics). In this regime, the upscale flow of energy feeds the mean zonal circulation. The unbalanced part of spectrum associated with the IG modes in this regime is mainly a projection of the tropical large-scale circulation features such as the Hadley and Walker circulations. In the midlatitudes, the gradient wind balance within the stratospheric polar vortex and orographic waves due to large-scale orography of Antarctica and Greenland also contribute to the large-scale IG spectrum.

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**Fig. 11.** Energy distribution as a function of the zonal wavenumber $k$ in (a) DJF 2016/17 and (b) July 2014. Input data are operational ECMWF analyses on (b) 134 levels were under 5 Pa and (b) all 137 model levels. All meridional and vertical modes are summed up.

**Fig. 12.** As in Fig. 11, but for ERA-Interim for July using (a) 57 model depth with the model top at 0.8 hPa and (c) 36 model depth with the top level at around 96 hPa.
The evidence of these properties in physical space was provided in Zagar et al. (2015). The slope of energy spectra in this regime is in ERA-Interim data close to −1 (Table 2). It can be noticed in Table 2 that L36 data are characterized by a somewhat shallower spectrum than the L57 data, which can be attributed to a smaller variability in the stratosphere.

2) The synoptic-scale weather regime is found in ERA data between the scales around 3000 and about 400 km. Dynamics in this regime involves both the Rossby and IG waves. The regime starts at $k = 7$ where the Rossby waves dominate and a majority of the unbalanced circulation is contributed by the tropics. As we move downscale, the IG waves become relatively more energetic, leading eventually to the crossing of the Rossby and IG wave spectra at $k_c = 47$ (~430-km scale). The scale varies with season and the analyzed vertical model depth. The slope of the total energy spectrum in ERA-Interim is −3 with the balanced component steeper and IG part shallower than −3 (Table 2).

3) The mesoscale regime beyond the crossing scale of the Rossby and IG waves. In this regime, the temperature and wind perturbations are primarily associated with IG waves. In other words, circulation is dominated by flow divergence and deformation. The average amount of global energy in this regime is very small, not reaching above 1 J kg$^{-1}$ of energy at a scale of 400 km. The average slope of the mesoscale energy spectrum in ERA-Interim is somewhat steeper than −3, and it is on average between the spectrum of Rossby waves with a slope steeper than −3 and the spectrum of the IG modes with a slope around −2.5.

The portion of energy in Rossby and IG modes as a function of scale is characterized by a nearly constant ratio of energy in the two types of modes in wave-numbers $k = 1$–6 with around 95% of energy being associated with balanced dynamics (not shown). From $k = 7$ the percentage of Rossby wave energy steadily reduces.

d. The IG spectrum in high-resolution operational analyses

In contrast to ERA-Interim, the operational analyses maintain the −3 slope for the Rossby modes for all $k > 6$ (Table 2 and Fig. 11). Moreover, for any particular month in the period from 2014 to the present, the balanced spectrum has a slope of $-3 \pm 0.05$ (not shown).

![Figure 13](image-url)
For large scales \((k \approx 6)\), the balanced spectrum slope is \(-1.1\) to \(-1.2\), that is, similar to L57 reanalyses data in spite of their different vertical depth.

The global IG spectrum derived from recent 0000 UTC ECMWF analyses is shown in Fig. 15. In comparison to the IG spectrum for the ERA-Interim data in Fig. 14, Fig. 15 presents variability simulated by a more advanced model and data assimilation system that represents a deeper layer of the atmosphere (up to 1 Pa; i.e., 80 km). Correspondingly, the amplitude of IG energy is greater and the spectrum is shallower in Fig. 15 than in Fig. 14. Similar to Fig. 14, we divide the global IG spectrum in three parts with their slopes listed in Table 2. The shallowest part of the IG spectrum corresponds to scales greater than about 3000 km. As discussed above and in Zagar et al. (2015), large-scale unbalanced circulation is associated primarily with tropical circulations and with the role of orography and stratospheric vortex in the extratropics.

Table 2 shows that recent analyses have smaller \(k_c\) than ERA-Interim and the difference in scale is 100–200 km. Based on the L137 dataset and sensitivity to the vertical depth, we define the crossing scale \(L_c\) for the current whole atmosphere ECMWF analyses to be at 500–600 km. Then the synoptic-scale regime is defined between about 3000 km \((k = 7)\) and 500–600 km \((k \approx 35)\). As the slope of the IG spectrum in this regime is \(-1.5\) to \(-1.6\), we can consider unbalanced dynamics in this range of scales well resolved by the current ECMWF analyses and forecasts. The amount of the global IG energy in this range reduces from about 100 J kg\(^{-1}\) at \(k = 7\) to about 10 J kg\(^{-1}\) at \(k = 30\). These values make about 10% and 50% of the total energy in these scales, respectively. The large relative increase of IG energy is associated with a reduction in balanced energy by two orders of magnitude.

At horizontal scales below 500 km, the IG energy dominates over energy due to Rossby waves. Also, the IG spectrum becomes somewhat steeper and the estimated fit for the 2015/16 analyses is close to \(-2\) (Table 2). This suggests that in these scales the current analyses’ data do not contain sufficient variability associated with unbalanced dynamics.

In Fig. 16, we use the new definition of the cutoff scale for the IG regime to distinguish the temperature and wind perturbations in the synoptic and mesoscale regimes. Figure 16a can be compared with Fig. 5a to notice the wind-field perturbations of the IG wave that were not shown in the previous section. Another IG wave with significant amplitude is found over New Zealand associated with the relief. The amplitude of this orographic gravity wave over the southern island increases in the forecast during the rest of the day (not shown) when the large gravity wave fluxes were measured (Smith et al. 2016). The IG waves make a majority of the perturbations in Fig. 16a. In contrast, the Rossby waves contribute most to the perturbations in the synoptic scales (Fig. 16b). [Notice that we used \(k = 30\) for the truncation but values \(k = 25\) or 35 result in almost the same figure.]

The true values of IG energy across the whole spectrum are not available but the state-of-the-art global analyses can be considered its reliable approximation at
scales greater than 500 km. According to Fig. 15, the globally integrated IG wave energy in the synoptic scales varies from a few up to several tens of joules per kilogram or even 100 J kg\(^{-1}\). Such estimates depend on the analyzed depth of the global atmosphere and vary by season and latitude.

The orders of magnitudes of IG energy agree with the first estimates by VanZandt (1982). Observational studies are mostly based on temperature observations and thus provide the estimates of IG wave potential energy. For example, Tsuda et al. (2000) calculated the global distribution of potential energy of the mesoscale gravity waves from the GPS occultation data at 20–30-km height and found monthly mean values of potential energy of up to 10 J kg\(^{-1}\). In contrast, our spectra contain both potential and kinetic energy. In Fig. 17 we show the monthly mean total IG energy for the planetary and synoptic regimes in operational ECMWF analyses with the whole model depth and L137 levels. We computed the synoptic range for \(7 \leq k \leq 25\) in agreement with Table 2 where the ranges were defined more conservatively to allow comparison between various datasets. Taking the range \(7 \leq k \leq 25\) does not affect the presented variability picture. Figure 17 shows the annual cycle of the large-scale unbalanced circulation and global synoptic activity. On large scales, two maxima of the semiannual cycle in DJF and JJA are up to 30% greater for the eastward component than for the westward component. At synoptic scales, the average monthly level of IG energy varies between 30 and 60 J kg\(^{-1}\) and the annual maximum in January appears significantly greater than the secondary maximum in July (Fig. 17b). This is most likely associated with the more active IG wave sources in the midlatitudes during the NH winter. Consequently, the energy distribution of the westward-propagating IG waves has a sinusoidal shape with the maximum energy in January and the energy minimum in July. The dominance of the eastward IG component over the westward component, which is largely contributed by the tropical synoptic-scale systems, is present in both DJF and JJA seasons.

5. Discussion

Throughout this paper, we have discussed distinct advantages of the decomposition of global data using Hough harmonics in contrast to the spherical harmonics. The spherical harmonics are the eigensolutions of the global barotropic vorticity equation, whereas the Hough harmonics are the eigenfunctions of the horizontal part of the linearized primitive equations on the sphere. Their coupling to the vertical structure equation is through the equivalent depth. The main advantage of the Hough harmonics is the interpretation of circulation in terms of the two kinds of linear waves: the quasi-geostrophic (or Rossby type) and inertia–gravity (or unbalanced) modes. Much of our understanding of atmospheric dynamics comes from considering these two regimes although their discussion has traditionally been carried out by using different sets of simplified equations. A disadvantage of the modal decomposition is that the horizontal decomposition can be performed only for vertical modes (i.e., for prescribed equivalent depths), which makes it difficult to localize the analysis to particular layers such as the upper troposphere or the stratosphere as possible by using the spherical harmonics.

In contrast to many studies that rely on temperature observations, the decomposition into Hough modes involves both wind and temperature perturbations of IG waves across many scales. Validation of the spectrum of IG waves in NWP and climate models can profit from the proposed diagnostic of IG waves in the models that can now be directly compared with the observed gravity wave fluxes such as in the DEEPWAVE campaign (Fritts et al. 2016).
Another advantage of using the unbalanced and balanced fields instead of vorticity and divergence is the discussion of tropical unbalanced circulations. Namely, large-scale tropical waves such as the Kelvin wave project partially to the rotational and partially to the divergent component and their discussion by using the spherical harmonics is not possible. In other words, spectra of the global divergent kinetic energy at any time and level are largely contributed by tropical divergent circulation but do not represent the equatorial waves (e.g., Kelvin waves or mixed Rossby–gravity waves).

The case study of IG waves associated with jet suggests that in many cases the IG waves may be produced by model dynamics; in other words, data assimilation does not introduce major new details associated with the life cycle of IG waves. This is not surprising given a lack of mesoscale observations and the smoothing role of the background-error covariances in data assimilation. The mesoscale high-resolution models aim to represent a part of the spectrum at scales of 100 km and smaller that are not well described by the global models. Indeed, many studies confirm that an increased resolution provides the energy spectra closer to $-5/3$ (i.e., more variance associated with gravity wave dynamics) (e.g., Wedi 2014). The decomposition of mesoscale NWP model energy spectra into rotational and divergent components showed that divergent energy makes around 50% of the kinetic energy spectrum in the free troposphere in midlatitudes (Blazica et al. 2013). A similar ratio was reported by Lindborg (2007) for the aircraft data from the upper troposphere and lower stratosphere. Here we notice that the spectra of kinetic energy in limited domains are partly filtered (e.g., by detrending) and the filtering effect may differ depending on the latitude and height (e.g., Frehlich and Sharman 2008; Blazica et al. 2015).

A typical mesoscale NWP model is tuned to produce optimal forecasts rather than to follow the $-5/3$ law. In contrast to operational NWP models that are concerned with the initial state and accurate short-range forecasts, high-resolution research models are often reported to exhibit the energy spectra with a slope following the $-5/3$ law in a well-developed simulation after the spinup stage (e.g., Skamarock 2004; Kitamura and Matsuda 2010; Waite and Snyder 2013; Mirzaei et al. 2014). A similar discussion applies to the global NWP models in which physical parameterizations affect all scales of motions and can modify energy spectra and nonlinear spectral fluxes to a great extent. For example, Malardel and Wedi (2016) found that the bottom boundary condition, the dissipation in the turbulent boundary layer, and the flow interaction with topography play a dominant role for the energy transfer in the ECMWF model simulations.

Presented results show that quasigeostrophic, vorticity dominated dynamics is characterized by a $-3$ slope all the way to the smallest analyzed scale. In other words, our decomposition does not support the idea that quasigeostrophic dynamics alone can produce spectra with a $-5/3$ slope through the forward energy cascade (e.g., Tung and Orlando 2003). Rather, the shallower spectrum of unbalanced motions is associated with the IG waves generated by the synoptic-scale flow, flow interaction with topography, and nonlinear processes including the downscale cascade of nonlinearly interacting IG waves and Rossby–IG wave interactions (e.g., Dewan 1979; VanZandt 1982; Bartello 1995).

Large scales ($1 \leq k \leq 6$) appear to be similarly well represented by the ERA-Interim data and the latest ECMWF analyses. Beyond the scale of around 3000 km ($k > 6$), ERA-Interim has the total spectrum with a slope.

FIG. 17. Monthly averaged energy associated with IG modes between July 2015 and July 2016. Presented is the average IG energy across the range of zonal wavenumbers: (a) $k = 1–6$ and (b) $k = 7–25$.
close to $-3$. This means that both balanced and IG circulations are underrepresented compared to current data, especially in summer. This is likely due to the lower horizontal and vertical resolutions and changes in physical parameterizations which largely control the IG spectrum.

We also showed that the energy spectrum of large-scale unbalanced circulation has a slope close to $-1$. Although a discussion of the exact nature of this spectrum is beyond the scope of this paper we notice that the 1D spectrum of the outgoing longwave radiation data has a slope close to $-1$ across synoptic scales (figure not shown). Such data have been used as a proxy of the deep tropical convection that maintains the tropical planetary-scale waves (Hendon and Wheeler 2008).

6. Conclusions

The most relevant findings of this paper are the following:

1) It is advantageous to use the Hough harmonics for the decomposition of global 3D data in terms of eigenfunctions of the linearized primitive equations—the Rossby and inertia–gravity waves. The decomposition provides the global total wave energy spectrum associated with quasigeostrophic and unbalanced dynamics.

2) A method for the scale-dependent quantification of temperature and wind perturbations associated with the inertia–gravity waves is presented. It is shown that despite analyzing circulation data only at selected processing times, the linear wave features readily persist.

3) The energy spectra of the Rossby waves in the operational 2014/16 ECMWF analyses are characterized by a $-3$ slope for all zonal wavenumbers greater than wavenumber 6. In contrast, ERA-Interim has energy spectra of the Rossby waves with slopes steeper than $-3$, suggestive of a lack of variability in balanced circulation.

4) The high-resolution global analyses represent unbalanced circulation in the range of zonal scales between 3000 and about 500 km relatively well. The associated energy spectrum has a slope very close to $-5/3$. The total energy in this synoptic-scale regime is still predominantly balanced so that its spectrum does not deviate remarkably from $-3$.

5) The change of the dominant dynamical regime takes place at the scale of around 500 km (about zonal wavenumber 35), where dynamics dominated by Rossby waves gradually becomes dynamics dominated by the inertia–gravity waves. The exact scale depends on the vertical depth, season, and the model.

6) Estimates of gravity wave fluxes and sources from global analyses may have large uncertainties due to applied assumptions on the cutoff scale. It is proposed that the crossing scale between the Rossby and IG energy spectra in the global, whole atmosphere data be considered in the filtering of the inertia–gravity waves in the models.

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